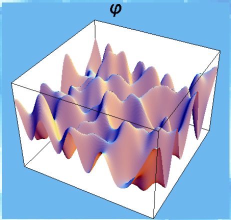
Jean-François Dufaux CITA

Gravitational wave background from preheating after inflation



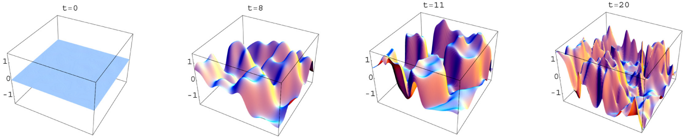
J.F.D, Bergman, Felder, Kofman and Uzan

arXiv:0707.0875 [astro-ph]

Preheating and primordial gravity waves

Inflation $\phi(t)$ no particle $\hookrightarrow$ **CMB, ...**	$\longrightarrow$	Reheating BANG! "Dark Age" $\hookrightarrow$ **????**	$\longrightarrow$	Hot Big Bang model Particles in thermal equilibrium $\hookrightarrow$ **BBN, ...**

In many models, the inflaton decays in a violent and highly inhomogeneous way



$\Rightarrow$ Production of gravity waves, carrying relic information about this epoch

Preheating: explosive and non-perturbative production of highly inhomogeneous, non-thermal fluctuations of the inflaton and other fields

$n_k >> 1 \Rightarrow$ non-linear classical random fields $\Rightarrow$ lattice simulations

"Rescattering": highly non-perturbative and non-linear stage

Turbulent interactions between classical waves

Thermalisation: occurs on much longer time scales

Complementarity with gravity waves from inflation

Inflation: Relevant for *high* energy inflation

Quantum fluctuations amplified to super-Hubble scales: $\Omega_{\rm gw}^{\rm Inf} \propto \frac{V_{\rm Inf}}{M_p^4}$

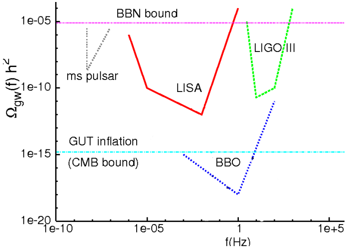
Reheating: Relevant for *low* energy inflation

Classical emission from time-dependent inhomogeneities at sub-Hubble scales

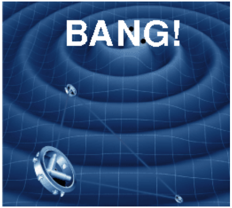
$$f_0 = \frac{a_p}{a_0} f_p \simeq \frac{f_p}{H_p}\sqrt{\frac{H_p}{M_p}}\, 4.5\, 10^{11}\,{\rm Hz} \sim \frac{V_{\rm Inf}^{1/4}}{10^8\,{\rm GeV}}\, 1\,{\rm Hz}$$

GW interferometers: f_0 from $10^{-5}\,{\rm Hz}$ (LISA) to $10^2\,{\rm Hz}$ (LIGO)

Sensitivities ($h^2\,\Omega_{\rm gw}(f)$)



Big Bang Observer



Gravity waves production from preheating

Different works, with different numerical methods (and results):

[Khlebnikov, Tkachev '97], **[Easther, Lim '06]**, **[Easther, Giblin, Lim '06]**, **[Garcia-Bellido, Figueroa '07]**, **[Garcia-Bellido, Figueroa, Sastre '07]**

Ex.: "Weinberg formula":

$$\frac{dE_{\rm gw}}{d\Omega d\omega} = 2\,G\,\Lambda_{ij,lm}(\hat{\mathbf{k}})\,\omega^2\;T_{ij}(\mathbf{k},\omega)\;T_{lm}(\mathbf{k},\omega)$$

for wave-zone approximation (localized source) in Minkowski background

Here: extended source in expanding universe

We developed a method to study, numerically and analytically, GW production from stochastic media of dynamical scalar fields in an expanding universe

$T_{\mu\nu} = \partial_\mu\phi\,\partial_\nu\phi - g_{\mu\nu}\;\left(\frac{1}{2}\,g^{\rho\sigma}\,\partial_\rho\phi\,\partial_\sigma\phi + V(\phi)\right)$

$\delta\phi(\mathbf{x},t) \sim \bar{\phi}(t)$ is not a small perturbation

Background: $G_{\mu\nu} = 8\pi G\,\langle T_{\mu\nu}\rangle \quad \Rightarrow \quad a(t)$

Linear response to inhomogeneities: $\delta G_{\mu\nu} = 8\pi G\,\left(T_{\mu\nu} - \langle T_{\mu\nu}\rangle\right)$

Gravity waves emission from inhomogeneous scalar fields backgrounds

$$ds^2 = a^2(\tau)\,\left[-d\tau^2 + (\delta_{ij} + h_{ij})\; dx^i dx^j\right] \quad \text{with} \quad \partial_i h_{ij} = h_{ii} = 0$$

Go to Fourier space and take $\bar{h}_{ij} = a\, h_{ij}$:

$$\bar{h}''_{ij}(\mathbf{k}) + \left(k^2 - \frac{a''}{a}\right)\, \bar{h}_{ij}(\mathbf{k}) = 16\pi G\, a\, \Pi^{\mathrm{TT}}_{ij}(\mathbf{k})$$

where $\Pi^{\mathrm{TT}}_{ij} = [\partial_i\phi\,\partial_j\phi]^{\mathrm{TT}} - \frac{1}{3}\,\langle(\nabla\phi)^2\rangle\,\bar{h}_{ij}$

Modes inside Hubble radius: $G\,\langle(\nabla\phi)^2\rangle \,\sim\, a''/a \,\sim\, a^2 H^2 << k^2$

$$\Rightarrow \quad \bar{h}_{ij}(\tau,\mathbf{k}) = \frac{16\pi G}{k}\int_{\tau_i}^{\tau} d\tau'\, \sin\left[k\,(\tau-\tau')\right]\, a(\tau')\, T^{\mathrm{TT}}_{ij}(\tau',\mathbf{k})$$

where the source is the transverse-traceless part of the stress-energy tensor:

$$T^{\mathrm{TT}}_{ij}(\mathbf{k}) = \mathcal{O}_{ij,lm}(\hat{\mathbf{k}})\; T_{lm}(\mathbf{k}) = \mathcal{O}_{ij,lm}(\hat{\mathbf{k}})\;\{\partial_l\phi\,\partial_m\phi\}\,(\mathbf{k})$$

where $\mathcal{O}_{ij,lm}(\hat{\mathbf{k}}) = P_{il}(\hat{\mathbf{k}})\,P_{jm}(\hat{\mathbf{k}}) - \frac{1}{2}\,P_{ij}(\hat{\mathbf{k}})\,P_{lm}(\hat{\mathbf{k}})$ with $P_{ij}(\hat{\mathbf{k}}) = \delta_{ij} - \hat{k}_i\,\hat{k}_j$

Spectrum of energy density in gravity waves: Numerics

$$\rho_{\rm gw} = \frac{1}{32\pi G}\,\langle \dot h_{ij}(t,\mathbf{x})\,\dot h_{ij}(t,\mathbf{x})\rangle = \frac{4\pi G}{a^4}\,\frac{1}{V}\int d\mathbf{k}\sum_{i,j}$$

$$\left\{\left|\int_{\tau_i}^{\tau} d\tau'\,\cos(k\,\tau')\,a(\tau')\,T_{ij}^{\rm TT}(\tau',\mathbf{k})\right|^2 + \left|\int_{\tau_i}^{\tau} d\tau'\,\sin(k\,\tau')\,a(\tau')\,T_{ij}^{\rm TT}(\tau',\mathbf{k})\right|^2\right\}$$

$$h^2\,\Omega_{\rm gw}(f) = \left(\frac{h^2}{\rho_c}\,\frac{d\rho_{\rm gw}}{d\ln f}\right)_0 = \left(\frac{1}{\rho_{\rm tot}}\,\frac{d\rho_{\rm gw}}{d\ln k}\right)_p\,\left(\frac{a_j}{a_*}\right)^{1-3w}\,\left(\frac{g_*}{g_0}\right)^{-1/3}\,h^2\,\Omega_{\rm rad}$$

Ex: $\mathbf{V} = \lambda\phi^4 + \mathbf{g}^2\,\phi^2\,\chi^2$ $\quad(\lambda \sim 10^{-14}\ ,\quad V_{\rm Inf}^{1/4} \sim 10^{15}\,{\rm GeV})$

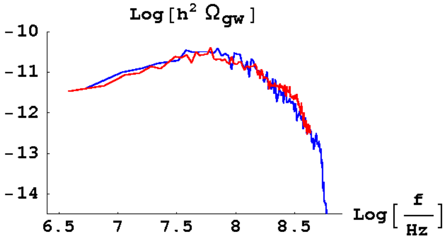
GW spectrum today: $\longrightarrow$

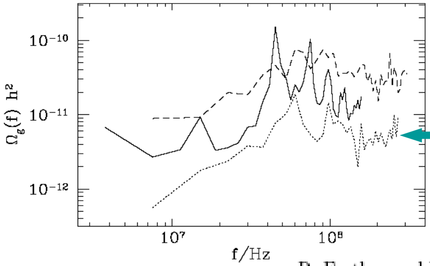
$h^2\,\Omega_{\rm gw}(f) = \left(\frac{h^2}{\rho_c}\,\frac{d\rho_{\rm gw}}{d\ln f}\right)_0$

For different g^2:

Max. $h^2\,\Omega_{\rm gw}^{\rm peak} \sim 10^{-9}$

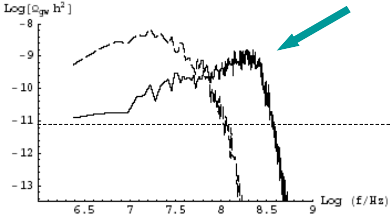
at $f_0^{\rm peak} \sim 10^6\,{\rm Hz}$



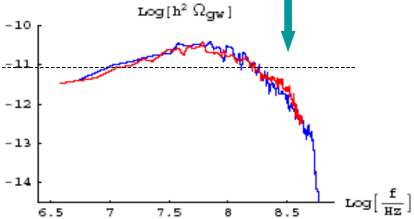


S. Y. Khlebnikov and I. I. Tkachev, [arXiv:hep-ph/9701423].

R. Easther and E. A. Lim, arXiv:astro-ph/0601617.



Our results



GW from random scalar fields media: Analytics

Ensemble average: $\langle \bar{h}'_{ij}(\mathbf{k},\tau)\, \bar{h}'^{*}_{ij}(\mathbf{k}',\tau)\rangle \;\Rightarrow\; \rho_{\rm gw}$

$$\bar{h}_{ij}(\tau,\mathbf{k}) = \tfrac{16\pi G}{k} \int_{\tau_i}^{\tau} d\tau' \;\sin\left[k\,(\tau-\tau')\right]\; a(\tau')\; T^{\rm TT}_{ij}(\tau',\mathbf{k})$$

$$T^{\rm TT}_{ij}(\mathbf{k}) = \mathcal{O}_{ij,lm}(\hat{\mathbf{k}})\;\{\partial_l\phi\,\partial_m\phi\}\,(\mathbf{k}) = \mathcal{O}_{ij,lm}(\hat{\mathbf{k}}) \int \tfrac{d\mathbf{p}}{(2\pi)^{3/2}}\, p_l\, p_m\, \phi(\mathbf{p})\,\phi(\mathbf{k}-\mathbf{p})$$

$$\langle T^{\rm TT}_{ij}(\tau',\mathbf{k})\; T^{\rm TT*}_{ij}(\tau'',\mathbf{k}')\rangle = \mathcal{O}_{ij,lm}(\hat{\mathbf{k}})\,\mathcal{O}_{ij,rs}(\hat{\mathbf{k}}') \int \tfrac{d\mathbf{p}}{(2\pi)^{3/2}} \int \tfrac{d\mathbf{p}'}{(2\pi)^{3/2}}$$
$$p_l\, p_m\, p'_r\, p'_s\; \langle \hat{\chi}(\mathbf{p},\tau')\,\hat{\chi}(\mathbf{k}-\mathbf{p},\tau')\,\hat{\chi}^*(\mathbf{p}',\tau'')\,\hat{\chi}^*(\mathbf{k}'-\mathbf{p}',\tau'')\rangle$$

Gaussian random fields: $\langle\chi\,\chi\,\chi\,\chi\rangle \;\Rightarrow\; \langle\chi\,\chi\rangle$

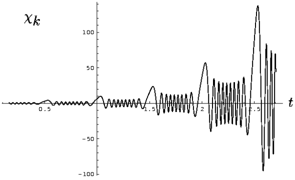
Linear preheating stage: $\hat{\chi}(\mathbf{p},\tau) = \chi_p(\tau)\,\hat{a}_{\mathbf{p}} + \chi^*_p(\tau)\,\hat{a}^{+}_{-\mathbf{p}}$

$$\langle 0|\hat{\chi}(\mathbf{p},\tau')\,\hat{\chi}^{+}(\mathbf{p}',\tau'')|0\rangle = \chi_p(\tau')\,\chi^*_p(\tau'')\,\delta^{(3)}(\mathbf{p}-\mathbf{p}')$$

$$\frac{d\rho_{\rm gw}}{d\ln k} = \frac{2Gk^3}{\pi a^4}\int \frac{d\mathbf{p}}{(2\pi)^3}\; p^4\, \sin^4(\hat{\mathbf{k}},\hat{\mathbf{p}})$$
$$\left\{\left|\int^{\tau} d\tau'\; \cos\left(k\,\tau'\right)\; a(\tau')\,\chi_p(\tau')\,\chi_{|\mathbf{k}-\mathbf{p}|}(\tau')\right|^2 + \left|\int^{\tau} d\tau'\; \sin\left(k\,\tau'\right)\; a(\tau')\,\chi_p(\tau')\,\chi_{|\mathbf{k}-\mathbf{p}|}(\tau')\right.\right.$$

Ex.: Preheating by parametric resonance $\quad V = \lambda\phi^4 + g^2\phi^2\chi^2$

$$\ddot{\chi}_k + \omega_k^2(t)\,\chi_k = 0 \quad \text{with} \quad \omega_k^2(t) = K^2 + q\,\bar{\phi}(t) \qquad (q = g^2/\lambda >> 1)$$



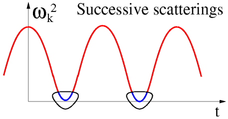
Particle production ($\dot{\omega}_k > \omega_k^2$) in very small intervals around $\bar{\phi}(t) = 0$

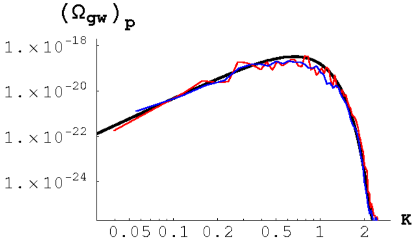
Only resonant momenta amplified: $K_* \sim q^{1/4}$

[Kofman, Linde, Starobinsky '97]

$$\chi_k^j(t) \simeq \frac{\alpha_k^j}{\sqrt{2\omega_k}}\, e^{-i\int^t \omega_k dt'} + \frac{\beta_k^j}{\sqrt{2\omega_k}}\, e^{i\int^t \omega_k dt'}$$

matched with **exact solution around** $\bar{\phi} \simeq 0$

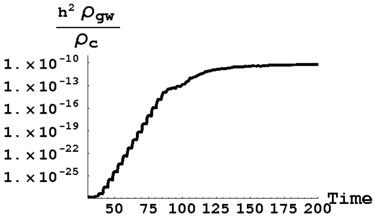




$\longrightarrow$ Spectrum of $\left(\frac{1}{\rho_{\rm tot}}\frac{d\rho_{\rm gw}}{d\ln k}\right)_p$ during preheating. The lattice results (color) oscillate around the (ensemble averaged) analytical curve (bold)

Accumulation of $\rho_{\rm gw}$ with time $\longrightarrow$ (Numerics)

Different stages of GW production



No GW production from scalar field waves (analytics)

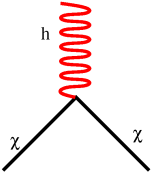
$$\rho_{\rm gw} \propto \int d^3\mathbf{p}\; p^4\; \sin^4(\hat{\mathbf{k}},\hat{\mathbf{p}}) \left| \int^{\tau} d\tau'\; e^{\pm ik\tau'}\; a(\tau')\,\chi_p(\tau')\,\chi_{|\mathbf{k}-\mathbf{p}|}(\tau') \right|^2$$

Klein-Gordon in Minkowski: $\chi_p(\tau) \propto e^{\pm i\omega_p\tau}$ with $\omega_p^2 = p^2 + m^2$

$\hookrightarrow \int^{\tau} d\tau'\; e^{i\,(\pm\,\omega_p \pm \omega_{|\mathbf{k}-\mathbf{p}|} \pm k)\,\tau'} \longrightarrow \omega_p + \omega_{|\mathbf{k}-\mathbf{p}|} = k \longrightarrow \mathbf{p} \parallel \mathbf{k}$

$\hookrightarrow \rho_{\rm gw} = 0$: $\chi + \chi \rightarrow h_{ij}$ forbidden by helicity conservation

Generalisation: No GW production from:
Preheating when $\omega_p^2 = p^2 + g^2\,\phi^2(t)$ adiabatic,
well-developed **turbulence** and **thermal bath**
of **scalar fields** (Different for vector fields)



Inflaton fragmentation and "bubble" collision See also [Felder, Kofman '06]

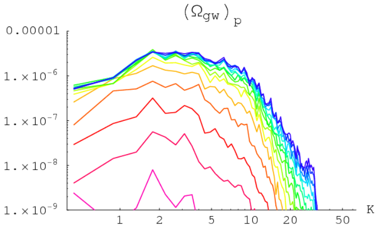
Present day frequency and amplitude of GW spectrum's peak:

$$f_{\text{peak}} \approx \frac{4 \times 10^{10}\,\text{Hz}}{R_* \, \rho_p^{1/4}} \quad , \quad h^2 \, \Omega_{\text{gw}}^{\text{peak}} \approx 10^{-6} \, (R_* \, H_p)^2$$

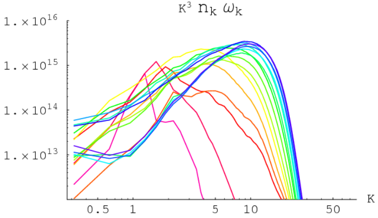
$R_* = a_p / k_*$: Characteristic (physical) size of the field inhomogeneities at the time of GW production

k_*: Typical (comoving) momentum amplified by preheating
(Here: $k_* \sim \sqrt{g}\,\lambda^{1/4} \, M_{\text{Pl}}$ is a non monotomic function of the parameters)

(Similar to GW from colliding bubbles from first order phase transition)



Evolution of the **gravity waves** spectrum at the time of production: $\left(\frac{1}{\rho_{\text{tot}}} \, \frac{d\rho_{\text{gw}}}{d \ln k} \right)_p$



Spectrum of **scalar fields** energy density per logarithmic momentum interval

Expectations for preheating after hybrid inflation

See also [Garcia-Bellido et al '07]

$$f_{\rm peak} \approx \frac{4 \times 10^{10}\,{\rm Hz}}{R_*\,\rho_p^{1/4}} \quad , \quad h^2\,\Omega_{\rm gw}^{\rm peak} \approx 10^{-6}\,(R_*\,H_p)^2 \qquad (R_* = a_p/k_*)$$

Chaotic inflation models: Different models amplify different typical sizes R_*

Ex.: $V = m^2\,\phi^2 + g^2\,\phi^2\,\chi^2 \quad \rightarrow \quad R_* \sim a_p/\sqrt{g\,m\,\Phi_0}$: smaller than for $\lambda\,\phi^4$

Generally: $\Omega_{\rm gw}^{\rm peak} < 10^{-9}$, $f_{\rm peak} > 10^6\,{\rm Hz}$

Hybrid inflation models: May occur at much lower energy scales

Ex.: $V = \frac{\lambda}{4}\,\left(|\sigma|^2 - v^2\right) + \frac{g^2}{2}\,\phi^2\,|\sigma|^2 + V_{\rm sr}(\phi)$

For $g^2 = 2\lambda$ and small $\dot{\phi}_{\rm init}$:

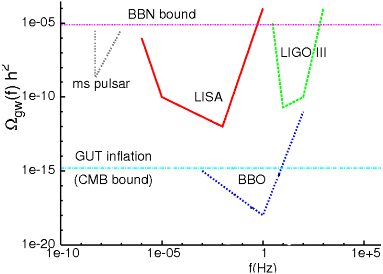
$$V_{\rm eff} = \frac{\lambda}{12}\,v^4 - \frac{\lambda}{3}\,v\,\sigma^3 + \frac{\lambda}{4}\,\sigma^4$$

$$f_{\rm peak} \sim \lambda^{3/4}\,10^{10}\,{\rm Hz}$$

$$h^2\,\Omega_{\rm gw}^{\rm peak} \sim \frac{2\times10^{-6}}{\lambda}\left(\frac{v}{M_{\rm Pl}}\right)^2$$

Ex.: $\lambda = 10^{-11}$ and $v = 10^{12}\,{\rm GeV}$:

$h^2\,\Omega_{\rm gw}^{\rm peak} \sim 2 \times 10^{-9}$ at $f_{\rm peak} \sim 50\,{\rm Hz}$



Conclusions

Preheating leads to large, time-dependent field inhomogeneities which act as classical source of gravitational radiation

We developed a formalism to calculate, numerically and analytically, GW production from random media of dynamical scalar fields in expanding universe

Applicable to other cosmological situations, such as phase transitions

For chaotic inflation:

- $\Omega_{\rm gw}^{\rm peak}$ depends in a simple way on typical resonant momentum
- Detailed analytical check possible
- Frequency too high to be detected

For hybrid inflation: could be observable by GW interferometers, but may require small coupling constants. Further work needed.

Other models ? Extra fields ? Vector fields and turbulence ?