Polarized Relic Gravitational Waves

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based on:
T.K., G. Gogoberidze, and B.Ratra, PRL 2005, 2007 (in preparation),
G. Gogoberidze, T.K., and A. Kosowsky, PRD (accepted) 2007

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Sussex University
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Gravitational Waves Astronomy?

More details talk by S. Rowan

- Advantage
  - Connection with High Energy Physics –
    The best laboratory to test the energy scales EVEN near the Plank scale
- Disadvantage
  - Direct detection is too complicated

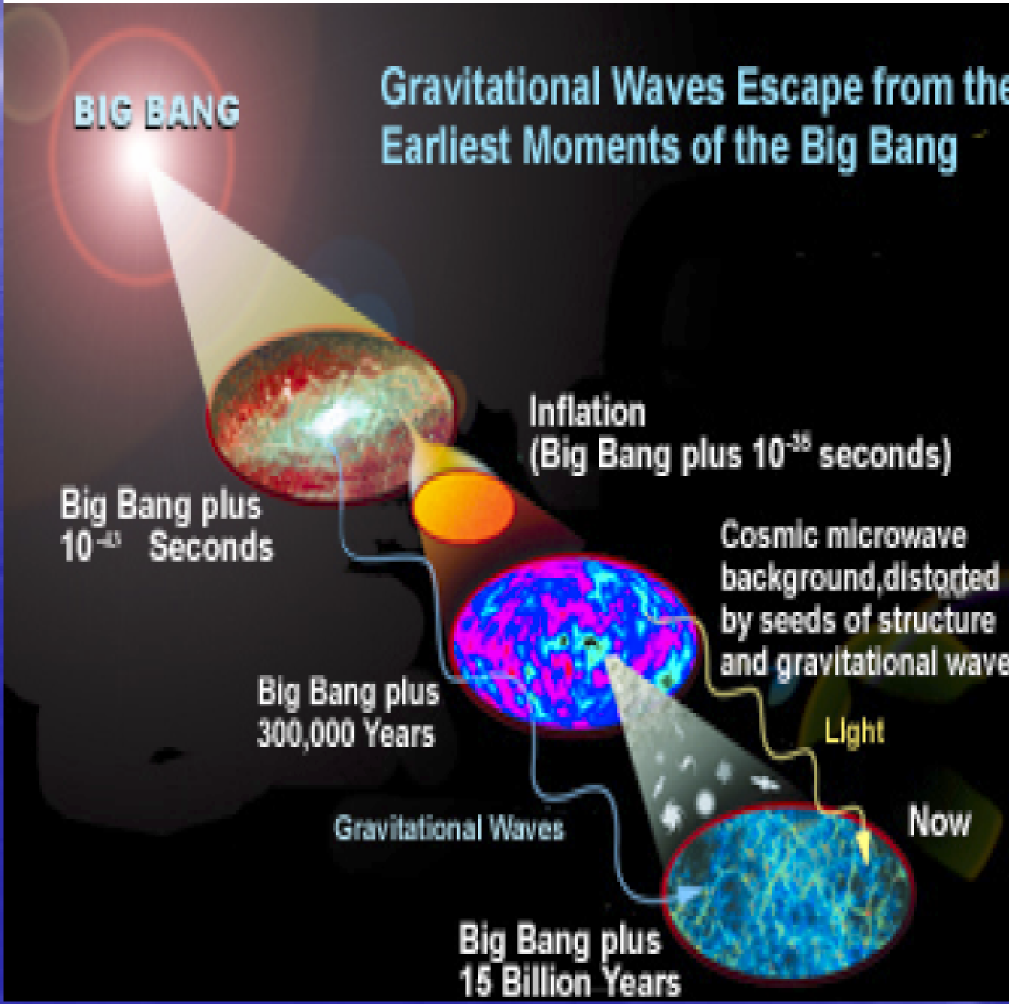
Outline

- Relic gravitational waves sources
  - Inflation
    - quantum fluctuations
    - Inflationary generated magnetic fields
  - during phase transitions
    - Bubble collisions
    - Topological defects (strings)
    - Primordial turbulence
    - Magnetic field
- Polarized gravitational waves imprints on CMB
  - Polarization anisotropies
  - Parity odd cross correlation spectra
- Polarized gravitational waves from phase transitions
  - Generation from parity violating sources
  - Helical turbulence
  - MHD inverse cascade
  - Future prospects for gravitational waves direct detection

Motivations

Precise cosmological observations allow to understand the physical processes in the very early Universe

- Since GWs propagate freely after being generate, they keep the information on their source
- Detection of GWs could open a window into new physics (parity symmetry violation, quantum gravity models, physics of phase transitions).
- If relic GWs will be detected – how it would be possible to identify their source?
  - – Characteristic frequency & spectrum shape
  - – Importance to connect GWs characteristics with theoretical models of the early Universe and physics of the source.



Tensor perturbations cosmological gravitational waves

- Cosmological GWs – linear tensor perturbations with respect to Friedman-Lemaitre-Robertson-Walker metric

  $$g_{ik}=a^2(\delta_{ik} + h_{ik})$$

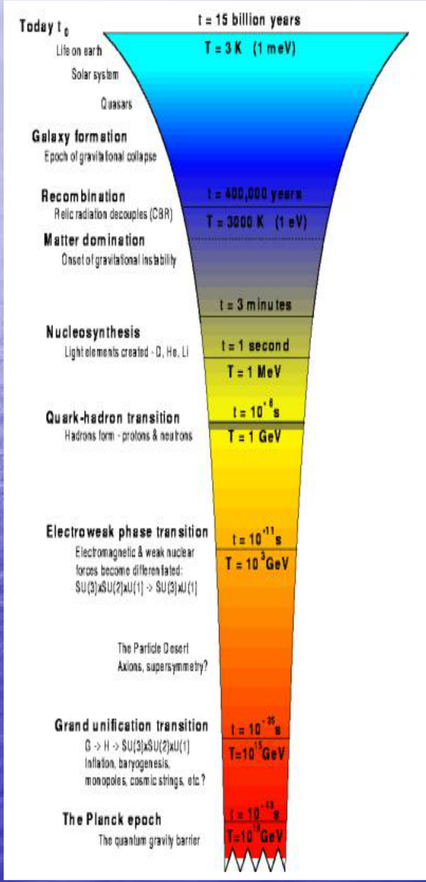
- GWs are gauge-invariant perturbations governed by Einstein equations:

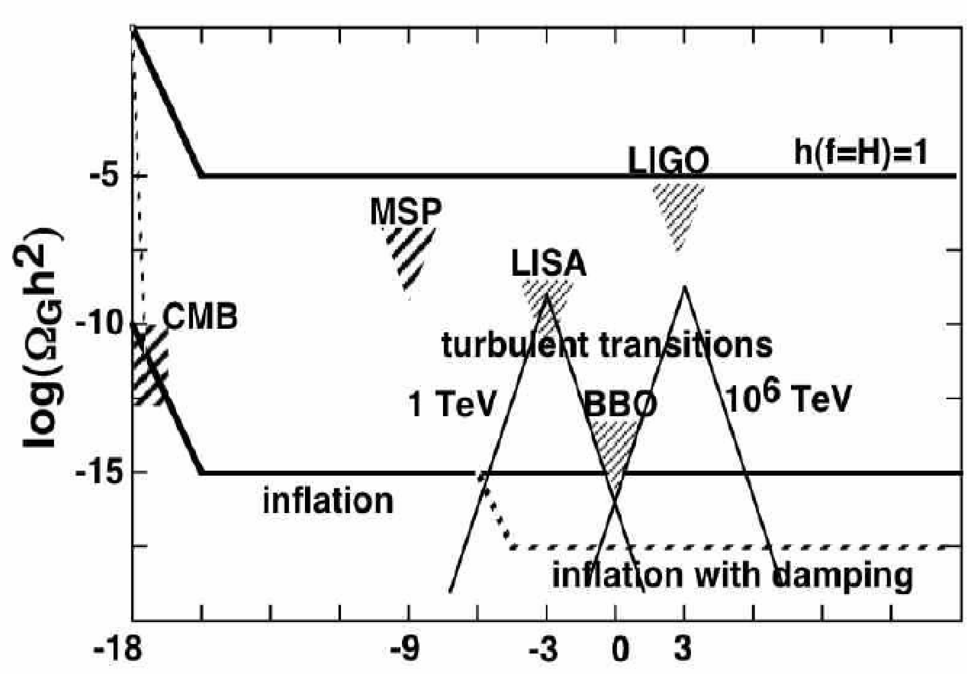
  $$G_{ik}=8\pi\, GT_{ik}$$

- Tensor mode of perturbations $h_{\mu\nu}= \delta g_{\mu\nu}$
  - no analogy under Newtonian gravity
  - gauge-invariant
  - transverse, traceless $h_{\mu\nu}$
  - symmetric $h_{\mu\nu} = h_{\nu\mu}$
    - gauge choice $\rightarrow h_{0i} = h_{0i} =0 = h_{00} \rightarrow$ 9 degrees
    - $h_{ij} = h_{ji} \rightarrow$ 6 degrees
    - $\sum h_{ii} =0 \rightarrow$ 5 degrees
    - $h_{ij}\, k^j=0 \rightarrow$ 2 degrees
- Two degrees of freedom represent two degrees of GWs polarization

Mukhanov, Feldman, & Brandenberger 1992

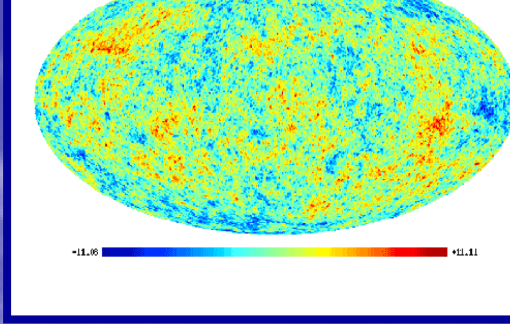
Relic gravitational waves background





From C. Hogan 2006

Polarized gravitational waves vs. CMB



- Polarized GWs might be generated from quantum fluctuations if the gravitational Chern-Simons term (responsible for mirror symmetry violation) is present.
- Such GWs induce specific parity-odd cross correlations of CMB anisotropies
  - *Lue, Wang, and Kamionkowski, 1999*
  - *Lyth, Quinbay, and Rodriguez, 2005*
  - *Satoh, Kanno, and Soda, 2007*
- If mirror symmetry violation is significant these effects are possibly detectable via CMB data.
- CMB anisotropy affecting polarized GWs might be generated by others parity violating sources present in the early Universe (i.e., inflationary helical magnetic field)
  - *Caprini, Durrer, and T.K. 2004*

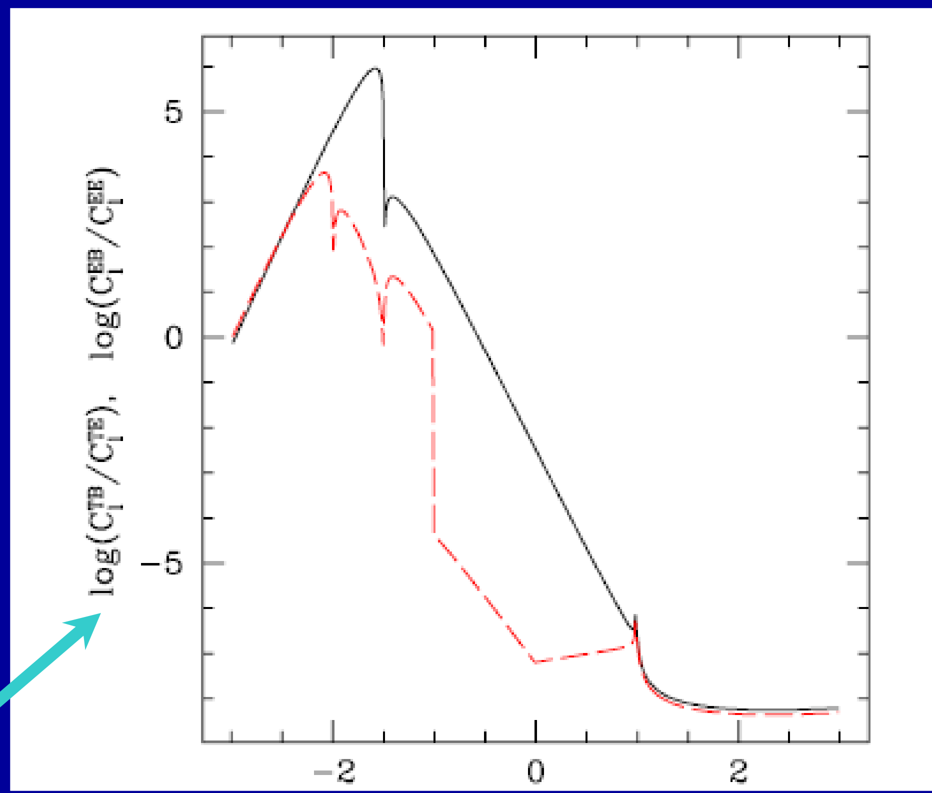
Polarized gravitational waves vs. CMB (continued)

- If CMB measurements would show non-zero cross correlations between temperature and B-(magnetic-type) polarization anisotropies, or/and E-(electric-type) and B-polarization – this indicates possible parity violation
  - *Lue, Wang, & Kamionkowski, 1999*
  - *Pogosian, Vachaspati & Winitzki, 2002,T.K. & Ratra 2005 (vector mode*
  - *Caprini, Durrer, & T.K. 2004*
  - *Saito, Ichiki, & Taruya, 2007*
- CMB data of parity-odd power spectra can be served as a test for parity symmetry breaking scales and magnitudes
- Polarized GWs might leave also specific signatures on CMB parity-even power spectra, leading to some CMB anomalies
  - *Alexander & Martin, 2005*
  - *Liu, Lee, & Ng, 2006*
  - *Feng, Li, Xia, & Chen, 2006*
  - *Seto & Taruya, 2007*

Example: testing magnetic helicity through CMB anisotropies

Primordial helicity generation:

Cornawll 1997, Giovannini & Shaposhnikov 1998, Caroll & Field 2000, Giovannini 2000, Vachaspati 2001, Sigl 2002 , Christensson, Hindmarsh, & Brandenburg 2003, Campaneli & Gianotti 2006, Sorbo 2006



Caprini, Durrer, & T.K. 2004

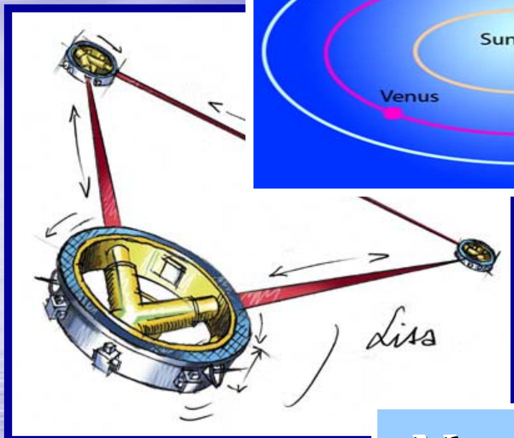
CMB anisotropy parity odd power spectra (tensor mode) might reflect the presence of primordial magnetic helicity

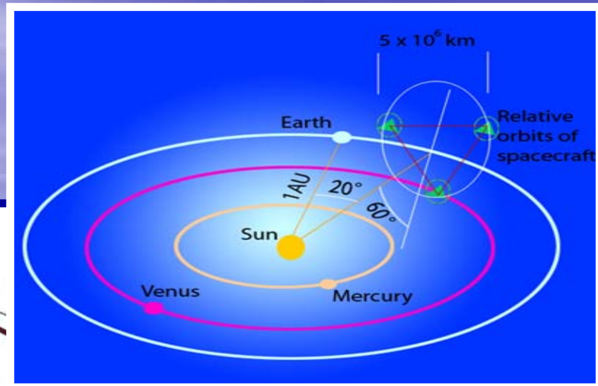
C_l^{TB}/C_l^{TE} (black); C_l^{EB}/C_l^{EE} (red)

$l=50$, $n_S=-3$

Even for a primordial magnetic field with maximal helicity such effects may be detectable if the current magnetic field amplitude is at least 10^{-9} – 10^{-10} Gauss on Mpc scales.

Direct detection of polarized relic gravitational waves



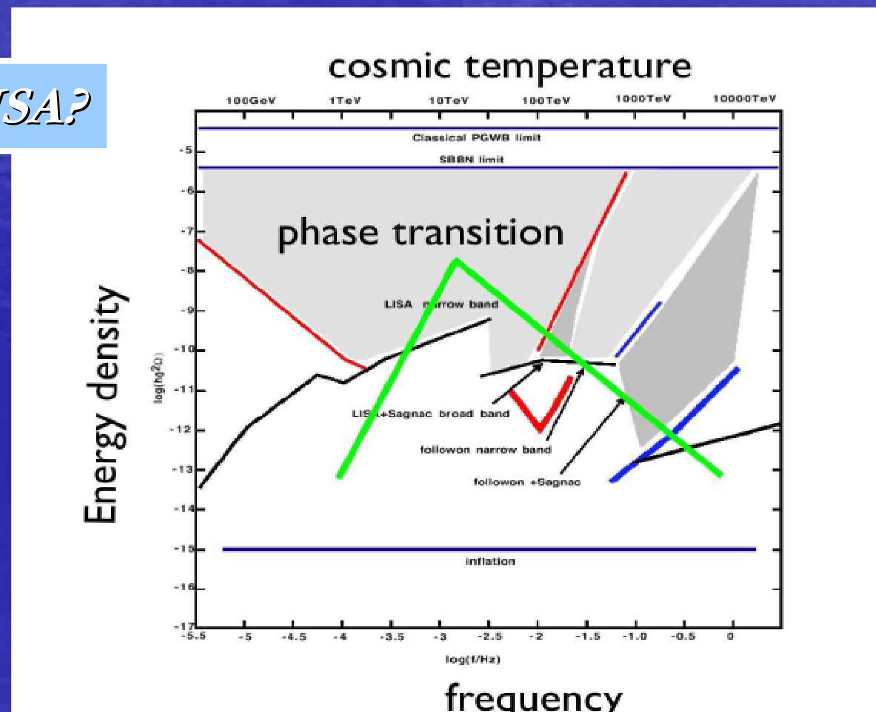


Space based missions *Seto 2006*

The primary scientific goal of the Laser Interferometer Space Antenna (LISA) mission is to detect and observe GWs from massive black holes and galactic binaries with periods in the range of a few seconds to a few hours, i.e. in the frequency range 10^{-4} to 10^{-1} Hz.

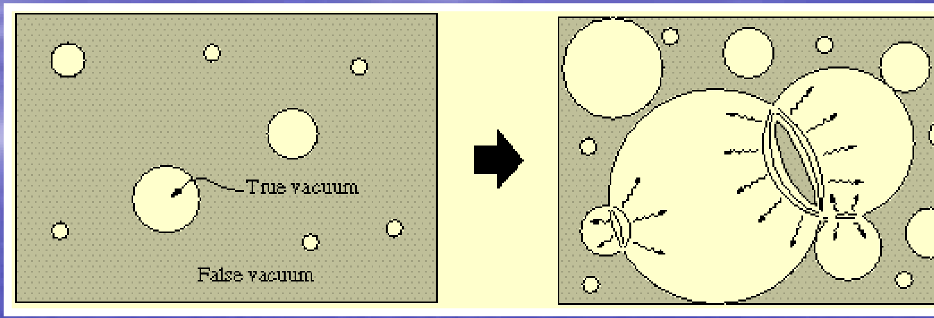
New Physics from LISA?

- Hubble frequency $f_0=10^{-4}$Hz (T/1Tev)
- Large Hadron Collider (LHC) ←→ relic GWs;
- LISA's peak sensitivity corresponds to ≈ 1/10 of Hubble horizon at 1 Tev energy scale



GWs from phase transitions

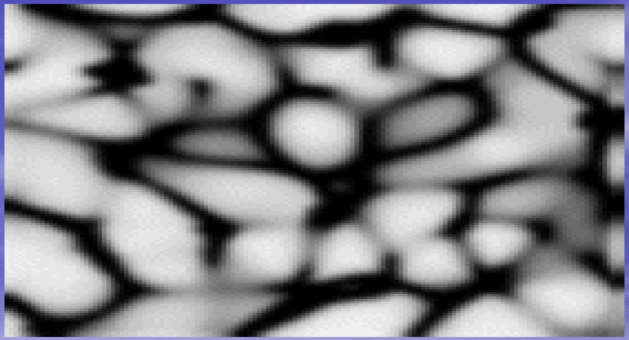
- Pioneering :
  - Witten 1984, Hogan 1984
- Earlier 90's
  - Turner & Wilczek, 1990
  - Kosowsky, Turner, & Watkins, 1992
  - Kamionkowsky, Kosowsky, & Turner, 1994



$$\nabla^2 h_{ij}(\mathbf{x},t) - \frac{\partial^2}{\partial t^2} h_{ij}(\mathbf{x},t) = -16\pi G S_{ij}(\mathbf{x},t).$$

- Bubbles collisions and nucleation
- Turbulence
  - COSMO-07 Talks by
    F. Dufaux, R. Durrer, L. Kofman

GWs by Primordial Turbulence



- Bubble walls collision and nucleation induce turbulent motions in primordial plasma.
- These motions might have non-zero helicity
- Turbulent eddy's characteristic size is related to the Hubble scale and bubbles number
- Turbulence is determined by Reynolds and Mach numbers and the Universe temperature when turbulence occurs
- GWs generation depends also on turbulence duration and stirring scale

We assume that in the early Universe at time t_{in} vacuum energy ρ_{vac} is converted into turbulent energy with an efficiency κ over a time scale τ_{stir} on a characteristic length scale L_S.

$$S_{ij}(\mathbf{x},t) = T_{ij}(\mathbf{x},t) - \frac{1}{3}\delta_{ij}T^k_k(\mathbf{x},t),$$

$$h_{ij}(\mathbf{x},t) = 4G\int \mathrm{d}^3\mathbf{x}'\frac{S_{ij}(\mathbf{x}',t-|\mathbf{x}-\mathbf{x}'|)}{|\mathbf{x}-\mathbf{x}'|}.$$

Main assumption on the turbulence model:

Stationary developed case – Kolmogoroff's hypothesis applies

- Even accounting for inevitable decay – the emitted GWs spectrum will be close to that from stationary turbulence

Justification: *(Proudman 1975): If the turbulence is decaying additiona terms proportional to time derivatives appear. But since the decay time τ is at least several times larger than the turnover time, then the additiona terms proportional to $1/\tau_d$ can be safely neglected*

Goldstein 1976

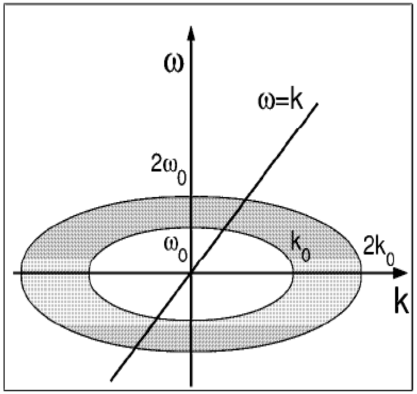


FIG. 1: To the determination of the characteristic frequency of the GWs generated by turbulence. Dark area contains mai part of turbulent energy.

<u>Analogy: acoustic waves generation by turbulence</u>

- Eddies length l_0 and velocity v_0
- Eddies characteristic frequency v_0/ l_0
- Eddies characteristic wave-number $1/ l_0$
- Because $v_0 /c<1$, the dark area is stretched along k axis.
- GWs generating turbulent elements lie along $k=\omega$ line, so ω_{GW} is given by eddy inverse turn-over time v_0/ l_0 .

Non-helical Kolmogoroff like turbulence: spectrum

- Turbulent motion two point correlation

$$R_{ij}(\mathbf{r},\tau) \equiv \langle v_i(\mathbf{x},t) v_j(\mathbf{x}+\mathbf{r},t+\tau)\rangle$$

- Power spectrum (space and time auto-correlations)

$$F_{ij}(\mathbf{k},\tau) = \frac{E_k}{4\pi k^2}\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right) f(\eta_k,\tau),$$

Kolmogoroff 1941

$$E_k = C_K \varepsilon^{2/3} k^{-5/3}, \quad \text{for} \quad k_0 < k < k_d,$$

$$f(\eta_k,\tau) = \exp\left(-\frac{\pi}{4}\eta_k^2\tau^2\right).$$

Kraichnan 1964

$$\eta_k = \frac{1}{\sqrt{2\pi}}\varepsilon^{1/3} k^{2/3}.$$

Relic GWs energy density and degree of polarization

$$\rho(\mathbf{x},t) = \frac{1}{32\pi G}\langle \partial_t h_{ij}(\mathbf{x},t)\partial_t h_{ij}(\mathbf{x},t)\rangle = \frac{G}{2\pi}\int \mathrm{d}^3\mathbf{x}'\mathrm{d}^3\mathbf{x}''\frac{\langle \partial_t S_{ij}(\mathbf{x}',t')\partial_t S_{ij}(\mathbf{x}'',t'')\rangle}{|\mathbf{x}-\mathbf{x}'||\mathbf{x}-\mathbf{x}''|},$$

GWs energy density

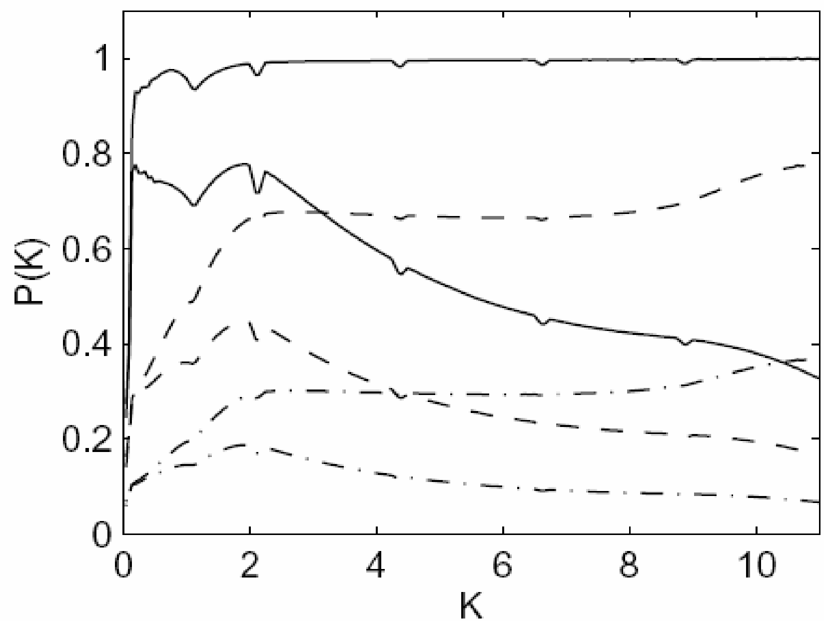
GWs source S_{ij} has helical structure

$$\langle h^{\star}_{ij}(\mathbf{k},t)h_{lm}(\mathbf{k}',t)\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}-\mathbf{k}')\left[\mathcal{M}_{ijlm}H(k,t) + i\mathcal{A}_{ijlm}\mathcal{H}(k,t)\right].$$

GWs two-point correlation function

$$\mathcal{P}(k) = \frac{\langle h^{+\star}(\mathbf{k})h^{+}(\mathbf{k}') - h^{-\star}(\mathbf{k})h^{-}(\mathbf{k}')\rangle}{\langle h^{+\star}(\mathbf{k})h^{+}(\mathbf{k}') + h^{-\star}(\mathbf{k})h^{-}(\mathbf{k}')\rangle} = \frac{\mathcal{H}(k)}{H(k)}.$$

GWs degree of circular polarization



T.K., Gogoberidze, & Ratra 2005

Asymptotical solutions (zero-helicity regime, forward cascade)

Low frequency regime: $h_c(f)$ » $f^{1/2}$

Intermediate regime: $h_c(f)$ » $f^{--13/4}$

High frequency regime: exp suppression of the power

Parameters:

τ_T turbulence lasting time;

k_0 stirring scale;

$\varepsilon = v_0^3\, k_0$ energy dissipation rate

$M^3=\varepsilon/k_0$; $M = v_0/c$ - Mach number

$R^{3/4}=k_d/k_0$ - Reynolds number

$$\kappa \sim v_0^3 G \rho l_0^2 = M^3 \frac{\rho}{\rho_1} \frac{l_0^2}{L_H^2},$$

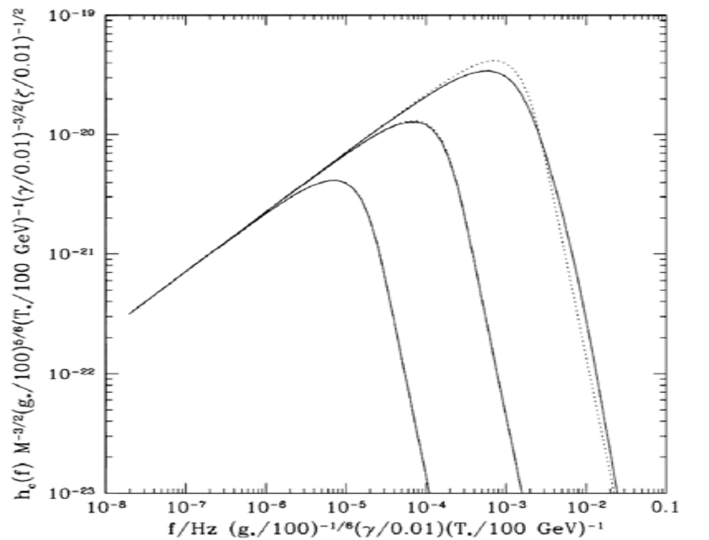


FIG. 1: The spectrum of gravitational radiation from turbulence. The three solid lines are for different Mach numbers, M = 0.01, M = 0.1, and M = 1 from lowest to highest amplitude. Note that these three cases have also been scaled by a fact $M^{-3/2}$ for display, since this is how the low-frequency tail scales with M. The dotted lines, which are virtually indistinguish from the solid lines except for the M = 1 case, show the $k = 0$ approximation to the gravitational wave source.

Efficiency of GWs generation is defined as a ratio between the GWs energy density and dissipated energy of turbulence

Gogoberidze, T.K., & Kosowsky 2007

α– the ratio between the false vacuum energy and plasma thermal energy.

β – time variation rate of bubbles nucleation

Relic GWs Spectrum (hydro-turbulence)

- Peak amplitude:

$$h_c(f) = 1.62 \times 10^{-18} \left(\frac{T_*}{100\,\mathrm{GeV}}\right) \left(\frac{g_*}{100}\right)^{-5/6} \left(\frac{\gamma}{0.01}\right)^{3/2} \left(\frac{\zeta}{0.01}\right)^{1/2} \left[k_0^3 f H_{ijij}(2\pi f, 2\pi f)\right]^{1/2}.$$

Converting the radiation frequency ω_* at time of phase transitions to the today frequency f

$$f = 1.55 \times 10^{-3}\,\mathrm{Hz} \left(\frac{\omega_*}{k_0}\right) \left(\frac{g_*}{100}\right)^{1/6} \left(\frac{\gamma}{0.01}\right)^{-1} \left(\frac{T_*}{100\,\mathrm{GeV}}\right),$$

- γ is the stirring scale's fraction of the Hubble length and ζ is the turbulence duration's fraction of the Hubble length.

$$\gamma H_*^{-1} = 2\pi/k_0, \qquad \zeta H_*^{-1} = \tau_T;$$

Results for kinetic (hydro) turbulence

- LISA has a 5σ sensitivity to stochastic backgrounds of below $h_c=10^{--23}$ between frequencies 10^{-3} and 10^{-2} Hz, and decreasing to around $h_c=10^{--20}$ at 10^{--4} Hz, for one year of integration.

- Turbulence with a Mach number M=1 would be a factor of 1000 larger than the LISA detection threshold at the peak frequency around 10^{--3} Hz. For a Mach number M=0.1, the peak amplitude decreases by a factor of 100. However, the peak frequency also shifts to 10^{--4} Hz, at which point LISA's sensitivity has declined greatly.

Future space-based interferometers could be configured to give strain sensitivities comparable to LISA, but with a frequency window between 10^{-4} and 10^{-6} Hz. Such an experiment would easily detect turbulence at the electroweak scale with a Mach number M=0.1, and would even lirt with a detection at M=0.01

- GWs power spectra are sharply peaked at the frequency which is determined by the largest eddy turn-over time
- GWs become observable only for the strong enough first-order phase transitions

if helical MHD inverse cascade turbulence is present

Cristensoon, Hindmarsh, & Brandenburg 2003,
Banerjee & Jedamzik 2004

- the kinetic energy might be transferred to large scales, and presuming helicity presence, a primordial magnetic field is generated which induces an additional GWs signal
- The peak frequency of this secondary GWs is shifted at low frequency range
- This allows to make GWs observable even if phase transitions occur at high energies
- Another advantages
  - the maximal length scale is (could be) now comparable with Hubble horizon
  - the duration time of turbulence and correspondently the amplitude of the signal are changed

MHD helical (inverse cascade) turbulence model

Biskamp & Mueller 1999

Christensson, Hindmarsh, & Brandenburg 2003

- Dynamics of MHD turbulence is dominated by Alfven waves whereas the magnetosonic waves are known to play a passive role
- Because the magnetic and kinetic parts of wave energy are equal for Alfven waves we assume $v_0 \sim b_0$, where $b_0 = B_0 / [4\pi(\rho+P)]^{1/2}$ is the characteristic magnetic field perturbation in velocity units.
- As it is known MHD turbulence even if is isotropic on large scales becomes locally anisotropic in smaller scales
- But because GWs are mainly generated by low frequency perturbations for the study of GW generation we assume global isotropy.
- Fourier transform of the magnetic field two-point auto-correlation function

$$F_{ij}^{M}(\mathbf{k},\tau) = \left[P_{ij}(\mathbf{k})\frac{E_k^M(t)}{4\pi k^2} + i\varepsilon_{ijl}\frac{k_l}{k}\frac{H_k(t)}{8\pi k}\right] f(\eta_k,\tau),$$

helical MHD turbulence evolution – two main stages

- First (direct cascade) stage
  - Duration $\tau_{S0}=s_0\ \tau_0\ (\tau_0^{-1}=v_0\ k_0)$
  - Initial helicity is small
  - Praudman 75: selective decay effect - turbulence can be treated as stationary during $\tau_{S0}/2$ period
  - Both Reynolds (usual and magnetic) numbers are large enough
  - Magnetic energy transfer reflects kinetic energy transfer

$$E_k^M = C_K \varepsilon^{2/3} k^{-5/3}, \quad \text{for} \quad k_0 < k < k_d,$$

- Second (unverse cascade) stage
  - At the end of the first stage helicity reaches its max value *Banerjee & Jedamzik 2004*
  - Eddy turn-over and cascade time-scales evolve

$$\tau_{to} \sim \tau_1 (1 + t/\tau_1),$$

$$\tau_{cas} \sim \tau_1 (1 + t/\tau_1)^2.$$

$$\tau_1^{-1} = k_0 v_1$$

  - velocity and magnetic field perturbations v_1 and b_1 determine magnetic and kinetic energy (now time dependent) transfers

$$E_k^M = C_1 v_1^2 / k_0, \quad \text{for} \quad k_s < k < k_0,$$

$$H_k^M = 2C_1 v_1^2 / (k_0 k), \quad \text{for} \quad k_s < k < k_0.$$

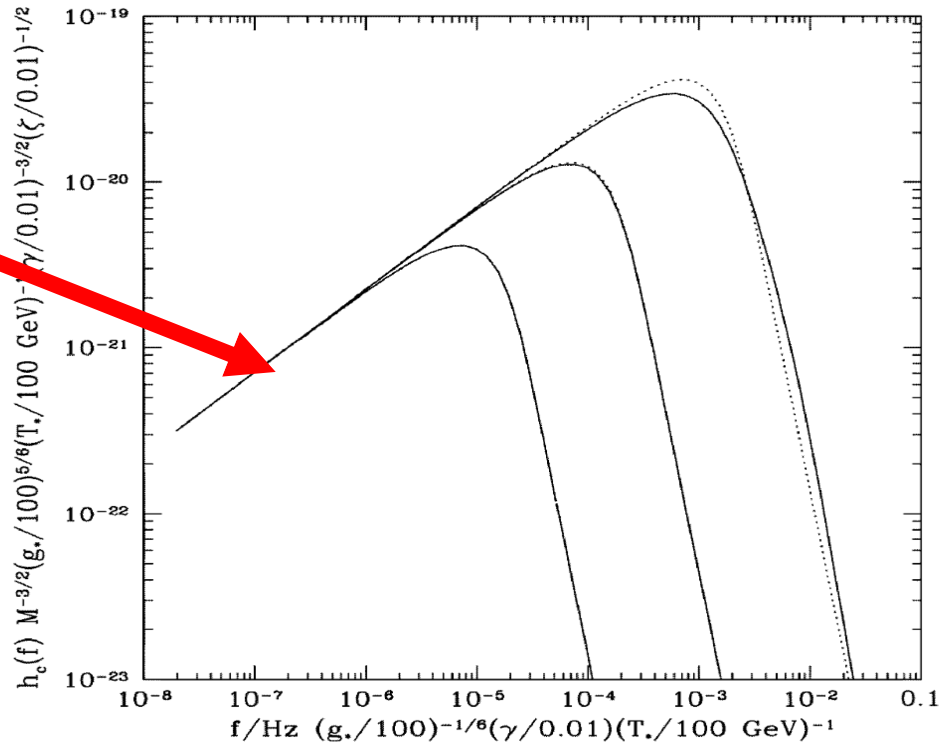
$$k^{-1} = \xi_M(t) = (1 + t/\tau_1)^{1/2} / k_0$$

GWs by MHD helical turbulence model

PRELIMENARY

On-going project T.K., Gogoberidze, and Ratr

secondary – turbulent magnetic field induced peak. The amplitude of this peak could be higher that first peak ones, because inverse cascade lasts longer period.



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Conclusions

- Polarized GWs from phase transitions are possibly detectable
- For strong enough helical sources during inflations – GWs are detectable through CMB precise data (parity-odd spectra)
- GWs polarization measurements can be used as a test for particle physics and cosmological models beyond standard scenarios
- MHD inverse cascade could play a crucial role for GWs generation
- Future GW astronomy would open a window onto new physics

THANKS