Non-linear Interactions in a cosmological background in the DGP braneworld model

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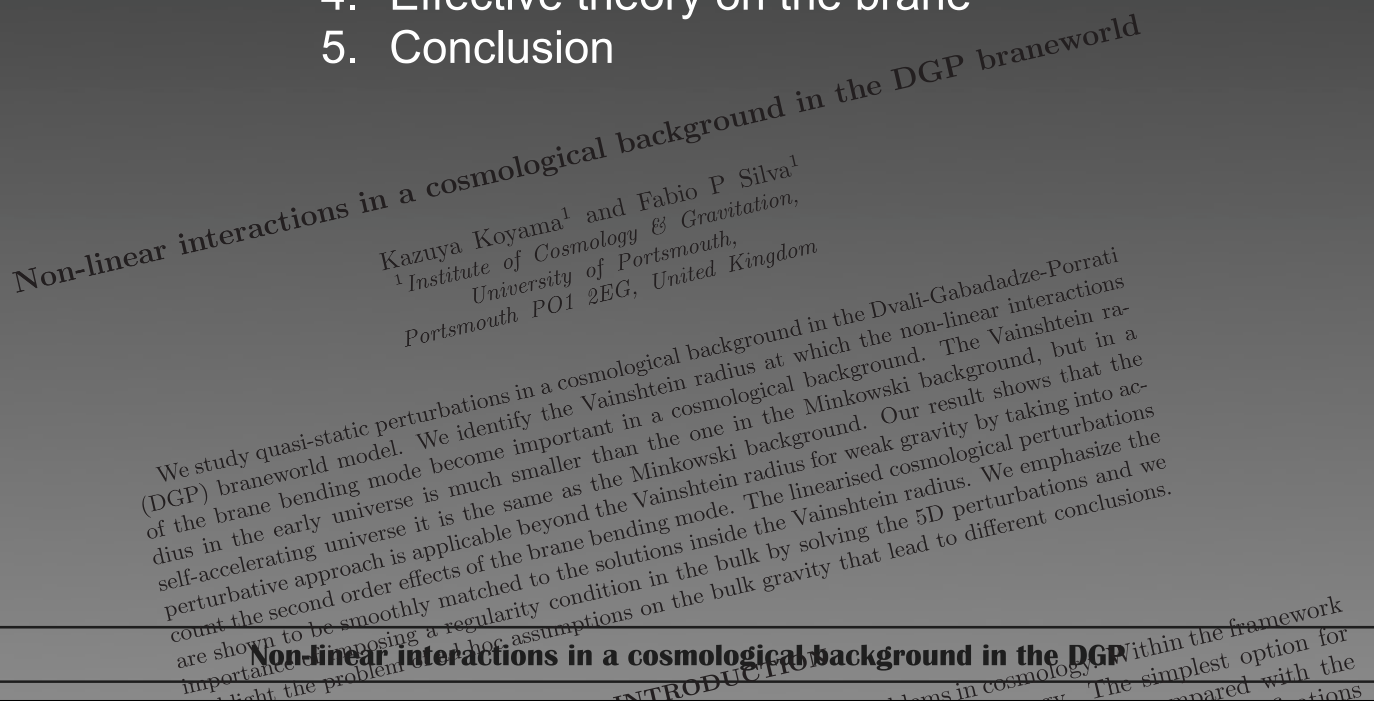
Non-linear interactions in a cosmological background in the DGP braneworld

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We study quasi-static perturbations in a cosmological background in the Dvali-Gabadadze-Porrati (DGP) braneworld model. We identify the Vainshtein radius at which the non-linear interactions of the brane bending mode become important in a cosmological background. The Vainshtein radius in the early universe is much smaller than the one in the Minkowski background, but in a self-accelerating universe it is the same as the Minkowski background. Our result shows that the perturbative approach is applicable beyond the Vainshtein radius for weak gravity by taking into account the second order effects of the brane bending mode. The linearised cosmological perturbations are shown to be smoothly matched to the solutions inside the Vainshtein radius. We emphasize the importance of imposing a regularity condition in the bulk by solving the 5D perturbations and we highlight the problem of ad hoc assumptions on the bulk gravity that lead to different conclusions.

The Dvali-Gabadadze-Porrati Model

4D Gravity on a Brane in 5D Minkowski Space, hep-th/0005016

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}^{(5)}R + \frac{1}{2\kappa_4^2}\int d^4x\sqrt{-\gamma}R + \int d^4x\sqrt{-\gamma}\mathcal{L}_m$$

$$r_c = \frac{\kappa^2}{2\kappa_4^2}$$

$$\oplus\frac{H}{r_c} = H^2 - \frac{\kappa_4^2}{3}\rho, \quad \dot{\rho} + 3H(\rho + p)\rho = 0$$

self-accelerating branch : $H_\infty \rightarrow \frac{1}{r_c}$

The Ghost in The Brane

Motivation

The self-accelerating solution contains a ghost.

↓

Space-time instability

Shown rigorously by studying linearized gravity.
Non-linear interactions might obscure this conclusion.

Arguments against the validity of the linearized analysis have been raised but only for perturbations around Minkowski space-time.

Thus it is important to study non-linear interactions in a cosmological background.

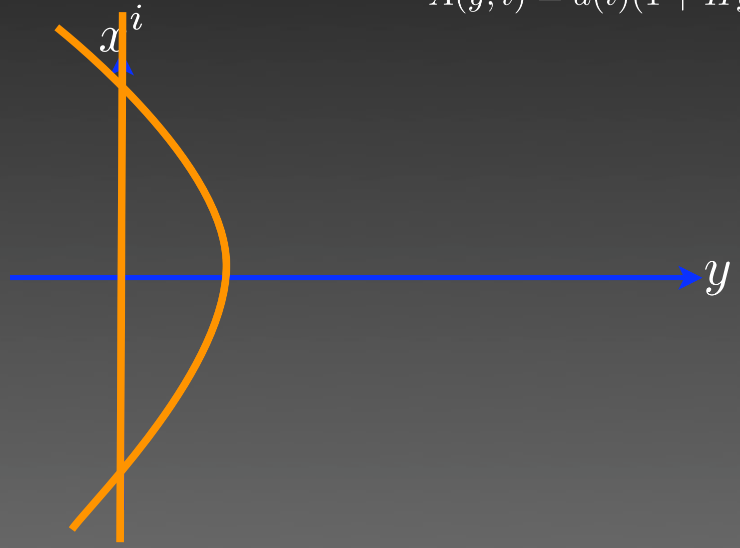
We did *not* solve the full non-linear problem.
We properly solved the linearised perturbations in 5D spacetime (weak-gravity) and we took into account the second order effects of the brane bending mode.

Quasi-Static Perturbations

The Line Element

$$ds^2 = -(1+2\mathcal{A})N(t,y)^2dt^2 + (1+2\mathcal{R})A(t,y)^2\delta_{ij}dx^idx^j + (1+2A_{yy})dy^2$$

$$A(y,t) = a(t)(1 \mp Hy), \quad N(y,t) = 1 \mp H\left(1+\frac{\dot{H}}{H^2}\right)y$$



$$ds^2 = -(1+2\Psi)N(t,y)^2dt^2 + (1+2\Phi)A(t,y)^2\delta_{ij}dx^idx^j + 2r_c\varphi_{,i}dx^idy + (1+2G)dy^2$$

brane-bending mode

Quasi-Static Perturbations
Junction Conditions

$$K_{\mu\nu} - K g_{\mu\nu} = -\frac{\kappa^2}{2} T_{\mu\nu} + r_c G_{\mu\nu}$$

$$K_{\mu\nu} = \frac{1}{2N} \left(\partial_y g_{\mu\nu} - \nabla_\mu N_\nu - \nabla_\nu N_\mu \right)$$

$$\begin{aligned} g_{\mu y} &\equiv N_\mu = g_{\mu\nu} N^\nu \\ g_{yy} &\equiv N^2 + g_{\mu\nu} N^\mu N^\nu \end{aligned}$$

(t,t) component :

$$\frac{2}{a^2} \nabla^2 \Phi = -\kappa_4^2 \delta\rho + \frac{1}{a^2} \nabla^2 \varphi - \frac{3}{r_c} \Phi'$$

spatial components :

$$\begin{aligned} \Phi + \Psi &= \varphi \\ \Psi' + 2\Phi' &= 0 \end{aligned}$$

Quasi-Static Perturbations
Solutions in the Bulk

Einstein Equations

$$\Psi'' + \frac{\nabla^2}{A^2}\Psi - \left(\frac{N'}{N}\right)\frac{r_c}{A^2}\nabla^2\varphi = 0$$

$$\mathcal{A} = \Psi - \frac{N'}{N} r_c \varphi = \left[c_1 (1 \mp Hy)^{\pm k/aH} + \cancel{c_2 (1 \mp Hy)^{\mp k/aH}}\right]$$

Regularity Condition

Quasi-Static Perturbations
Equations on the Brane

$$\frac{2}{a^2}\nabla^2\Phi = -\kappa_4^2\delta\rho + \frac{1}{a^2}\nabla^2\varphi$$
$$\Psi + \Phi = \varphi$$

Structure Formation

These Equations form the basis for the study of structure formation tests in this model.

$$3\beta(t)\frac{\nabla^2}{a^2}\varphi + \frac{r_c^2}{a^4}\left[(\nabla^2\varphi)^2 - (\nabla_i\nabla_j\varphi)^2\right] = \kappa_4^2\delta\rho$$

$$\beta(t) = 1 - \frac{2r_c}{3}\left(2\frac{A'}{A} + \frac{N'}{N}\right) = 1 \pm 2Hr_c\left(1 + \frac{\dot{H}}{3H^2}\right)$$

Solutions On The Brane

The Ghost

BD scalar has the wrong sign if $\omega < -3/2$

this means $\beta < 0$

which happens for H.rc > 1/2 (on the self accelerating branch)

which is the condition for the existence of the ghost

e.g. Koyama (2005)

Linearised solutions

$$\frac{\nabla^2}{a^2}\Phi = -\frac{\kappa_4^2}{2}\left(1-\frac{1}{3\beta}\right)\delta\rho$$

$$\frac{\nabla^2}{a^2}\Psi = \frac{\kappa_4^2}{2}\left(1+\frac{1}{3\beta}\right)\delta\rho$$

Brans-Dicke with $\omega = \frac{3}{2}(\beta - 1)$

Solutions On The Brane

The effects of non-linear interactions

TOO DIFFICULT

spherical symmetry

$$\frac{d\varphi}{dr} = \frac{r_g}{r^2}\Delta(r)$$

$$\Delta(r) = \frac{2}{3\beta}\left(\frac{r}{r_*}\right)^3\left(\sqrt{1+\left(\frac{r_*}{r}\right)^3}-1\right)$$

$$r_*^3 = \frac{8r_c^2 r_g}{9\beta^2}$$

Solutions On The Brane

$$\Phi = \frac{r_g}{2r} + \frac{\varphi}{2}$$
$$\Psi = -\frac{r_g}{2r} + \frac{\varphi}{2}$$

Early universe:

$\beta^2 >> 1$

$r_* << 1$

$\omega \longrightarrow \infty$

(we can trust the linearized solution down to small scales)

Self accelerating universe:

$\beta^2 = 1$ $r_* = r_V$

$r < r_*$

$$\Phi = \frac{r_g}{2r} + \frac{1}{\beta}\sqrt{\frac{\beta^2 r_g r}{2r_c^2}}$$
$$\Psi = -\frac{r_g}{2r} + \frac{1}{\beta}\sqrt{\frac{\beta^2 r_g r}{2r_c^2}}$$

Same Results

Lue & Starkman (2004)

Different Results

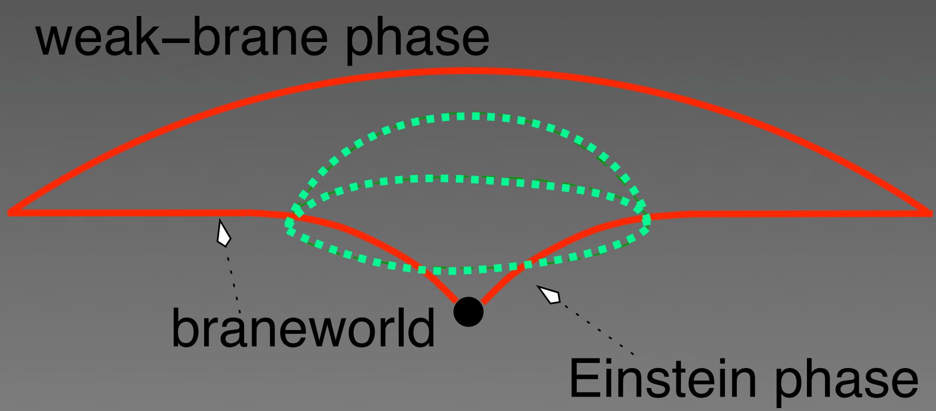
Gabadadze & Iglesias (2005) ?

$r > r_*$

$$\Phi = \frac{r_g}{2r}\left(1 - \frac{1}{3\beta}\right)$$
$$\Psi = -\frac{r_g}{2r}\left(1 + \frac{1}{3\beta}\right)$$

In Conclusion

Solutions On The Brane



In Conclusion

We studied quasi-static perturbations in a cosmological background in the DGP braneworld. Solving the bulk metric perturbations and imposing a regularity condition, we got a closed set of equations on the brane.

At linearised level the theory is decribed by the BD theory with $\omega = 3(\beta - 1)/2$

The non-linear interactions of the brane bending mode come into play at the Vainshtein radius, given by $r_*^3 = r_V^3/\beta^2$

On scales smaller than the Vainshtein radius the solution approaches 4D GR.

Our solutions agree with previous results both on a Minkowski and a Friedmann background.

We checked the consistency of our equations using effective equations on the brane.

On the Ghost problem:

Since we showed that the linearised analysis makes sense (for scales greater than the Vainshtein radius) we still find the ghost on the self-accelerated universe.

Usually one expects an instant instability of the spacetime in the presence of the ghost. However, in this case, it is not so obvious that the ghost leads to that classically or quantum mechanically. Furthermore, non-linear interactions of the brane bending mode would become important if such instabilities arise.

Work in Progress:
- numerically solve the full non-linear problem
- numerically find solutions for very large scales (with S. Seahra and A. Cardoso)

Appendix

Neglected Terms:

$\frac{aH}{k} << 1$ **sub-Horizon perturbations ($k >> aH$)**

∂_t **quasi-static approximation**

$\frac{A'}{A}\Phi'$ **negligible because** $\Phi' \sim k\Phi << \nabla^2\Phi \sim k^2\Phi$

$O(\varphi^n), \; n > 2$ **negligible because higher order terms are much smaller than the linear terms, being suppressed by Planck's Mass:**

$$O(\varphi^2) \sim (r_c \varphi_{,r})^2 \sim \frac{r_g}{r} \qquad \left(\frac{r_g}{r}\right)^2 \sim \frac{1}{M_p^4}$$

Appendix
Dissimilar Results

Gabadadze and Iglesias, hep-th/0407049

r* | rc | r

4D Einstein | **4D** assymptotically approaches 5D | **5D** Linearized Solution

$$M_{eff} \sim M \left(\frac{r_g}{r_c}\right)^{1/3}$$

BUT

ad hoc metric:

$$ds^2 = -\mathrm{e}^{-\lambda}dt^2 + \mathrm{e}^{\lambda}dr^2 + r^2 d\Omega^2 + \gamma\, dr dy + \mathrm{e}^{\sigma} dy^2$$

and they didn't solve the bulk

Work in Progress

Check if this solution violates the regularity condition in the bulk