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Analyzing WMAP Observation by Quantum Gravity

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with Shinichi Horata, Naoshi Sugiyama, and Tetsuyuki Yukawa

arXiv:0705.3490[astro-ph],
Phys. Rev. D74 (2006) 123502, astro-ph/0607586
and
“Focus on Quantum Gravity Research”
(Nova Science Publisher, NY, 2006), Chap.1

Motivation

WMAP has established the inflationary scenario of the universe.
Various cosmological parameters have been determined precisely.

But, basic problems are remained:

- **What is the inflaton field?**
- **What is the inflaton potential?**

WMAP determined the initial conditions for cosmological perturbation theory.

But, its origin is not understood yet.

The aim of this talk is
to show an inflationary scenario of quantum gravity origin
consistent with WMAP observation
without introducing any artificial field.

The Model of Quantum Gravity

Our model is based on 3 fundamental conditions:

- quantum diffeomorphism invariance
  (=conformal invariance)
- finiteness
  (=renormalizability, no BH singularity)
- 4 space-time dimensions

➔**These restrict gravitational action.**

Renormalizable quantum gravity

$$I = \int d^4x\sqrt{-g}\left\{-\frac{1}{t^2}C^2_{\mu\nu\lambda\sigma} - bG_4 + \frac{m^2_{\text{pl}}}{16\pi}R - \Lambda\right\} + I_{\text{Matter}}$$

dimensionless

Weyl action Euler density

Conformal inv. actions

R^2 is forbidden by Wess-Zumino condition.

Perturbation about conformal flat ($C_{\mu\nu\lambda\sigma} = 0$):

$$g_{\mu\nu} = e^{2\phi}\left(\hat{g}_{\mu\nu} + t h_{\mu\nu} + \cdots\right), \quad tr(h) = 0$$

Conformal mode (non-perturbative)

Traceless tensor mode (perturbative)

Non-perturbative formulation of QG

recent development and essential point! K.H., hep-th/0203260

Dynamics of Weyl action (traceless mode)

Asymptotic Freedom (AF)

$$t_r^2(p) = \frac{1}{\beta_0 \log(p^2/\underline{\underline{\Lambda_{QG}^2}})}, \qquad \beta = -\beta_0 t_r^3$$

Consequence of AF 1

New dynamical scale Λ_{QG} ⇨ space-time phase transition from quantum to classical

Consequence of AF 2

At very high energies ($t_r \to 0$), $\underline{\underline{C_{\mu\nu\lambda\sigma} \to 0}}$

(cf. $F^a_{\mu\nu} \to 0 \ (g \to 0)$ in QCD)

⬇

Singularity with divergent Riemann curvature is excluded quantum mechanically

➔toward resolution of information loss problem!

Consequence of AF 3

In very early universe,
fluctuations of conformal mode become dominant
→ Exact Conformal Symmetry

Initial fluctuations are scalar-like and scale-invariant
&
Tensor mode is small

Agreement with the observation

Inflation induced by quantum gravity

$$Z = \int [dg\cdots]_{\underline{g}} \exp(iI)$$

$$= \int [d\phi dh\cdots]_{\underline{\hat{g}}} \exp(\underline{iS(\phi)} + iI)$$

Starobinsky 1980,
K.H. and Yukawa
[astro-ph/0401070]

Jacobian = Wess-Zumino action

Dynamics of conformal mode is induced from the measure

$$S(\phi) = -\frac{b_1}{(4\pi)^2}\int d^4x\sqrt{-\hat{g}}\left\{\underline{2\phi\hat{\Delta}_4\phi} + \left(G_4 - \frac{2}{3}\hat{\nabla}^2\hat{R}\right)\phi\right\} + O(\phi^3)$$

Kinetic term

higher order of t_r^2

Conformal Field Theory (CFT) at $t_r^2 \to 0$

t_r^2 **measures a deviation from CFT**

(exact) $0 < t_r^2 < \infty$ (broken)

Planck mass $m_{\rm pl}$ >> Dynamical scale $\Lambda_{\rm QG}$

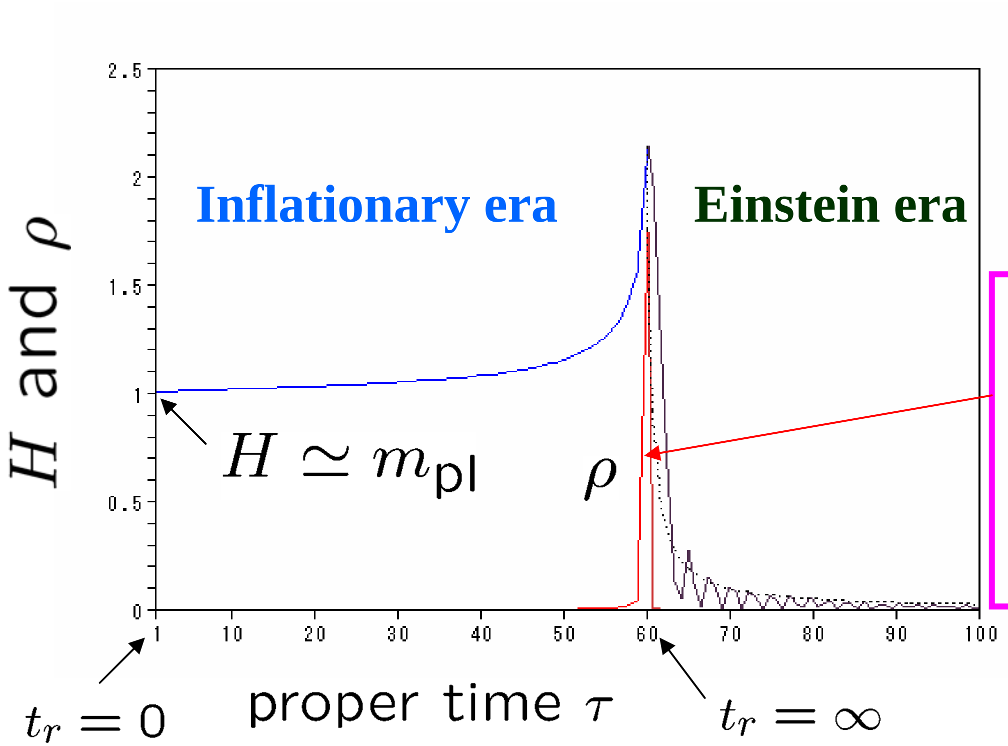
Inflation starts at the Planck scale and ends at the dynamical scale

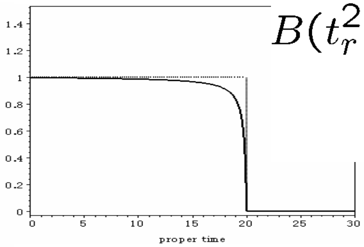
Wess-Zumino action

Einstein action

$$\begin{cases} b_1 B(\tau)\left(\dddot{H} + 7H\ddot{H} + 4\dot{H}^2 + 18H^2\dot{H} + 6H^4\right) - 3\pi m_{\rm pl}^2\left(\dot{H} + 2H^2\right) = 0 \\ b_1 B(\tau)\left(2H\ddot{H} - \dot{H}^2 + 6H^2\dot{H} + 3H^4\right) - 3\pi m_{\rm pl}^2 H^2 + 8\pi^2\rho = 0 \end{cases}$$

dynamical factor





$$B(t_r^2) = 1 - a_1 t_r^2(p) + \cdots = \frac{1}{1 + a_1 t_r^2(\tau)}$$

$(p \to 1/\tau)$

Big Bang $(t_r = \infty)$**:**

extra degrees of freedom in higher-derivative gravitational fields shift to matter fields ρ .

$$H = \dot{a}(\tau)/a(\tau) = \dot{\phi}(\tau)$$

Evolutional scenario

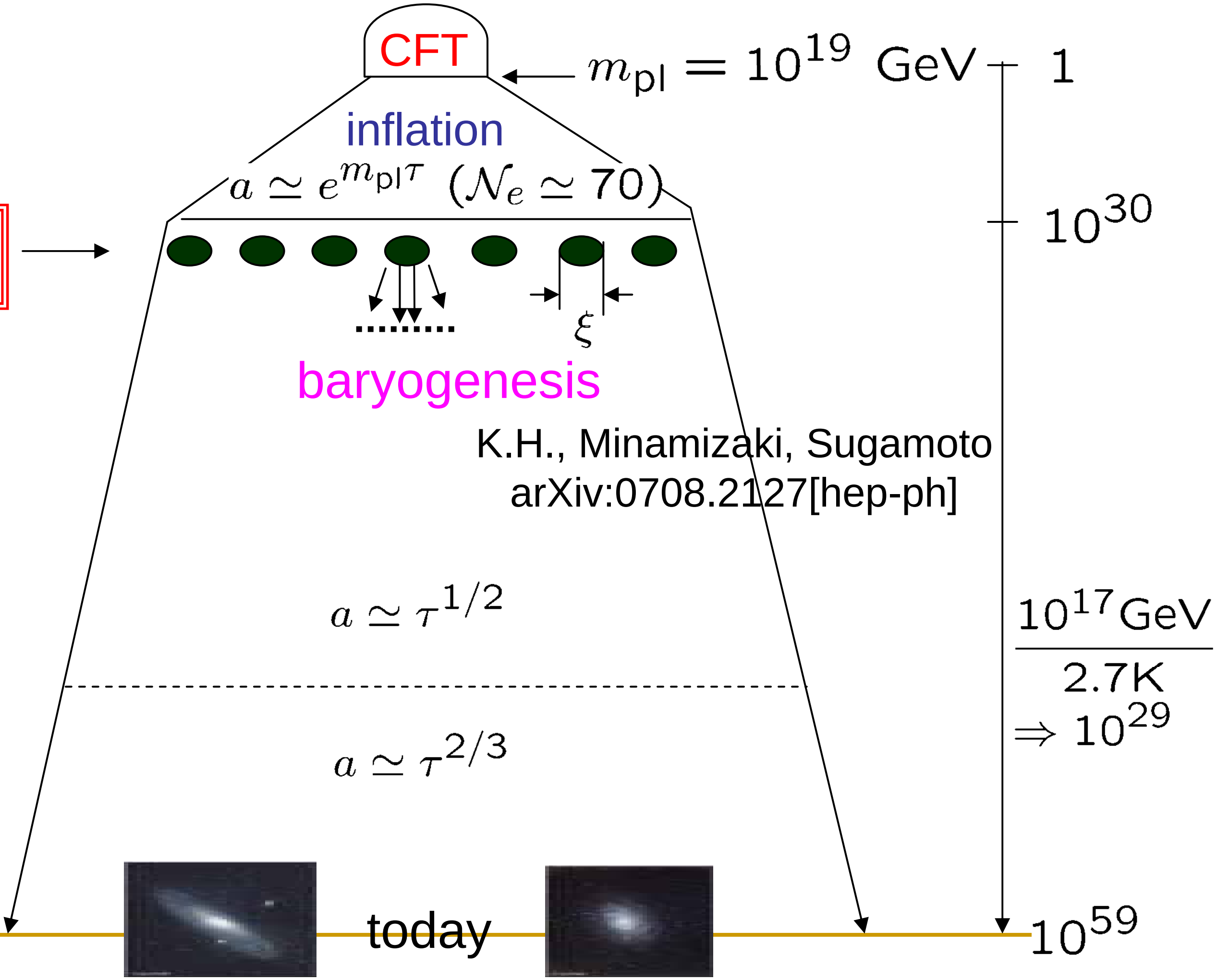
Number of e-foldings

$$\mathcal{N}_e = \log \frac{a(\tau_\Lambda)}{a(\tau_{\rm pl})} \simeq \frac{m_{\rm pl}}{\Lambda_{\rm QG}}$$

➔ $\boxed{\Lambda_{\rm QG} \simeq 10^{17}{\rm GeV}}$

correlation length:

$$\xi = 1/\Lambda_{\rm QG}$$



K.H., Minamizaki, Sugamoto
arXiv:0708.2127[hep-ph]

**Planck length
at Planck time
grows up to
the Hubble distance
today**

Calculation of CMB Multipoles

The evolution of scalar curvature fluctuation

$$\left.\frac{\delta R}{R}\right|_{\tau_{\mathrm{pl}}} \Longrightarrow \left.\frac{\delta R}{R}\right|_{\tau_\Lambda^-} \Longrightarrow \left.\frac{\delta \rho}{\rho}\right|_{\tau_\Lambda^+} \Longrightarrow \left.\frac{\delta T}{T}\right|_{\mathrm{today}}$$

(CFT)

Solve evolution equations on the inflation

Big Bang

Solve cosmological perturbation theory (CMBFAST)

gives the initial conditions of CMBFAST

Simple estimation of the amplitude

$$\frac{\delta R}{R} \sim \frac{E^2}{12 m_{\mathrm{pl}}^2}$$

At the big bang $E \sim \Lambda_{\mathrm{QG}}$ ➔ $\left.\frac{\delta R}{R}\right|_{\tau_\Lambda} \sim 10^{-5}$

de Sitter curvature

Linear perturbation is applicable for $m_{\mathrm{pl}} < E < \Lambda_{\mathrm{QG}}$

Scalar perturbations

Gauge invariant variables: $\underbrace{\Phi, \quad \Psi}_{\text{gravitational potentials}}, \quad (\underbrace{\delta\rho, \quad v}_{\text{determined by gravitational potentials}})$

Scalar equation

$$
\begin{aligned}
&\frac{b_1}{8\pi^2} B_0(\tau) \Big\{ -2\partial_\eta^4 \Phi - 2\partial_\eta \phi \partial_\eta^3 \Phi + \left(-8\partial_\eta^2 \phi + \frac{10}{3} \not{\partial}^2 \right) \partial_\eta^2 \Phi \\
&\qquad + \left(-12\partial_\eta^3 \phi + \frac{10}{3} \partial_\eta \phi \not{\partial}^2 \right) \partial_\eta \Phi + \left(\frac{16}{3} \partial_\eta^2 \phi - \frac{4}{3} \not{\partial}^2 \right) \not{\partial}^2 \Phi \\
&\qquad + 2\partial_\eta \phi \partial_\eta^3 \Psi + \left(8\partial_\eta^2 \phi + \frac{2}{3} \not{\partial}^2 \right) \partial_\eta^2 \Psi + \left(12\partial_\eta^3 \phi - \frac{10}{3} \partial_\eta \phi \not{\partial}^2 \right) \partial_\eta \Psi \\
&\qquad + \left(-\frac{16}{3} \partial_\eta^2 \phi - \frac{2}{3} \not{\partial}^2 \right) \not{\partial}^2 \Psi \Big\} \\
&+ M_{\rm P}^2 e^{2\phi} \Big\{ 6\partial_\eta^2 \Phi + 18\partial_\eta \phi \partial_\eta \Phi - 4\not{\partial}^2 \Phi - 6\partial_\eta \phi \partial_\eta \Psi \\
&\qquad\qquad + \left(12\partial_\eta^2 \phi + 12\partial_\eta \phi \partial_\eta \phi - 2\not{\partial}^2 \right) \Psi \Big\} = 0.
\end{aligned}
$$

Constraint equation

$$
\begin{aligned}
&\frac{b_1}{8\pi^2} B_0(\tau) \Big\{ \frac{4}{3} \partial_\eta^2 \Phi + 4\partial_\eta \phi \partial_\eta \Phi + \left(\frac{28}{3} \partial_\eta^2 \phi - \frac{8}{3} \partial_\eta \phi \partial_\eta \phi - \frac{8}{9} \not{\partial}^2 \right) \Phi \\
&\qquad\qquad - \frac{4}{3} \partial_\eta \phi \partial_\eta \Psi + \left(-\frac{4}{3} \partial_\eta^2 \phi + \frac{8}{3} \partial_\eta \phi \partial_\eta \phi - \frac{4}{9} \not{\partial}^2 \right) \Psi \Big\} \\
&+ \frac{2}{t_r^2(\tau)} \left\{ 4\partial_\eta^2 \Phi - \frac{4}{3} \not{\partial}^2 \Phi - 4\partial_\eta^2 \Psi + \frac{4}{3} \not{\partial}^2 \Psi \right\} \\
&+ M_{\rm P}^2 e^{2\phi} \left\{ -2\Phi - 2\Psi \right\} = 0.
\end{aligned}
$$

$$
\left\{
\begin{array}{ll}
\text{initially } (t_r = 0) & \Phi = \Psi \\
\text{finally } (t_r = \infty) & \Phi = -\Psi
\end{array}
\right.
$$

Spectrum of quantum gravity (2-pt. function)

Initial QG spectrum at Planck time
= CFT spectrum (scale invariant)

$$P_S(k) = A\left(\frac{k}{m}\right)^{n_s-1}$$

Scalar spectral index

coeff. of Wess-Zumino action
(dependent on matter contents)

$$n_s = 5-8\frac{1-\sqrt{1-2/b_1}}{1-\sqrt{1-4/b_1}} = 1+2/b_1+4/b_1^2+O(1/b_1^3) \qquad b_1 > 4$$

HZ spectrum

Size of fluctuation we consider
is Planck length at Planck time

➔at the transition point, the size is much more extended than the correlation length ξ

➔not disturbed by the dynamics of transition

$$m = a(\tau_{\mathrm{pl}})m_{\mathrm{pl}}$$

comoving Planck const.

$$m = 0.05\ \mathrm{Mpc}^{-1}$$

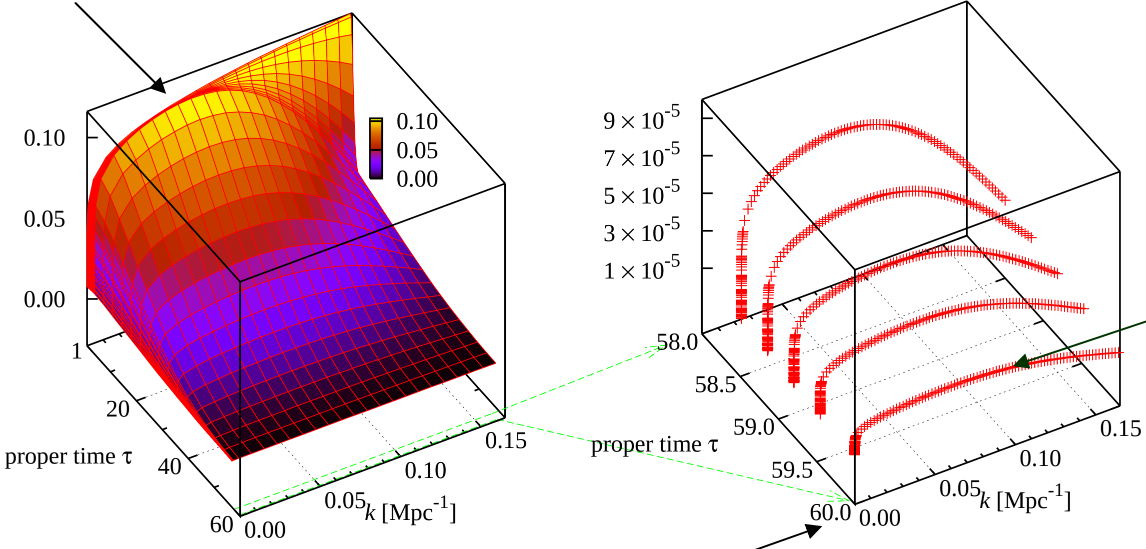
➔ $a(\tau_{\mathrm{pl}}) \simeq 10^{-59}$

consistent with the evolutional scenario

We can see the Planck scale phenomena directly!

Initial CFT spectrum

Bardeen Potential $\Phi(b_1=10, m=0.05)$



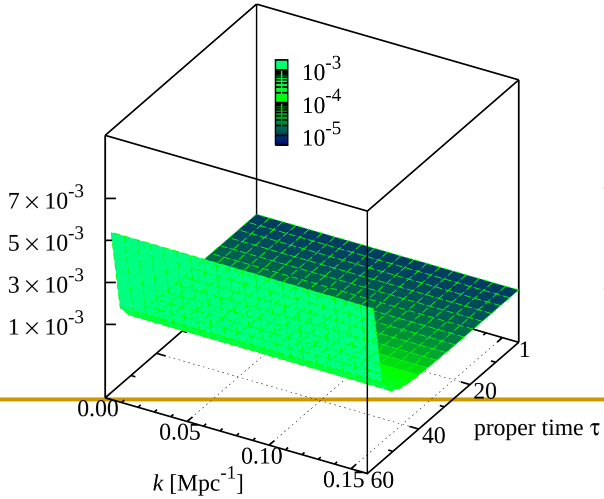
Scalar fluctuation (Bardeen potential) gradually decreases during inflation

Primordial scalar spectrum (red tilt for k > m)

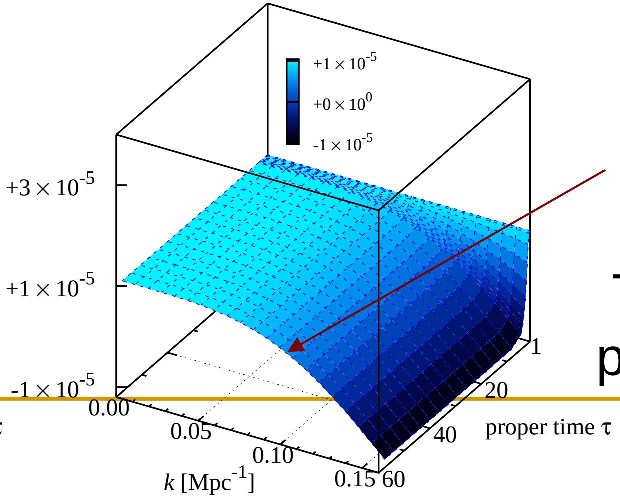
Big Bang

Initial conditions for CMBFAST

Vector Perturbation $(b_1=10, m=0.05)$



Tensor Perturbation $(b_1=10, m=0.05)$



Primordial tensor spectrum

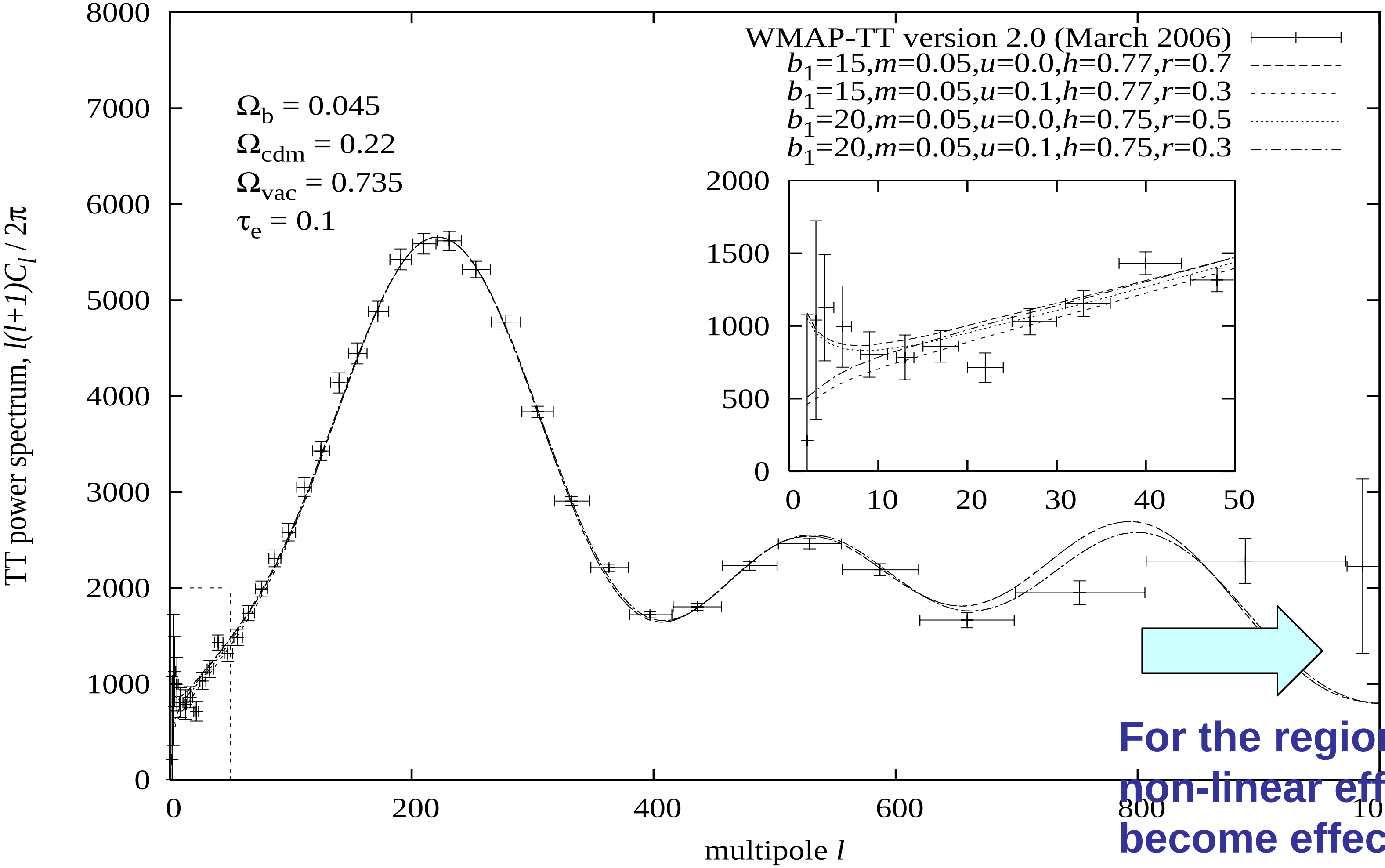
Tensor fluctuation is preserved to be small

CMB Multipoles

low-multipole damping factor

$$P(k) \to P(k)\frac{k^2}{k^2 + u\lambda^2}$$

$$\lambda = a(\tau_{\mathrm{pl}})\Lambda_{\mathrm{QG}}$$



For the region $k > 2m$ non-linear effects (CFT) become effective.

(in progress)

cosmological parameters adjusted properly

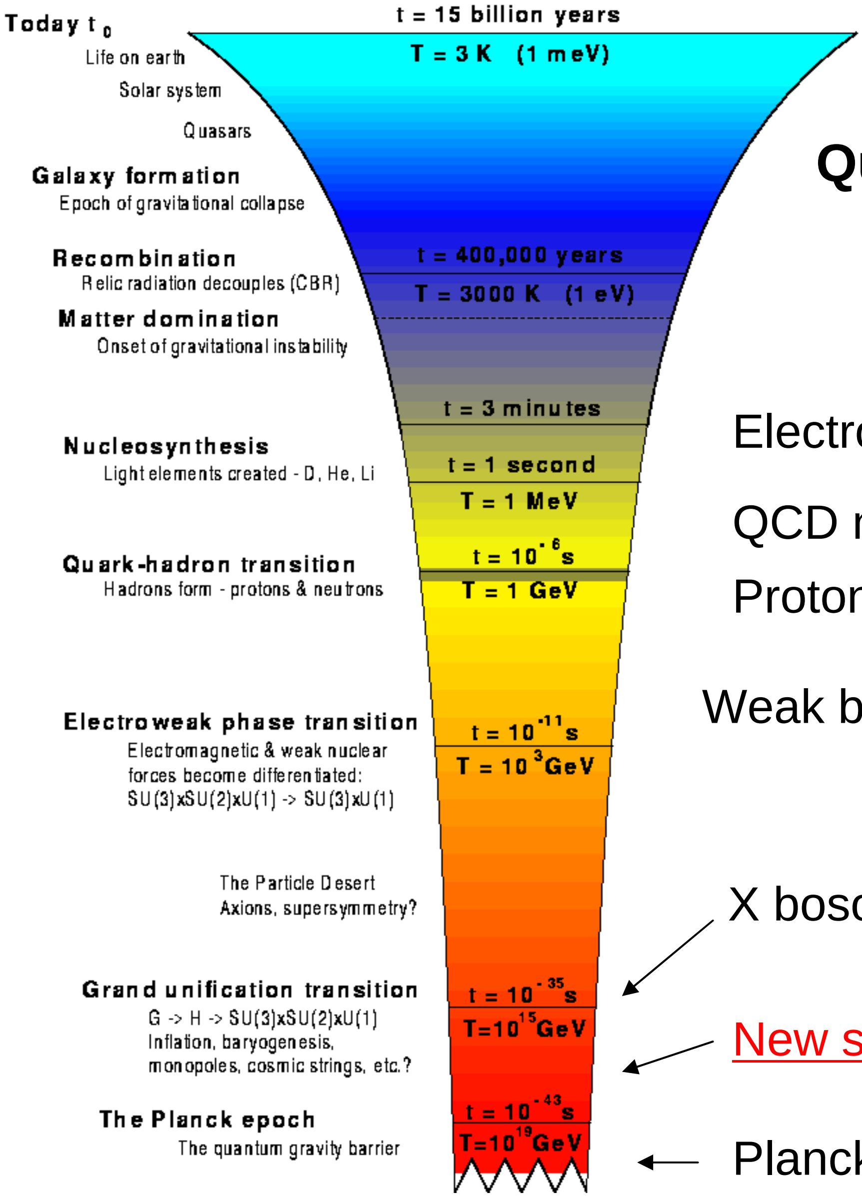
Summary

- **Asymptotic freedom of traceless tensor mode**
  - ➔indicates the existence of novel dynamical scale:

$$\Lambda_{\rm QG} \simeq 10^{17}{\rm GeV}$$

  - ➔space-time phase transition (=big bang) at this scale.
- **Repulsive force in quantum gravity**
  - ➔induces inflation.
  - ➔number of e-foldins is given by $\mathcal{N}_e \simeq m_{\rm pl}/\Lambda_{\rm QG}$
- **Quantum gravity spectrum**
  - ➔given by conformal field theory (=non-perturbative formulation of quantum gravity).
  - ➔scalar fluctuation decreases during inflation and the amplitude at the big bang is estimated as $\delta R/R \sim \Lambda_{\rm QG}^2/12m_{\rm pl}^2$
  - ➔CMB spectrum is consistent with WMAP.

Scales in the history of universe



Quantum gravity is a real physical target in 21 century

Electron mass $m_e = 0.5\ \text{MeV}$ (QED)

QCD mass scale $\Lambda_{\text{QCD}} \simeq 300\text{MeV}$

Proton mass $m_P = 1\ \text{GeV}$

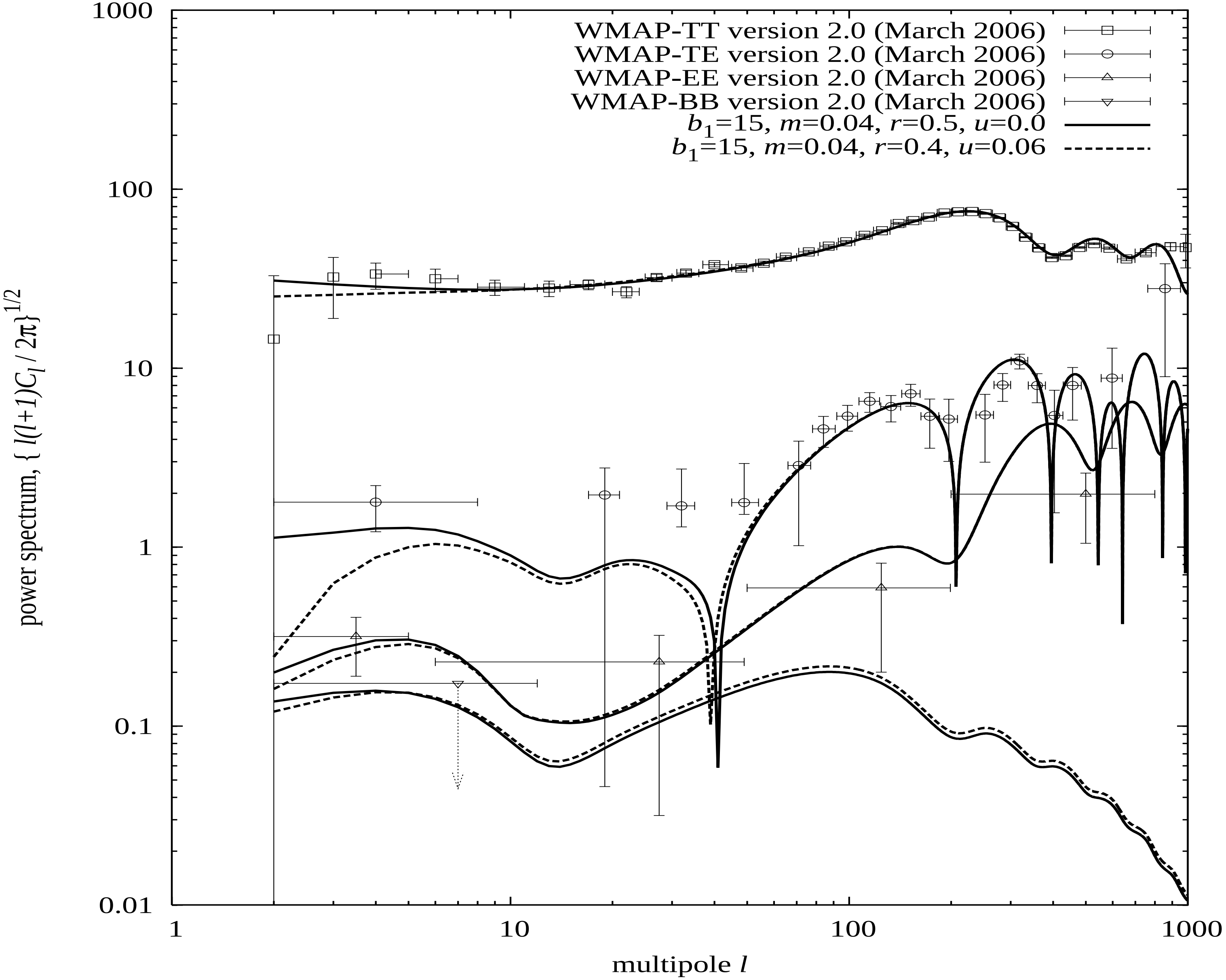
Weak boson mass $m_W = 80\ \text{GeV}$ (EW theory)

X boson mass $m_X = 10^{16}\ \text{GeV}$ (GUT)

New scale $\Lambda_{\text{QG}} \simeq 10^{17}\text{GeV}$ (Quantum Gravity)

Planck mass $m_{\text{pl}} = 10^{19}\ \text{GeV}$

Appendix



WMAP-TT version 2.0 (March 2006)
WMAP-TE version 2.0 (March 2006)
WMAP-EE version 2.0 (March 2006)
WMAP-BB version 2.0 (March 2006)
b_1=15, m=0.04, r=0.5, u=0.0
b_1=15, m=0.04, r=0.4, u=0.06
power spectrum, $\{ l(l+1)C_l / 2\pi\}^{1/2}$
multipole l
1000
100
10
1
0.1
0.01
1
10
100
1000

Tensor and Vector perturbations

Gauge invariant variables: $h_{ij}^{\mathrm{TT}}, \ \Upsilon_i$

Tensor equation

$$
\begin{aligned}
&-\frac{2}{t_\tau^2(\tau)}\left\{\partial_\eta^4 h_{ij}^{\mathrm{TT}} - 2\not\partial^2\partial_\eta^2 h_{ij}^{\mathrm{TT}} + \not\partial^4 h_{ij}^{\mathrm{TT}}\right\} \\
&+\frac{b_1}{8\pi^2}B_0(\tau)\Bigg\{\left(\frac{1}{3}\partial_\eta^2\phi + \frac{4}{3}\partial_\eta\phi\partial_\eta\phi\right)\partial_\eta^2 h_{ij}^{\mathrm{TT}} + \left(\frac{1}{3}\partial_\eta^3\phi + \frac{8}{3}\partial_\eta^2\phi\partial_\eta\phi\right)\partial_\eta h_{ij}^{\mathrm{TT}} \\
&\qquad\qquad + \left(-\frac{7}{3}\partial_\eta^2\phi + \frac{2}{3}\partial_\eta\phi\partial_\eta\phi\right)\not\partial^2 h_{ij}^{\mathrm{TT}}\Bigg\} \\
&+M_{\mathrm{P}}^2 e^{2\phi}\left\{-\frac{1}{2}\partial_\eta^2 h_{ij}^{\mathrm{TT}} - \partial_\eta\phi\partial_\eta h_{ij}^{\mathrm{TT}} + \frac{1}{2}\not\partial^2 h_{ij}^{\mathrm{TT}}\right\} = 0.
\end{aligned}
$$

Vector equation

$$
\begin{aligned}
&\frac{2}{t_\tau^2(\tau)}\left\{\partial_\eta^3\Upsilon_i - \partial_\eta\not\partial^2\Upsilon_i\right\} \\
&-\frac{b_1}{8\pi^2}B_0(\tau)\left\{\left(\frac{1}{3}\partial_\eta^2\phi + \frac{4}{3}\partial_\eta\phi\partial_\eta\phi\right)\partial_\eta\Upsilon_i + \left(\frac{1}{3}\partial_\eta^3\phi + \frac{8}{3}\partial_\eta^2\phi\partial_\eta\phi\right)\Upsilon_i\right\} \\
&+M_{\mathrm{P}}^2 e^{2\phi}\left\{\frac{1}{2}\partial_\eta\Upsilon_i + \partial_\eta\phi\Upsilon_i\right\} = 0.
\end{aligned}
$$

Running coupling constant $\beta_t = -\beta_0 t_r^3$

[asymptotic freedom]

$$\begin{aligned} \mathcal{L}_{\rm eff} &= -\left\{ \frac{1}{t_r^2} - 2\beta_0\phi + \beta_0 \log\left(\frac{k^2}{\mu^2}\right) + \cdots \right\} C^2_{\mu\nu\lambda\sigma} \\ &= -\frac{1}{t_r^2(p)} C^2_{\mu\nu\lambda\sigma} \end{aligned}$$

k :comoving momentum defined on $\hat{g}_{\mu\nu} = \eta_{\mu\nu}$

where

$$t_r^2(p) = \frac{1}{\beta_0 \log(p^2/\Lambda^2_{\rm QG})},$$

Physical momentum : $p = k/a$ with $a = e^{\phi}$

Dynamical scale : $\Lambda_{\rm QG} = \mu e^{-1/2\beta_0 t_r^2}$

Conformal mode increasing => running coupling getting large!

Einstein phase ($E < \Lambda_{\rm QG}$)

Low energy effective action (derivative expansion)

$$I_{\rm low} = \int d^4x \sqrt{-g}\,\{\mathcal{L}_2 + \mathcal{L}_4 + \cdots\}$$

tree + 1-loop

tree

$$\mathcal{L}_2 = \frac{M_{\rm P}^2}{2} R + \mathcal{L}_2^{\rm Matter}$$

cf. chiral perturbation theory

Here, we restrict effective action up to the fourth order, and thus using lowest Einstein's equation $M_{\rm P}^2 R_{\mu\nu} = T_{\mu\nu}^{\rm Matter}$, a variety of four-derivative actions is reduced, which is merely given by

$$\mathcal{L}_4 = \frac{\alpha}{(4\pi)^2} R^{\mu\nu} R_{\mu\nu}$$

with running effect

$$\alpha(E) = \alpha_0 + \zeta \log(E^2/\Lambda_{\rm QG}^2)$$

$\alpha_0 \ (> 0)$

:phenomenologically determined

$$\zeta = \frac{N_{\rm X}}{120} + \frac{N_{\rm A}}{10} + \cdots \ (> 0)$$

Higher-derivative terms are irrelevant!

Wheeler-DeWitt Equations of Conformal Algebra

Conformal algebra and Physical states (on cylinder $R \times S^3$):

$$[Q_M, Q_N^\dagger] = 2\delta_{MN} H + 2R_{MN}$$

Antoniadis-Mazur-Mottola
Horata-K.H., K.H.

special conf. transfs.   Hamiltonian   rotation on S^3

M, N = vector index of SO(4)

$$Q_M|\mathrm{phys}\rangle = H|\mathrm{phys}\rangle = R_{MN}|\mathrm{phys}\rangle = 0$$

Conformal inv. vacuum = physical state satisfying $Q_M^\dagger|\Omega\rangle = 0$

Physical operators: $\mathrm{e}^{\gamma_0\phi}$ cosmological const.

$\mathcal{R}(\partial\phi)\mathrm{e}^{\gamma_2\phi}$, scalar curvature

$\cdots$

Conformal charge: $\gamma_n = 2b_1 - 2\sqrt{b_1^2 - (4-n)b_1} = 4 - n + 0(1/b_1)$

$\Longrightarrow$ scaling behavior of physical operators