The left-right symmetric seesaw mechanism

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Neutrino masses are limited by cosmological bounds to be

\[ \sum_i m_{\nu_i} \lesssim 1 \text{ eV}. \]

If this mass is only due to the Higgs mechanism, the corresponding Yukawa coupling would be

\[ y = \frac{m_\nu}{\nu} \sim 10^{-12}. \]
Seesaw mechanism

This issue can be elegantly resolved by adding a Majorana mass term for the right-handed neutrino to the Lagrangian

$$\mathcal{L} \ni \frac{1}{2} \bar{\nu}_R m_N (\nu_R)^c + \text{h.c.}$$

what leads to the following mass matrix

$$\begin{pmatrix} \bar{\nu}_L \nonumber \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D^T & m_N \end{pmatrix} \begin{pmatrix} \nu_L \nonumber \end{pmatrix}$$

and after block diagonalization in leading order ($m_N \gg m_D$)

$$\begin{pmatrix} \bar{\nu} \nonumber \\ \bar{N} \end{pmatrix} \begin{pmatrix} -m_D \frac{1}{m_N} m_D^T & 0 \\ 0 & m_N \end{pmatrix} \begin{pmatrix} \nu \nonumber \end{pmatrix}.$$
The smallness of the neutrino masses in the type I seesaw

\[ m_\nu = -m_D \frac{1}{m_N} m_D^T \]

can be explained by a large (GUT) scale

\[ m_N \sim 10^{14} \text{ GeV}, \quad m_\nu \sim 1 \text{ eV}, \quad m_D \sim 100 \text{ GeV}. \]

Additionally, since the Majorana mass term is lepton number violating and CP-violating, this mechanism provides the possibility to explain the baryon asymmetry of the Universe (BAU) via leptogenesis.
The seesaw mechanism provides all prerequisites to produce a baryon asymmetry

- When the temperature of the Universe drops below the mass of the right-handed neutrinos, the neutrinos become over-abundant and decay out-of-equilibrium
- This decay is lepton number violating \((m_N \neq 0)\)
- This decay is CP-violating \((m_N \neq m^*_N)\)
- The sphaleron process converts the lepton asymmetry into a baryon asymmetry
The baryon-to-photon ratio, $\eta_B = (6.1 \pm 0.2) \times 10^{-10}$, can in leading order be parametrized as

$$\eta_B = 3 \times 10^{-2} \eta \epsilon_N$$

where $\eta$ denotes the efficiency factor of the decays of the lightest right-handed neutrino and $\epsilon_N$ the CP asymmetry in its decays into leptons and Higgs particles

$$\epsilon_N = \frac{\Gamma(N_1 \rightarrow l H) - \Gamma(N_1 \rightarrow \bar{l} H^*)}{\Gamma(N_1 \rightarrow l H) + \Gamma(N_1 \rightarrow \bar{l} H^*)}.$$
Leptogenesis

**Fukugita, Yanagida, ’86**

The L and CP violating part of the decay rate results from a cross term between the tree level decay amplitude and the following loop diagrams

(a) \[ \bar{H}_u \rightarrow H^+_H L^f \]

(b) \[ \bar{H}_u \rightarrow H^+_H L^f \]

and can in the limit \( m_{N_1} \ll m_{N_2} \) be written

\[
\epsilon_N = \frac{3m_{N_1}}{16\pi v^2} \text{Im}[(m_D^\dagger m_{\nu} m_D^*)_{11}] \cdot \frac{(m_D^\dagger m_D)_{11}}{(m_D^\dagger m_D)_{11}}.
\]
Decay asymmetry

**Davidson, Ibarra, ’02**

In the type I seesaw model, the decay asymmetry fulfills the Davidson-Ibarra-bound

\[ \epsilon_N < \frac{3m_{N_1} \sqrt{|\Delta m_{atm}^2|}}{16\pi v^2}, \quad \sqrt{|\Delta m_{atm}^2|} \approx 0.05 \text{ eV}. \]

This leads to the fact that for viable leptogenesis, \( \epsilon_N \sim 10^{-7} \), a lower bound on the mass of the lightest right-handed is given by

\[ m_{N_1} \gtrsim 10^8 \text{GeV} \]

for typical efficiency factors of \( \eta \lesssim 1 \). Besides, this bound is not easily saturated.
In supersymmetric models, the decay of the gravitino into the LSP poses constraints on the reheating temperature (dark matter over-abundance, BBN constraints). Depending on the parameters of the mSUGRA model this bound lies in the range

\[ T_R \lesssim 10^7 \text{GeV to } T_R \lesssim 10^{10} \text{GeV} \leftrightarrow m_{N_1} > 10^8 \text{GeV}. \]
Hierarchies in Yukawa couplings and tuning

Motivated by GUT models, one expects a similar hierarchical structure for the Yukawa coupling of the neutrino as found for the other fermions

\[ y_{up} \sim y_{\nu}, \quad y_{down} \sim y_{l} \]

Since the neutrino mass matrix has only a mild hierarchy (at least between the two largest elements) and

\[ m_{\nu} = -m_{D} \frac{1}{m_{N}} m_{D}^{T} \]

the Majorana mass \( m_{N} \) needs to have the doubled hierarchy of \( m_{D} \).

Hence, hierarchical neutrino Yukawa couplings seem to be unnatural in the seesaw framework.
Open questions in the seesaw mechanism

**Hierarchy in the neutrino Yukawa coupling**

A hierarchy in the Yukawa couplings, as expected from GUT models requires the doubled hierarchy in the Majorana mass term, what is an unnatural situation.

**Gravitino bound**

In supersymmetric models, thermal leptogenesis requires rather high reheating temperatures, that can lead to over-closure of the Universe with gravitinos.
Mass generation by the seesaw mechanism

The left-right symmetric framework is based on the gauge group

\[ SU(2)_L \times SU(2)_R \times SU(3)_{\text{color}} \times U(1)_{B-L} \]

and contains the following (color singlet) Higgs fields

\[ \Phi(2, 2, 0) \]
\[ \Delta_L(3, 1, -2) \]
\[ \Delta_R(1, 3, 2) \]

By spontaneous symmetry breaking, the neutral components of the Higgs fields obtain vacuum expectation values

\[ \langle \Phi^0 \rangle = v \]
\[ \langle \Delta^0_L \rangle = v_L \]
\[ \langle \Delta^0_R \rangle = v_R \]

The Lagrangian is given by

\[ \mathcal{L} \supset f^{\alpha\beta} R_\alpha^T C_i \tau_2 \Delta_R R_\beta + y^{\alpha\beta} \bar{R}_\alpha \Phi L_\beta + f^{\alpha\beta} L_\alpha^T C_i \tau_2 \Delta_L L_\beta + \text{h.c.} \]
The left-right symmetry imposes in this case $m_D = m_D^T$ and the seesaw relation reads

$$m_\nu = m_N \frac{v_L}{v_R} - m_D \frac{1}{m_N} m_D = m_\nu^{\text{II}} + m_\nu^{\text{I}}.$$ 

Strategy: Use $m_D = m_{up}$ motivated by GUTs/prejudice. The remaining parameters of the model are then:

- The lightest neutrinos mass $m_0$
- The hierarchy (normal/inverted) of the light neutrinos
- Five Majorana phases (three more than in pure type I)
- The ratio $v_R/v_L$

This seesaw relation is invertible for given $m_\nu$ and $m_D$ and leads to $2^n$ solutions for $m_N$ in the case of $n$ flavors (with same low energy phenomenology).

*Akhemdov, Frigerio, ’05*
Small $v_R/v_L$

In the one-flavor case the solution has a two-fold ambiguity

$$m_N = \frac{m_\nu}{2} \frac{v_R}{v_L} \pm \sqrt{\frac{m_\nu^2}{4} \left( \frac{v_R}{v_L} \right)^2 + m_D^2 \frac{v_R}{v_L}}$$

cancellation: $|m_I| \approx |m_{II}| \gg |m_\nu|$, domination: $m_I \approx m_\nu$ or $m_{II} \approx m_\nu$
Inversion formula

In the three-flavor case, the solutions can be given in closed form what requires to find the roots of a quartic equation.

Analogously to the one-flavor case, the solution in the three-flavor case have a eight-fold ambiguity ($y = y_{up}$).

Pure type II ’+++’: Gravitino problem for $v_R/v_L > 10^{21}$. 
The pure type I solution ('− − −') shows a large spread in the right-handed masses and a strong suppression of mixing angles.

Leptogenesis possible? Fine-tuning needed?
In addition there are six mixed cases.

'− − +': Gravitino bound eventually fulfilled. Leptogenesis possible? Stability for $\nu_R/\nu_L > 10^{18}$?
Hambye, Senjanovic, ’03
Antusch, King, ’04

The L and CP violating part of the decay rate results from a cross term between the tree level decay amplitude and the following loop diagrams

and the Higgs triplet leads to an additional contribution.
Leptogenesis

Antusch, King, ’04

In the limit $m_{N_1} \ll m_{N_2}, m_\Delta$ the asymmetry can be written

$$
\epsilon_{N_1} = \frac{3m_{N_1}}{16\pi v^2} \text{Im}[(m_D^\dagger (m_{I\nu} + m_{II\nu}) m_D^*)_{11}] \cdot \frac{1}{(m_D^\dagger m_D)_{11}}.
$$

Due to the modified seesaw formula, the DI-bound is avoided, but leads to the slightly weaker bound

$$
\epsilon_{N_1} \sim \frac{3m_{N_1} m_{\nu,\text{max}}}{16\pi v^2}
$$

and hence

$$
m_{N_1} \gtrsim 3 \times 10^7 \text{GeV}.
$$

This bound is only slightly better than in the pure type I case, but easier to saturate due to additional Majorana phases.
For example, in the one-flavor case, the relative phase $\kappa$ between $m_D$ and $m_\nu$ cannot be removed and can even lead to leptogenesis for the type II dominated solution.

$$\epsilon_N = \frac{3}{16\pi} \frac{m_\nu m_N}{v^2} \sin(4\kappa), \quad \eta_B = 1.7 \times 10^{-6} \text{ eV} \frac{m_\nu m_N^2}{|m_D|^2 v^2} \sin(4\kappa).$$

Thus, it is possible to saturate the Antusch/King bound and to reproduce the observed baryon asymmetry e.g. with the values ($\kappa = \pi/8$)

$$|y| = 10^{-4}, \quad m_0 = 0.1 \text{ eV}, \quad \frac{v_R}{v_L} = 1.7 \times 10^{20}.$$
The same mechanism is operative in the two-flavor case in the limit

$$\frac{4 m_{D,1}^2}{m_0^2} \ll \frac{v_R}{v_L} \ll \frac{4 m_{D,2}^2}{m_0^2},$$

for the two solutions of type '±' as long as the second eigenvalue of the Yukawa coupling $y_2 > 5 \times 10^{-4}$. 
The numerical evaluation for the three-flavor case gives for the solution ‘− − +’ ( \( m_0 = 0.1 \) eV, \( y = y_u \) ). Leptogenesis is generally viable for the four solutions of type ‘± ± +’.
Stability measure

Akhmedov, Blennow, Häggren, T.K., Ohlsson, ’06

To quantify tuning we use the following stability measure

\[
Q = \left| \frac{\det m_N}{\det m_\nu} \right|^{1/3} \sqrt{\sum_{k,l=1}^{12} \left( \frac{\partial m_l}{\partial M_k} \right)^2}.
\]

where \( m_l \) and \( M_k \) determine the light and heavy neutrino mass matrices according to

\[
m_N = \sum_k (M_k + iM_{k+N}) T_k, \quad m_\nu = \sum_k (m_k + i m_{k+N}) T_k,
\]

and \( T_k, k \in [1, 6] \), form a normalized basis of the complex symmetric \( 3 \times 3 \) matrices.
The qualitative analysis mostly depends on the fact the $y = y_{up}$ has a large hierarchy.
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\[ v_{R}/v_{L} \]  

\[ Q \]  

'± ± −': Unstable, no leptogenesis
Summary

The qualitative analysis mostly depends on the fact the $y = y_{up}$ has a large hierarchy.

'± − +', $v_R/v_L \gg 10^{23}$: Unstable
The qualitative analysis mostly depends on the fact the $y = y_{up}$ has a large hierarchy.

\[ Q \propto \frac{v_R}{v_L} \]

$\pm ++$, $v_R/v_L \gg 10^{23}$: Gravitino bound violated
The qualitative analysis mostly depends on the fact that $y = y_{up}$ has a large hierarchy.

'$\pm \pm +$, $10^{16} \ll v_R/v_L \ll 10^{23}$: Sweet spot
In case of hierarchical Yukawa coupling for the neutrinos, the left-right symmetric type I+II framework constitutes a model that compared to the pure type I framework has the following properties:

**GUT embedding**

A large hierarchy in the neutrino Yukawa coupling is naturally implemented. The required tuning is reduced from $Q \sim 10^5$ to $Q \sim 10^2$.

**SUSY embedding - Gravitino bound**

The lower bound on the lightest right-handed neutrino mass is slightly relaxed from $m_{N_1} > 5 \times 10^8$ GeV to $m_{N_1} > 1 \times 10^8$ and the Antusch/King bound can be easily saturated using the additional Majorana phases.