Backreaction from Linear Perturbations

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Motivation

The Dark Energy Problem

- Three main probes for dark energy: CMB, LSS and SNIa
- Convergence at $\Omega_\Lambda \approx 0.7$

The Dark Energy Problem

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❖ Backreaction
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Summary and Prospects

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- Many proposed mechanisms
  - Λ, quintessence and related models, braneworld theories, modified gravity etc.
  - All involve new physics and suffer from the coincidence problem: Why now?

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  - All involve new physics and suffer from the coincidence problem: Why now?
- “Backreaction” gives potential solution to coincidence problem – acceleration driven by structure formation
- Based on non-commutability of time-derivatives and spatial averaging – so $\partial_t \langle \rho \rangle \neq \langle \partial_t \rho \rangle$ etc.
- We are using the wrong Einstein equations

Backreaction

- In simplest form addresses only local evidence and not CMB
- Construct large-scale EFE by averaging local equations
- Modifications can act in principle as a dark energy

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- Many previous studies (e.g. Buchert 98,00,01; Wetterich 02; Räsänen 03,04; Kolb *et. al.* 06; Kasai, Asada and Futamase 06, Tanaka and Futamase 07; Schwarz and Li 07); but few quantitive estimates

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- Effect from linear perturbations (e.g. Wetterich, Räsänen) $\approx 10^{-5}$
- Vandervald, Flanagan and Wasserman 07: Reconstructed overall impact $\approx 10^{-5}$
- Aim: To predict size of backreaction from Boltzmann codes and test Vandervald *et. al.*'s estimate.

Backreaction

Formalism

- Employ ADM 3+1 split with vanishing shift vector, non-vanishing lapse function, $ds^2 = -\alpha^2 dt^2 + h_{ij} dx^i dx^j$
- Average equations across slices of equal time with normal n^μ

Formalism

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- Local equations:
  - Hamiltonian constraint: $\mathcal{R} + K^2 - K^i_j K^j_i = 16\pi G\rho + 2\Lambda$
  - Evolution equation: $\frac{1}{\alpha}\dot{K}_{ij} = 8\pi G S_{ij} + 4\pi G h_{ij}\left(\rho - S\right) + \Lambda h_{ij} + 2K_{in}K^n_j - KK_{ij} - \mathcal{R}_{ij} + \frac{1}{\alpha}D_i D_j \alpha$.
  - $T_{\mu\nu}$ separated w.r.t. n^μ; K_{ij} the extrinsic curvature

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- Average $\langle A\rangle = \frac{1}{V}\int A\sqrt{-h}d^3x$, scale factor $3\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} = \frac{\dot{V}}{V}$, commutation relation $\left\langle \dot{A}\right\rangle = \frac{\partial}{\partial t}\langle A\rangle + 3\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}\langle A\rangle - \langle A\alpha K\rangle$

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- Averaged equations:
  - Friedmann equation:
    $$\left(\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}\right)^2 = \frac{8\pi G}{3} \left\langle \alpha^2 \rho \right\rangle + \frac{\Lambda}{3} \left\langle \alpha^2 \right\rangle - \frac{1}{6} \left(\mathcal{Q}_{\mathcal{D}} + \mathcal{R}_{\mathcal{D}} \right)$$
  - Raychaudhuri equation:
    $$\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} = -\frac{4\pi G}{3} \left\langle \alpha^2 (\rho + S) \right\rangle + \frac{\Lambda}{3} \left\langle \alpha^2 \right\rangle + \frac{1}{3} \left(\mathcal{Q}_{\mathcal{D}} + \mathcal{P}_{\mathcal{D}} \right)$$
  - $\mathcal{Q}_{\mathcal{D}}$ is the kinematical backreaction, $\mathcal{P}_{\mathcal{D}}$ the dynamical backreaction and $\mathcal{R}_{\mathcal{D}}$ a contribution from the intrinsic curvature of the slices

Link to Perturbation Theory

- Why Newtonian gauge?

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  - Complementary to synchronous results

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  - Easily implemented into Boltzmann code

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  - Drawback: “integrability relation” has complicated source term
- $\rho = T_{\mu\nu} n^\mu n^\nu = \overline{\rho}(1+\delta)(u_\mu n^\nu)^2$ giving extra correction $\mathcal{T}_\mathcal{D}$

Link to Perturbation Theory

- Result:
  - Friedmann equation:
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  - $\mathcal{T}_{\mathcal{D}} = \frac{8\pi G}{3}\overline{\rho}\left\langle 2\phi\delta + a^2 v^2\right\rangle$
  - $\mathcal{P}_{\mathcal{D}} = -6\frac{\dot a}{a}\left\langle \dot\phi\phi\right\rangle - 3\left\langle \dot\phi^2\right\rangle + \frac{2}{a^2}\left\langle \phi\nabla^2\phi - (\nabla\phi)^2\right\rangle$
  - $\mathcal{Q}_{\mathcal{D}} = 6\left\langle \dot\phi^2\right\rangle$
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- Assume large domain (ergodicity) $\Rightarrow$ convert spatial averages to ensemble averages of form $\int \mathcal{P}(k)A(t,k)B^*(t,k)\frac{dk}{k}$

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Linear Regime

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- Employ CMBEasy to evaluate backreaction terms; confirm with modified CMBFast
- Use WMAPIII concordance for ΛCDM; model Einstein de Sitter universe with $h = 0.41$, $\Omega_m = 1$, $\Omega_b = 0.05$

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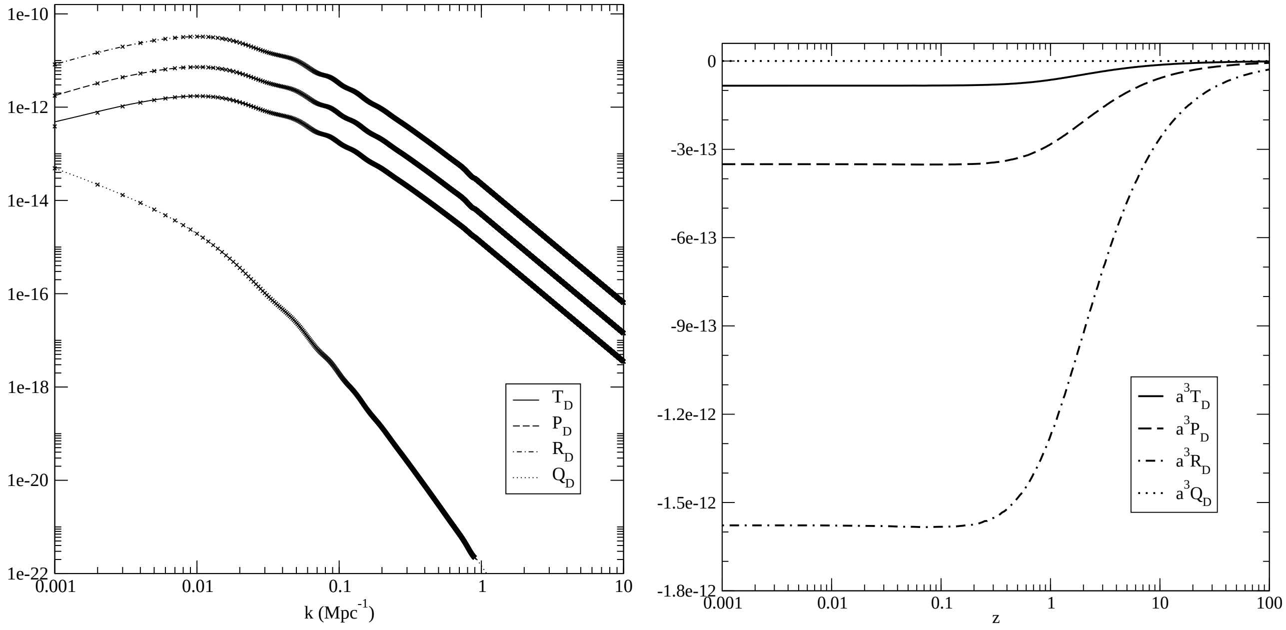

Linear Regime

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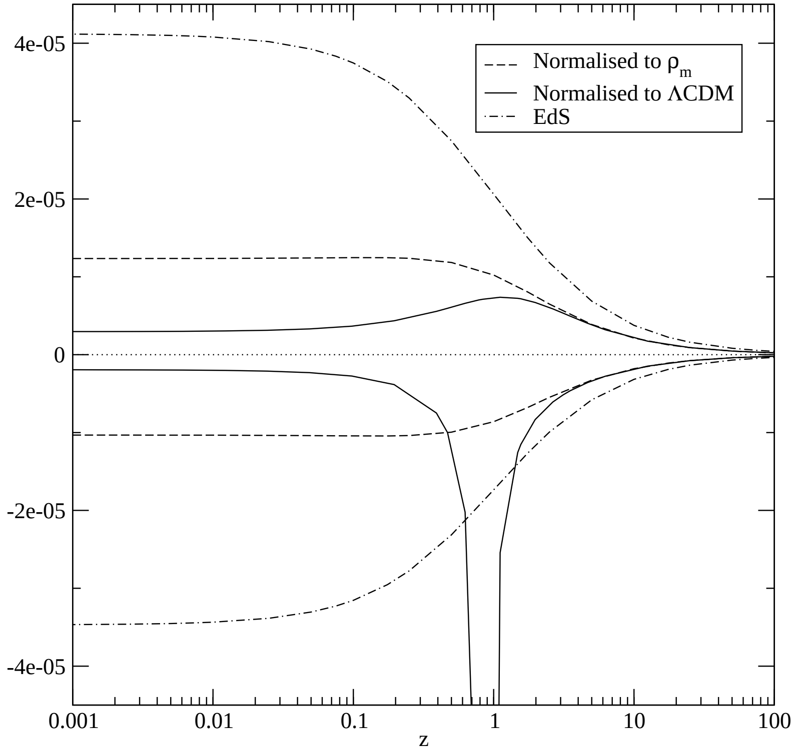
- Modifications normalised to standard matter contribution/perturbative prediction
- $\approx 10^{-5}$ impact as predicted



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- Modified Halofit estimates nonlinear matter power spectrum $\Rightarrow$ approximate ϕ, $\dot{\phi}$, v^2

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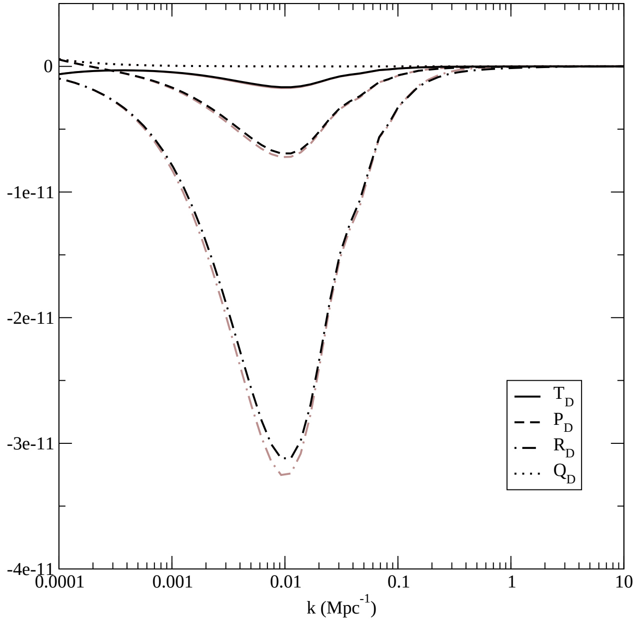

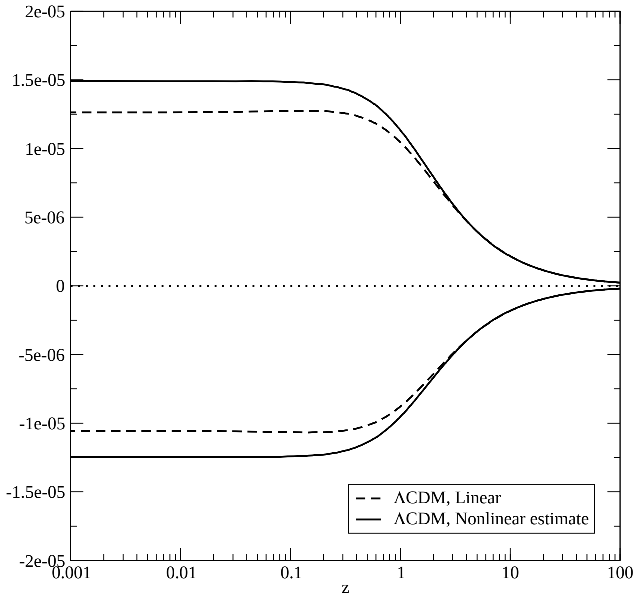

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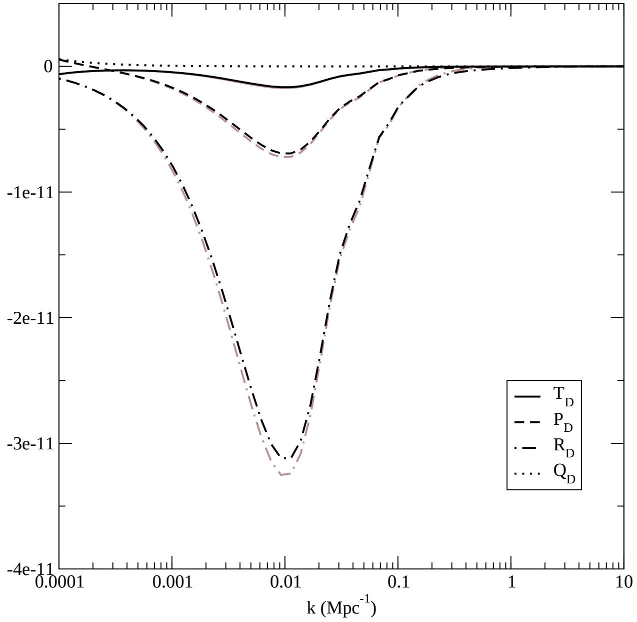

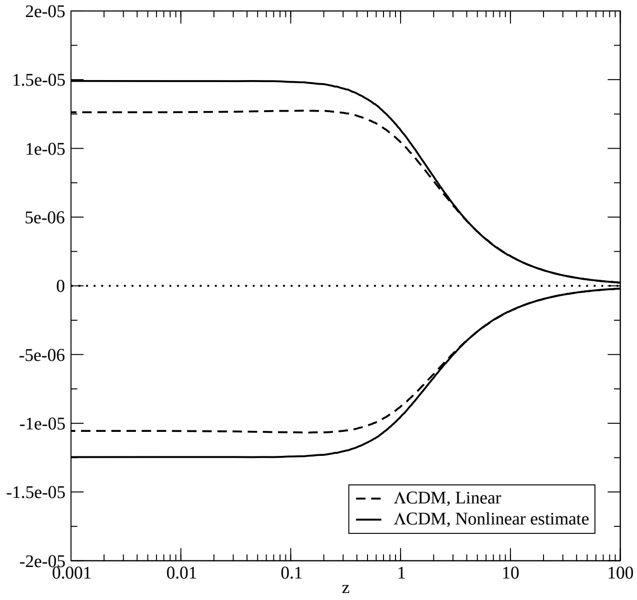

- c.f. estimate of Vandervald *et. al.*
- Impact is to *decelerate* (c.f. Tanaka and Futamase)

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  - So future study into properly averaging realistic inhomogeneous models will be very interesting