Inflationary primordial perturbations during sharp transitions into radiation domination

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Motivation

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- Single field slow-roll inflation, $(\rho + p) = \dot{\phi}^2 \ll 1$.

  On superhorizon scales, $k \ll aH$, $\mathcal{R} = \mathrm{Constant}$.

- For an instantaneous phase transition

$$\boxed{[\mathcal{R}_\mathrm{k}]_-^+ = 0\,,\, [\Phi_\mathrm{k}]_-^+ = 0\,,}$$

  for $\phi_\mathrm{e}(x,t) = \mathrm{Constant}$.

Israel (66), Derruelle and Mukhanov (95).

- During radiation domination, denoting $\mathcal{A}_\mathcal{R} \equiv \mathcal{R}_\mathrm{k}|_{k=a_e H_e}$

$$\Phi_\mathrm{k} \simeq 0 \quad \longrightarrow \quad \frac{2}{3}\mathcal{R}_\mathrm{k} \sim \frac{2}{3}\mathcal{A}_\mathcal{R}\left(\frac{k}{a_e H_e}\right)^{1/2-\nu}, \mathrm{for}\; k \ll a_e H_e$$

$$\longrightarrow \quad \frac{2}{\sqrt{c_\mathrm{s}}}\frac{\mathcal{A}_\mathcal{R}}{\tau^2}\sin\left(kc_\mathrm{s}\left(\tau - \tau_\mathrm{e}\right)\right)\,, \mathrm{for}\; k \gg a_e H_e$$

$\Longrightarrow$ Formation of PBHs for $k \leq a_e H_e$

Lyth, Malik, Sasaki and IZ (06), IZ, Green, Malik and Sasaki (07).

Gauge invariant linear perturbations

- The perturbed metric in the longitudinal gauge can be written

$$ds^2 = a^2(\tau)\left[-(1+2\Psi)\,d\tau^2 + (1+2\Phi)\,\delta_{ij}dx^i dx^j\right]$$

Ψ and Φ are the gauge invariant Bardeen potentials

Bardeen (80)

τ is the conformal time, $a d\tau = dt$

- $\delta T^{\mu\nu}$ for a perfect fluid determined by $\delta\rho$, δp and v^i
- The constraint equations

$$\begin{aligned} \Phi + \Psi &= 0 \\ \nabla^2\Phi &= 4\pi G\delta\rho_{\rm C} \end{aligned}$$

$\Longrightarrow$ Φ peculiar gravitational field

The evolution of the perturbations

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- The evolution of Φ is given by

$$(\rho+p)\,a^2\left[\frac{1}{(\rho+p)\,a^2}f_{\mathrm{k}}'\right]' - 4\pi G\,(\rho+p)\,a^2 f_{\mathrm{k}} = -4\pi G a^2\left(\frac{p'}{\rho'}a\delta\rho_{\mathrm{C}} + a\delta\bar{p}\right)$$

Sasaki (83)

where $f = a\Phi$ and $\delta\bar{p} = \delta p - (p'/\rho')\delta\rho$.

- Defining the velocity of "sound" as $c_{\mathrm{s}}^2 \equiv \delta p_{\mathrm{C}}/\delta\rho_{\mathrm{C}}$

$$\boxed{f_{\mathrm{k}}'' - \left[\frac{(\rho+p)'}{(\rho+p)} + 2\mathcal{H}\right] f_{\mathrm{k}}' + \left[k^2 c_{\mathrm{s}}^2\,(\tau) - 4\pi G\,(\rho+p)\,a^2\right] f_{\mathrm{k}} = 0}$$

Inflation $\Longrightarrow$ $c_{\mathrm{s}}^2 = 1$, $(\rho+p) \ll 1$

Radiation $\Longrightarrow$ $c_{\mathrm{s}}^2 = 1/3$, $(\rho+p) = 4/3\,\rho$

The perturbations during inflation

- Single field slow-roll inflation, $(\rho + p) = a^2 \phi'^2 \ll 1$

$$\epsilon_{\rm H} = \frac{\phi'^2}{2\mathcal{H}^2} \ , \ \eta_{\rm H} = 1 - \frac{1}{\mathcal{H}} \frac{\phi''}{\phi'}$$

- For $\epsilon_{\rm H}$ and $\eta_{\rm H} = \text{constant}$, Φ obeys Bessel differential equation

$$\boxed{\Phi_{\rm k} = \frac{\sqrt{\pi}}{4k^{3/2}} \frac{(\rho + p)^{1/2}}{M_p^2} (-k\tau)^{1/2} H_\nu^{(1)} (-k\tau)}$$

where $\nu \simeq \frac{1}{2} + 2\epsilon_{\rm H} - \eta_{\rm H}$ for $\epsilon_{\rm H}\, , \eta_{\rm H}$ small, and $\tau = -1/[\mathcal{H}(1 - \epsilon_H)]$.
The amplitude of Φ is

$$\boxed{\Phi \sim (H^2/\dot{\phi})\epsilon_H \ll 1} \qquad (1)$$

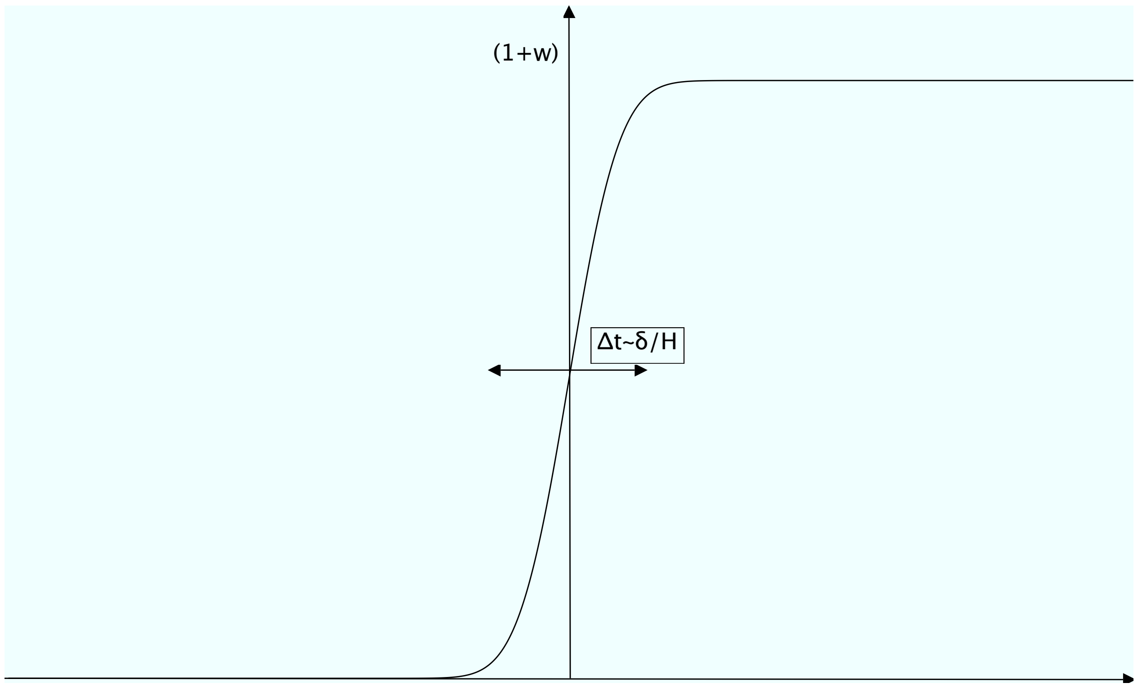
- The curvature perturbation $\mathcal{R}$

$$\mathcal{R}_{\rm k} = -\frac{2}{3} \frac{\mathcal{H}^{-1}\Phi'_{\rm k} + \Phi_{\rm k}}{1 + w} - \Phi_{\rm k} = -\frac{\sqrt{\pi}}{2k^{3/2}} \frac{H^2}{\dot{\phi}} (-k\tau)^{3/2} H_{\nu+1}^{(1)} (-k\tau)$$

Lyth and Stweart (92), Stweart and Lyth (93)

The transition into radiation domination I

$$f_{\rm k}'' - \left[\frac{(\rho+p)'}{(\rho+p)} + 2\mathcal{H}\right] f_{\rm k}' + \left[k^2 c_{\rm s}^2 - 4\pi G\,(\rho+p)\,a^2\right] f_{\rm k} = 0$$



where the equation of state is $p = w(t)\rho$.

On scales $k \gg \mathcal{H}/\delta(1+w)$, Φ is approximately given by

$$\Phi_{\rm k}\,(\tau) \sim \frac{(\rho+p)^{1/2}}{M_p^2}\,{\rm e}^{-ikc_s\tau}$$

The transition into radiation domination II

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- On scales $k \ll \mathcal{H}/\delta(1+w)$

$$f''_{\rm k} - \left[\frac{(\rho+p)'}{(\rho+p)} + 2\mathcal{H}\right] f'_{\rm k} \simeq 0$$

and then

$$\frac{f'_{\rm k}}{a^2(\rho+p)} = \text{Constant} \quad \Longrightarrow \quad \mathcal{R}_{\rm k} \simeq \text{Constant}$$

$$\Longrightarrow \quad \Phi_k \simeq \text{Constant}$$

- The initial conditions for radiation domination are

$$\begin{aligned} \Phi_{\rm rad} &\simeq \Phi_{\rm inf} \\ \left(\mathcal{H}^{-1}\Phi'_{\rm rad} + \Phi_{\rm rad}\right) &\simeq \frac{(\rho+p)_{\rm rad}}{(\rho+p)_{\rm inf}} \left(\mathcal{H}^{-1}\Phi'_{\rm inf} + \Phi_{\rm inf}\right) \end{aligned}$$

Radiation domination evolution

$$\Phi_{\mathrm{k}}(\tau) = \tau^{-3/2}\left[C_1 J_{3/2}(kc_{\mathrm{s}}\tau) + C_2 Y_{3/2}(kc_{\mathrm{s}}\tau)\right]$$

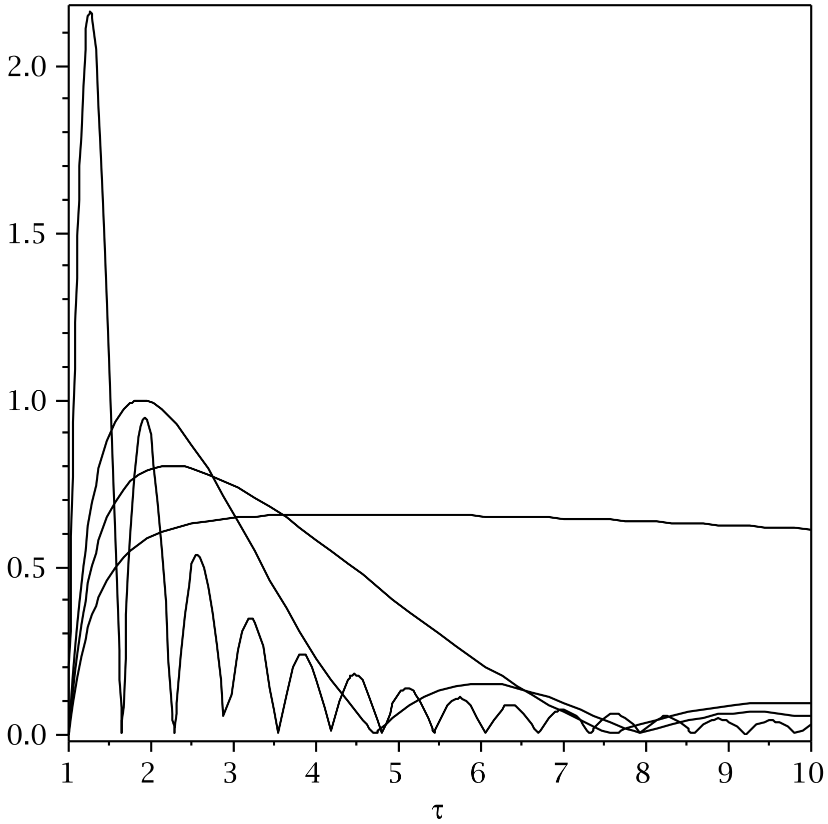

Figure 1: $|\Phi_{\mathrm{k}}|$ in $\mathcal{A}_{\mathcal{R}} \equiv (\mathcal{R}_k)_{k=a_e H_e}$ units for $kc_s/a_e H_e = 5, 1, 0.6, 0.1$. The conformal time τ is given in τ_{e} units

Conclusions

- For a rapid transition from slow-roll into radiation domination we expect that Φ will oscillate, with a relative large amplitude , for a certain range of scales inside the horizon at the end of inflation.

- This may have caused the formation of PBHs, and therefore it may have left a "relic" of the transition epoch.