# Solar System Constraints on $f(\mathcal{G})$ Dark Energy

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(in preparation)

related work: arXiv:0704.0175

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**Dark energy and Quadratic Gravity**

- Supernovae measurements indicate expansion of universe is accelerating.
- Could be explained by cosmological constant,
  but suffers from fine-tuning and coincidence problems.
- Maybe need to change gravity instead?

E.g. Use quadratic curvature Gauss-Bonnet term $\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$

$$\mathcal{L} = \sqrt{-g}\left[\frac{R}{2} + f(\mathcal{G})\right] \qquad \longleftrightarrow \qquad \mathcal{L} = \sqrt{-g}\left[\frac{R}{2} + f'(\phi)\mathcal{G} + f(\phi) - \phi f'(\phi)\right]$$

[Nojiri and Odintsov (2005)]

scalar field fixed at $\phi = \mathcal{G}$ (no extra degrees of freedom)

BUT, what about the evidence for general relativity?
– no use explaining dark energy if can't explain other gravity measurements!

- What does the solar system tell us about gravity?
- Can it significantly constrain such dark energy models?

**Post-Newtonian approximation**

In solar system, can use (to leading order) linearised metric

$$ds^2 = -(1 + 2\Phi/c^2)(cdt)^2 + (1 - 2\Psi/c^2)dx_i dx^i + \cdots$$

Einstein equations

$$\Delta\Phi = \Delta\Psi = 4\pi G\rho_m$$

with $\Delta X = \sum_i X_{,ii}$

Usual, general relativity, solution for point mass $m$ (the sun)

$$\Phi = \Psi = -\frac{Gm}{r} \lesssim 10^{-5}$$

- Gauss-Bonnet corrections second order in curvature,
  – expect to be automatically insignificant
- no extra degrees of freedom (unlike Brans-Dicke),
  – expect no corrections to light bending measurements

$\Rightarrow$ Suggests no significant conflict with solar system observations

However not so simple: consider, for example, $f = C\,\mathcal{G}^n$

[$n = 0$: cosmological constant, $n = 1$: trivial, $\Omega_{\mathcal{G}} \equiv 0$]

**Cosmology**

Friedmann equation $1 = \Omega_m + \Omega_{\mathcal{G}}$ with $\mathcal{G} = 24H^2(\dot{H} + H^2)$ and

$$\Omega_{\mathcal{G}} = -8H\partial_t f'(\mathcal{G}) + \frac{-f(\mathcal{G}) + \mathcal{G}f'(\mathcal{G})}{3H^2} \sim (1-n)CH^{4n-2}$$

To have any hope of explaining dark energy, will need $\Omega_{\mathcal{G}} \sim 1$

$\Rightarrow C \sim H_0^{2-4n}$, which is typically very large (for $n > 1/2$)

$\Rightarrow$ cannot ignore Gauss-Bonnet corrections

Expect that corrections need to be small to satisfy gravity experiments.

– Perturb Einstein solution to obtain approx $f(\mathcal{G})$ one, so $\Phi \approx -Gm/r$, etc.

**Field equations**

Making no assumptions about size of $f$, find to leading order

$$\Delta\Phi = 4\pi G\rho_m + f(\mathcal{G}) - \mathcal{G}f'(\mathcal{G}) - 4\mathcal{D}(f'(\mathcal{G}), \Phi + \Psi) + \cdots$$

$$\Delta\Psi = 4\pi G\rho_m + \frac{\mathcal{G}f'(\mathcal{G}) - f(\mathcal{G})}{2} - 4\mathcal{D}(f'(\mathcal{G}), \Psi) + \cdots$$

where $\mathcal{D}(X, Y) = \sum_{i,j} X_{,ij}Y_{,ij} - \Delta X \Delta Y$

find $\mathcal{G} = 8\mathcal{D}(\Phi, \Psi) \approx 8\mathcal{D}\left(\frac{Gm}{r}, \frac{Gm}{r}\right) = 48\frac{(Gm)^2}{r^6}$

For $f = C\,\mathcal{G}^n$, find to leading order

$$\Phi = -\frac{Gm}{r} - 2\frac{48^n(n-1)n}{3(2n-1)} C\frac{(Gm)^{2n-1}}{r^{6n-3}}, \qquad \Psi = -\frac{Gm}{r} - \frac{48^n(n-1)n}{3(2n-1)} C\frac{(Gm)^{2n-1}}{r^{6n-3}}$$

- Non-Newtonian, and typically large, corrections
- Standard Parametrised Post-Newtonian (PPN) analysis not applicable

**Planetary motion**

For gravitational acceleration $g_{\text{acc}} = -Gm/r^2$,
period of elliptical planetary orbit is $2\pi\sqrt{a^3/Gm}$ ($a =$ semi-major axis)

GB corrections alter effective sun's mass felt by planets ($r_g = Gm/c^2$)

$$g_{\text{acc}} = -\frac{d\Phi}{dr} = -\frac{Gm}{r^2}\left[1 + 2nC48^n(n-1)\frac{r_g^{2n-2}}{r^{6n-4}}\right] \equiv -\frac{G(m+\delta m)}{r^2}$$

Bounded by uncertainty in $a$: $\frac{1}{3}\frac{\delta m}{m} < \frac{\delta a}{a}$

For $n > 1$, strongest bound from Mercury ($a \approx 57 \times 10^6\,\text{km}$, $\delta a \approx 0.1\,\text{m}$)

$$|C| \lesssim 5 \times 10^{-43}\,\text{km}^{-2}\,(4 \times 10^{44}\,\text{km}^4)^n \qquad \Rightarrow \qquad \Omega_{\mathcal{G}} \lesssim 8 \times 10^{-44}$$

In fact, using all planets, find $\Omega_{\mathcal{G}} \lesssim 0.1$ for $n \gtrsim 0.08$

Note solar system constraints directly applicable to cosmological scales,
since theory has no extra degrees of freedom

Hence $\mathcal{L} = \sqrt{-g}\left[\frac{R}{2} + C\,\mathcal{G}^n\right]$ gravity mostly ruled out

**Possible Exceptions:**

$n = 2/3$ leading corrections to $\Phi$, $\Psi$ cancel

Need higher order expansion (expect to be ruled out too)

$n \lesssim 0.08$ satisfies planetary tests of Newton's law, but

$$\delta g_{\text{acc}} \propto \frac{n r^{2-6n}}{(Gm)^{1-2n}}$$

increases as $m \to 0$ (for $n < 1/2$, $n \neq 0$)

Gravitational field of, e.g. $\nu$ will be huge!

$n = 0$ identical to cosmological constant

More general theory: $$f(\mathcal{G}) = \sum_n C_n \mathcal{G}^n$$

Each term gives correction to Newton potential $\Phi$ with different $r$ dependence

$\Rightarrow$ can bound most $\Omega_{\mathcal{G}}^{(n)} \ll 1$ separately

Corrections could cancel, but must occur for
every choice of $r$ (and $m$) for which there is a
planet, moon, laboratory experiment, light-bending measurement, . . .

**Summary**

- Correct analysis of solar system gravity provides powerful probe of higher curvature gravity modifications
- $f(\mathcal{G})$ dark energy appears to be ruled out
- Except for cosmological constant: $f =$ const.
  (and possibly $f(\mathcal{G}) \propto \mathcal{G}^{2/3}$)