Baryonic Acoustic Oscillations via the Renormalization Group

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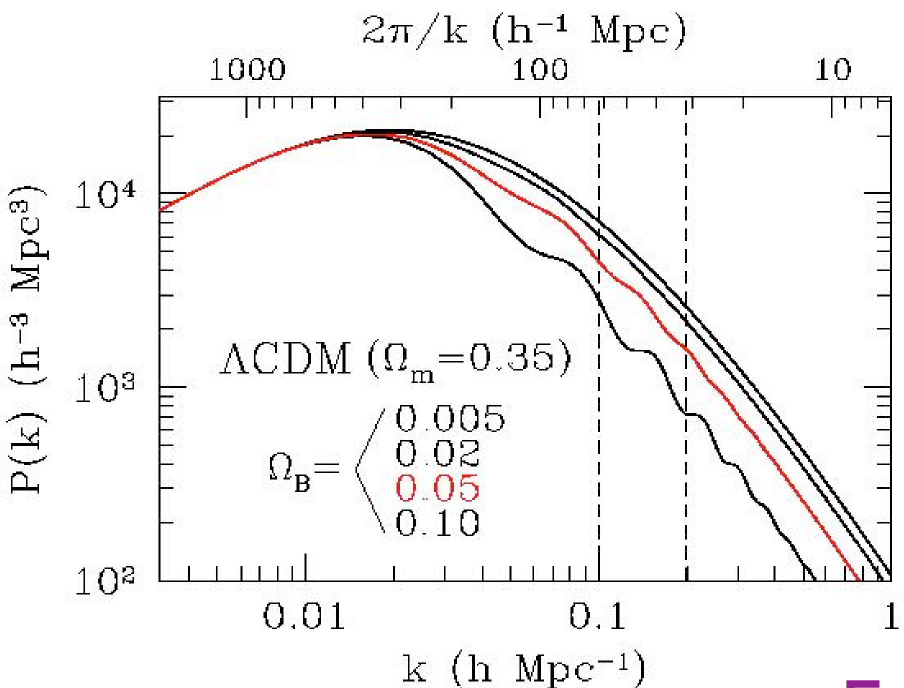
- Motivations: non-linear effects on the power spectrum in the BAO range
- RG approach: The emergence of an intrinsic UV cutoff
- Results

based on astro-ph/0702653, astro-ph/0703563 (JCAP), with Sabino Matarrese

A standard ruler: Baryonic Acoustic Oscillations

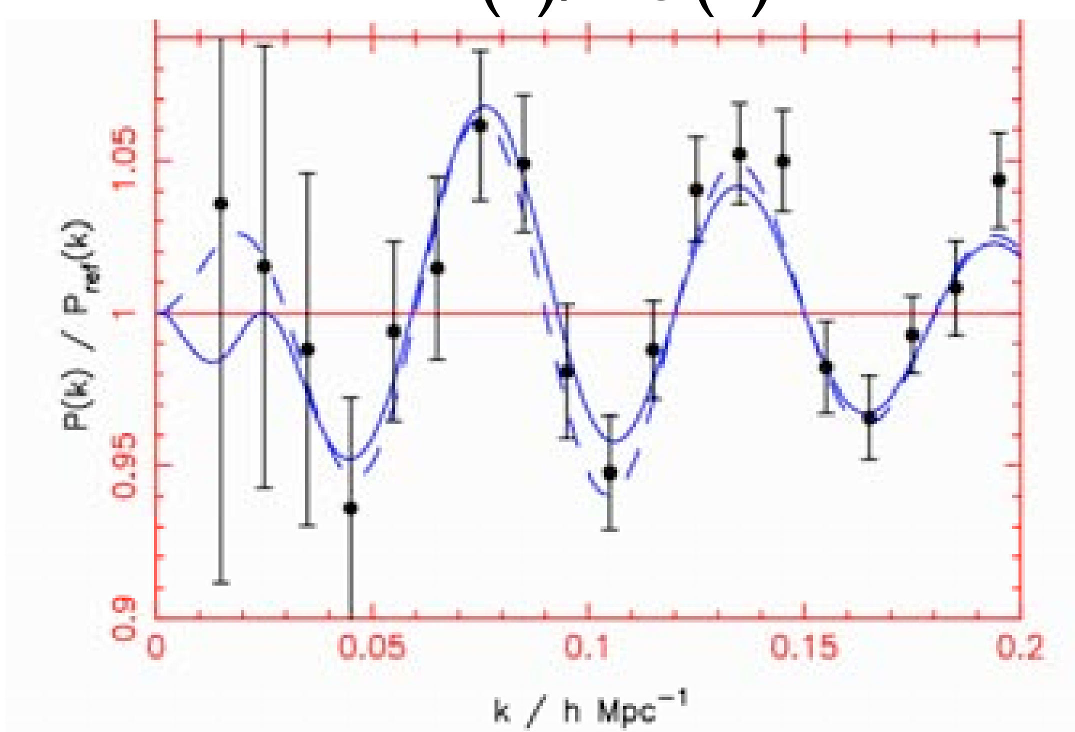


peaks are a small effect.
require large surveys to detect.

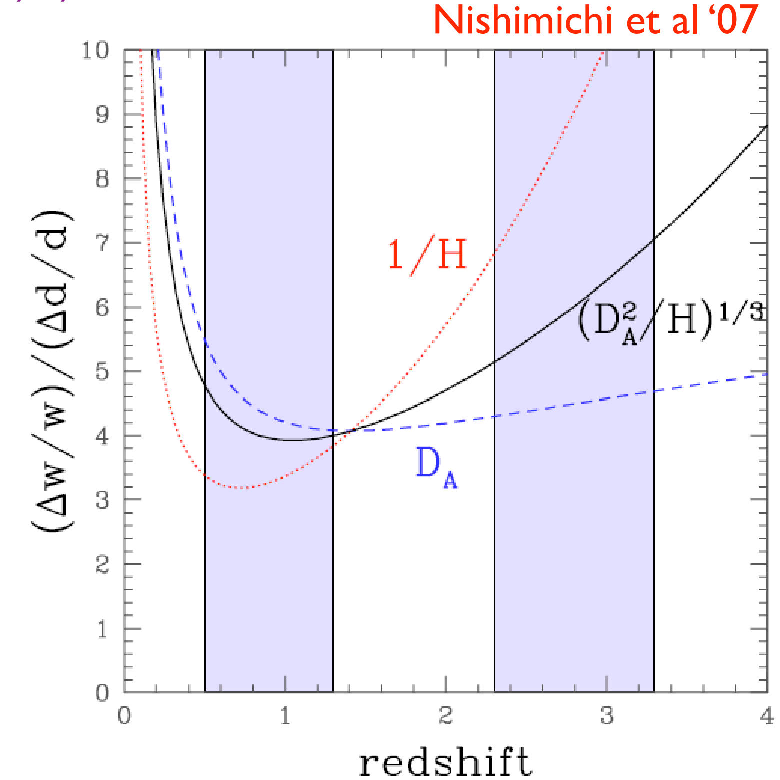
Ex.: BAO from WFMOS

(2M galaxies at 0.5<z<1)3)

P(k)/Pref(k)



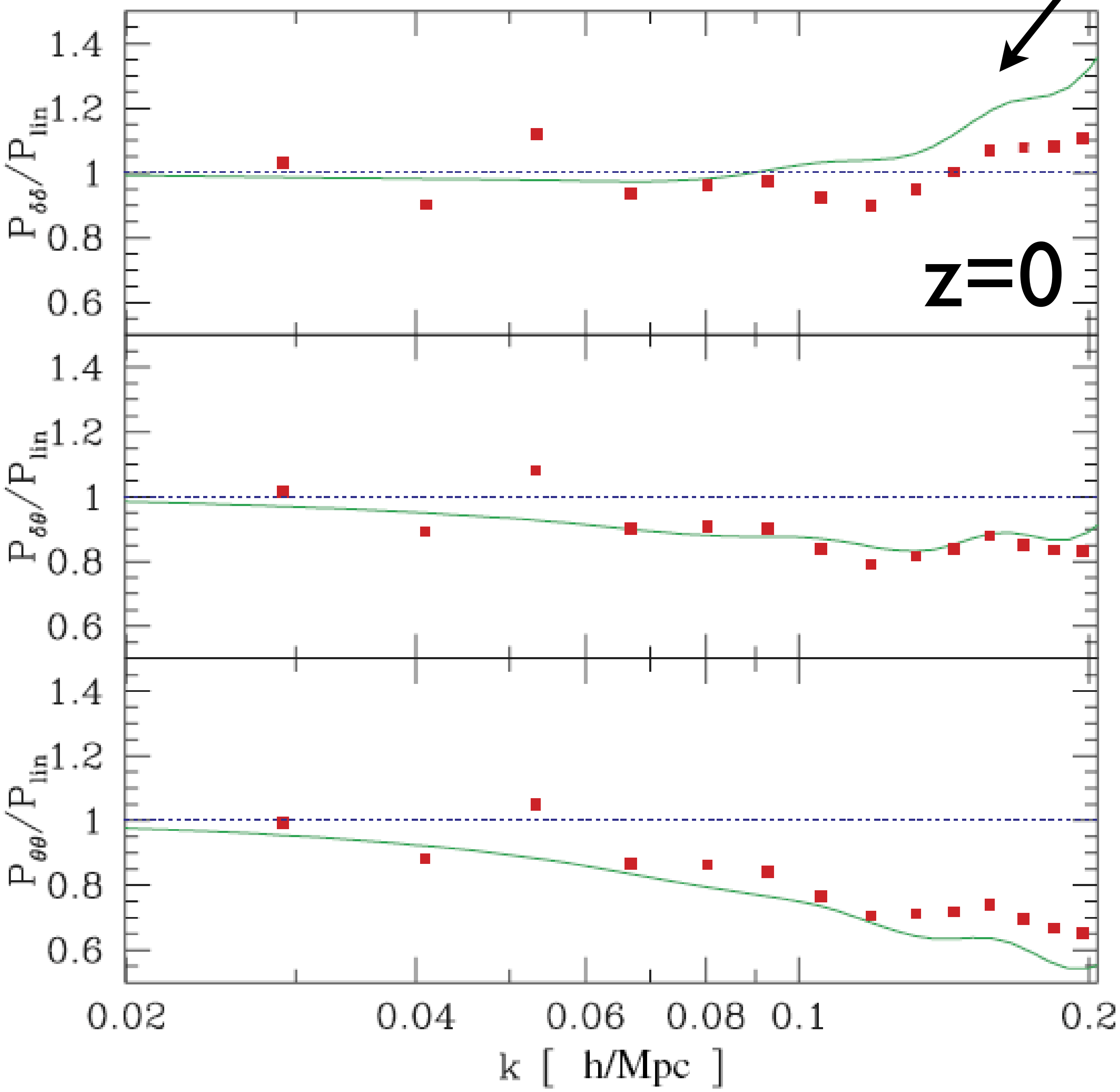
Nishimichi et al '07



Goal: predict the LSS power spectrum to % accuracy

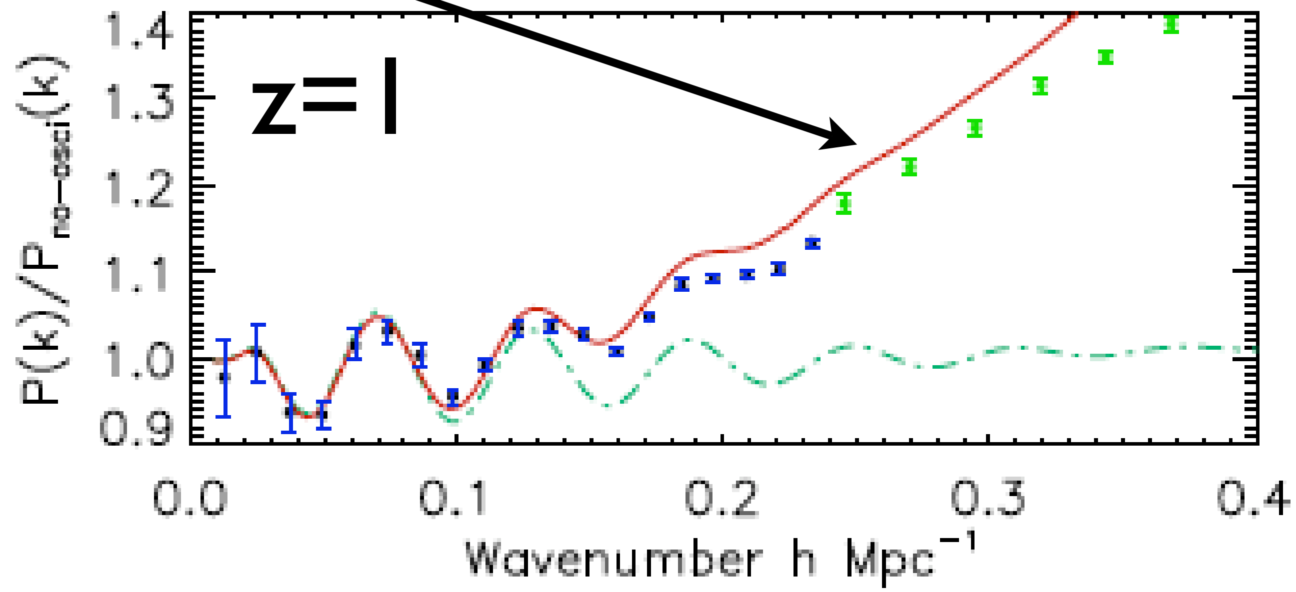
Present Status: Pert. Theory

1-loop PT



Scoccimarro, '04

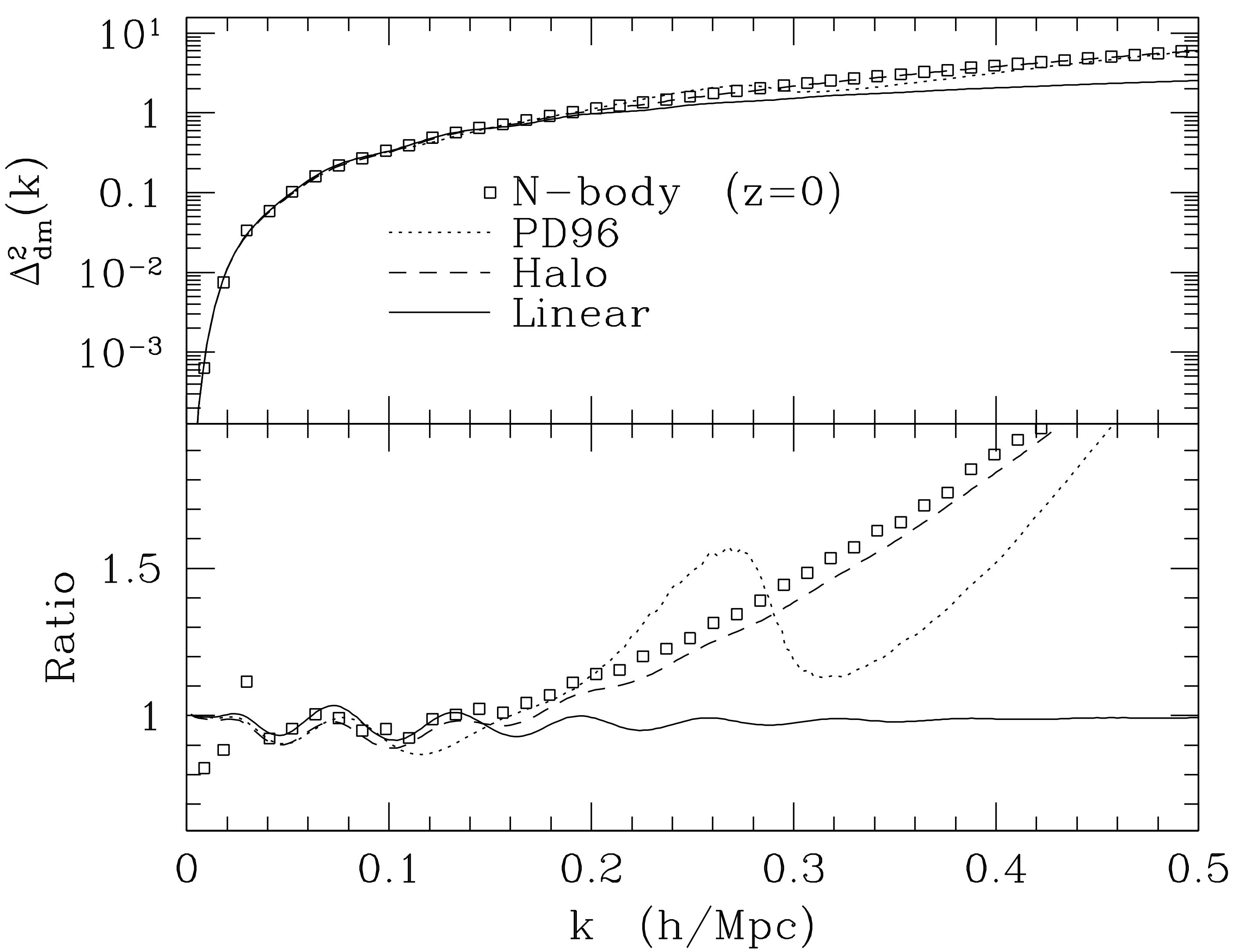
Jeong Komatsu, '06



Non-linearities becomes more and more

relevant in the DE-sensitive range 0<z<1

Standard Approach: N-body simulations+fitting functions



Huff et al, '06

5-10% discrepancies between fitting functions and simulations

(real space ⟶ redshift space, not trivial)

Goals

- Improve Pert. Theory towards lower z and higher k
- Study the effect of non-linearities on baryonic acoustic oscillations

Compact Perturbation Theory

Crocce, Scoccimarro '05

The hydrodynamical equations for density and velocity perturbations,

$$\frac{\partial\,\delta}{\partial\,\tau} + \nabla\cdot[(1+\delta)\mathbf{v}] = 0\,, \qquad \frac{\partial\,\mathbf{v}}{\partial\,\tau} + \mathcal{H}\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla\phi\,,$$

can be written in a compact form (we assume an EdS model):

$$(\delta_{ab}\partial_\eta + \Omega_{ab})\,\varphi_b(\eta,\mathbf{k}) = e^\eta \gamma_{abc}(\mathbf{k},\,-\mathbf{k_1},\,-\mathbf{k_2})\,\varphi_b(\eta,\mathbf{k_1})\,\varphi_c(\eta,\mathbf{k_2})$$

where $\begin{pmatrix} \varphi_1(\eta,\mathbf{k}) \\ \varphi_2(\eta,\mathbf{k}) \end{pmatrix} \equiv e^{-\eta} \begin{pmatrix} \delta(\eta\,,\mathbf{k}) \\ -\theta(\eta\,,\mathbf{k})/\mathcal{H} \end{pmatrix}$ $\qquad \eta = \log\frac{a}{a_{in}} \qquad \mathbf{\Omega} = \begin{pmatrix} 1 & -1 \\ -3/2 & 3/2 \end{pmatrix}$

and the only non-zero components of the vertex are

$$\gamma_{121}(\mathbf{k_1},\,\mathbf{k_2},\,\mathbf{k_3}) = \gamma_{112}(\mathbf{k_1},\,\mathbf{k_3},\,\mathbf{k_2}) = \delta_D(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})\,\frac{(\mathbf{k_2}+\mathbf{k_3})\cdot\mathbf{k_2}}{2k_2^2}$$

$$\gamma_{222}(\mathbf{k_1},\,\mathbf{k_2},\,\mathbf{k_3}) = \delta_D(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})\,\frac{|\mathbf{k_2}+\mathbf{k_3}|^2\,\mathbf{k_2}\cdot\mathbf{k_3}}{2\,k_2^2\,k_3^2}$$

The initial 'time' $\eta=0$ corresponds to $z=z_{in}$, chosen well inside the linear epoch.
In practice, we take $z_{in}=80$.

Extension to other cosmologies

$$\eta = \log \frac{a}{a_{in}} \longrightarrow \eta = \log \frac{D^+}{D^+_{in}}$$

Bouchet et al. '92
Bernardeau '94
Nusser et al '98

where D^+ is the linear growth factor

An action principle

Matarrese, M.P., '07

The fluid equations can be derived by varying the action:

$$S = \int d\eta_1 d\eta_2\, \chi_a\, g^{-1}_{ab}\, \varphi_b - \int d\eta\, e^{\eta}\, \gamma_{abc}\, \chi_a\, \varphi_b\, \varphi_c$$

where the auxiliary field $\chi_a(\eta, \mathbf{k})$ has been introduced and $g_{ab}(\eta_1, \eta_2)$ is the retarded propagator:

$$(\delta_{ab}\partial_\eta + \Omega_{ab})\, g_{bc}(\eta, \eta') = \delta_{ac}\, \delta_D(\eta - \eta')$$

so that $\varphi^0_a(\eta, \mathbf{k}) = g_{ab}(\eta, \eta')\varphi^0_b(\eta', \mathbf{k})$ is the solution of the ***linear*** equation

Explicitly, one finds: $\mathbf{g}(\eta_1, \eta_2) = \begin{cases} \mathbf{B} + \mathbf{A}\, e^{-5/2(\eta_1 - \eta_2)} & \eta_1 > \eta_2 \\ 0 & \eta_1 < \eta_2 \end{cases}$

$$\mathbf{B} = \frac{1}{5}\begin{pmatrix} 3 & 2 \\ 3 & 2 \end{pmatrix} \qquad \mathbf{A} = \frac{1}{5}\begin{pmatrix} 2 & -2 \\ -3 & 3 \end{pmatrix}$$

growing mode ($\mathbf{B}$) — decaying mode ($\mathbf{A}\, e^{-5/2(\eta_1 - \eta_2)}$)

Initial conditions: $\varphi^0_b(\eta', \mathbf{k}) \propto u_b = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ (growing mode), $\begin{pmatrix} 1 \\ -3/2 \end{pmatrix}$ (decaying mode)

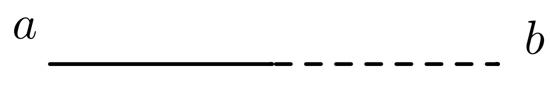
The generating functional

$$Z[\mathbf{J},\, \mathbf{\Lambda}] = \int \mathcal{D}\varphi\, \mathcal{D}\chi \exp\left\{ \int d\eta_1 d\eta_2 \left[-\frac{1}{2}\, \chi\, \mathbf{g}^{-\mathbf{1}} \mathbf{P^L} \mathbf{g^T}^{-\mathbf{1}} \chi + i\, \chi\, \mathbf{g}^{-\mathbf{1}}\, \varphi \right] - i \int d\eta\, [\mathbf{e}^\eta \gamma\, \chi \varphi \varphi - \mathbf{J} \varphi - \mathbf{\Lambda} \chi] \right\}$$

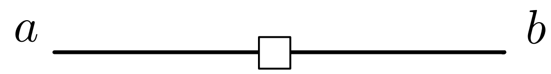
valid for <u>Gaussian</u> initial conditions, encoded in the power spectrum: $P^L_{ab}(\eta,\, \eta';\, \mathbf{k}) \equiv \left(\mathbf{g}(\eta)\mathbf{P^0}(\mathbf{k})\mathbf{g}^T(\eta')\right)_{ab}$

Derivatives of Z w.r.t. the sources J and Λ give all the N-point correlation functions (power spectrum, bispectrum, ...) and the full propagator

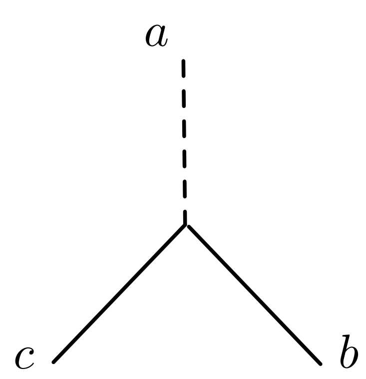
Compact Diagrammar



propagator (linear growth factor): $-i\, g_{ab}(\eta_a,\, \eta_b)$

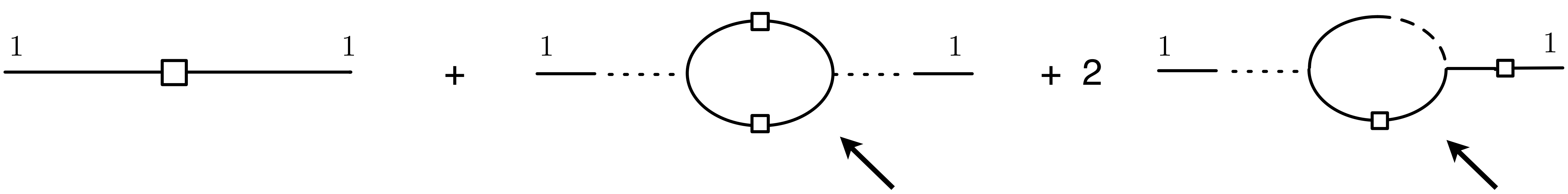


power spectrum: $P^L_{ab}(\eta_a,\, \eta_b;\, \mathbf{k})$



interaction vertex: $-i\, e^{\eta}\, \gamma_{abc}(\mathbf{k_a},\, \mathbf{k_b},\, \mathbf{k_c})$

Example: 1-loop correction to the density power spectrum:



a.k.a. "P22"

a.k.a. "P13"

All known results in cosmological perturbation theory are expressible in terms of diagrams in which only a trilinear fundamental interaction appears

Beyond perturbation theory: the renormalization group

Inspired by applications of Wilsonian RG to field theory : the RG parameter is momentum

Modify the primordial ($z=z_{in}$) power spectrum as: $P^0_\lambda(k) = P^0(k)\,\Theta(\lambda - k)$ (step function)

then, plug it into the generating functional: $Z[\mathbf{J},\,\mathbf{\Lambda}] \longrightarrow Z_\lambda[\mathbf{J},\,\mathbf{\Lambda}]$

$$Z_\lambda[\mathbf{J},\,\mathbf{\Lambda}] = \int \mathcal{D}\varphi\,\mathcal{D}\chi\,\exp\left\{\int d\eta_1 d\eta_2 \left[-\frac{1}{2}\chi\,\mathbf{g}^{-1}\mathbf{P}^{\mathbf{L}}_\lambda\mathbf{g}^{\mathbf{T}^{-1}}\chi + i\,\chi\,\mathbf{g}^{-1}\varphi\right] - i\int d\eta\,[\mathbf{e}^\eta\gamma\,\chi\varphi\varphi - \mathbf{J}\varphi - \mathbf{\Lambda}\chi]\right\}$$

The evolution from $\lambda = 0$ to $\lambda = \infty$ can be described non-perturbatively by RG equations:

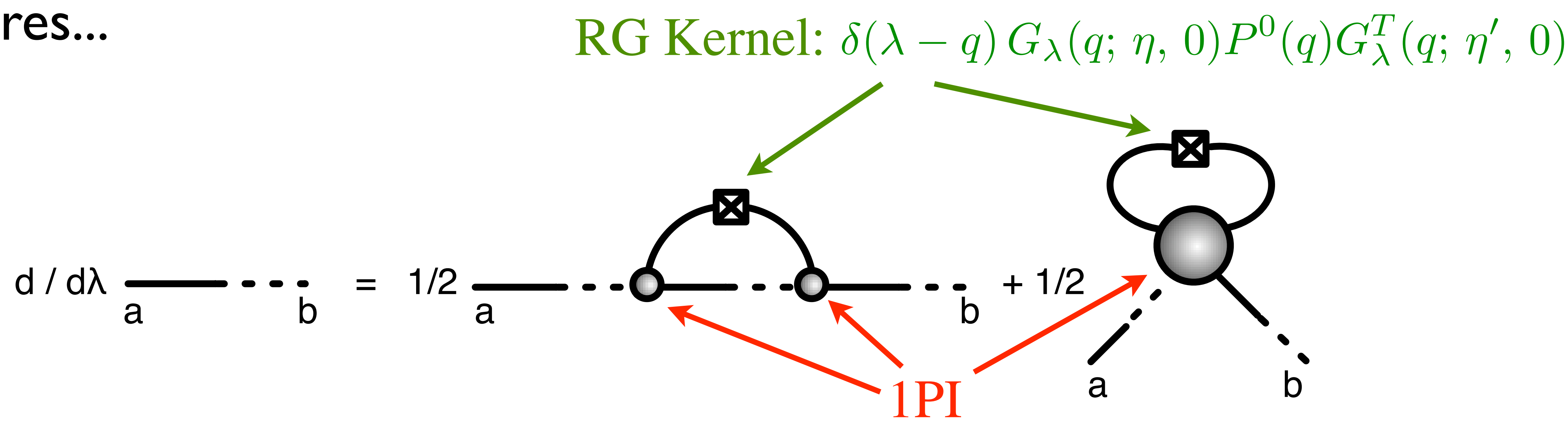
$$\frac{\partial}{\partial\lambda}Z_\lambda = \int d\eta\,d\eta'\left[\frac{1}{2}\frac{\partial}{\partial\lambda}\left(g^{-1}P^L_\lambda g^{-1\,T}\right)_{ab}\frac{\delta^2 Z_\lambda}{\delta\Lambda_b\delta\Lambda_a}\right]$$

The propagator

$$\delta^{(3)}(\mathbf{k}+\mathbf{k}')\, G_{\lambda,ab}(k;\, \eta_a,\, \eta_b) = -\frac{\delta^2 W_\lambda[J,\,\Lambda]}{\delta J_a(\mathbf{k},\, \eta_a)\delta\Lambda_b(\mathbf{k}',\, \eta_b)} \qquad W_\lambda[J,\,\Lambda] = -i\log Z_\lambda[J,\,\Lambda]$$

$$\frac{\partial}{\partial\lambda}\frac{\delta^2 W_\lambda}{\delta J_a\,\delta\Lambda_b} = \frac{1}{2}\int d\eta_c d\eta_d\, d^3\mathbf{q}\;\delta(\lambda-q)\left(g^{-1}P^L {g^{-1}}^T\right)_{cd}\frac{\delta^4 W_\lambda}{\delta J_a\,\delta\Lambda_b\,\delta\Lambda_c\,\delta\Lambda_d}$$

in pictures...



infinite tower of RGE's

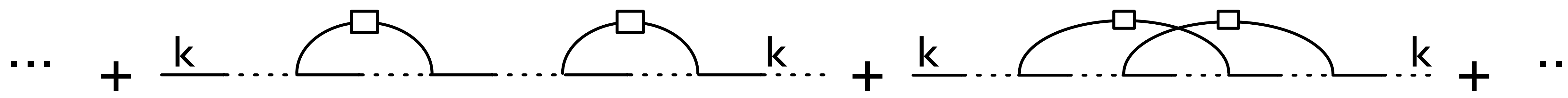
Approximation: full, 1PI, vertices ⟶ tree-level vertices

Large momentum, k >> λ: it can be integrated analytically!

$$G_{\lambda,\,ab}(k;\eta_a,\,\eta_b) = g_{ab}(\eta_a,\,\eta_b)\exp\left[-\frac{k^2\sigma_\lambda^2}{2}(e^{\eta_a}-e^{\eta_b})^2\right]$$

where $\sigma_\lambda^2 = \frac{1}{3}\int d^3\mathbf{q}\frac{P^0(q)}{q^2}\theta(\lambda-q)$

in perturbation theory, it can be obtained by summing the infinite series of chain diagrams (Crocce Scoccimarro, '06)



physically, it represents the effect of multiple interactions of the k-mode with the surrounding modes

$$G \sim e^{-\frac{k^2\sigma^2}{2}e^{2\eta}}$$

`coherence momentum' $k_{ch} = (\sigma\, e^{\eta})^{-1} \simeq 0.15\,\mathrm{h\,Mpc}^{-1}$

in the BAO range!

A self-generated UV cutoff

Inserting this result in the expression for the RG kernel, we get:

$$\mathcal{K}_\lambda(q;\,\eta,\,\eta') = \delta(\lambda - q)\,P^0(q)\,\exp\left[-\frac{q^2\sigma_\lambda^2}{2}\left((e^\eta - 1)^2 + (e^{\eta'} - 1)^2\right)\right]$$

The effect of modes with momenta larger than $\sigma_\lambda^{-1}(e^\eta - 1)^{-1}$ is exponentially screened.

The UV is much better behaved than one would guess from `usual' perturbation theory!!

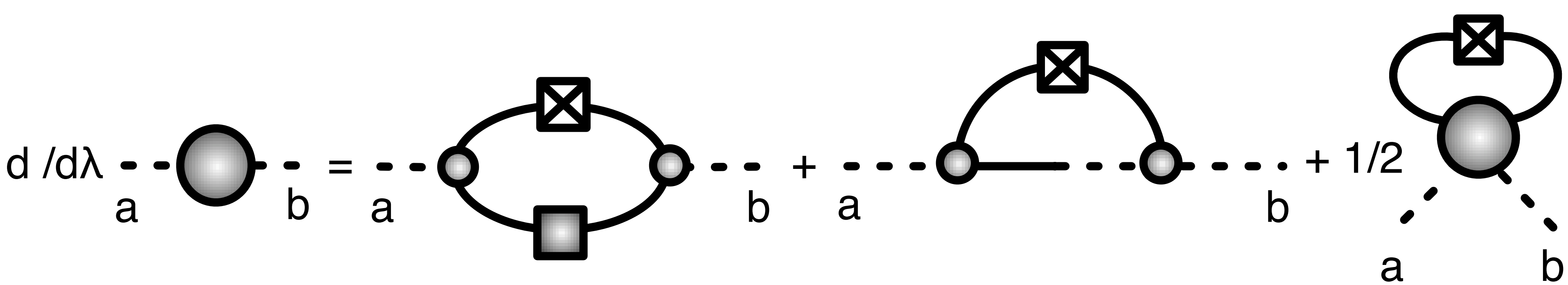
$$\mathcal{K}_\lambda(q;\,\eta,\,\eta') \longrightarrow \delta(\lambda - q)\,P^0(q)\,(1 + O(q^2\sigma^2 e^{2\eta}))$$

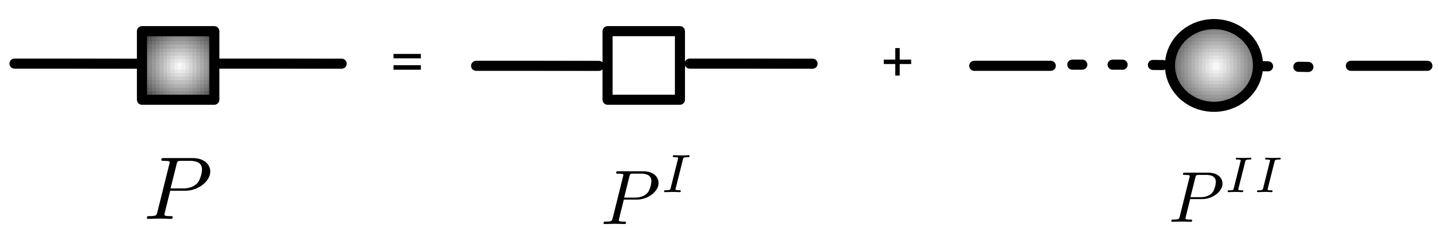
The power spectrum

The full PS has the structure: $P_{ab} = P^{I}_{ab} + P^{II}_{ab}$

with $$P^{I}_{ab}(k;\eta_a,\eta_b) = G_{ac}(k;\eta_a,0)G_{bd}(k;\eta_b,0)P^{0}_{cd}(k)$$

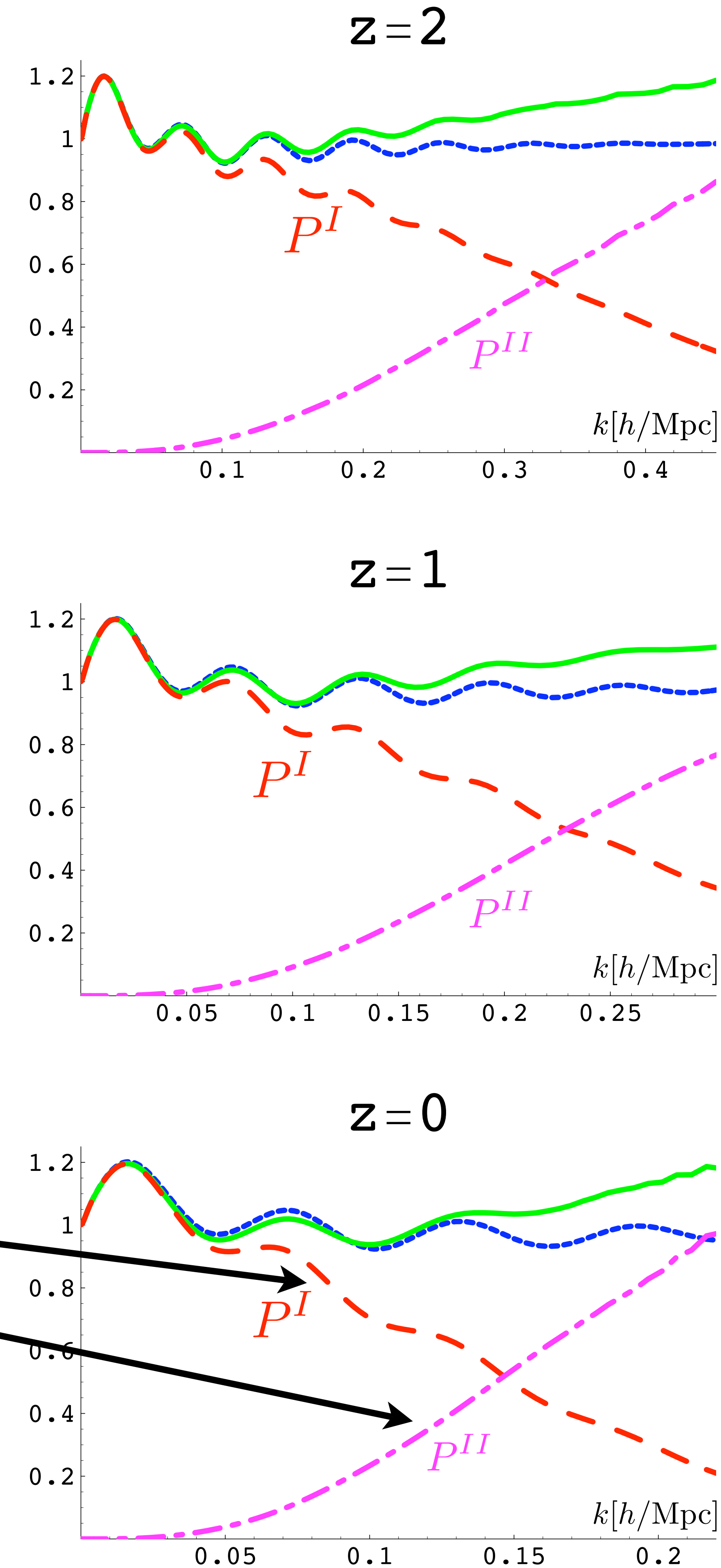
$$P^{II}_{ab}(k;\eta_a,\eta_b) = \int_0^{\eta_a} ds_1 \int_0^{\eta_b} ds_2\, G_{ac}(k;\eta_a,s_1)G_{bd}(k;\eta_b,s_2)\Phi_{cd}(k;s_1,s_2)$$





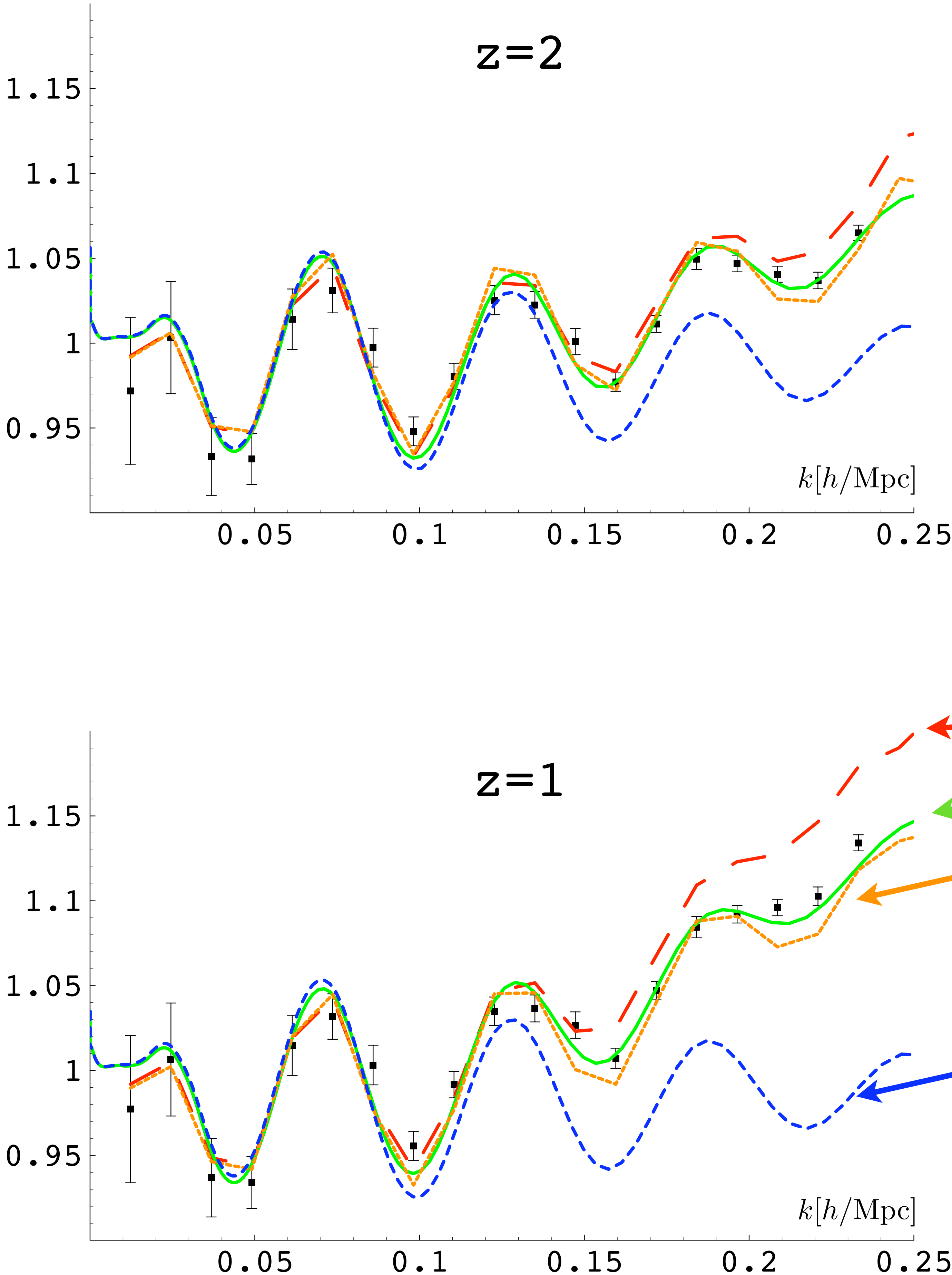
again, tree-level vertices...

$$\partial_\lambda \Phi_{ab,\,\lambda}(k;\, s_1,\, s_2) = 4\, e^{s_1+s_2} \int d^3\mathbf{q}\, \delta(\lambda - q) P^I_{dc,\lambda}(q;\, s_1,\, s_2) \times$$
$$P_{fe,\lambda}(|\mathbf{q}-\mathbf{k}|;\, s_1,\, s_2)\, \gamma_{adf}(\mathbf{k}, -\mathbf{q}, -\mathbf{k}+\mathbf{q}) \gamma_{bce}(-\mathbf{k}, \mathbf{q}, \mathbf{k}-\mathbf{q})$$



accuracy of linear theory up to k~0.12 h/Mpc is fortuitous: cancellation between two large non-linear effects

comparison with other approaches



ΛCDM model

Ω_Λ =0.73, Ω_b=0.043 h=0.7

1-loop PT

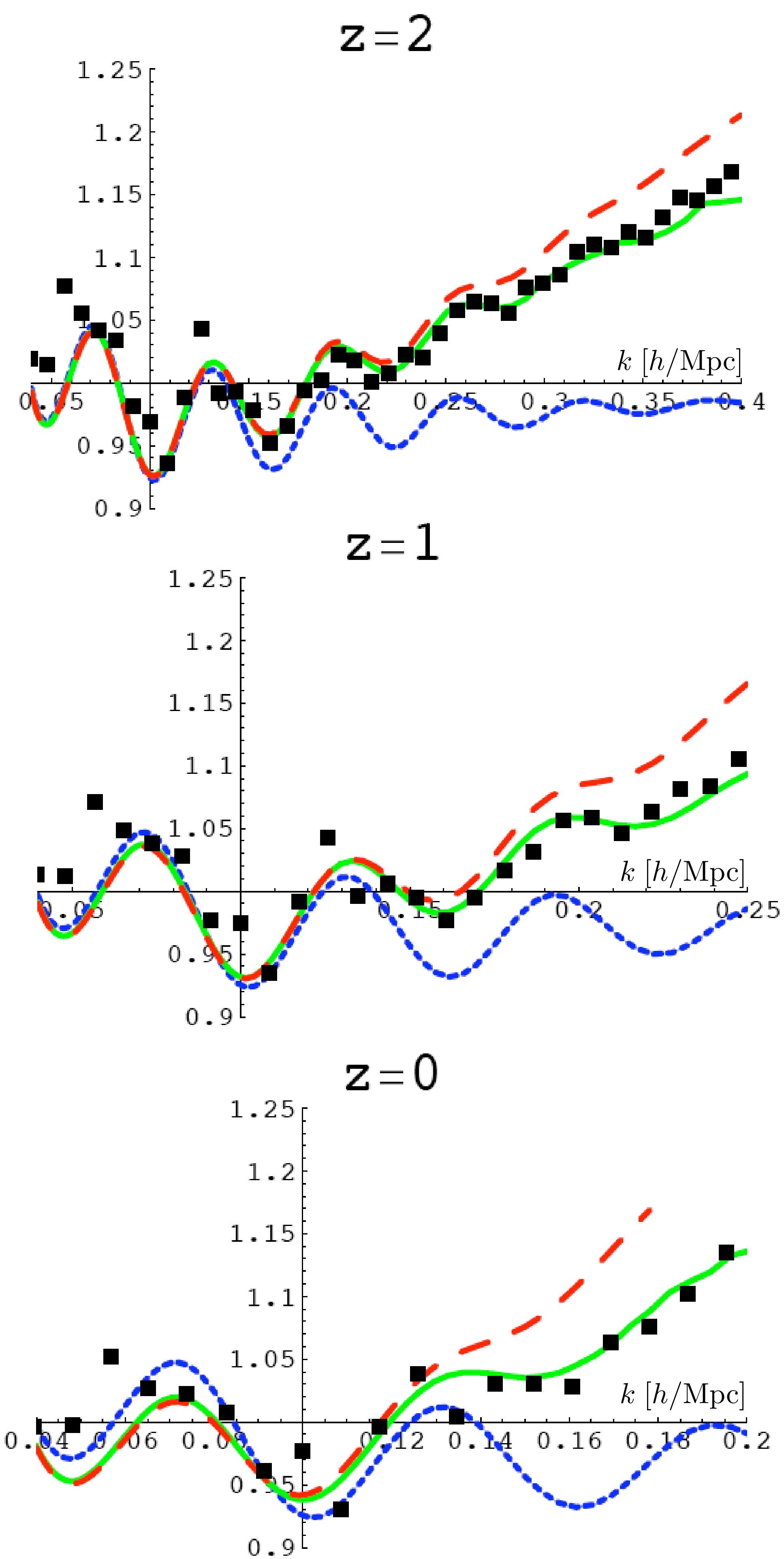
RG (Matarrese, M.P.)

Halo model

Linear theory

data from N-body simulations of

Jeong and Komatsu, astro-ph/0604075



ΛCDM model

$\Omega_\Lambda = 0.7$, $\Omega_b = 0.046$ $h = 0.72$

data from N-body simulations of
Huff et al, Astrop. Phys. 26, 351 (2007)

Conclusions

RG techniques can be conveniently applied to cosmology; Exact RG equations can be derived for any kind of correlation function; Systematic approximation schemes, based on truncations of the full hierarchy of equations, can be applied, borrowing the experience from field theory.

A simple approximation scheme already shows the emergence of an intrinsic UV cutoff in the RG running.

RG-improved results on the power spectrum agree with existing N-body simulations better than any other approach (see also Crocce and Scoccimarro, arXiv: 0704.2783).

Future lines of development include: including the running of the vertex, computing the bispectrum, non-gaussian initial conditions.