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Curvature perturbation from Ekpyrotic collapse with multiple fields

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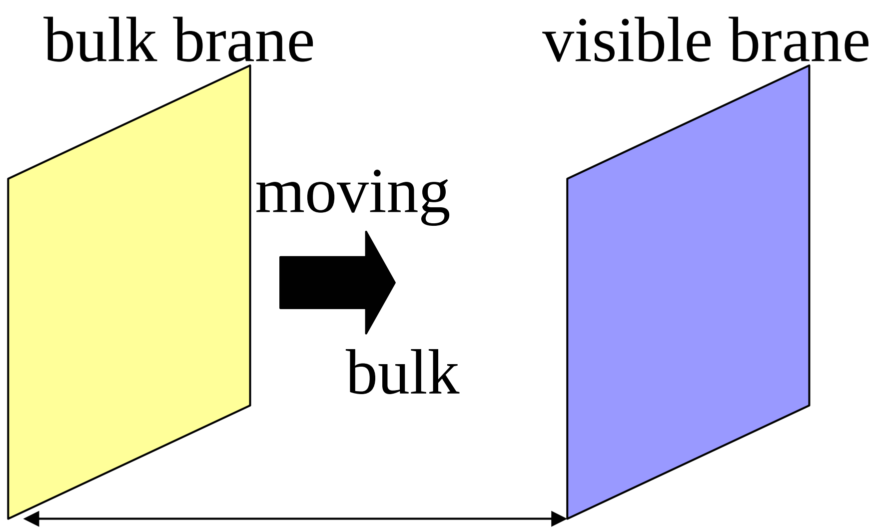
1.Introduction
Ekpyrotic scenario

Khoury, Ovrut, Steinhardt, Turok `01

Hot big-bang universe is produced
by the collision of branes!



separation is parametrized
by a scalar field Φ

- Set up is based on heterotic M theory
  Lucas, Ovrut, Waldram `98
- Fluctuation is generated in the contracting phase

  bulk brane moves
  toward visible brane
- Collision of two branes thermalise the visible brane

Primordial curvature perturbations

In contracting phase, cosmology on the brane is described by 4d effective scalar field theory with potential

$$V(\phi) = -V_0 \exp\left(-\sqrt{\frac{2}{p}}\frac{\phi}{M_p}\right)$$

- Spectrum

Lyth `02

$$n = 1 + \frac{2}{1-p} \underset{p \to 0}{\longrightarrow} 3$$

strongly blue tilted

- comoving curvature perturbation remains constant for adiabatic perturbations on large scales

⇒ strongly blue tilted spectrum for single field model in which 4d effective theory is valid

Is it impossible to obtain scale-invariant spectrum?

2.New Ekpyrotic scenario

Lehners, McFadden, Turok, Steinhardt `07
Buchbinder, Khoury, Ovrut `07,   Creminelli, Senatore `07

Considering non-adiabatic perturbations

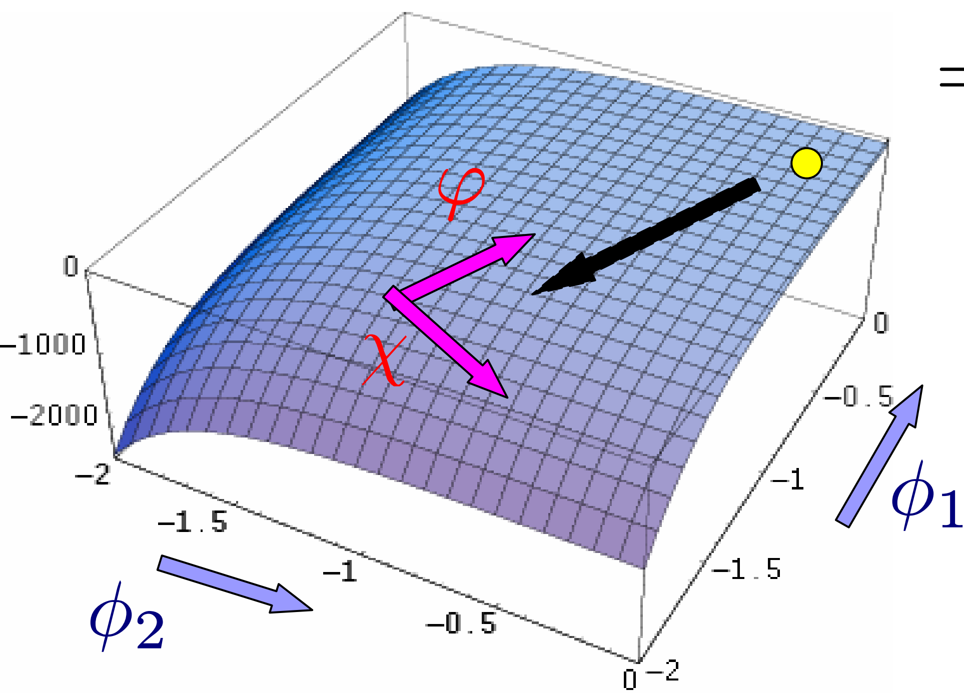
- Model   (2 scalar case)

$$V(\phi_1, \phi_2) = -V_1 e^{-c_1\phi_1} - V_2 e^{-c_2\phi_2}$$

$$= -\underline{U(\chi)}\, e^{-c\varphi} \qquad c_1, c_2 \gg 1$$

$$U_0\left[1 + \frac{c^2}{2}(\chi - \chi_0)^2 + \cdots\right]$$

$$c = \frac{c_1 c_2}{\sqrt{c_1^2 + c_2^2}} < c_1, c_2$$



Background dynamics

Contracting phase is characterised by scaling solutions!!

- Scaling solution supported by a single field $\phi_i\ (i=1,2)$

$$a=(-t)^{p_i}, \quad \phi_i=\frac{2}{c_i}\ln(-t)-\frac{1}{c_i}\ln\left(\frac{p_i(1-3p_i)}{V_i}\right)$$

with $p_i=\frac{2}{c_i^2}$

cf. old ekpyrotic model

- Multi-field scaling solution

assisted contraction

Finelli `02

$$a=(-t)^{p}, \quad \phi_i=\frac{2}{c_i}\ln(-t)-\frac{1}{c_i}\ln\left(\frac{2(1-3p)}{c_i^2 V_i}\right)$$

with $p=\frac{2}{c^2}, \quad c=\frac{c_1 c_2}{\sqrt{c_1^2+c_2^2}}$ if $c^2>6$

cf. assisted inflation (N-flation)

Generation of entropy perturbations

- evolution eq. for entropy field

$$\ddot{\delta s} + 3H\dot{\delta s} + \frac{k^2}{a^2}\delta s + (V_{,ss} - \dot{\theta}^2)\delta s = \underline{-2\frac{\dot{\theta}}{\dot{r}}}\left[\dot{r}\dot{\delta r} - \left(\frac{\dot{r}^3}{2H} + \ddot{r}\right)\delta r\right]$$

coupling with adiabatic field

If we choose the background as

- multi-field scaling solution (B) $\theta = \arctan\frac{c_2}{c_1}$
- single field scaling solution (B1, B2) $\theta = \frac{\pi}{2}, \quad 0$

→
- adiabatic and entropy fields are decoupled
- possible to quantise the independent fluctuations

- spectral index

B: $n_{\delta s} = 2p$ scale-invariant for $p \to 0$ B1,B2: $n_{\delta s} = 2$ blue

3.Generation of curvature perturbations

Koyama, Wands `07, Koyama, SM, Wands `07

- motivations
  - Is the multi-field scaling solution realised naturally?
  - How to convert the scale-invariant isocurvature perturbations to comoving curvature perturbations?

    In the previous works, they rely on additional process

    ex) sudden change of potential, bouncing, etc.
  - Is it possible to evaluate the curvature perturbations quantitatively, if it is generated ?

Stability analysis

Koyama, Wands `07

- Phase space variables

$$x_i \equiv \frac{\dot{\phi}_i}{\sqrt{6}H}, \quad y_i \equiv \frac{\sqrt{V_i e^{-c_i\phi_i}}}{\sqrt{3}H}$$

$i = 1, 2$

constrained by

$$\sum_j x_j^2 - \sum_j y_j^2 = 1$$

- FIxed points

A: $\sum_j x_j^2 = 1, \quad y_j = 0$ (kinetic term dominant solution)

Bi: $x_i = \frac{c_i}{\sqrt{6}}, y_i = -\sqrt{\frac{c_i^2}{6} - 1}, x_j = y_j = 0, \ (\text{for} j \neq i)$

(single field dominant scaling solution)

B: $x_j = \frac{\sqrt{6}}{3p}\frac{1}{c_j}, y_j = -\sqrt{\frac{2}{c_j^2 p}\left(\frac{1}{3p} - 1\right)}$

(multi-field scaling solution)

- linear analysis

A: unstable Bi: stable B: saddle for $\begin{cases} c_i^2 > 6 \\ \sum c_i^{-2} < 1/6 \end{cases}$

Generation of adabatic perturbations (on large scales)

Koyama, SM, Wands `07

- Coupled evolution equation

$$\begin{pmatrix} \delta r \\ \delta s \end{pmatrix}'' + \begin{pmatrix} K_{rr} & K_{rs} \\ K_{sr} & K_{ss} \end{pmatrix} \begin{pmatrix} \delta r \\ \delta s \end{pmatrix}' + \begin{pmatrix} M_{rr} & M_{rs} \\ M_{sr} & M_{ss} \end{pmatrix} \begin{pmatrix} \delta r \\ \delta s \end{pmatrix} = 0$$

$$K_{rs} = K_{sr} = M_{rs} = M_{sr} = 0$$ at B, Bi

- Initial condition

initial positions in the phase space slightly perturbed away from B

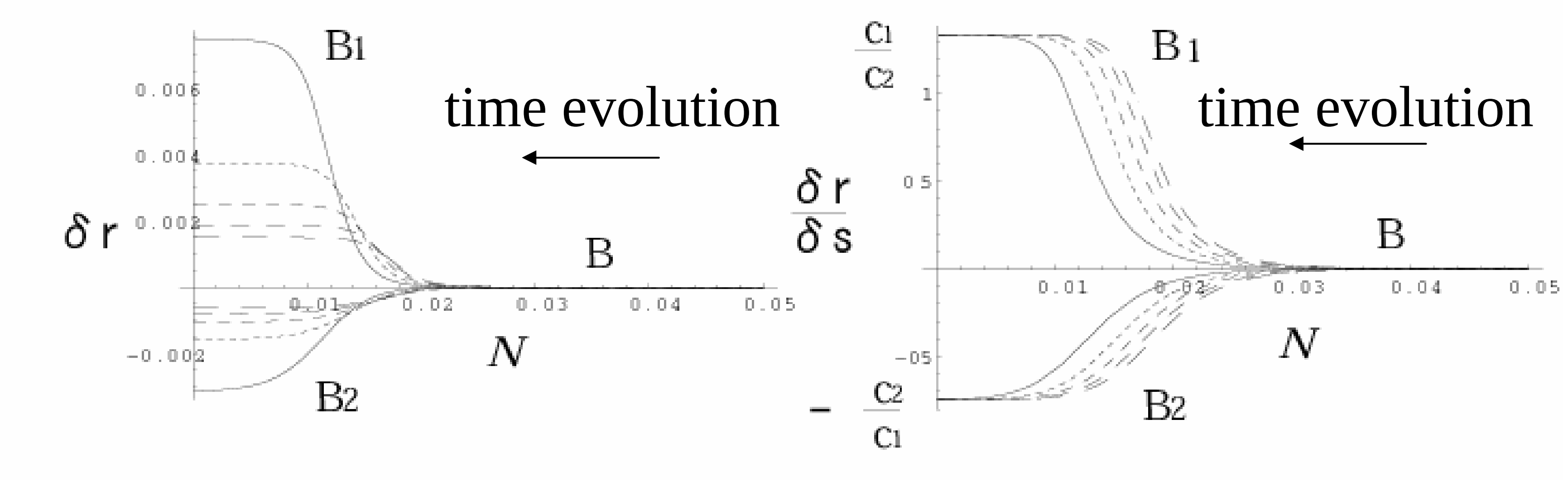
adiabatic and entropy field perturbations

$\delta r = 0$ ⟵ blue spectrum at B

$\delta s = \frac{1}{p} \left| \frac{H}{2\pi} \right|$ ⟵ Bunch-Davies vacuum

Numerical results

- generation of adiabatic perturbations from mixing



- ratio of adiabatic to entropy perturbations

$$\frac{\delta r}{\delta s} = \frac{c_1}{c_2}, \text{at } B_1 \qquad \frac{\delta r}{\delta s} = -\frac{c_2}{c_1}, \text{at } B_2$$

This is independent of the path of the fields or the parameters!

Amplitude of curvature perturbations

- comoving curvature perturbations after transition

$$\begin{cases} B \longrightarrow B1 & \mathcal{R}_c \equiv \frac{H}{\dot\phi_1}\delta r = \frac{1}{c_1}\delta r = \frac{\delta\chi}{\sqrt{c_1^2+c_2^2}} \\ B \longrightarrow B2 & \mathcal{R}_c \equiv \frac{H}{\dot\phi_2}\delta r = \frac{1}{c_2}\delta r = \frac{\delta\chi}{\sqrt{c_1^2+c_2^2}} \end{cases}$$

same for both background

where $\delta\chi = \frac{c_1\delta\phi_1 - c_2\delta\phi_2}{\sqrt{c_1^2+c_2^2}}$

- evolution of $\delta\chi$ at multi-field scaling sol.

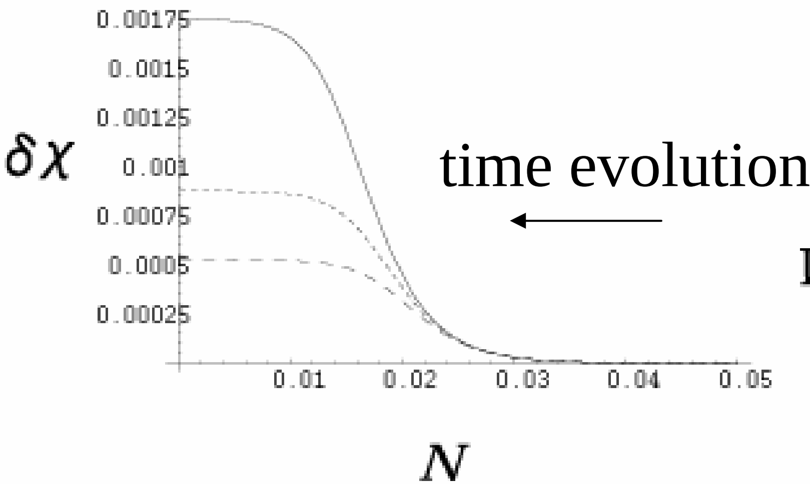
$$\delta\chi = \frac{1}{p}\left|\frac{H}{2\pi}\right| \qquad \left(p = \frac{2}{c^2}\right)$$

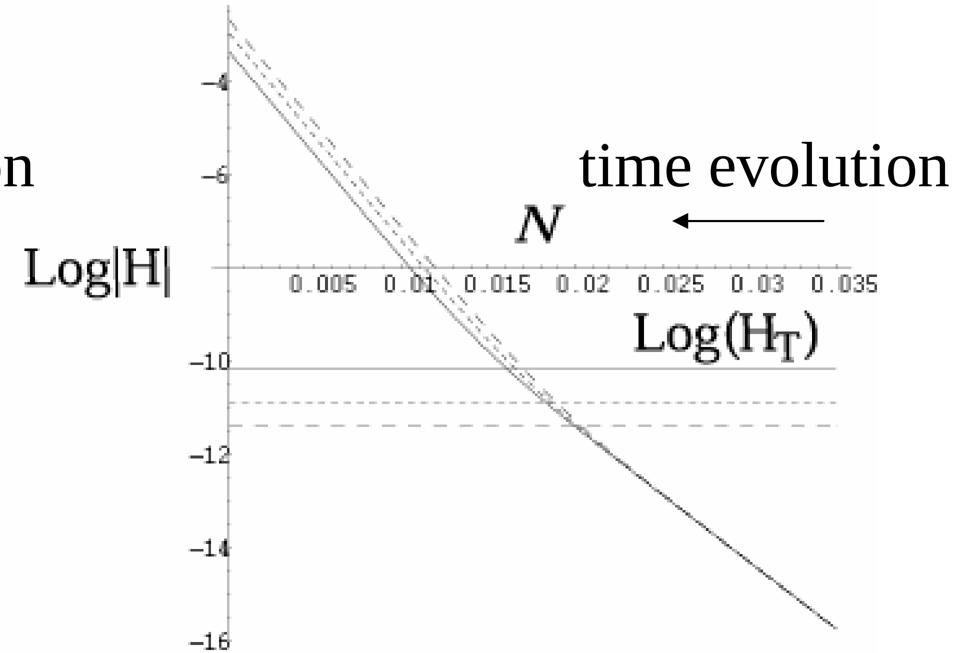
$$\mathcal{R}_c = \frac{c^2}{2\sqrt{c_1^2+c_2^2}}\left|\frac{H}{2\pi}\right|_T$$

assuming sudden transition

Numerical results (2)

- validity of sudden transition approximation





$$|H| = \frac{4\pi}{c^2}\delta\chi$$

$$\dot{H} = -\frac{1}{2}(\dot{\phi_1}^2 + \dot{\phi_2}^2)$$

H_T constructed from the amplitude of $\delta\chi$ can reproduce H at transition well!

5.Conclusion $V = -V_1 e^{-c_1\phi_1} - V_2 e^{-c_2\phi_2}$

- Cosmological perturbations from ekpyrotic collapse
- Scale-invariant isocurvature perturbations are generated during multi-field scaling solutions
- Unstable modes drives the multi-field scaling solutions to single field dominated scaling solutions
- Transition automatically converts the initial isocurvature perturbations to curvature perturbations

$$\mathcal{R}_c = \frac{c^2}{2\sqrt{c_1^2+c_2^2}} \left| \frac{H}{2\pi} \right|_T$$

- Large non-Gaussianity is generated during transition

Koyama-SM-Vernizzi-Wands (in preparation)

$$f_{NL} = \frac{5}{12} c_1^2$$

Discussions

- Spectrum is slightly blue for pure exponential potentials $n = 2p, \quad p \ll 1$

  -deviation from exponential potential

- How to realise the non-singular bounce?

  -relying on ghost condensation

- How to realise initial to be near the saddle point

  - relying on the cyclic scenario?

Buchbinder, Khoury, Ovrut (arXive:0706.3903)