The left-right symmetric seesaw mechanism

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Akhmedov, Blennow, Hällgren, T.K., Ohlsson,
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Mass generation by the seesaw mechanism

Neutrino masses are limited by cosmological bounds to be

$$\sum_i m_{\nu_i} \lesssim 1 \text{ eV}.$$

If this mass is only due to the Higgs mechanism, the corresponding Yukawa coupling would be

$$y = \frac{m_\nu}{v} \sim 10^{-12}.$$

Seesaw mechanism

This issue can be elegantly resolved by adding a Majorana mass term for the right-handed neutrino to the Lagrangian

$$\mathcal{L} \ni \frac{1}{2}\overline{\nu_R}\, m_N \,(\nu_R)^c + \quad \text{h.c.}$$

what leads to the following mass matrix

$$\left(\overline{(\nu_L)^c} \quad \overline{\nu_R}\right) \begin{pmatrix} 0 & m_D \\ m_D^T & m_N \end{pmatrix} \begin{pmatrix} \nu_L \\ (\nu_R)^c \end{pmatrix}$$

and after block diagonalization in leading order ($m_N \gg m_D$)

$$\left(\overline{\nu^c} \quad \overline{N}\right) \begin{pmatrix} -m_D \frac{1}{m_N} m_D^T & 0 \\ 0 & m_N \end{pmatrix} \begin{pmatrix} \nu \\ N^c \end{pmatrix}.$$

Seesaw mechanism

The smallness of the neutrino masses in the type I seesaw

$$m_\nu = -m_D \frac{1}{m_N} m_D^T$$

can be explained by a large (GUT) scale

$$m_N \sim 10^{14}\ \mathrm{GeV}, \quad m_\nu \sim 1\ \mathrm{eV}, \quad m_D \sim 100\ \mathrm{GeV}.$$

Additionally, since the Majorana mass term is lepton number violating and CP-violating, this mechanism provides the possibility to explain the baryon asymmetry of the Universe (BAU) via leptogenesis.

Sakharov conditions

The seesaw mechanism provides all prerequisites to produce a baryon asymmetry

- When the temperature of the Universe drops below the mass of the right-handed neutrinos, the neutrinos become over-abundant and decay out-of-equilibrium
- This decay is lepton number violating ($m_N \neq 0$)
- This decay is CP-violating ($m_N \neq m_N^*$)
- The sphaleron process converts the lepton asymmetry into a baryon asymmetry

Leptogenesis

The baryon-to-photon ratio, $\eta_B = (6.1 \pm 0.2) \times 10^{-10}$, can in leading order be parametrized as

$$\eta_B = 3 \times 10^{-2} \eta \epsilon_N$$

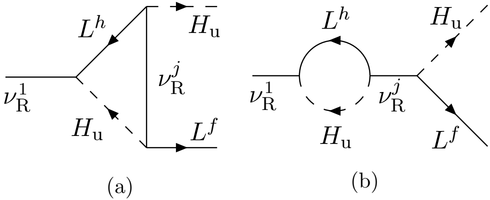
where η denotes the efficiency factor of the decays of the lightest right-handed neutrino and ϵ_N the CP asymmetry in its decays into leptons and Higgs particles

$$\epsilon_N = \frac{\Gamma(N_1 \rightarrow l\, H) - \Gamma(N_1 \rightarrow \bar{l}\, H^*)}{\Gamma(N_1 \rightarrow l\, H) + \Gamma(N_1 \rightarrow \bar{l}\, H^*)}\,.$$

Decay asymmetry

Fukugita, Yanagida, '86

The L and CP violating part of the decay rate results from a cross term between the tree level decay amplitude and the following loop diagrams



and can in the limit $m_{N_1} \ll m_{N_2}$ be written

$$\epsilon_N = \frac{3 m_{N_1}}{16\pi v^2} \frac{\mathrm{Im}[(m_D^\dagger m_\nu m_D^*)_{11}]}{(m_D^\dagger m_D)_{11}} .$$

Decay asymmetry

Davidson, Ibarra, '02

In the type I seesaw model, the decay asymmetry fulfills the Davidson-Ibarra-bound

$$\epsilon_N < \frac{3 m_{N_1} \sqrt{|\Delta m^2_{\rm atm}|}}{16\pi v^2}, \qquad \sqrt{|\Delta m^2_{\rm atm}|} \approx 0.05 \text{ eV}.$$

This leads to the fact that for viable leptogenesis, $\epsilon_N \sim 10^{-7}$, a lower bound on the mass of the lightest right-handed is given by

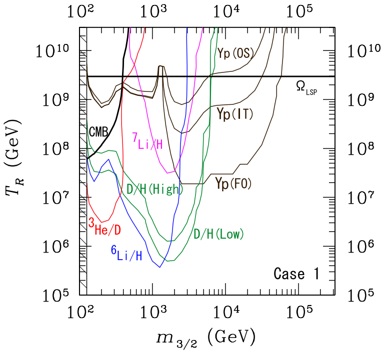
$$m_{N_1} \gtrsim 10^8 \mathrm{GeV}$$

for typical efficiency factors of $\eta \lesssim 1$. Besides, this bound is not easily saturated.

Gravitino bound in SUSY models

Moroi, Murayama, Yamaguchi, '93

Kawasaki, Kohri, Moroi, '05



In supersymmetric models, the decay of the gravitino into the LSP poses constraints on the reheating temperature (dark matter over-abundance, BBN constraints). Depending on the parameters of the mSUGRA model this bound lies in the range

$$T_R \lesssim 10^7 \mathrm{GeV} \text{ to } T_R \lesssim 10^{10} \mathrm{GeV} \leftrightarrow m_{N_1} > 10^8 \mathrm{GeV}.$$

Hierarchies in Yukawa couplings and tuning

Motivated by GUT models, one expects a similar hierarchical structure for the Yukawa coupling of the neutrino as found for the other fermions

$$y_{up} \sim y_{\nu}, \quad y_{down} \sim y_{l}$$

Since the neutrino mass matrix has only a mild hierarchy (at least between the two largest elements) and

$$m_{\nu} = -m_D \frac{1}{m_N} m_D^T$$

the Majorana mass m_N needs to have the doubled hierarchy of m_D.

Hence, hierarchical neutrino Yukawa couplings seem to be unnatural in the seesaw framework.

Open questions in the seesaw mechanism

Hierarchy in the neutrino Yukawa coupling

A hierarchy in the Yukawa couplings, as expected from GUT models requires the doubled hierarchy in the Majorana mass term, what is an unnatural situation.

Gravitino bound

In supersymmetric models, thermal leptogenesis requires rather high reheating temperatures, that can lead to over-closure of the Universe with gravitinos.

Mass generation by the seesaw mechanism

The left-right symmetric framework is based on the gauge group

$$SU(2)_L \times SU(2)_R \times SU(3)_{color} \times U(1)_{B-L}$$

and contains the following (color singlet) Higgs fields

- $\Phi(2,2,0)$
- $\Delta_L(3,1,-2)$
- $\Delta_R(1,3,2)$

- $\langle \Phi^0 \rangle = v$
- $\langle \Delta_L^0 \rangle = v_L$
- $\langle \Delta_R^0 \rangle = v_R$

By spontaneous symmetry breaking, the neutral components of the Higgs fields obtain vacuum expectation values

$$\mathcal{L} \supset f^{\alpha\beta} R_\alpha^T C i\tau_2 \Delta_R R_\beta + y^{\alpha\beta} \bar{R}_\alpha \Phi L_\beta + f^{\alpha\beta} L_\alpha^T C i\tau_2 \Delta_L L_\beta + \mathrm{h.c.}.$$

Mass generation by the seesaw mechanism

The left-right symmetry imposes in this case $m_D = m_D^T$ and the seesaw relation reads

$$m_\nu = m_N \frac{v_L}{v_R} - m_D \frac{1}{m_N} m_D = m_\nu^{\mathrm{II}} + m_\nu^{\mathrm{I}}.$$

Strategy: Use $m_D = m_{up}$ motivated by GUTs/prejudice. The remaining parameters of the model are then:

- The lightest neutrinos mass m_0
- The hierarchy (normal/inverted) of the light neutrinos
- Five Majorana phases (three more than in pure type I)
- The ratio v_R/v_L

This seesaw relation is invertible for given m_ν and m_D and leads to 2^n solutions for m_N in the case of n flavors (with same low energy phenomenology).

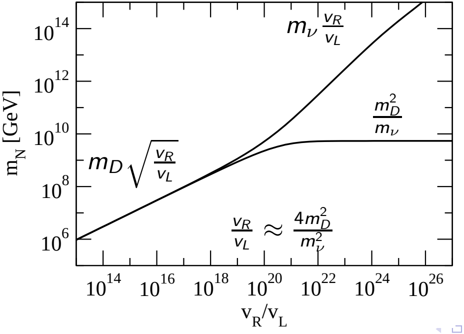
Akhemdov, Frigerio, '05

Small v_R/v_L

In the one-flavor case the solution has a two-fold ambiguity

$$m_N = \frac{m_\nu}{2}\frac{v_R}{v_L} \pm \sqrt{\frac{m_\nu^2}{4}\left(\frac{v_R}{v_L}\right)^2 + m_D^2\frac{v_R}{v_L}}$$

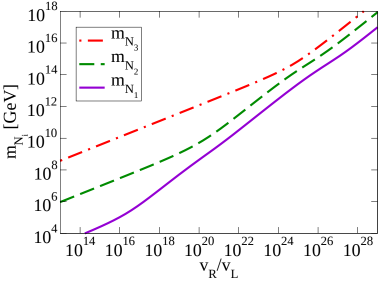
cancellation: $|m_{\mathrm{I}}| \approx |m_{\mathrm{II}}| \gg |m_\nu|$, domination: $m_{\mathrm{I}} \approx m_\nu$ or $m_{\mathrm{II}} \approx m_\nu$

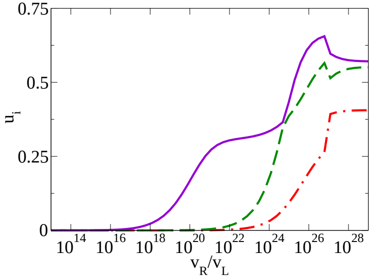


Inversion formula

In the three-flavor case, the solutions can be given in closed form what requires to find the roots of a quartic equation.

Analogously to the one-flavor case, the solution in the three-flavor case have a eight-fold ambiguity ($y = y_{up}$).

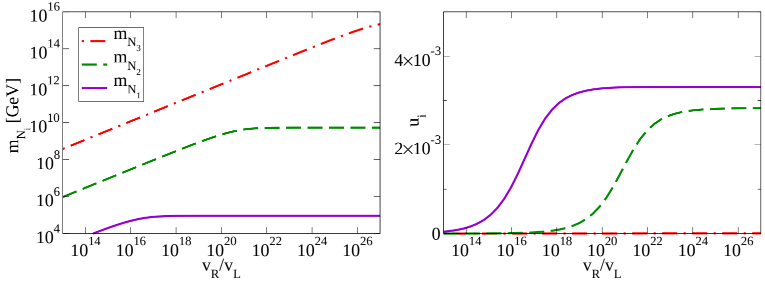




Pure type II '$+++$': Gravitino problem for $v_R/v_L > 10^{21}$.

Pure type I

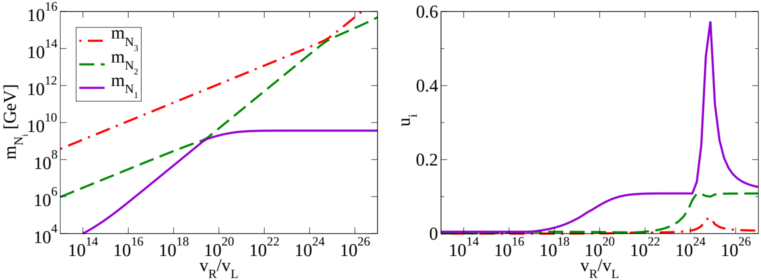
The pure type I solution ('$- - -$') shows a large spread in the right-handed masses and a strong suppression of mixing angles.



Leptogenesis possible? Fine-tuning needed?

Mixed solution

In addition there are six mixed cases.

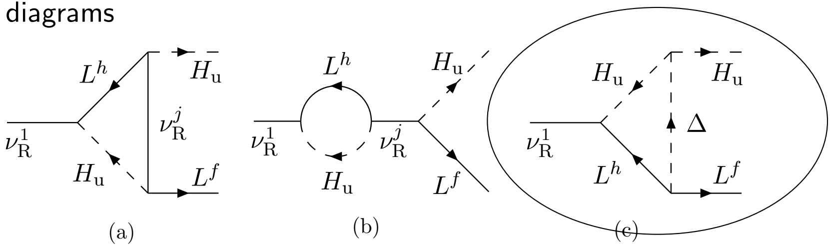


'$--+$': Gravitino bound eventually fulfilled. Leptogenesis possible? Stability for $v_R/v_L > 10^{18}$?

Decay asymmetry

Hambye, Senjanovic, '03
Antusch, King, '04

The L and CP violating part of the decay rate results from a cross term between the tree level decay amplitude and the following loop diagrams



and the Higgs triplet leads to an additional contribution.

Leptogenesis

ANTUSCH, KING, '04

In the limit $m_{N_1} \ll m_{N_2}, m_\Delta$ the asymmetry can be written

$$\epsilon_{N_1} = \frac{3 m_{N_1}}{16\pi v^2} \frac{\mathrm{Im}[(m_D^\dagger (m_\nu^{\mathrm{I}} + m_\nu^{\mathrm{II}}) m_D^*)_{11}]}{(m_D^\dagger m_D)_{11}} .$$

Due to the modified seesaw formula, the DI-bound is avoided, but leads to the slightly weaker bound

$$\epsilon_{N_1} \lesssim \frac{3 m_{N_1} m_{\nu,max}}{16\pi v^2}$$

and hence

$$m_{N_1} \gtrsim 3 \times 10^7 \mathrm{GeV}.$$

This bound is only slightly better than in the pure type I case, but easier to saturate due to additional Majorana phases.

Additional Majorana phases

Akhmedov, Blennow, Hällgren, T.K., Ohlsson, '06

For example, in the one-flavor case, the relative phase κ between m_D and m_ν cannot be removed and can even lead to leptogenesis for the type II dominated solution.

$$\epsilon_N = \frac{3}{16\pi}\frac{m_\nu m_N}{v^2}\sin(4\kappa)\,, \quad \eta_B = 1.7\times 10^{-6}\,\mathrm{eV}\,\frac{m_\nu m_N^2}{|m_D|^2\, v^2}\,\sin(4\kappa).$$

Thus, it is possible to saturate the Antusch/King bound and to reproduce the observed baryon asymmetry e.g. with the values ($\kappa = \pi/8$)

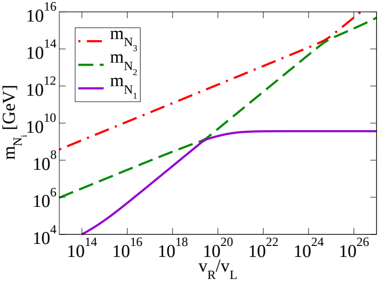
$$|y| = 10^{-4}\,, \qquad m_0 = 0.1\,\mathrm{eV}\,, \qquad \frac{v_R}{v_L} = 1.7\times 10^{20}.$$

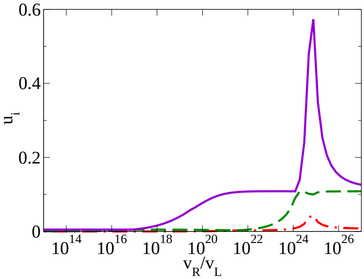
Leptogenesis

The same mechanism is operative in the two-flavor case in the limit

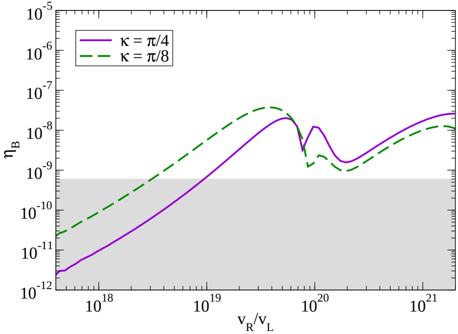
$$\frac{4\, m_{D,1}^2}{m_0^2} \ll \frac{v_R}{v_L} \ll \frac{4\, m_{D,2}^2}{m_0^2}\,,$$

for the two solutions of type '$\pm+$' as long as the second eigenvalue of the Yukawa coupling $y_2 > 5 \times 10^{-4}$.





Leptogenesis



The numerical evaluation for the three-flavor case gives for the solution '$- - +$' ($m_0 = 0.1$ eV, $y = y_u$). Leptogenesis is generally viable for the four solutions of type '$\pm\pm+$'.

Stability measure

AKHMEDOV, BLENNOW, HÄLLGREN, T.K., OHLSSON, '06

To quantify tuning we use the following stability measure

$$Q = \left| \frac{\det m_N}{\det m_\nu} \right|^{1/3} \sqrt{\sum_{k,l=1}^{12} \left(\frac{\partial m_l}{\partial M_k} \right)^2} \, .$$

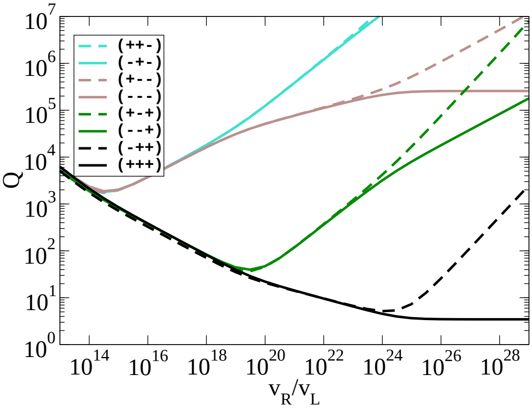
where m_l and M_k determine the light and heavy neutrino mass matrices according to

$$m_N = \sum_k (M_k + iM_{k+N}) T_k, \quad m_\nu = \sum_k (m_k + i m_{k+N}) T_k,$$

and T_k, $k \in [1, 6]$, form a normalized basis of the complex symmetric 3×3 matrices.

Summary

The qualitative analysis mostly depends on the fact the $y = y_{up}$ has a large hierarchy.



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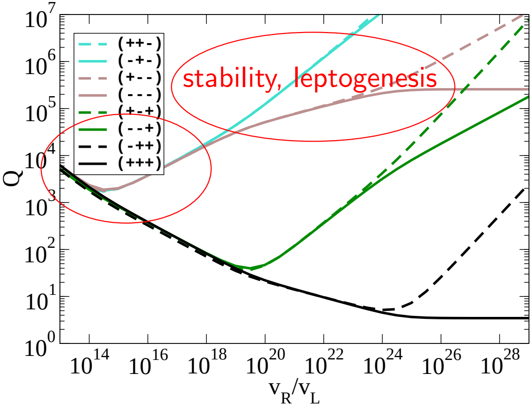

'$\pm \pm -$': Unstable, no leptogenesis

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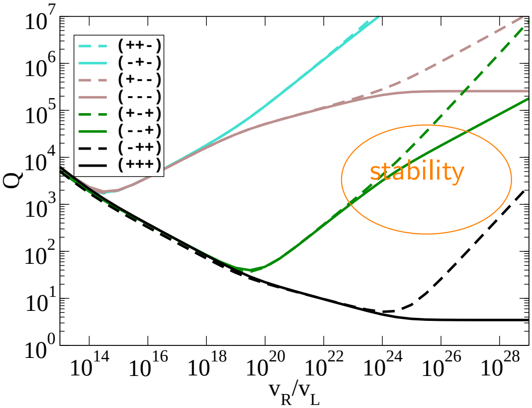

'$\pm - +$', $v_R/v_L \gg 10^{23}$: Unstable

Summary

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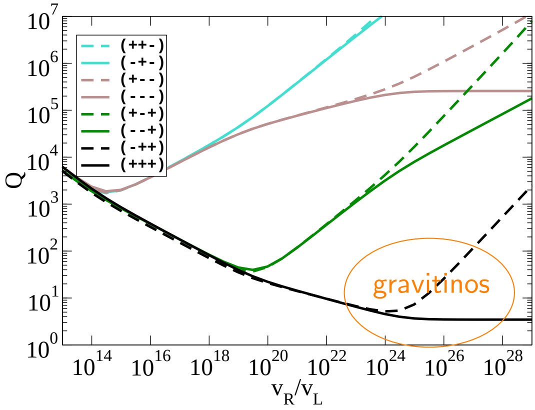

'$\pm ++$', $v_R/v_L \gg 10^{23}$: Gravitino bound violated

Summary

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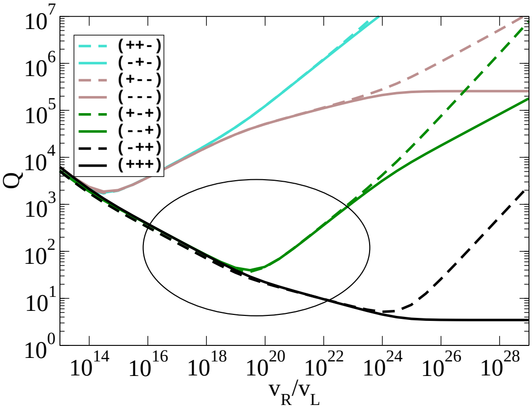

'$\pm\pm+$', $10^{16} \ll v_R/v_L \ll 10^{23}$: Sweet spot

Conclusion

In case of hierarchical Yukawa coupling for the neutrinos, the left-right symmetric type I+II framework constitutes a model that compared to the pure type I framework has the following properties:

GUT embedding

A large hierarchy in the neutrino Yukawa coupling is naturally implemented. The required tuning is reduced from $Q \sim 10^5$ to $Q \sim 10^2$.

SUSY embedding - Gravitino bound

The lower bound on the lightest right-handed neutrino mass is slightly relaxed from $m_{N_1} > 5 \times 10^8$ GeV to $m_{N_1} > 1 \times 10^8$ and the Antusch/King bound can be easily saturated using the additional Majorana phases.