Motivations: non-linear effects on the power spectrum in the BAO range

RG approach: The emergence of an intrinsic UV cutoff

Results

based on astro-ph/0702653, astro-ph/0703563 (JCAP), with Sabino Matarrese
A standard ruler: Baryonic Acoustic Oscillations

peaks are a small effect. require large surveys to detect.

Ex.: BAO from WFMOS
(2M galaxies at 0.5<z<1)

Goal: predict the LSS power spectrum to % accuracy
Non-linearities becomes more and more relevant in the DE-sensitive range $0 < z < 1$.
Standard Approach: N-body simulations+fitting functions

5-10% discrepancies between fitting functions and simulations

(real space -> redshift space, not trivial)
Goals

• Improve Pert. Theory towards lower z and higher k

• Study the effect of non-linearities on baryonic acoustic oscillations
The hydrodynamical equations for density and velocity perturbations,

\[
\frac{\partial \delta}{\partial \tau} + \nabla \cdot [(1 + \delta)v] = 0, \\
\frac{\partial v}{\partial \tau} + H v + (v \cdot \nabla)v = -\nabla \phi,
\]

can be written in a compact form (we assume an EdS model):

\[
(\delta_{ab} \partial_\eta + \Omega_{ab}) \varphi_b(\eta, k) = e^\eta \gamma_{abc}(k, -k_1, -k_2) \varphi_b(\eta, k_1) \varphi_c(\eta, k_2)
\]

where

\[
\begin{pmatrix}
\varphi_1(\eta, k) \\
\varphi_2(\eta, k)
\end{pmatrix}
\equiv e^{-\eta}
\begin{pmatrix}
\delta(\eta, k) \\
-\theta(\eta, k)/H
\end{pmatrix} \\
\eta = \log \frac{a}{a_{in}} \\
\Omega = \begin{pmatrix}
1 & -1 \\
-\frac{3}{2} & \frac{3}{2}
\end{pmatrix}
\]

and the only non-zero components of the vertex are

\[
\gamma_{121}(k_1, k_2, k_3) = \gamma_{112}(k_1, k_3, k_2) = \delta_D(k_1 + k_2 + k_3) \frac{(k_2 + k_3) \cdot k_2}{2k_2^2} \\
\gamma_{222}(k_1, k_2, k_3) = \delta_D(k_1 + k_2 + k_3) \frac{|k_2 + k_3|^2 k_2 \cdot k_3}{2k_2^2 k_3^2}
\]

Compact Perturbation Theory

Crocce, Scoccimarro ‘05
The initial ‘time’ $\eta=0$ corresponds to $z=z_{in}$, chosen well inside the linear epoch. In practice, we take $z_{in}=80$.

Extension to other cosmologies

$$\eta = \log \frac{a}{a_{in}} \rightarrow \eta = \log \frac{D^+}{D_{in}^+}$$

where $D^+$ is the linear growth factor

Bouchet et al. '92
Bernardeau '94
Nusser et al. '98
An action principle

The fluid equations can be derived by varying the action:

\[ S = \int d\eta_1 d\eta_2 \chi_a g_{ab}^{-1} \varphi_b - \int d\eta e^\eta \gamma_{abc} \chi_a \varphi_b \varphi_c \]

where the auxiliary field \( \chi_a(\eta, k) \) has been introduced and \( g_{ab}(\eta_1, \eta_2) \) is the retarded propagator:

\[ (\delta_{ab} \partial_\eta + \Omega_{ab}) g_{bc}(\eta, \eta') = \delta_{ac} \delta_D(\eta - \eta') \]

so that \( \varphi^0_a(\eta, k) = g_{ab}(\eta, \eta') \varphi^0_b(\eta', k) \) is the solution of the linear equation.

Explicitly, one finds:

\[
g(\eta_1, \eta_2) = \begin{cases} B + A e^{-5/2(\eta_1 - \eta_2)} & \eta_1 > \eta_2 \\ 0 & \eta_1 < \eta_2 \end{cases}
\]

\[
B = \frac{1}{5} \begin{pmatrix} 3 & 2 \\ 3 & 2 \end{pmatrix}, \quad A = \frac{1}{5} \begin{pmatrix} 2 & -2 \\ -3 & 3 \end{pmatrix}
\]

Initial conditions:

\[ \varphi^0_b(\eta', k) \propto u_b = \begin{pmatrix} 1 & 1 \\ 1 & -3/2 \end{pmatrix} \]
The generating functional

\[ Z[J, \Lambda] = \int \mathcal{D}\varphi \mathcal{D}\chi \exp \left\{ \int d\eta_1 d\eta_2 \left[ -\frac{1}{2} \chi g^{-1} P L g^{-1} T \chi + i \chi g^{-1} \varphi \right] - i \int d\eta [e^{\eta} \gamma \chi \varphi - J \varphi - \Lambda \chi] \right\} \]

valid for \textbf{Gaussian} initial conditions, encoded in the power spectrum:

\[ P_{ab}^L(\eta, \eta'; k) \equiv (g(\eta) P^0(k) g^T(\eta'))_{ab} \]

Derivatives of \( Z \) w.r.t. the sources \( J \) and \( \Lambda \) give all the N-point correlation functions (power spectrum, bispectrum, ...) and the full propagator.
Compact Diagrammar

- **propagator (linear growth factor):**
  \[-i \, g_{ab}(\eta_a, \eta_b)\]

- **power spectrum:**
  \[P^L_{ab}(\eta_a, \eta_b; k)\]

- **interaction vertex:**
  \[-i \, e^n \, \gamma_{abc}(k_a, k_b, k_c)\]

Example: 1-loop correction to the density power spectrum:

\[\text{a.k.a. “P22”}\]
\[\text{a.k.a. “P13”}\]

All known results in cosmological perturbation theory are expressible in terms of diagrams in which only a trilinear fundamental interaction appears.
Beyond perturbation theory: the renormalization group

Inspired by applications of Wilsonian RG to field theory: the RG parameter is momentum

Modify the primordial ($z=z_{in}$) power spectrum as: $P^0_\chi(k) = P^0(k) \Theta(\lambda - k)$ (step function)

then, plug it into the generating functional: $Z[J, \Lambda] \rightarrow Z_\lambda[J, \Lambda]

$Z_\lambda[J, \Lambda] = \int D\varphi D\chi \exp \left\{ \int d\eta_1 d\eta_2 \left[ -\frac{1}{2} \chi g^{-1} P_L g^{T^{-1}} \chi + i \chi g^{-1} \varphi \right] - i \int d\eta [e^{\gamma \chi \varphi \varphi} - J \varphi - \Lambda \chi] \right\}

The evolution from $\lambda = 0$ to $\lambda = \infty$ can be described non-perturbatively by RG equations:

$$\frac{\partial}{\partial \lambda} Z_\lambda = \int d\eta d\eta' \left[ \frac{1}{2} \frac{\partial}{\partial \lambda} \left( g^{-1} P^L g^{-1} T \right)_{ab} \frac{\delta^2 Z_\lambda}{\delta \Lambda_b \delta \Lambda_a} \right]$$
The propagator

\[ \delta^{(3)}(k + k') G_{\lambda,ab}(k; \eta_a, \eta_b) = - \frac{\delta^2 W_\lambda[J, \Lambda]}{\delta J_a(k, \eta_a) \delta \Lambda_b(k', \eta_b)} \\]

\[ W_\lambda[J, \Lambda] = -i \log Z_\lambda[J, \Lambda] \]

\[ \frac{\partial}{\partial \lambda} \frac{\delta^2 W_\lambda}{\delta J_a \delta \Lambda_b} = \frac{1}{2} \int d\eta_c d\eta_d d^3q \delta(\lambda - q) \left( g^{-1} P^L g^{-1T} \right)_{cd} \frac{\delta^4 W_\lambda}{\delta J_a \delta \Lambda_b \delta \Lambda_c \delta \Lambda_d} \]

in pictures...

RG Kernel: \( \delta(\lambda - q) G_\lambda(q; \eta, 0) P^0(q) G^T_\lambda(q; \eta', 0) \)

infinite tower of RGE's
Approximation: full, 1PI, vertices $\rightarrow$ tree-level vertices

Large momentum, $k >> \lambda$: it can be integrated analytically!

\[
G_{\lambda, ab}(k; \eta_a, \eta_b) = g_{ab}(\eta_a, \eta_b) \exp \left[ -\frac{k^2 \sigma^2}{2} (e^{\eta_a} - e^{\eta_b})^2 \right]
\]

where \[
\sigma^2 = \frac{1}{3} \int d^3 q \frac{P^0(q)}{q^2} \theta(\lambda - q)
\]
in perturbation theory, it can be obtained by summing the infinite series of chain diagrams ([Crocce Scoccimarro, ’06](https://doi.org/10.1086/523205))

\[
\ldots + \frac{k}{\cdots} + \frac{k}{\cdots} + \frac{k}{\cdots} + \ldots
\]

physically, it represents the effect of multiple interactions of the k-mode with the surrounding modes

\[
G \sim e^{-\frac{k^2 \sigma^2}{2} e^{2\eta}}
\]

`coherence momentum’ \( k_{ch} = (\sigma e^\eta)^{-1} \approx 0.15 \, h \, \text{Mpc}^{-1} \)

in the BAO range!
A self-generated UV cutoff

Inserting this result in the expression for the RG kernel, we get:

$$\mathcal{K}_\lambda(q; \eta, \eta') = \delta(\lambda - q) P^0(q) \exp \left[ -\frac{q^2 \sigma^2}{2} \left( (e^\eta - 1)^2 + (e^{\eta'} - 1)^2 \right) \right]$$

The effect of modes with momenta larger than $\sigma^{-1}_\lambda (e^\eta - 1)^{-1}$ is exponentially screened.

The UV is much better behaved than one would guess from ‘usual’ perturbation theory!!

$$\mathcal{K}_\lambda(q; \eta, \eta') \longrightarrow \delta(\lambda - q) P^0(q) (1 + O(q^2 \sigma^2 e^{2\eta}))$$
The power spectrum

The full PS has the structure: \[ P_{ab} = P^I_{ab} + P^{II}_{ab} \]

with \[ P^I_{ab}(k; \eta_a, \eta_b) = G_{ac}(k; \eta_a, 0)G_{bd}(k; \eta_b, 0)P^0_{cd}(k) \]

\[ P^{II}_{ab}(k; \eta_a, \eta_b) = \int_0^{\eta_a} ds_1 \int_0^{\eta_b} ds_2 G_{ac}(k; \eta_a, s_1)G_{bd}(k; \eta_b, s_2)\Phi_{cd}(k; s_1, s_2) \]
\[ \partial_\lambda \Phi_{ab,\lambda}(k; s_1, s_2) = 4 e^{s_1+s_2} \int d^3q \delta(\lambda - q) P_{dc,\lambda}^I(q; s_1, s_2) \times P_{fe,\lambda}(|q - k|; s_1, s_2) \gamma_{adf}(k, -q, -k + q) \gamma_{bce}(-k, q, k - q) \]

again, tree-level vertices...

accuracy of linear theory up to \(k \sim 0.12 \, h/\text{Mpc}\) is fortuitous: cancellation between two large non-linear effects
comparison with other approaches

$\Lambda$CDM model
$\Omega_\Lambda = 0.73$, $\Omega_b = 0.043$, $h = 0.7$

data from N-body simulations of Jeong and Komatsu, astro-ph/0604075

\[ \Lambda \text{CDM model} \]
\[ \Omega_{\Lambda} = 0.7, \Omega_b = 0.046, \, h = 0.72 \]
Conclusions

RG techniques can be conveniently applied to cosmology; Exact RG equations can be derived for any kind of correlation function; Systematic approximation schemes, based on truncations of the full hierarchy of equations, can be applied, borrowing the experience from field theory.

A simple approximation scheme already shows the emergence of an intrinsic UV cutoff in the RG running.

RG-improved results on the power spectrum agree with existing N-body simulations better than any other approach (see also Crocce and Scoccimarro, arXiv: 0704.2783).

Future lines of development include: including the running of the vertex, computing the bispectrum, non-gaussian initial conditions.