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Non-Gaussianity from Preheating

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Curvature Perturbations

- Power spectrum of inflaton fluctuations (contribution from log scale)

$$\mathcal{P}_\phi(k) = \frac{H^2}{4\pi^2}$$

- Curvature perturbation at constant density

$$\zeta = \frac{H}{\dot\phi}\delta\phi = \frac{\delta T}{T}$$

- Temperature power spectrum, observed by COBE, WMAP, . . .

$$\mathcal{P}_T \approx (10^{-5})^2 = \frac{H^2}{\dot\phi^2}\mathcal{P}_\phi = \frac{H^4}{4\pi^2\dot\phi^2}$$

  - Linearized theory $\Rightarrow$ Gaussian perturbations

Non-Gaussianity

- Simplest assumption: Local non-Gaussianity

$$\zeta(x) = \zeta_0(x) - \frac{3}{5} f_{\rm NL} \left(\zeta_0(x)^2 - \langle \zeta_0(x)^2 \rangle \right)$$

- Three-point function

$$\begin{aligned} \langle \zeta(k_1)\zeta(k_2)\zeta(k_2) \rangle &= -\frac{5}{6} f_{\rm NL} \left[P(k_1)P(k_2) + \text{cyclic} \right] \\ &\times (2\pi)^3 \delta(k_1 + k_2 + k_3) \end{aligned}$$

- Use this as a *definition* of $f_{\rm NL}$
  - Generally $f_{\rm NL} = f_{\rm NL}(k_1, k_2, k_3)$
- Single field: $f_{\rm NL} \ll 1$ (Maldacena 2003)
- Observations: WMAP $-54 < f_{\rm NL} < 134$

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Preheating

- End of inflation: $V(\phi) \rightarrow$ radiation (=reheating)
  - Perturbative reheating (slow)
  - Parametric resonance (=preheating)
  - Phase transition (=tachyonic preheating)
- Last two highly nonlinear $\Rightarrow$ non-Gaussianity?
  - Local processes: Cannot *create* super-Hubble perturbations
  - Need a light scalar χ: Convert $\delta\chi \rightarrow \zeta$
- Difficult to compute
  - Non-equilibrium quantum fields coupled to gravity

Massless Preheating

- Chaotic inflation + massless scalar χ

$$V(\phi,\chi) = \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2, \quad \lambda \sim 10^{-14}$$

- During inflation $\chi = 0$, $\phi \gg M_{\rm Pl}$
  - Mass of χ: $m_\chi = g\phi$
- Compare with Hubble rate $H = \sqrt{V/3M_{\rm Pl}^2} = \sqrt{\lambda/12}\phi^2/M_{\rm Pl}$
  - If $g^2 \lesssim \lambda$, then χ is light ($m_\chi \lesssim H$)
  - Similar (Gaussian) perturbations at the end of inflation

$$\mathcal{P}_\chi \approx \mathcal{P}_\phi \approx \frac{H^2}{4\pi^2} \approx 10^{-12}$$

  - Will these be converted into ζ?
  - Jokinen&Mazumdar 2006: Yes!! – $f_{\rm NL} \sim O(1000)$

Parametric Resonance

- After inflation: Radiation domination $a \propto t^{1/2}$
- Inflaton zero mode $\ddot{\phi} + 3H\dot{\phi} + \lambda\phi^3 = 0$
  - Rescale the field $\phi = a^{-1}\tilde{\phi}$
  - Rescale time $dx = a^{-1}\lambda^{1/2}\tilde{\phi}_{\rm ini}dt$

$\Rightarrow$ $\boxed{\tilde{\phi}'' + \lambda\tilde{\phi}^3 = 0}$ $\Rightarrow$ $\boxed{\tilde{\phi}(x) = \tilde{\phi}_{\rm ini}{\rm cn}(x; 1/\sqrt{2})}$ (Jacobi cosine)

- Inhomogeneous χ modes $\chi_k = a^{-1}\tilde{\chi}_k$

$$\boxed{\tilde{\chi}_k'' + \left[\kappa^2 + \frac{g^2}{\lambda}{\rm cn}^2(x; 1/\sqrt{2})\right]\tilde{\chi}_k = 0, \quad \kappa^2 = \frac{k^2}{\lambda\tilde{\phi}_{\rm ini}^2}}$$

  - Lamé equation
  - Same as in Minkowski space (conformal invariance)

Lamé Equation

$$\tilde{\chi}_k'' + \left[\kappa^2 + \frac{g^2}{\lambda}\mathrm{cn}^2(x;1/\sqrt{2})\right]\tilde{\chi}_k = 0$$

- Periodic coefficients

$\Rightarrow$ Floquet theorem

  - Solutions of the form $\tilde{\chi}_k(x) = e^{\mu x} f(x)$ with periodic $f(x)$
  - If $Re\mu > 0$, grows exponentially = parametric resonance

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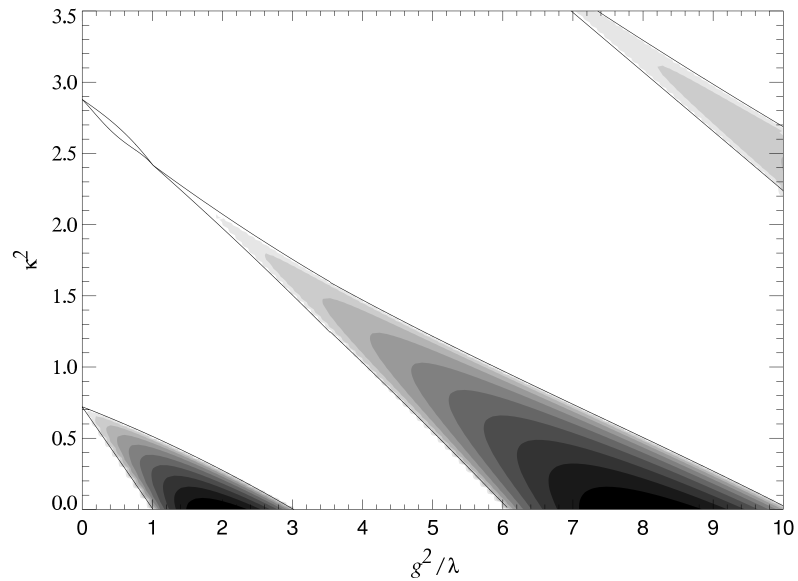
Instability Bands of the Lamé Equation

$$\tilde{\chi}_k'' + \left[\kappa^2 + \frac{g^2}{\lambda}\mathrm{cn}^2(x; 1/\sqrt{2})\right]\tilde{\chi}_k = 0$$



(Greene et al 1997)

Parametric Resonance

$$\tilde{\chi}_k'' + \left[\kappa^2 + \frac{g^2}{\lambda}\mathrm{cn}^2(x; 1/\sqrt{2})\right]\tilde{\chi}_k = 0$$

- Rapid energy transfer $\phi \to \chi$
- Resonance ends when dynamics becomes non-linear

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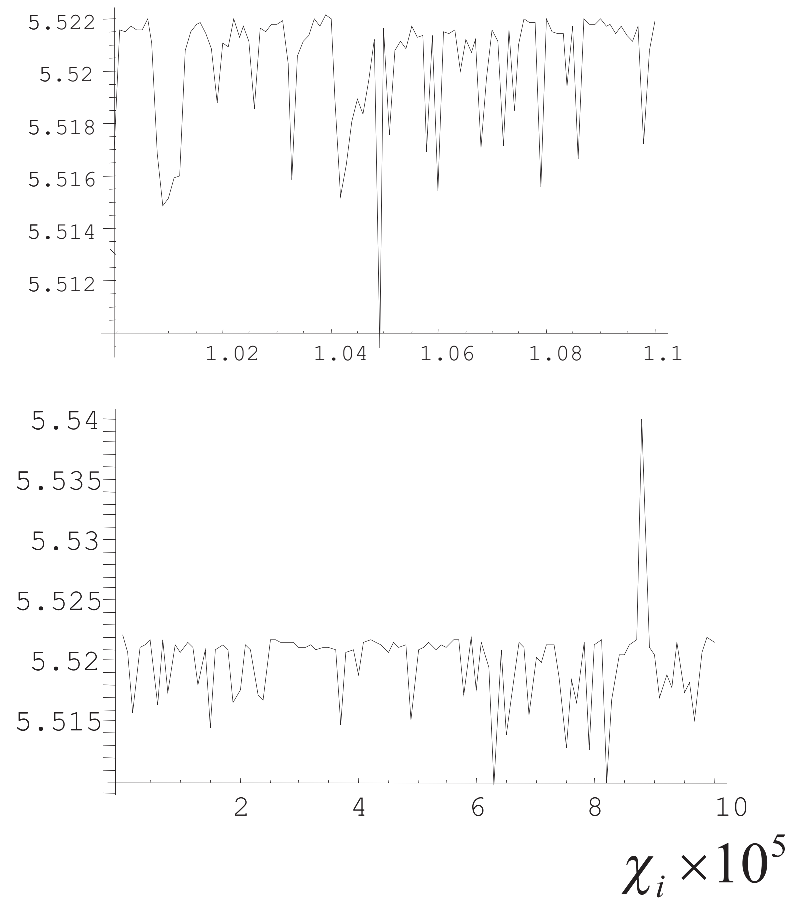
Separate Universes

- Curvature perturbation due to preheating?
  - Non-linearities: Linear theory, Hartree not enough
- Separate universe approximation
  - Regions separated by $d \gg 1/H$ treated as independent FRW universes, but with different initial data
  - Friedmann eq. $\Rightarrow a(t)$, $H(t)$
  - Curvature perturbation $\zeta = \delta \ln a|_H$ (constant H)
  - Non-linear, includes gravity, but ignores sub-Hubble dynamics
- Initial conditions:
  - Changes in $\phi \Rightarrow$ Usual inflationary perturbations
  - Changes in $\chi \Rightarrow$ New contribution

$\Rightarrow$ Need $N(\chi_{\rm ini}) \equiv \ln a|_H(\chi_{\rm ini})$

Earlier Results



(Bassett and Tanaka 2003)

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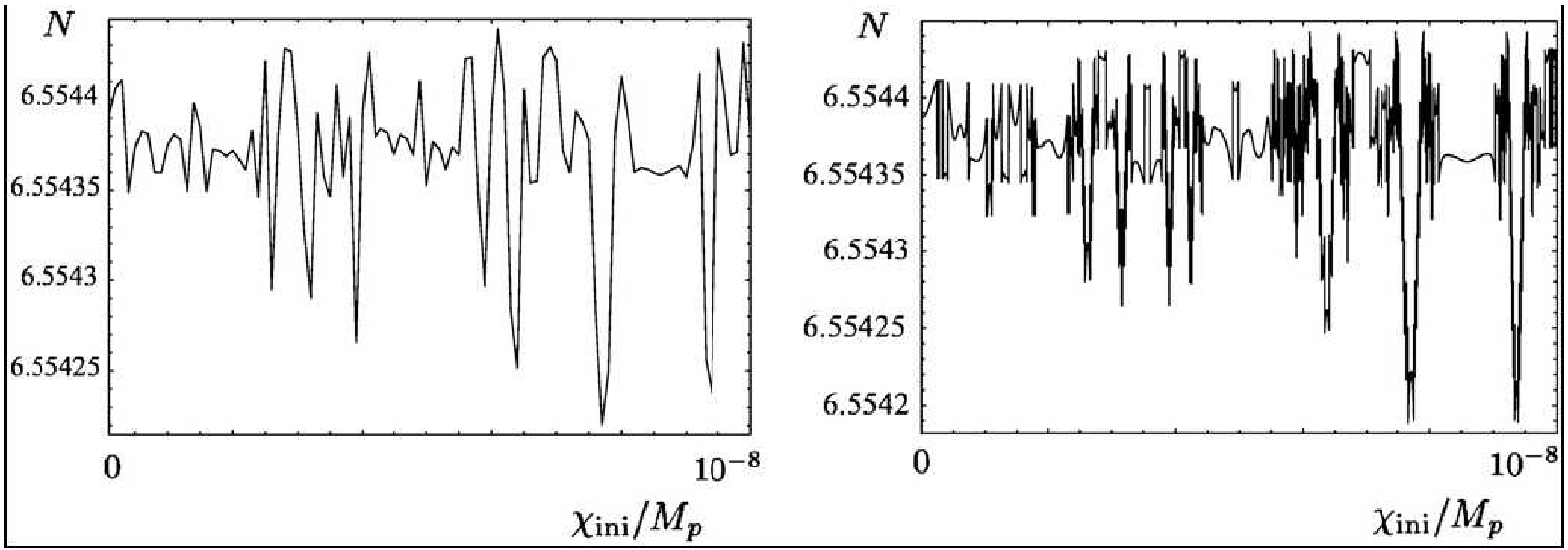
Earlier Results



(Suyama and Yokoyama 2006)

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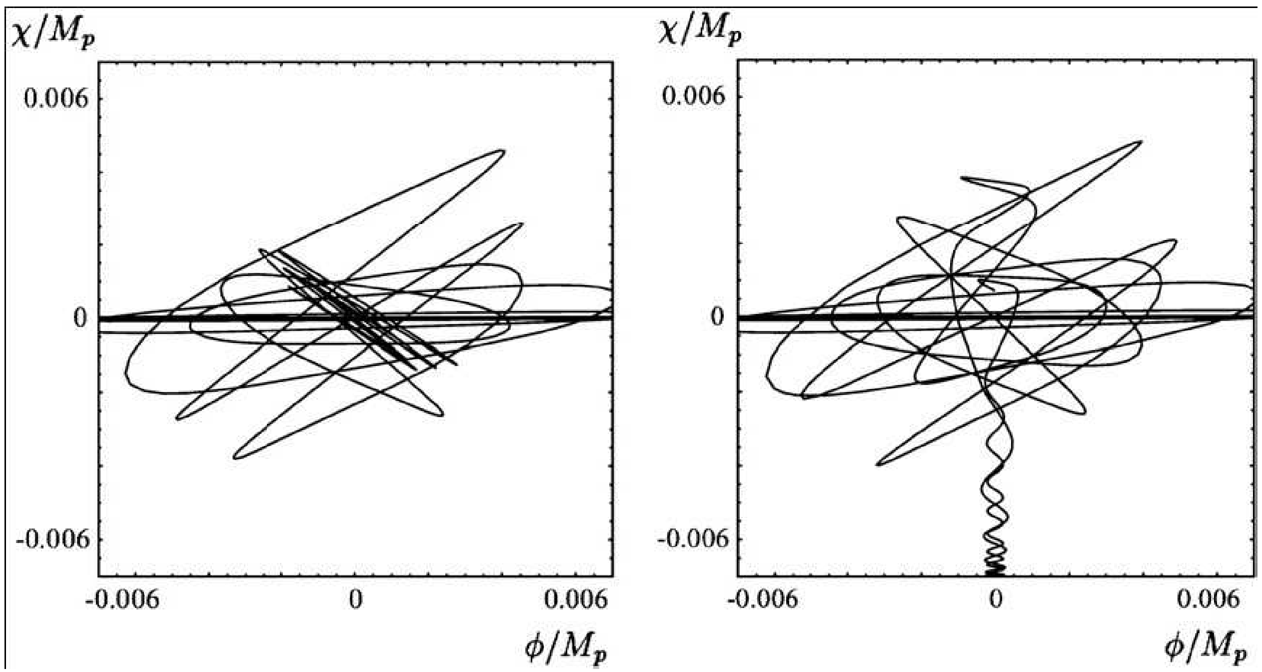
Earlier Results

$\chi_{ini} = 5.0 \times 10^{-9}$ $\chi_{ini} = 5.2 \times 10^{-9}$



(Suyama and Yokoyama 2006)

- Chaotic (?)

Inhomogeneous Fields

- Make ϕ and χ position-dependent

$$\begin{aligned} \ddot{\phi} + 3H\dot{\phi} - \frac{1}{a^2}\vec{\nabla}^2\phi + \lambda\phi^3 + g^2\chi^2\phi &= 0 \\ \ddot{\chi} + 3H\dot{\chi} - \frac{1}{a^2}\vec{\nabla}^2\chi + g^2\phi^2\chi &= 0 \end{aligned}$$

- Non-linear dynamics
- Standard approach to preheating (Khlebnikov& Tkachev, Prokopec&Roos)
- Does not allow to calculate ζ

Inhomogeneous Fields

- Make ϕ and χ position-dependent

$$\begin{aligned} \ddot{\phi} + 3H\dot{\phi} - \frac{1}{a^2}\vec{\nabla}^2\phi + \lambda\phi^3 + g^2\chi^2\phi &= 0 \\ \ddot{\chi} + 3H\dot{\chi} - \frac{1}{a^2}\vec{\nabla}^2\chi + g^2\phi^2\chi &= 0 \end{aligned}$$

- Couple to Friedmann equation with averaged energy density

$$\begin{aligned} H^2 &= \frac{1}{3M_{\rm Pl}^2 V}\int d^3x \left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\chi}^2 + \frac{1}{2}(\vec{\nabla}\phi)^2 + \frac{1}{2}(\vec{\nabla}\chi)^2 \right. \\ &\quad \left. + \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2\right) \end{aligned}$$

Inhomogeneous Fields

- Works if
  - the dynamicals scales are smaller than the lattice size $l \lesssim L$ (so it is described by the fields)
  - the lattice size is smaller than the horizon $L \ll 1/H$ (so the averaging makes sense)
  - the perturbations are measured at super-Hubble scale $k \ll H$ (so the separate universe approximation works)
- Non-local Friedmann eq: Tricky but doable

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Quantum Initial Conditions

- Start time: Early enough during inflation so that $m_\chi \ll H$
- Gaussian fluctuations with the same two-point function as the quantum vacuum (Khlebnikov&Tkachev)

$$\overline{|\chi_k|^2} = \overline{|\delta\phi_k|^2} = \frac{1}{V}\frac{1}{2k}$$

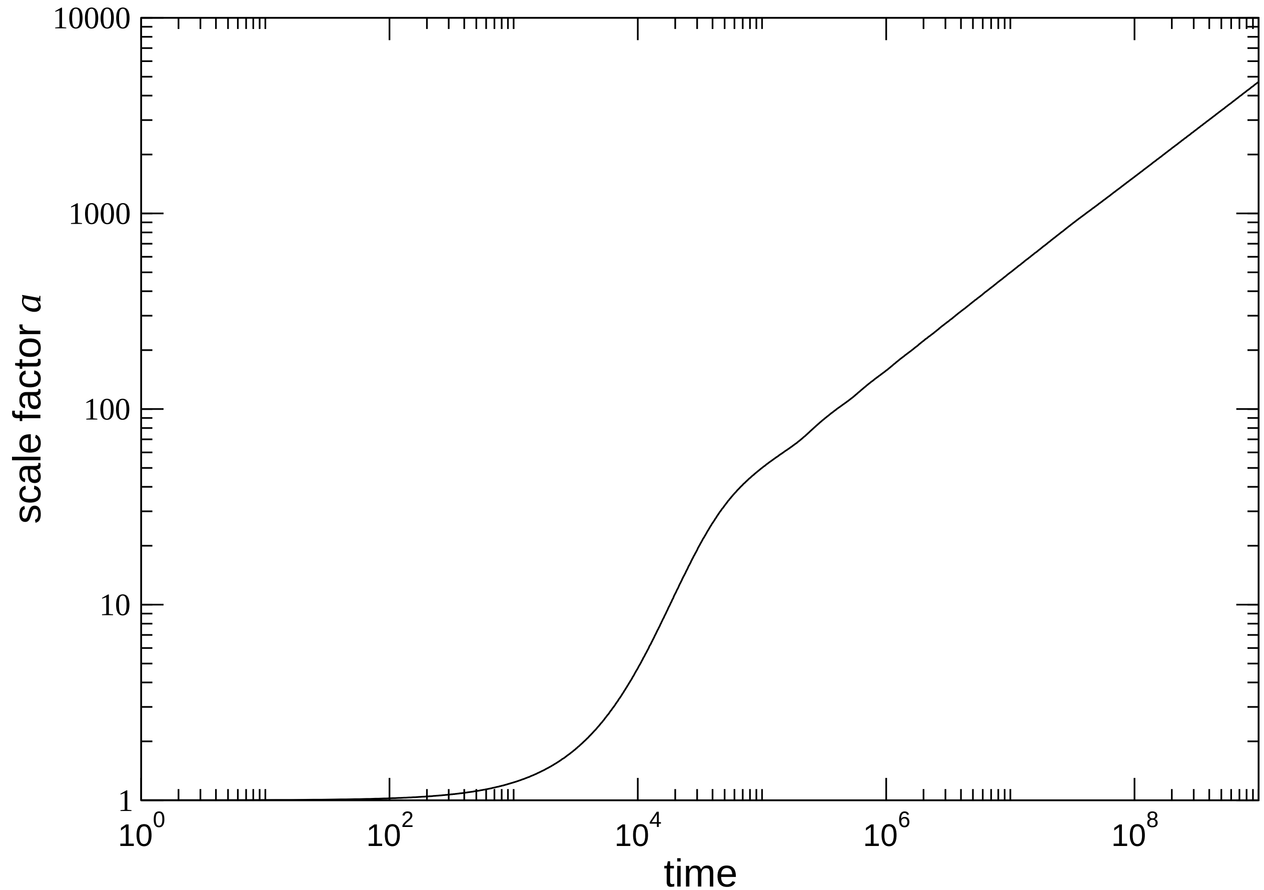
- Linear dynamics: quantum = classical
- Non-linear dynamics: quantum $\approx$ classical

Parameters

- Ultraviolet divergence $\rho_{\rm UV} \sim \delta x^{-4}$
  Must have $\rho_{\rm UV} \ll \rho_{\rm phys} \approx (\lambda/4)\phi^4$
  $\Rightarrow$ Condition on δx
- We chose:
  - $\lambda = 7 \times 10^{-14}$, $g^2/\lambda = 2.7$ (edge of first resonance)
  - Lattice sizes $32^2 \ldots 128^3$
  - $\delta x = 10^4$, $\delta\tau = 10^3$, $\phi_{\rm ini} = 5$ in Planck units

  $\Rightarrow$ $\rho_{\rm UV} \sim 10^{-16} \ll \rho_{\rm phys} \sim 10^{-11}$
- Statistics: Ten runs for each $\chi_{\rm ini}$

Inflation



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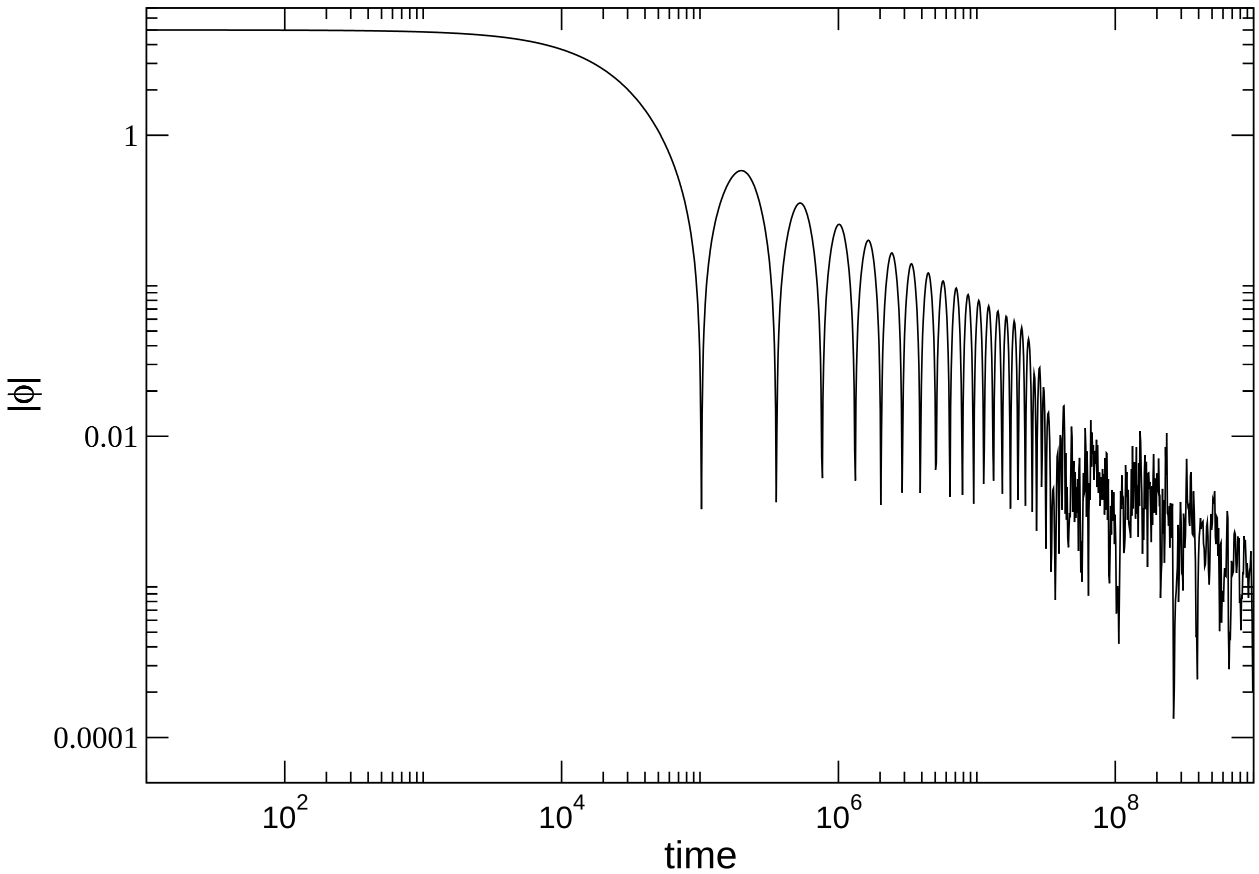
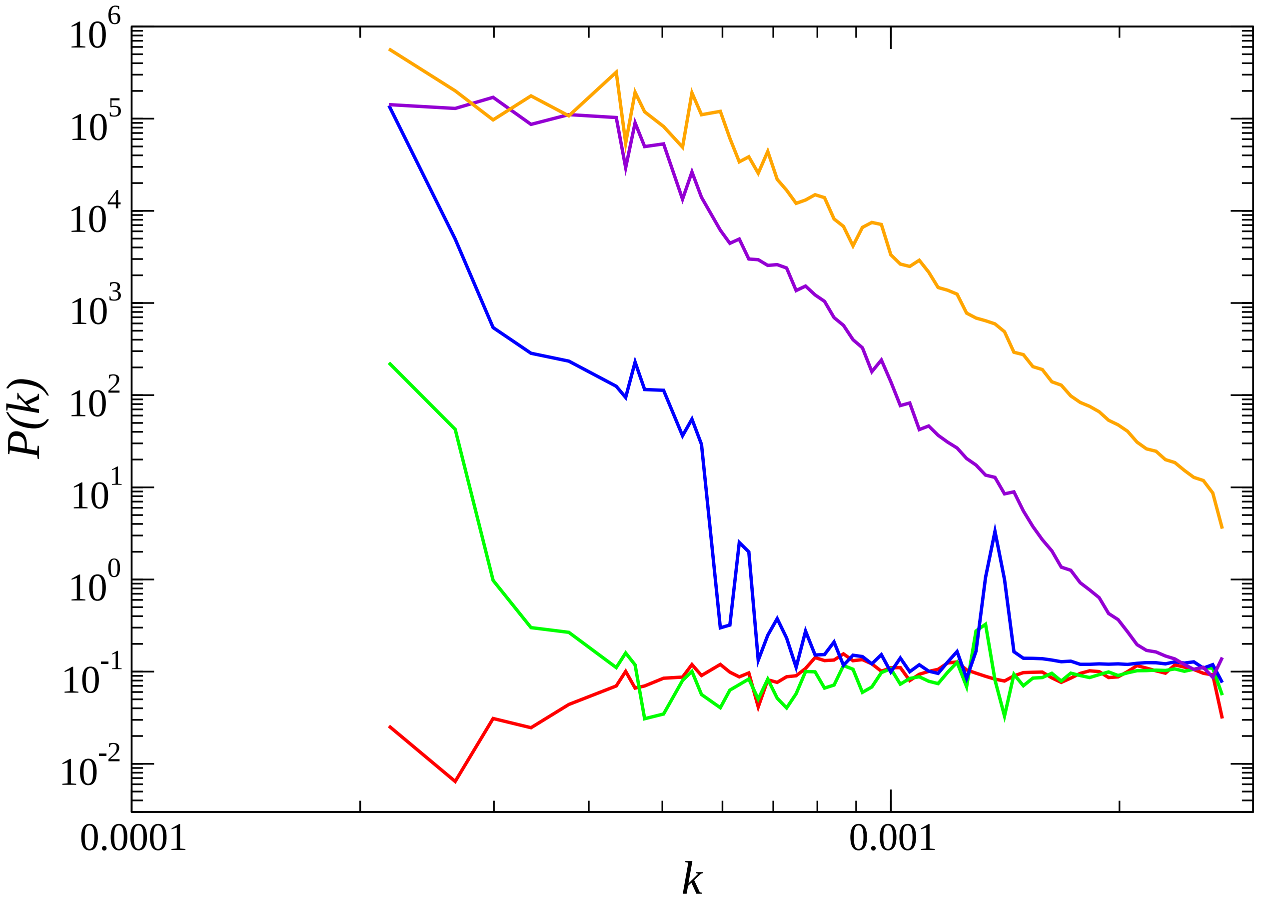
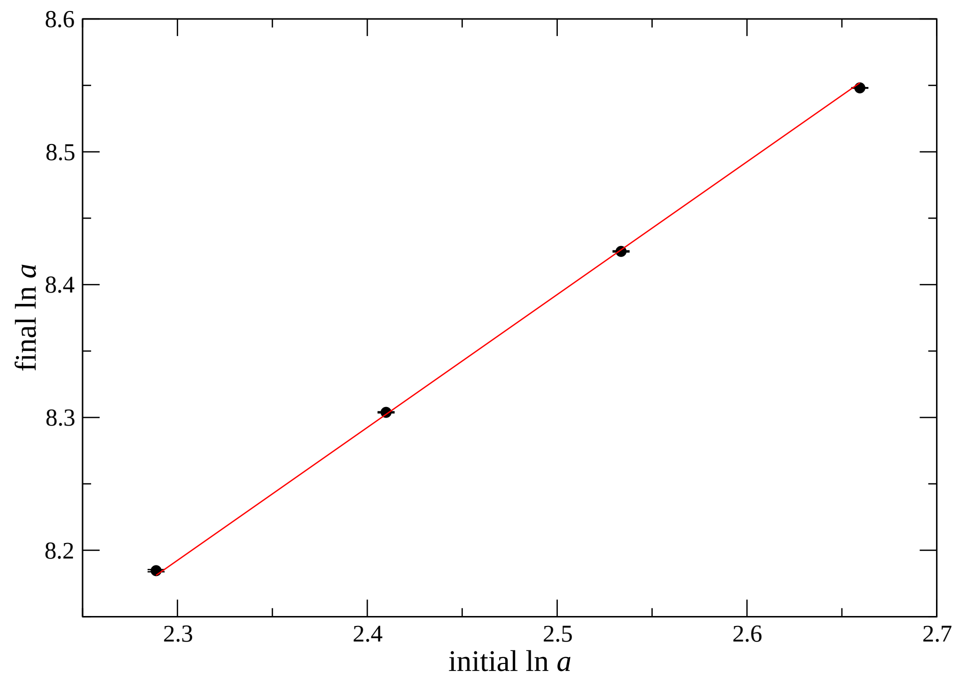
Inflaton Oscillations



Resonance



Accuracy Check



- Shift in $\ln a$ remains unchanged:
  - Measure of lattice spacing dependence
  - Calibrate results for even higher accuracy

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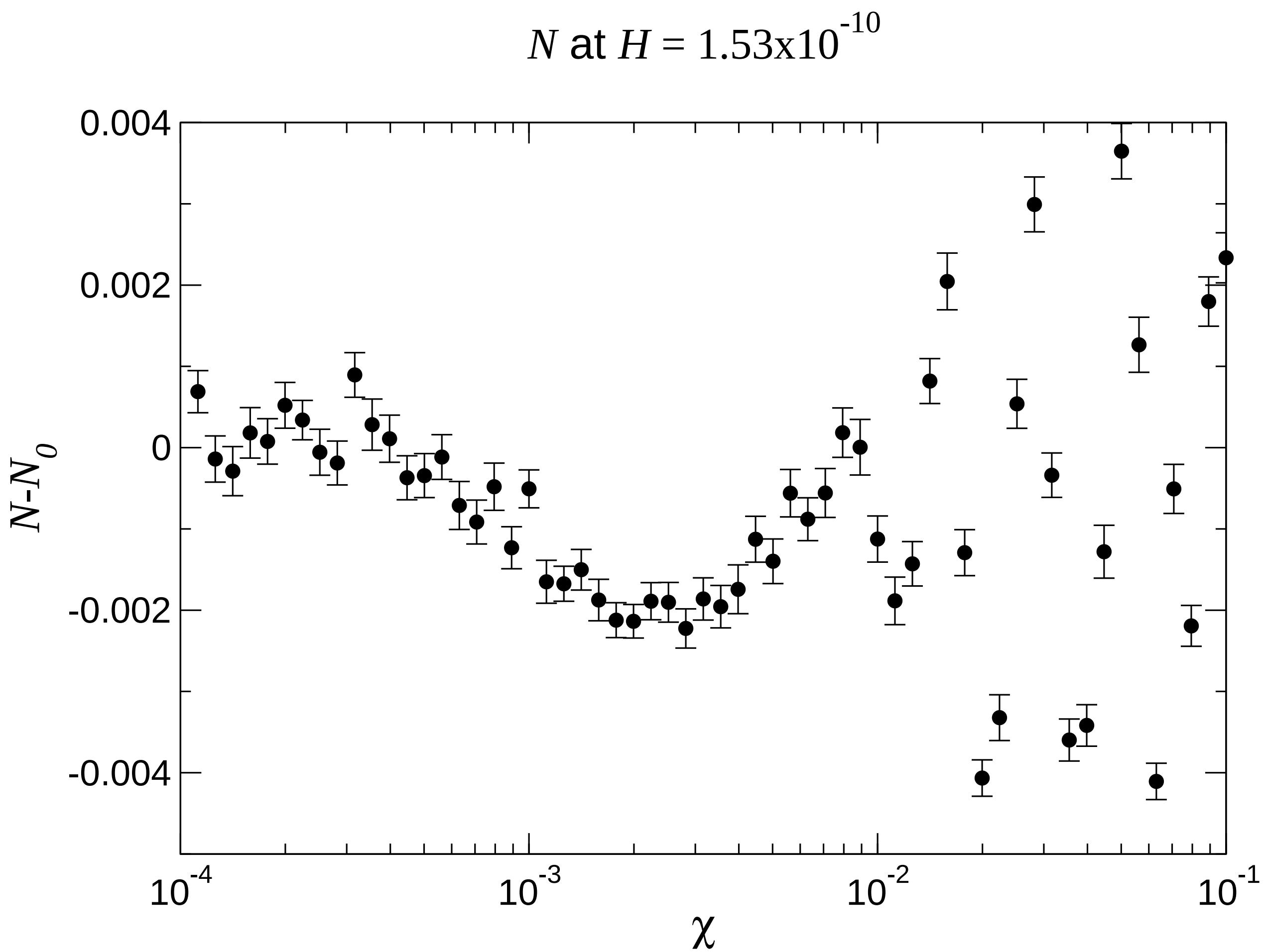
Dependence on χ



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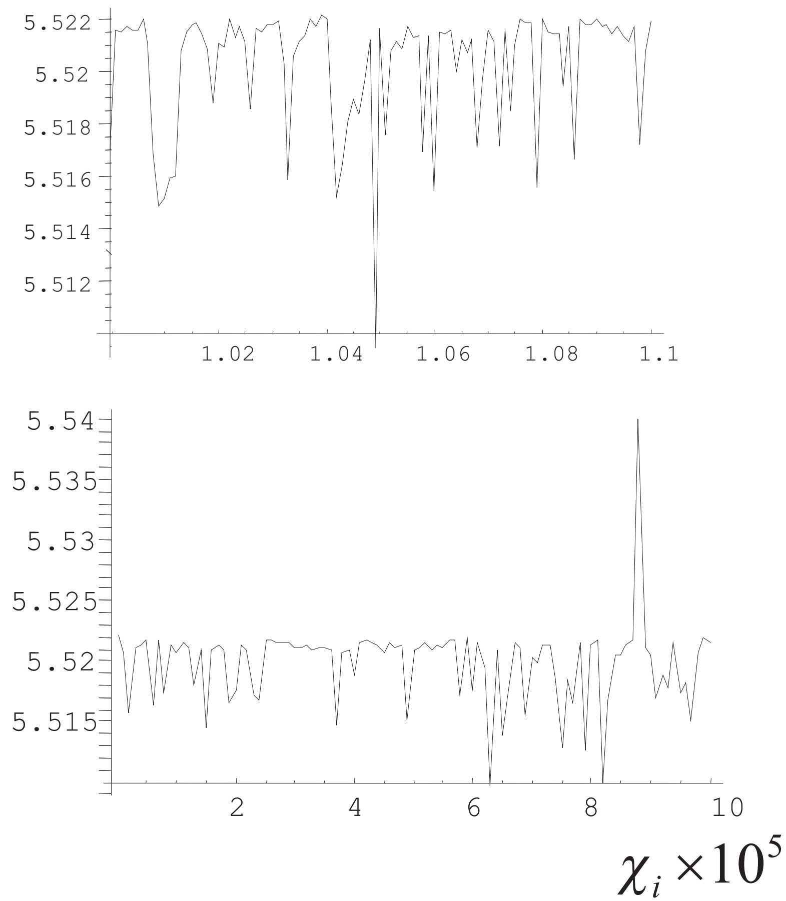
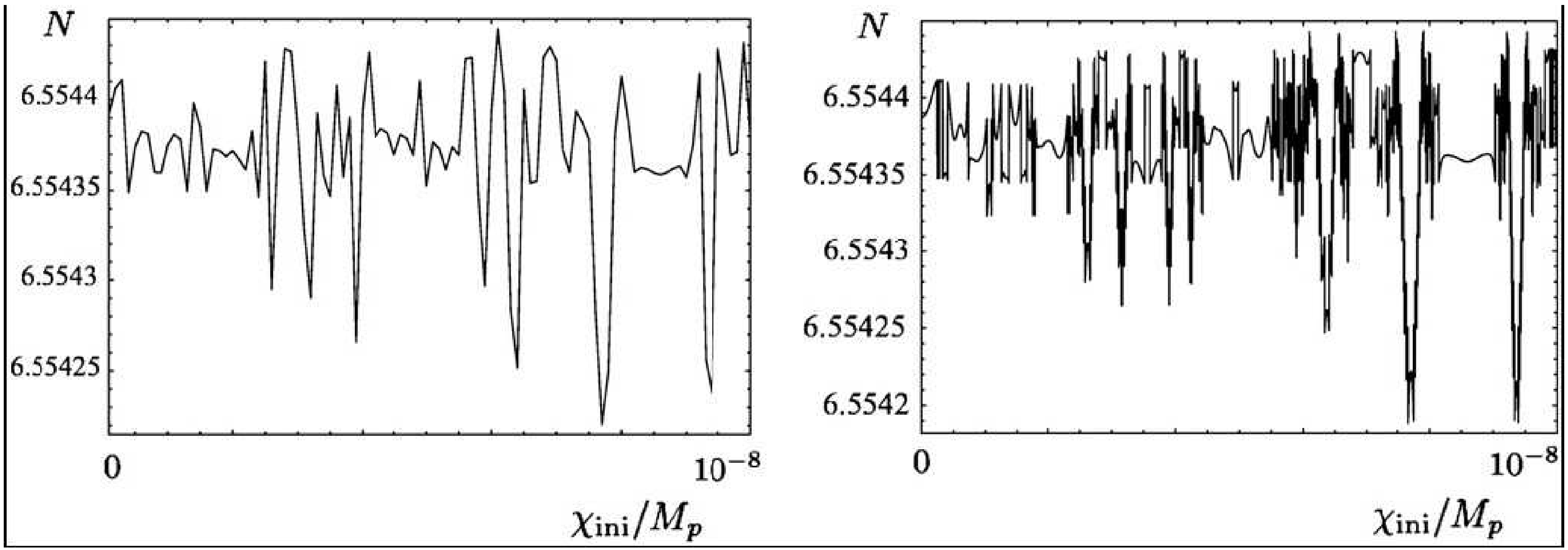
Earlier Results



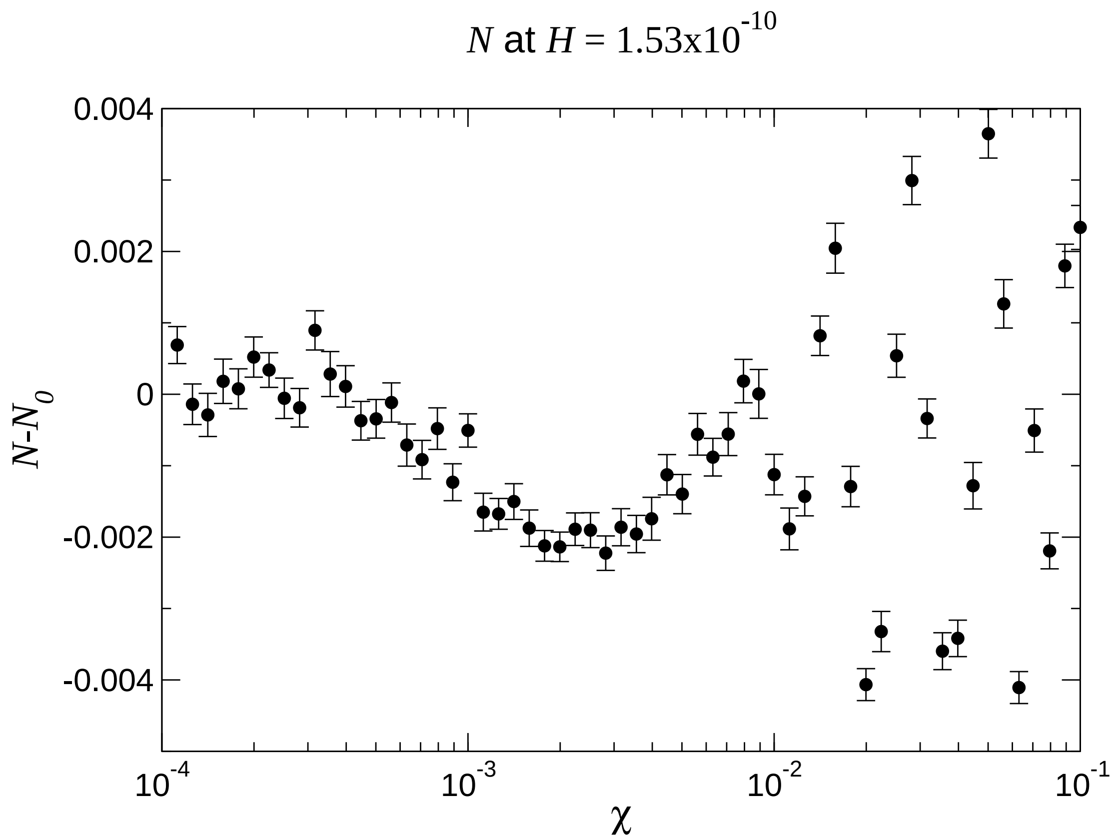
(Bassett and Tanaka 2003)

Earlier Results



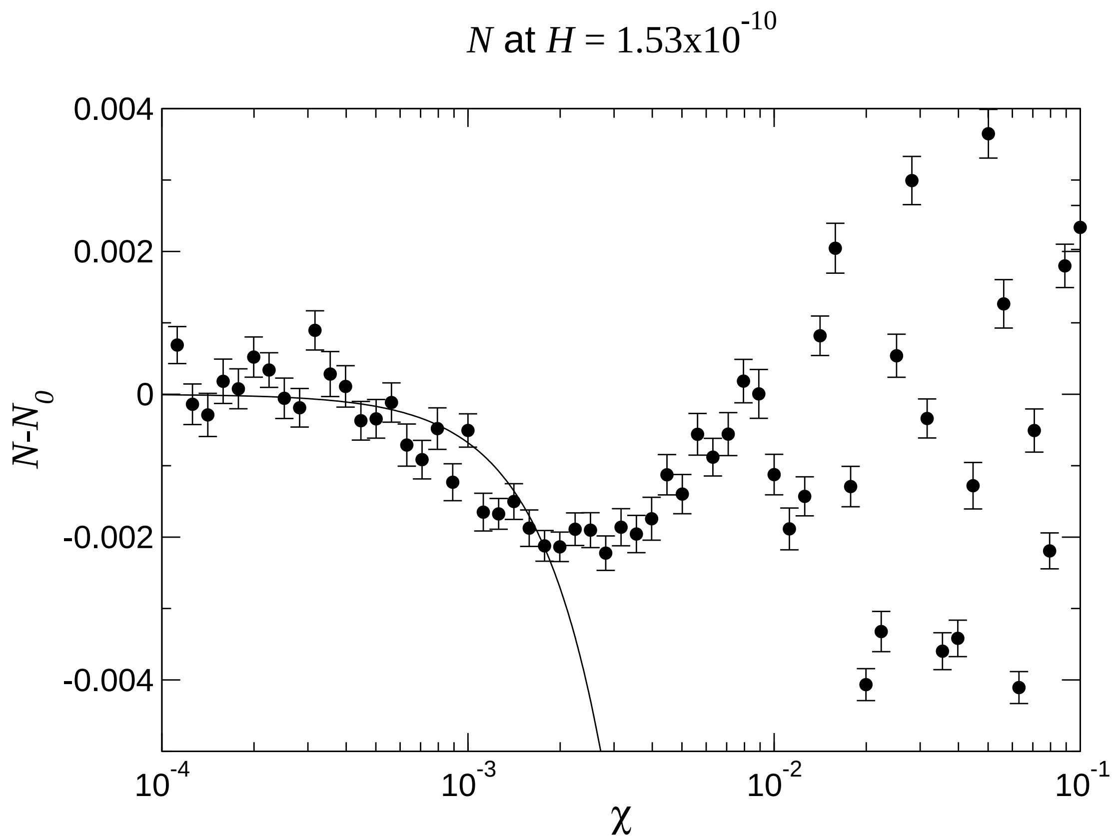
(Suyama and Yokoyama 2006)

Fitting $N(\chi)$



- Appears regular at small ξ
- Analyticity $\Rightarrow N(\chi) = N(0) + c\chi^2 + O(\chi^4)$

Fitting $N(\chi)$



- Fit $N(\chi) = N(0) + c\chi^2 + O(\chi^4)$
  $\Rightarrow c = -679 \pm 74$

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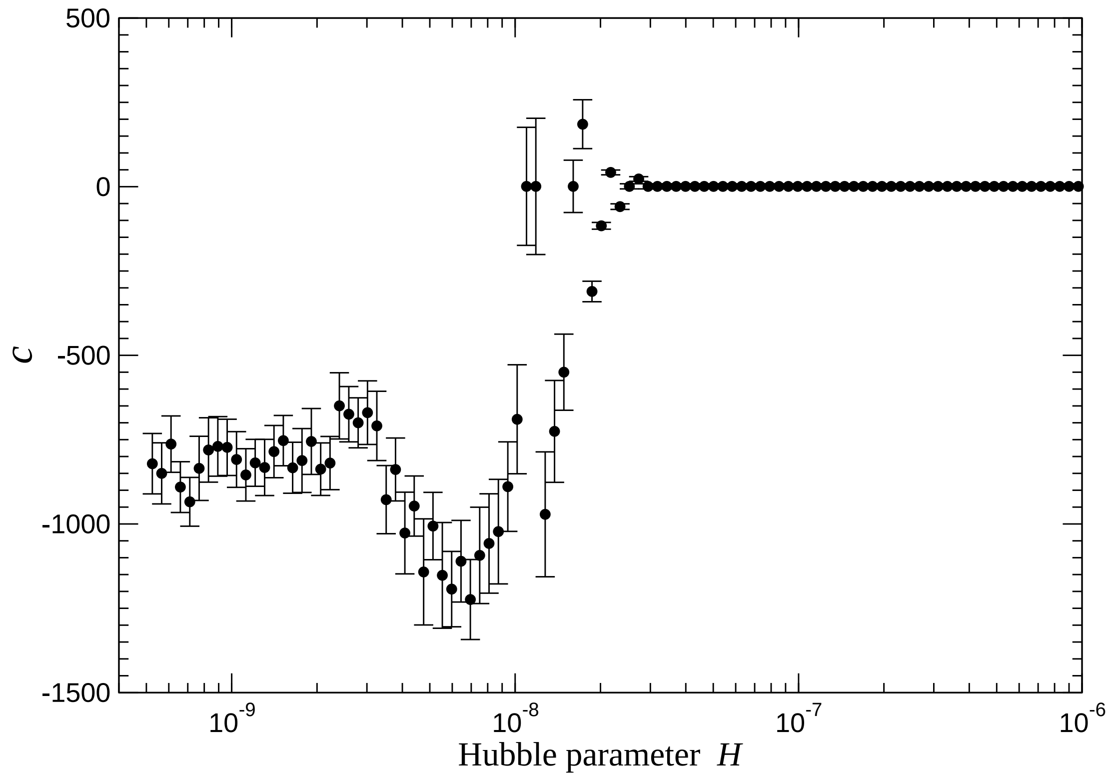
Coefficient of χ^2



- Coefficient c in $\boxed{N(\chi) = N(0) + c\chi^2}$ as a function of H for $g^2/\lambda = 2.7$

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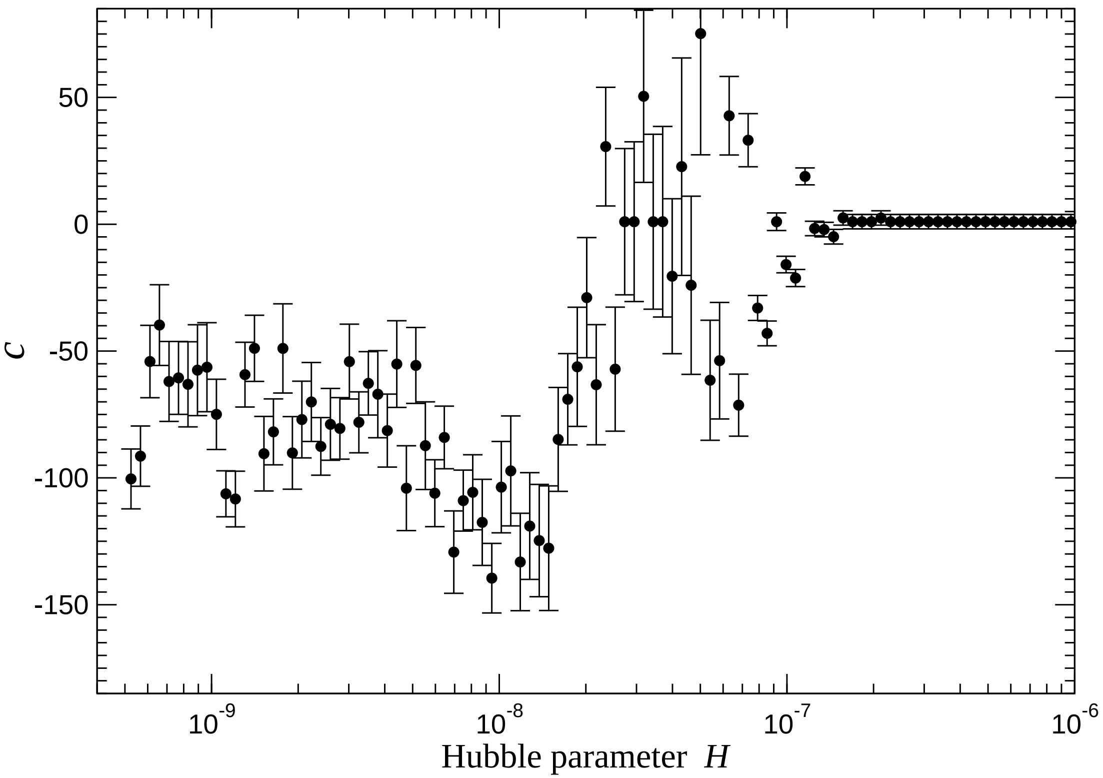
Coefficient of χ^2



- Coefficient c in $\boxed{N(\chi) = N(0) + c\chi^2}$ as a function of H for $g^2/\lambda = 2.0$

Calculating $f_{\rm NL}$

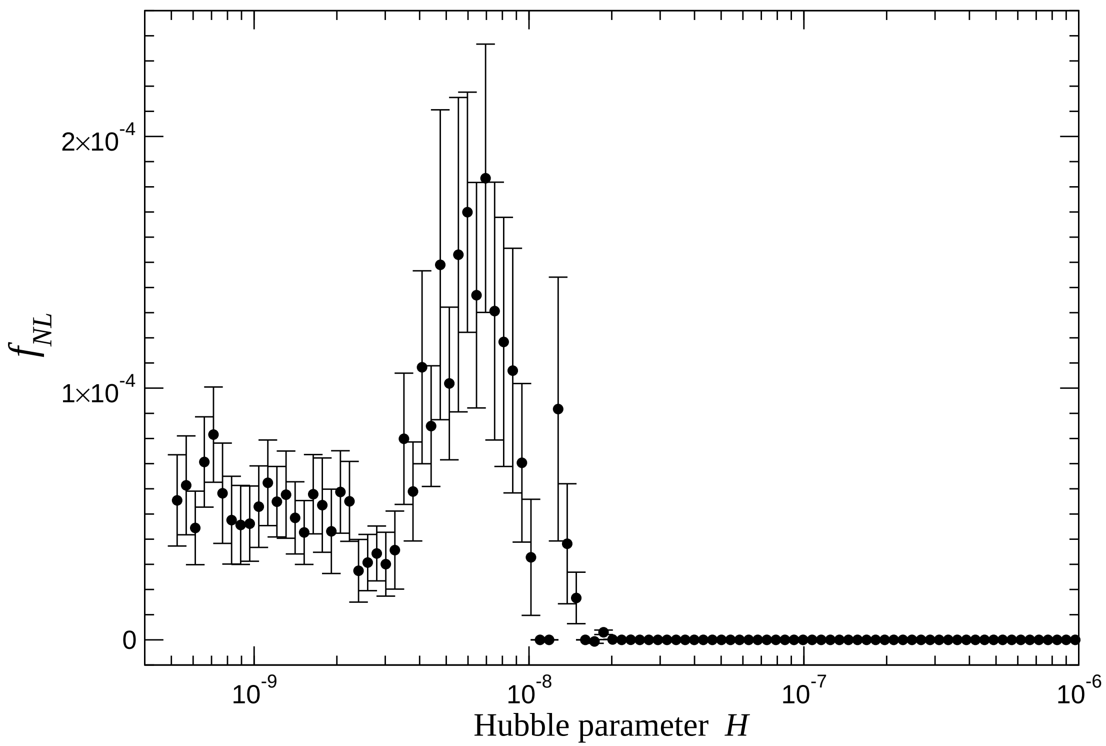
- Boubekeur&Lyth: $\zeta = \zeta_0 + c(\chi^2 - \langle\chi^2\rangle)$ gives

$$f_{NL} = -\frac{5}{6} c^3 \frac{\mathcal{P}_\chi^3}{\mathcal{P}_\zeta^2} \ln \frac{k}{H}$$

- Using $\mathcal{P}_\chi = (H^2/4\pi^2)$ and $\mathcal{P}_\zeta = (V/24\pi^2 \epsilon M_{\rm Pl}^4)$

$$\Rightarrow \quad f_{NL} = -\frac{40}{9\pi^2} c^3 \lambda M_{\rm Pl}^6 \ln \frac{k}{H}$$

The Value of f_{NL}



- Tiny value $f_{\mathrm{NL}} \sim 10^{-4}$ well below the observational bounds $-54 < f_{\mathrm{NL}} < 134$

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Conclusions

- Way to calculate non-Gaussianity due to non-equilibrium field dynamics
- Better behaved than the usual δN approach
- Massless preheating:
  - Tiny signal:
  - For $g^2/\lambda = 2.7$, $f_{\rm NL} \sim 10^{-4}$
  - For $g^2/\lambda = 2.0$, $f_{\rm NL} \sim 10^{-7}$
  - Scan g^2/λ to find $f_{\rm NL}(g^2/\lambda)$
- General method: Applies to any (bosonic) field dynamics
  - Phase transitions (hybrid inflation)
  - Topological defects
  - ...

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