Cosmological Phase Transition and Electromagnetic Field

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August 24, 2007, Cosmo07, Brighton

1. Introduction

embedded defects (unstable at zero temperature)

↓ finite temperature plasma ← early universe

stabilized ∵ asymmetry between charged scalar components and neutral ones

↓ low temperature, decoupling

core phase transition, decay

→ primordial magnetic field ?

⇒ Some kinds of effects on cosmic microwave background radiation could be observed.

2. Cosmic String

$$\phi \equiv \phi_1 + i\phi_2 = \eta\, e^{in\theta}$$

string solution

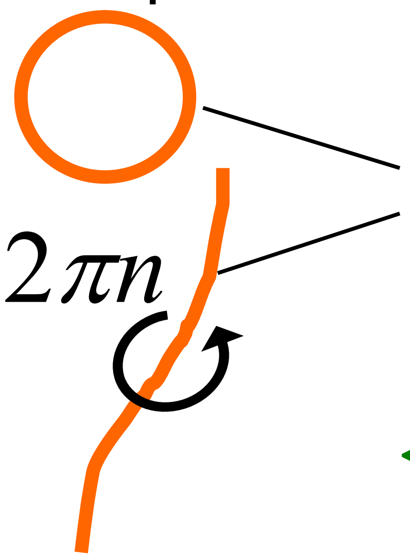
$$G\mu = \left(\frac{\eta}{M_{pl}}\right)^2$$

is the parameter which determines the magnitude of the string effect.

n : winding number

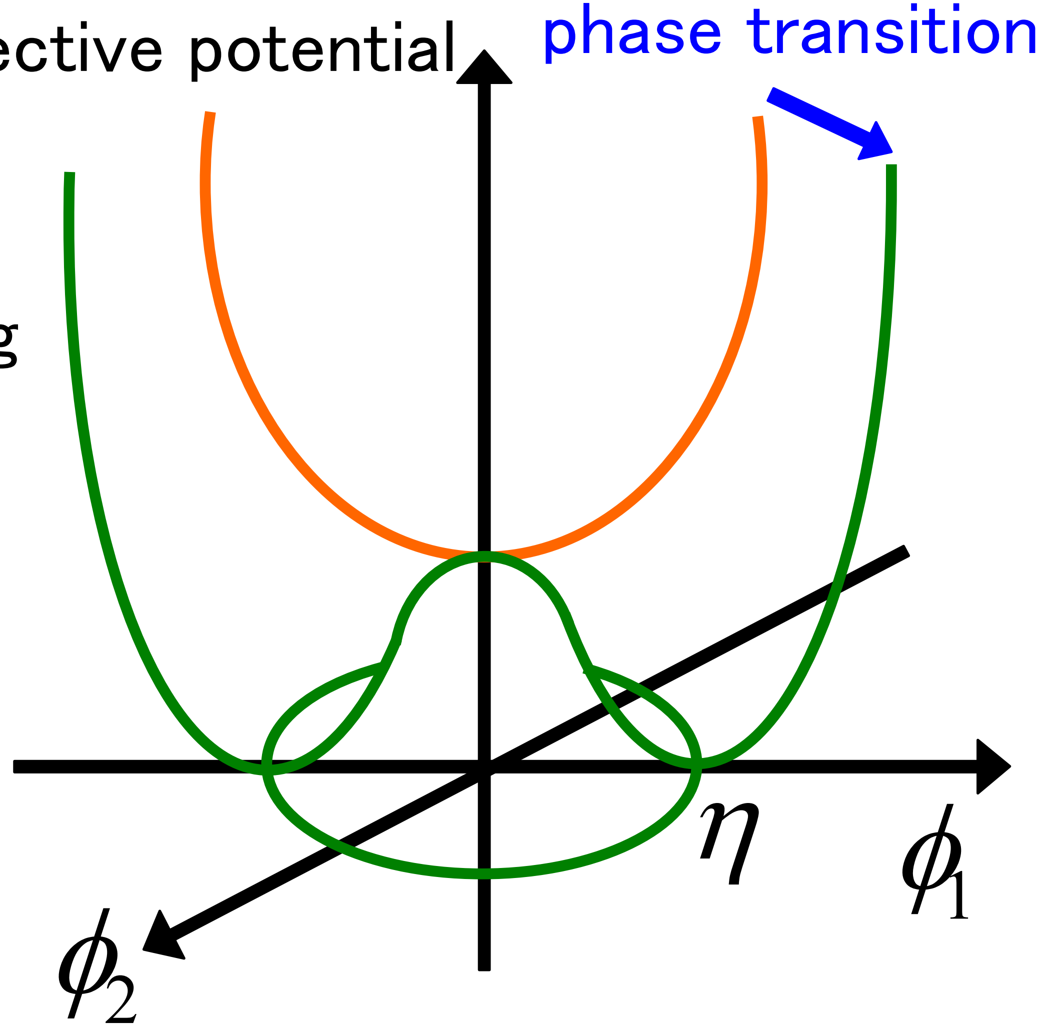
$\mu \approx \eta^2$: line energy density of string

loop



$$<\phi>=0$$

$$<\phi>=\eta$$



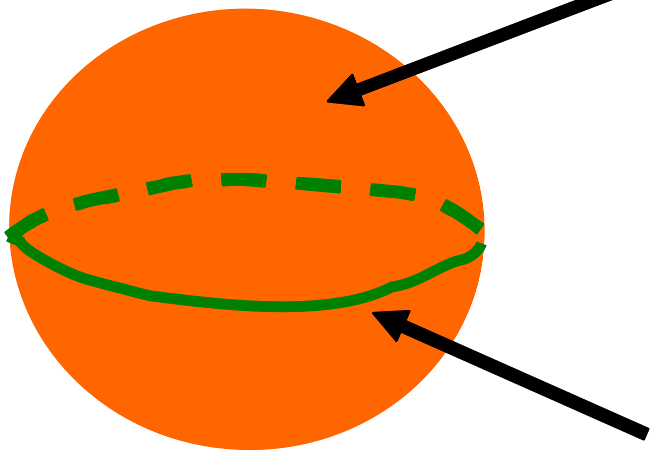
3. Embedded String

Although the configuration of embedded defects satisfies equations of motion, they are topologically, and in general also dynamically unstable.

ex.) three real scalar fields $V(\phi)=\frac{\lambda}{4}(\sum_{i=1}^{3}\phi_i^2-\eta^2)^2$

no string

$$S^2:\sum_{i=1}^{3}\phi_i^2=\eta^2$$



freezing out certain components

$(\phi_3=0)$

$$S^1:\sum_{i=1}^{2}\phi_i^2=\eta^2$$

string !

4. Pion String

one example of embedded global string

standard model of strong interaction

→ QCD phase transition $(\simeq 200\ \mathrm{MeV})$

Below the confinement scale, this model is described by a sigma model involving the sigma field σ and the three pions $\vec{\pi} = (\pi^0, \pi^1, \pi^2)$.

$$V_0 \equiv \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - \eta^2)^2$$

$$L_0 = \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma + \frac{1}{2}\partial_\mu \vec{\pi} \partial^\mu \vec{\pi} - V_0$$

cf. Although it is different from the cosmological scenario, following the Kibble-Zurek mechanism the pion strings are expected to be formed in LHC Pb – Pb collision experiments. These effects could be observable and bring distinction compared to conventional predictions.

Ref. H. Mao, Y. Li, MN, X. Zhang & T. Huang, 2005

5. Finite Temperature Effect

Introducing the coupling to electromagnetism, L_0 is rewritten as

$$L_0 = \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma + \frac{1}{2}\partial_\mu \pi^0 \partial^\mu \pi^0 + D_\mu^+ \pi^+ D^{\mu -} \pi^- - V_0 \ ,$$

where $D_\mu^\pm \equiv \partial_\mu \pm eA_\mu$ and charged fields are

$$\pi^\pm \equiv \frac{1}{\sqrt{2}}(\pi^1 \pm i\pi^2) \ .$$

When the background photon plasma can be regarded as a thermal bath, the interaction between charged fields and photon could be included into the effective potential. If the Lagrangian is thermally averaged, the first order term of photon field vanishes and the second order term becomes

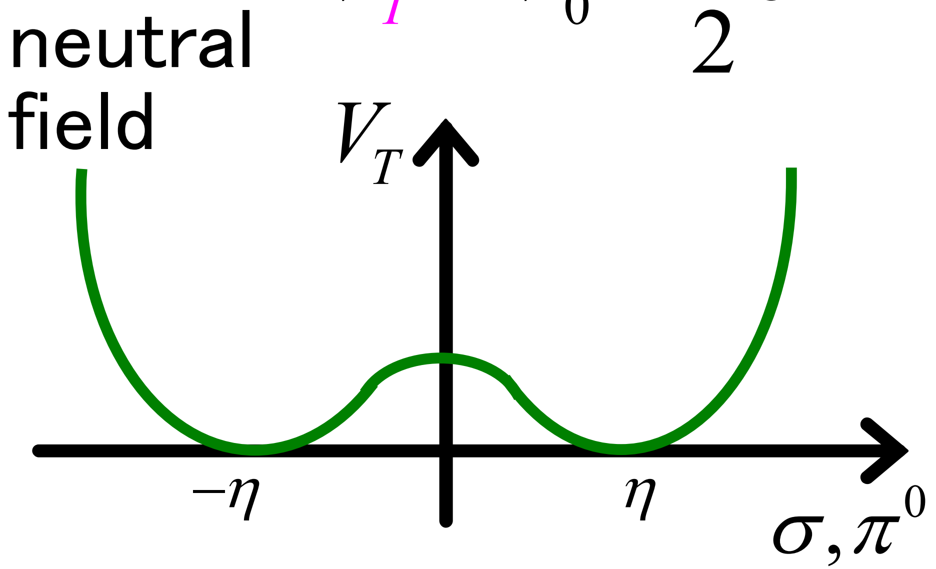
$$A_\mu A_\mu \rightarrow -\kappa T^2 . \quad \kappa \sim O(1)$$

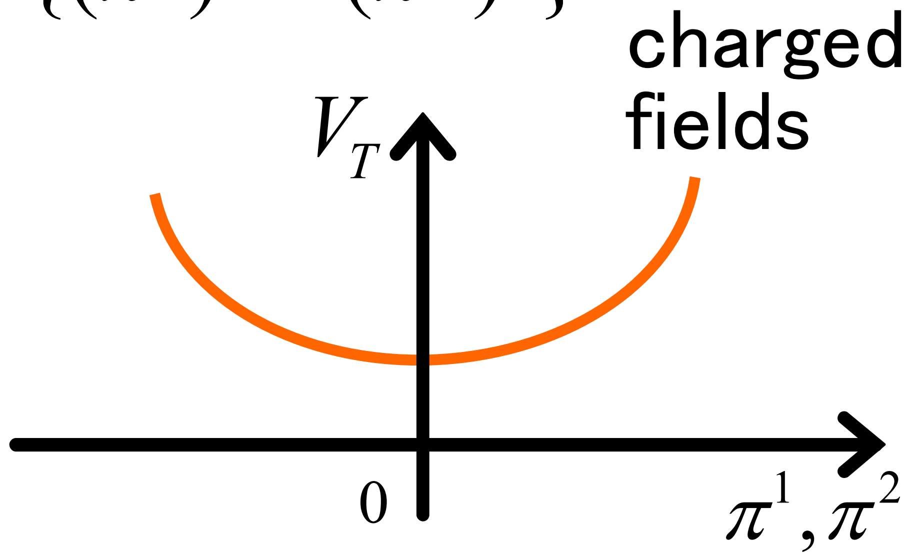
Ref. MN & R. Brandenberger, 2003

6. Effective Potential

Thus the vacuum manifold which is S^3 at zero temperature becomes S^1 at finite temperature.

$$V_T = V_0 + \frac{1}{2} e^2 \kappa T^2 \{(\pi^1)^2 + (\pi^2)^2\}$$





By analyzing the stability of the pion string solution under simple assumptions, the destabilization temperature can be calculated as $T_D = 2\lambda^{1/2}\kappa^{-1/2}e^{-1}\eta$.

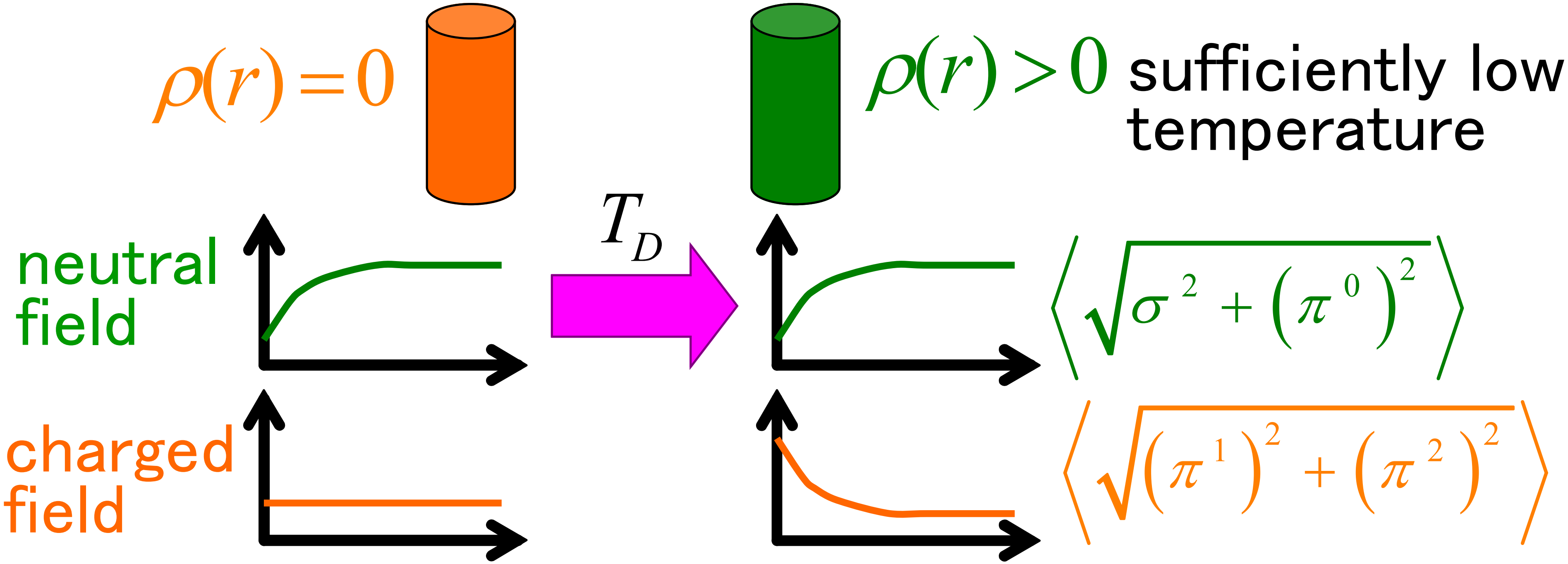
This is consistent with the results of numerical simulations.

7. Core Phase Transition

The results of numerical simulations show that even below T_D , the string does not decay. Actually the Higgs field has a finite expectation value at the string core and the neutral field configuration is not destroyed, which means that a core phase transition occurs.

∵ Since the winding number is a kind of topological charge, it must be conserved.

finite temperature effect domination



8. Superconductivity

After the core phase transition, charged fields have finite expectation value and the phase has a spatial gradient along the string so that the electric current will be generated.

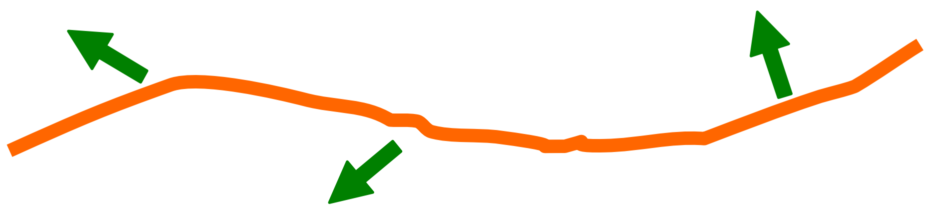
$$\sqrt{\left(\pi^1\right)^2+\left(\pi^2\right)^2}=\phi_c\left(x,y\right)e^{i\varphi_c(z,t)}$$

current amplitude $\sim e\dfrac{d\varphi_c}{dz}$

Davis & Shellard, 1989

loops → vorton

infinite strings

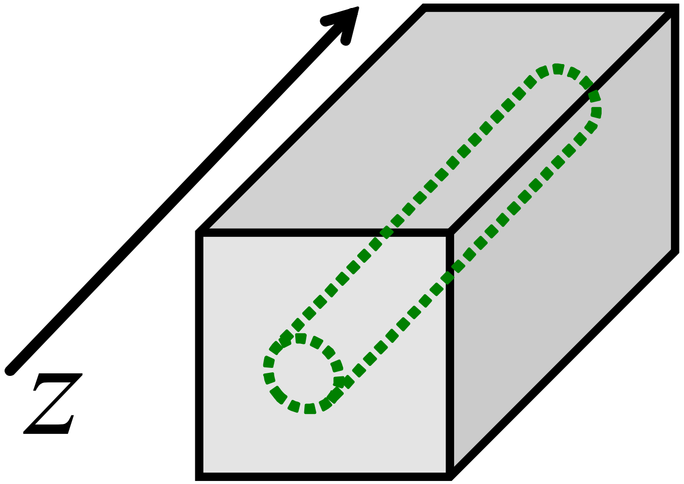


Infinitely long strings and/or loops of large curvature radius could show a filament-like spatial distribution feature.

⟺ astrophysical counterparts: gravitational lensing?

9. 3-dimensiocal simulation

A initially translation symmetric string evolution is solved in a 3-dimensional box and the distribution of φ_c shows the winding number appears in some cases.



$300^2 \times 600$

periodic boundary

The probability of winding number appearance is 20%.

$[-\pi, \pi]$: random

$600\eta^{-1}/T^{-1}$ steps

$\sim$16%

$$T = 10^{-2}\eta$$

φ_c when the winding number exists.

φ_c when there is no winding number.

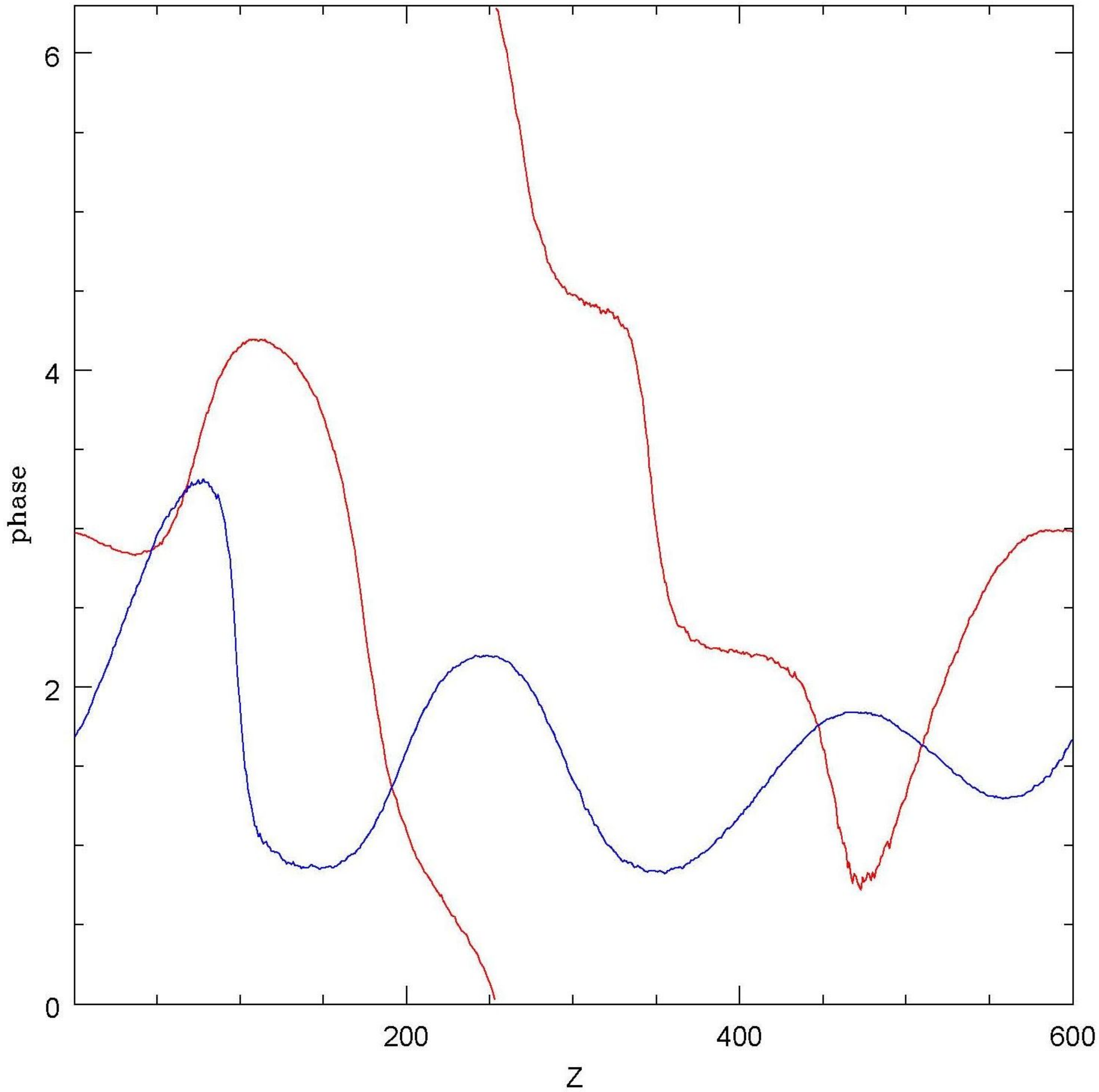
$$T = 10^{-2}\eta$$

φ_c

$$\frac{d\varphi_c}{dz} \neq 0$$

φ_c

no winding number



10. Interaction of Scalar Field with Electromagnetic Field

In general, the following type of interaction between the electromagnetic filed and a certain kind of field would appear in the Lagrangian when the anomaly or the Chern-Simons term is taken into account.

$$L = L_{EM} + L_{\text{int}} \;\; ; \;\; L_{\text{int}} = -\frac{1}{2} O_{\mu} A_{\nu} \varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$$

Then the equations of motion for electromagnetic field should be modified so that the generation of magnetic field and/or the polarization of background radiation would occur because of the field, O_{μ}.

$$\partial_{\mu} F^{\mu\nu} = 4\pi J^{\nu} + O_{\mu} \varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$$

ex.) axion, type IIB string model...

11. Magnetic Filed Generation from Pion Strings

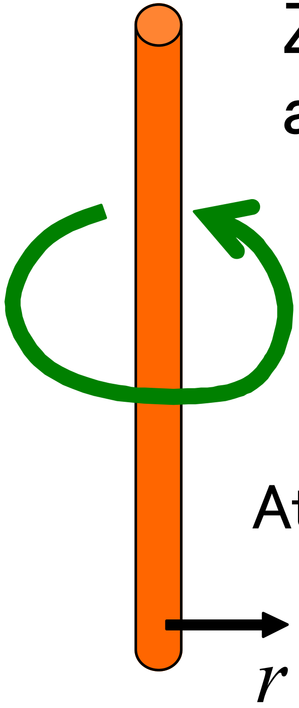
R. Brandenberger & X. Zhang, 1999

In case of the pion string, there exists an interaction between the pion field π^0 and the electromagnetic field.

$$L_{\mathrm{int}} = -\frac{N_c \alpha}{24\pi} \frac{\pi^0}{f_\pi} \varepsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}$$

$N_c = 3$

α : fine structure constant



Zero mode current appears within the string core and the azimuthal magnetic field is produced.

$$B_\theta = -N_c \frac{eT_{\mathrm{C}}}{2\pi} \left(\frac{r}{\delta_s} \right)^{\alpha/\pi} \frac{1}{r}$$

$\delta_s \approx f_\pi^{-1}$: string core radius

At the recombination, $B(T_{\mathrm{rec}}) \approx 10^{-23} \left(\frac{r}{\delta_s} \right)^{\alpha/\pi} \frac{1\mathrm{kpc}}{r}$ G .

$T = T_{\mathrm{C}}$

present

12. Helicity of Magnetic Field

If the twist and tangle of strings are biased when the CP violation exists, then the helicity of magnetic field is also biased so that its conservation leads to the generation of larger magnetic field amplitude.

helicity density : $\mathrm{H} = \frac{1}{V}\int_V d^3x \mathbf{A}\cdot\mathbf{B}$

After the phase transition,

$$\mathrm{H} \approx \varepsilon_{CP} 4 N_c^{\,2} \left(\frac{\xi_s}{T^{-1}}\right)^{-2+\alpha/\pi} \left(\frac{f_\pi}{T}\right)^{1+\alpha/\pi} T^3 \ .$$

ε_{CP} : CP violation strength ξ_s : correlation length of strings

It can be shown that the helicity is not erased and according to the most optimistic estimation, on the scale of 1pc, at the recombination, $B \approx 10^{-9}\ \mathrm{G}$.

13. Modified Maxwell Equation

Next it is considered how the same interaction affects the light propagation.

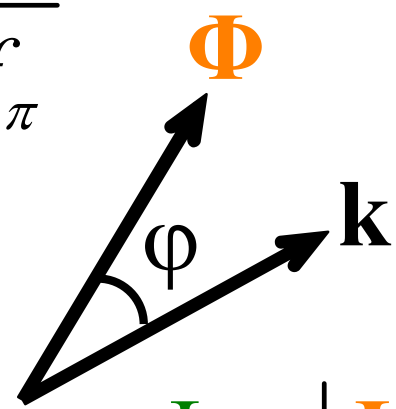
$$L_{\rm int} = -\frac{N_c \alpha}{24\pi}\frac{\pi^0}{f_\pi}\varepsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}$$

If the time evolution of string distribution can be neglected, then the equations without the current under the string background will be written as

$$\begin{cases} \nabla \mathbf{E} = -\mathbf{\Phi}\cdot\mathbf{B} \\ -\dfrac{\partial \mathbf{E}}{\partial t} + \nabla\times\mathbf{B} = \mathbf{\Phi}\times\mathbf{E} \end{cases} \qquad \mathbf{\Phi} \equiv \frac{N_c\alpha}{3\pi}\nabla\frac{\pi^0}{f_\pi}$$

and the dispersion relation becomes

$$k^2 = \omega^2 \pm \omega\Phi\cos\varphi\left(1-\frac{\Phi^2\sin^2\varphi}{\omega^2-k^2}\right)^{-1/2}.$$



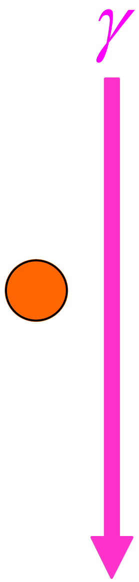
$$\Phi \equiv |\mathbf{\Phi}|$$

14. Rotation of Polarization Axis

Under the approximation that Φ is sufficiently small,

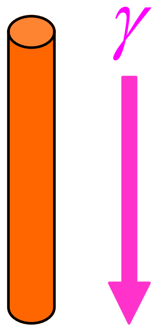
$$k \cong \omega \pm \frac{\Phi}{2}\cos\varphi \ .$$

Thus the rotation angle difference of polarization axis between left-handed polarization and right-handed one, $\Delta\psi$, can be estimated as follows dependent on the direction of the string axis to the line of sight.



When the light travels perpendicular to the string, $\Delta\psi \approx 10^{-3}$,

which is the maximum value. Note that the sign depends on which side the light passes.



When the light travels in parallel with the string $\Delta\psi \approx 0$.

- In any cases, the distance between the string and the light path is not so significant.

15. Summary

It is considered that the magnetic field generation by pion strings produced at the QCD phase transition and the interaction of this magnetic field with the cosmic background radiation.

- Magnetic field strength generated by pion strings depends on the distance to the string axis and it is sufficiently high when the effective α is large.
- In the case that the bulk helicity exists, although the amplitude of magnetic field would be enhanced, the correlation scale should be smaller.
- It is shown that the rotation of light polarization axis is caused by the string field and the spatial distribution of rotation angle directly traces the arrangement of the strings in our universe.