Vector modes generated by primordial density fluctuations

Teresa Hui-Ching Lu,
Kishore Ananda and Chris Clarkson
University of Cape Town

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- Generation of tensor modes from scalar modes has been studied by various people, for example, Ananda et al (gr-qc/0612013) [talk given by K. Ananda on Wed], Baumann et al (hep-th/0703290).
- Second order vector modes generated from the same mechanism during radiation era is the focuss here. See Mollerach et al Phy. Rev. D **69** 0603002 (2004) for the dust era.

Calculation overview

The Metric

- The perturbed metric:

$$\bar{g}_{\mu\nu} = g_{\mu\nu} + \epsilon\delta^{(1)}g_{\mu\nu} + \epsilon^2\delta^{(2)}g_{\mu\nu}$$

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- Our metric:

$$\begin{aligned} \bar{g}_{00} &= -a^2\left(1+2\Phi\right), \\ \bar{g}_{0i} &= -\frac{1}{2}a^2 S_i, \\ \bar{g}_{ij} &= a^2\left(1-2\Phi\right)\gamma_{ij}, \end{aligned}$$

with a longitudinal gauge and perfect fluid background. Also, $\partial^i S_i = 0$.

Einstein Field Equations

- Trace reversed Einstein field equations:

$$\bar{R}_{\alpha\beta} = 8\pi G \left(\bar{T}_{\alpha\beta} - \frac{1}{2} \bar{g}_{\alpha\beta} \bar{T} \right) = 8\pi G \bar{\Upsilon}_{\alpha\beta}.$$

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- Ricci tensor:

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- Ricci tensor:

$$\delta^{(2)}R_{ij} = \delta^{(2)}R^{\mathcal{V}}_{ij} + \delta^{(2)}R^{(S,S)}_{ij}.$$

- Trace reversed energy momentum tensor:

$$\delta^{(2)}\Upsilon_{ij} = -a^2(1-w)\Phi\delta^{(1)}\rho\gamma_{ij} + a^2(1+w)\rho\left(\partial_i v_{(1)}\right)\left(\partial_j v_{(1)}\right)$$

where $\partial_i v_{(1)}$ is the scalar part of the first order 3-velocity.

Mode extraction

- Extraction of the evolution equation:

$$\hat{\mathcal{V}}_i^{lm}\left(\partial_{(l}S'_{m)} + 2\mathcal{H}\partial_{(l}S_{m)}\right) = 2\hat{\mathcal{V}}_i^{lm}\Sigma_{lm}.$$

$$\begin{aligned}\hat{\mathcal{V}}_i^{lm} &= -\frac{2i}{(2\pi)^3}\int d^3\mathbf{k}'\ k'^{-2}\int d^3\mathbf{x}'\ k'^{l}\ e^{i\mathbf{k}'\cdot(\mathbf{x}-\mathbf{x}')} \\ &\times \left[e_i(\mathbf{k}')e^m(\mathbf{k}') + \bar{e}_i(\mathbf{k}')\bar{e}^m(\mathbf{k}')\right],\end{aligned}$$

$$\Sigma_{lm} = \delta^{(2)}R_{lm}^{(S,S)} + \delta^{(2)}\Upsilon_{lm}.$$

Second order vectors from scalars

First order scalar modes - I

- Equation of motion in Fourier space:

$$\Phi''(\mathbf{k},\eta)+3\mathcal{H}(1+c_s^2)\Phi'(\mathbf{k},\eta)+c_s^2k^2\Phi(\mathbf{k},\eta)=0.$$

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- Radiation era ($c_s^2 = w = 1/3$):

$$\Phi_r(\mathbf{k},\eta) = \frac{A_r(\mathbf{k})}{(k\eta)^3}\left[\sin\left(\frac{k\eta}{\sqrt{3}}\right) - \frac{k\eta}{\sqrt{3}}\cos\left(\frac{k\eta}{\sqrt{3}}\right)\right].$$

Modes oscillate after entering the Hubble radius ($k\eta = 1$).

First order scalar modes - II

- Power spectrum defined as:

$$\langle \Phi^*(\mathbf{k}_1,\eta)\Phi(\mathbf{k}_2,\eta)\rangle = \frac{2\pi^2}{k^3}\delta^3(\mathbf{k}_1-\mathbf{k}_2)\mathcal{P}_\Phi(k,\eta).$$

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- Relating Φ to curvature perturbation $\mathcal{R}$ can get amplitude on large scales:

$$A_r(k)^2 \approx \frac{216\pi^2}{k^3}\Delta^2_{\mathcal{R}}(k).$$

$\Delta^2_{\mathcal{R}}(k)$ is the power spectrum for the curvature perturbation $\mathcal{R}$.

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- $\Delta^2_{\mathcal{R}} \approx 2.4 \times 10^{-9}$ at a scale $k_{CMB} = 0.002\mathrm{Mpc}^{-1}$ (Spergel et al., astro-ph/0603449).

Second order vector modes

- Power spectrum of the induced vector mode $\mathcal{P}_{\mathcal{V}}(k,\eta)$ for radiation era:

$$\mathcal{P}_{\mathcal{V}}(k,\eta) = \frac{(243)^2}{4(k\eta)^4}\int_0^\infty dv \int_{|v-1|}^{v+1} du\, \mathcal{P}_\Phi(ku)\mathcal{P}_\Phi(kv)\mathcal{F}(u,v,x).$$

$\mathcal{F}(u,v,x)$ contains two time integrals. v is radial coordinate in k-space, u is angular coordinate and dimensionless variable $x = k\eta$.

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- 2nd order vector modes can only be generated from interaction of scalar modes.
- Choose scalar power spectrum of the form:

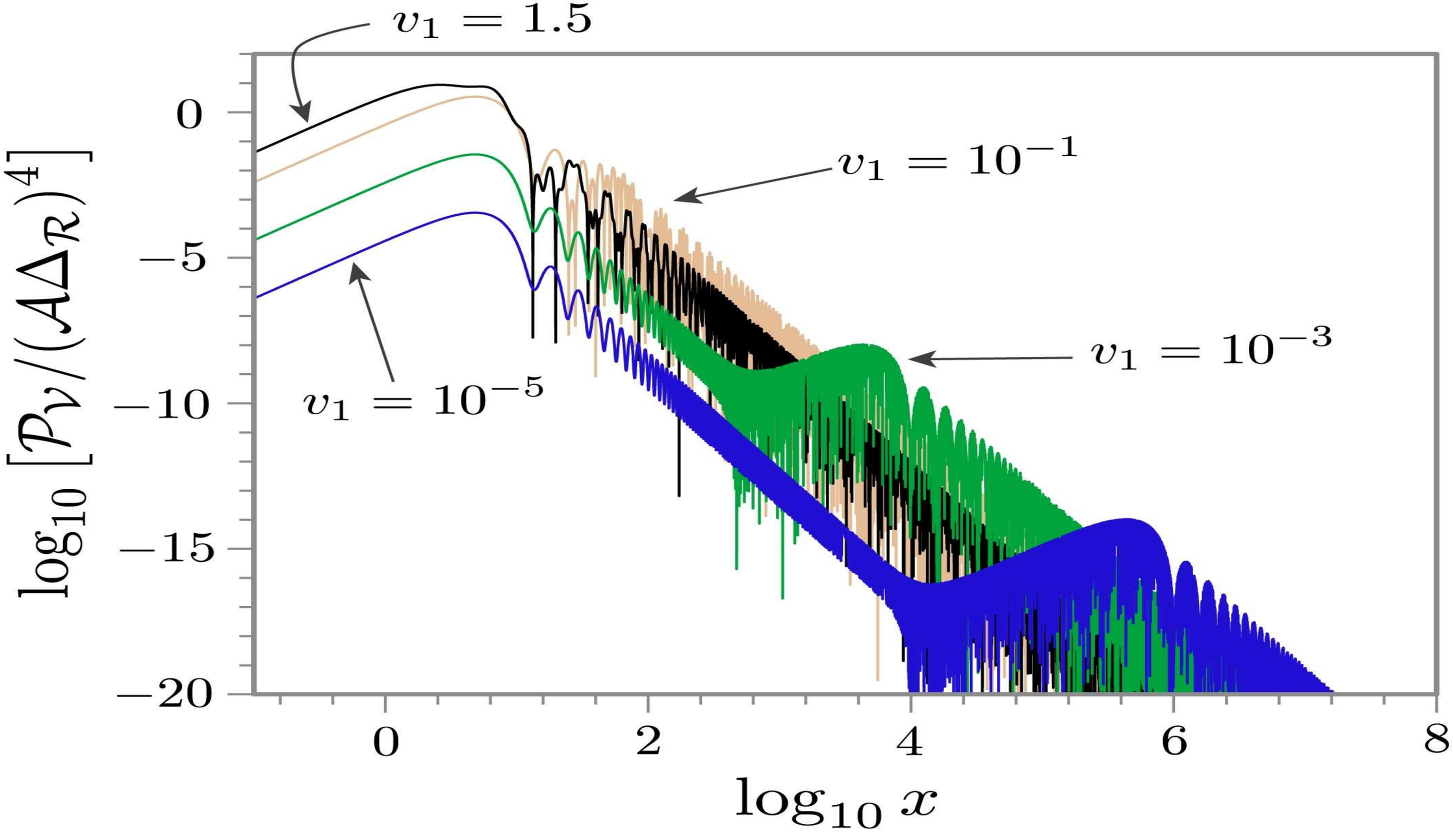
$$\mathcal{P}_\Phi(k) = \frac{4}{9}\mathcal{A}^2\Delta^2_{\mathcal{R}}(k_{CMB})\left\{\delta[\ln(k_1/k)] + \delta[\ln(k_2/k)]\right\},$$

∗ $\mathcal{A}$ is the amplitude relative to the measured spectrum.
∗ Vector modes produced in range

$$k_1 + k_2 > k > |k_1 - k_2|.$$

Vector modes generation - II

- Evolution of modes of wavenumber $k = k_2$. $v_1 = k_1/k$ must satisfy $0 < v_1 < 2$.



Outside Hubble radius modes grow $\sim \eta^2$. Inside Hubble radius modes decay $\sim \eta^{-4}$.

Knees produced at later time as long mode enter Hubble radius at $x \sim 1/v_1$.

Power law scalar modes

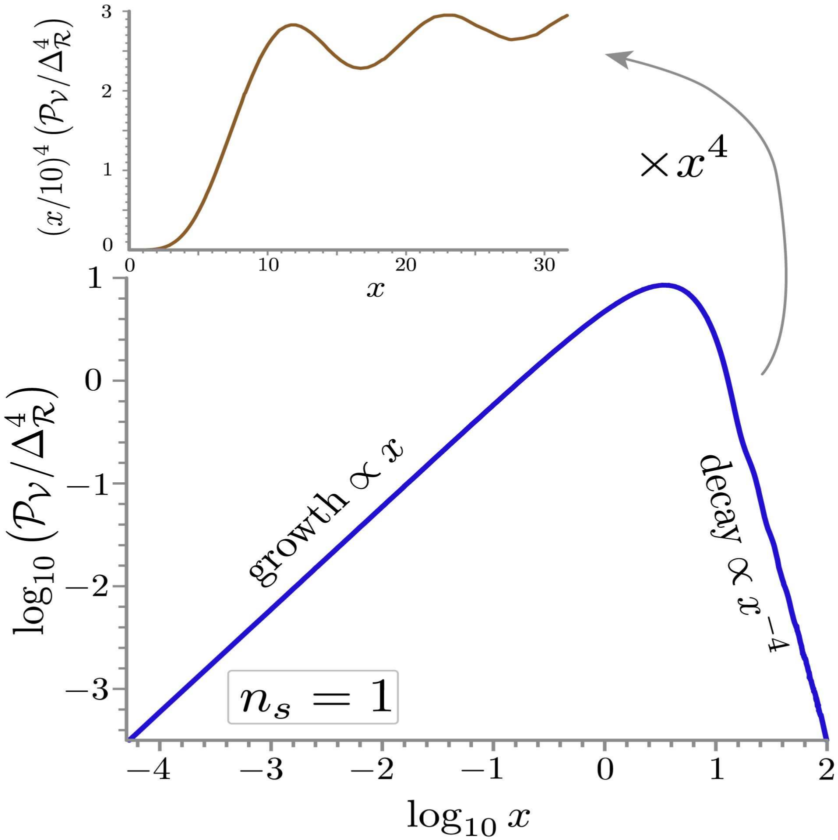
- Input power law scalar mode spectrum:

$$\mathcal{P}_{\Phi}(k) = \frac{4}{9}\Delta^2_{\mathcal{R}} \left(\frac{k}{k_{CMB}}\right)^{n_s-1} .$$

n_s is the index tells us the tilt of the spectrum relative to scale invariance $n_s = 1$.

Vector modes from power law scalar modes

- Two ways of reading this picture:
  - $*$ Fix k, then x is viewed as time η.
  - $*$ Fix η, then x is viewed as k.



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- A scalar mode produces vector modes as it enters the Hubble radius by interaction with scalar modes of differing wavelength.
- Given that linear order vector modes decay, these are the dominate vector modes during radiation era.
- THE END!!!