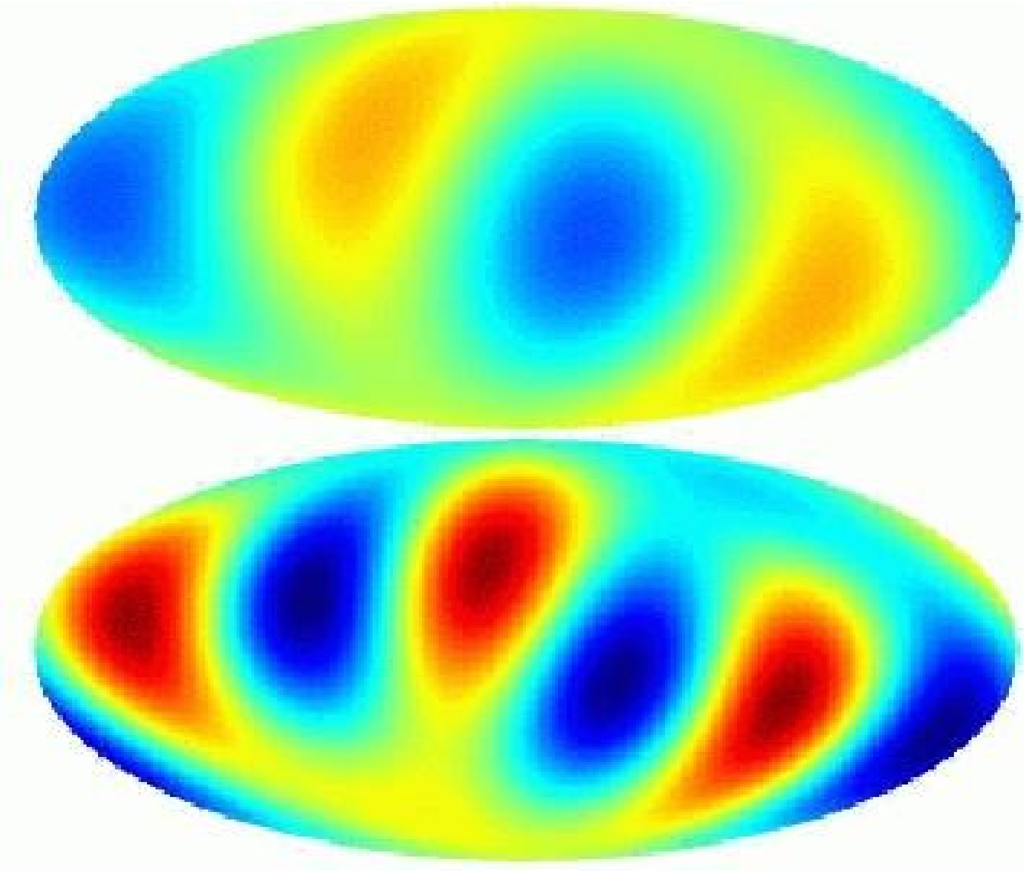
CORRELATIONS IN THE CMB DUE TO DARK ENERGY

RICHARD BATTYE

JODRELL BANK CENTRE FOR ASTROPHYSICS
UNIVERSITY OF MANCHESTER

IN COLLABORATION WITH ADAM MOSS (University of British Columbia)

MOTIVATION : LARGE SCALE CMB & DE



CORRELATIONS BETWEEN l=2,3

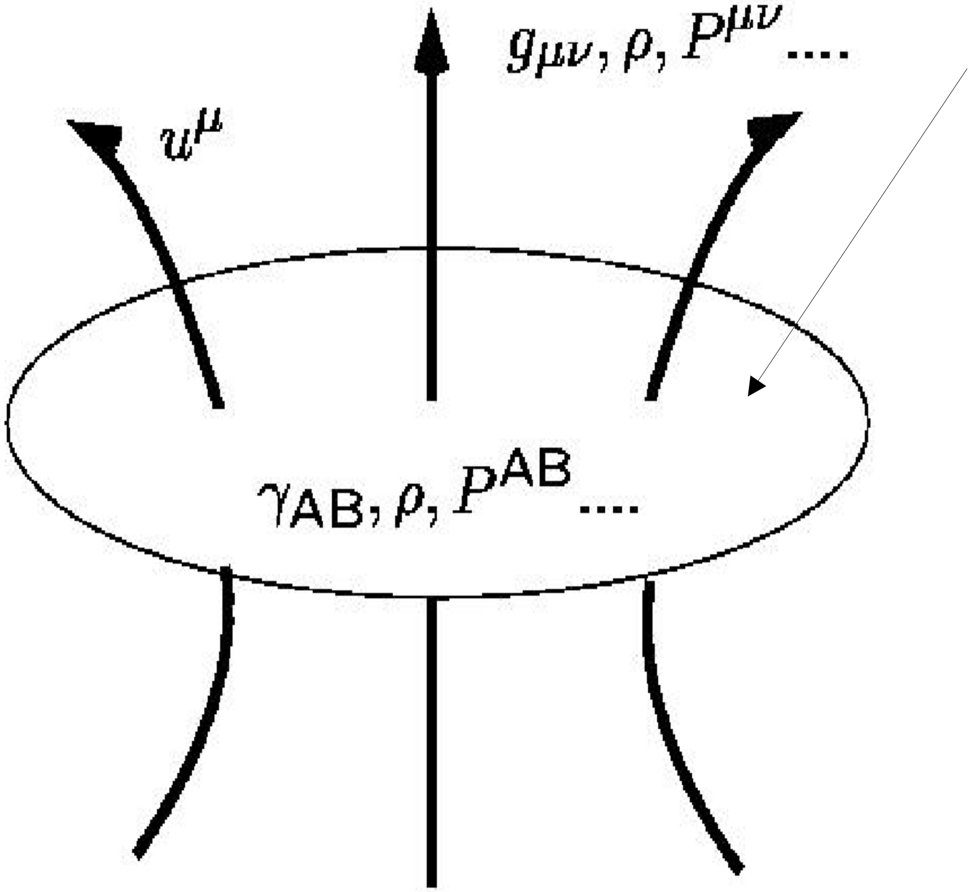
LARGE SCALE MODES
COME INSIDE THE HORIZON
AT LATE TIMES :
WHEN DARK ENERGY DOMINATES

↓

MAYBE THE TWO
IDEAS ARE RELATED

OTHER STATISTICS : N-S ASYMMETRY IN POWER SPECTRUM
"AXIS OF EVIL" ETC

GENERALIZED MATERIAL DESCRIPTION



PERPENDICULAR 3D SPACE

$$V_\mu = V^\perp u_\mu + V^{||}_\mu$$

WHERE $V^\perp = u^\mu V_\mu$

$$V^{||}_\mu = \gamma^\nu{}_\mu V_\nu$$

PROJECTION TENSOR

$$\gamma_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$$

FLOW LINES : u_μ

$$u^\mu u_\mu = -1$$

THEORY DEVELOPED BY CARTER AND
OTHERS IN 1970s TO MODEL NEUTRON STARS

LAGRANGIAN, EM TENSOR

ASSUMPTION :

$\mathcal{L}(\gamma_{\mu\nu}, \rho, P, ..)$

ACTION : $\mathcal{I} = \int |g|^{1/2} d^4x \, \mathcal{L}$

EM- TENSOR : $T_{\mu\nu} = -2|g|^{-1/2}\frac{\delta}{\delta g^{\mu\nu}}\left[|g|^{1/2}\mathcal{L}\right]$

STANDARD DEFINITIONS

RELATIVISTIC ELASTICITY TENSOR :

$$W_{\mu\nu\rho\sigma} = -2|g|^{-1/2}\frac{\delta}{\delta g_{\mu\nu}}\left[|g|^{1/2}T_{\rho\sigma}\right] \quad \Rightarrow \quad \delta T^{\mu\nu} = -\frac{1}{2}\left(W^{\mu\nu\rho\sigma} + T^{\mu\nu}g^{\rho\sigma}\right)\delta g_{\rho\sigma}$$

HENCE $W^{\mu\nu\rho\sigma}$ PARAMETERIZES FLUID PERTS

EULERIAN/LAGRANGIAN PERTURBATIONS

$$\delta g_{\mu\nu} = h_{\mu\nu} + 2\nabla_{(\mu}\xi_{\nu)}$$

EULERIAN LAGRANGIAN

BACKGROUND MEDIUM

ASSUME GAUGE CONDITIONS : $\xi^{\mu} u_{\mu} = 0 \quad u^{\nu} h_{\mu\nu} = 0$

PERTURBATIVE LAGRANGIAN

$$\begin{aligned}\mathcal{L}_{\mathrm{pert}} &= |g|^{-1/2}\delta^2\left[|g|^{1/2}\mathcal{L}\right] \\ &= \frac{1}{4}W^{\mu\nu\rho\sigma}\delta g_{\mu\nu}\delta g_{\rho\sigma}\end{aligned}$$

THE MOST GENERAL LAGRANGIAN FOR LINEARIZED ADIABATIC PERTURBATIONS

STANDARD ELASTICITY TENSOR

$$\left.\begin{aligned} T^{\mu\nu} &= \rho u^{\mu} u^{\nu} + P^{\mu\nu} \\ W^{\mu\nu\rho\sigma} &= E^{\mu\nu\rho\sigma} + P^{\mu\nu} u^{\rho} u^{\sigma} + P^{\rho\sigma} u^{\mu} u^{\nu} - P^{\mu\rho} u^{\rho} u^{\nu} \end{aligned}\right\}$$

3+1 SPLIT

$$- \quad P^{\mu\sigma} u^{\rho} u^{\nu} - P^{\nu\rho} u^{\mu} u^{\sigma} - P^{\nu\sigma} u^{\mu} u^{\rho} - \rho u^{\mu} u^{\nu} u^{\rho} u^{\sigma}$$

WHERE

$$E^{\mu\nu\rho\sigma} u_{\sigma} = 0 \qquad P^{\mu\nu} u_{\nu} = 0$$

$$E_{\mu\nu\rho\sigma} = E_{\rho\sigma\mu\nu} = E_{[\mu\nu][\rho\sigma]}$$

HENCE, ONLY SPATIAL COMPONENTS ARE NON-ZERO

PRESSURE TENSOR :
6 COMPONENTS

W IS ONE OF THEM

ELASTICITY TENSOR :
21 COMPONENTS

1 BULK MODULUS
20 SHEAR MODULI

(BUCHER & SPERGEL 1998,
BATTYE, BUCHER & SPERGEL 1999)

ISOTROPY

NB w=0, μ=0 IS CDM
w=-1 IS Λ

ISOTROPIC
TENSORS

$$P_{\mu\nu} = P\gamma_{\mu\nu} \qquad \gamma_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu$$

$$E^{\mu\nu\rho\sigma} = \left(B - \frac{1}{3}P\right)\gamma^{\mu\nu}\gamma^{\rho\sigma} + 2(\mu + P)\left(\gamma^{\mu(\rho}\gamma^{\sigma)\nu} - \frac{1}{3}\gamma^{\mu\nu}\gamma^{\rho\sigma}\right)$$

P = PRESSURE

$B = (\rho + P)\dfrac{dP}{d\rho}$ = BULK MODULUS

μ= SHEAR MODULUS

IN PRINCIPLE :

w COULD BE ANYTHING !

→ TWO PARAMETER
DE MODEL

SOUND SPEEDS

$$c_S^2 = \frac{dP}{d\rho} + \frac{4\mu}{3(\rho + P)}$$

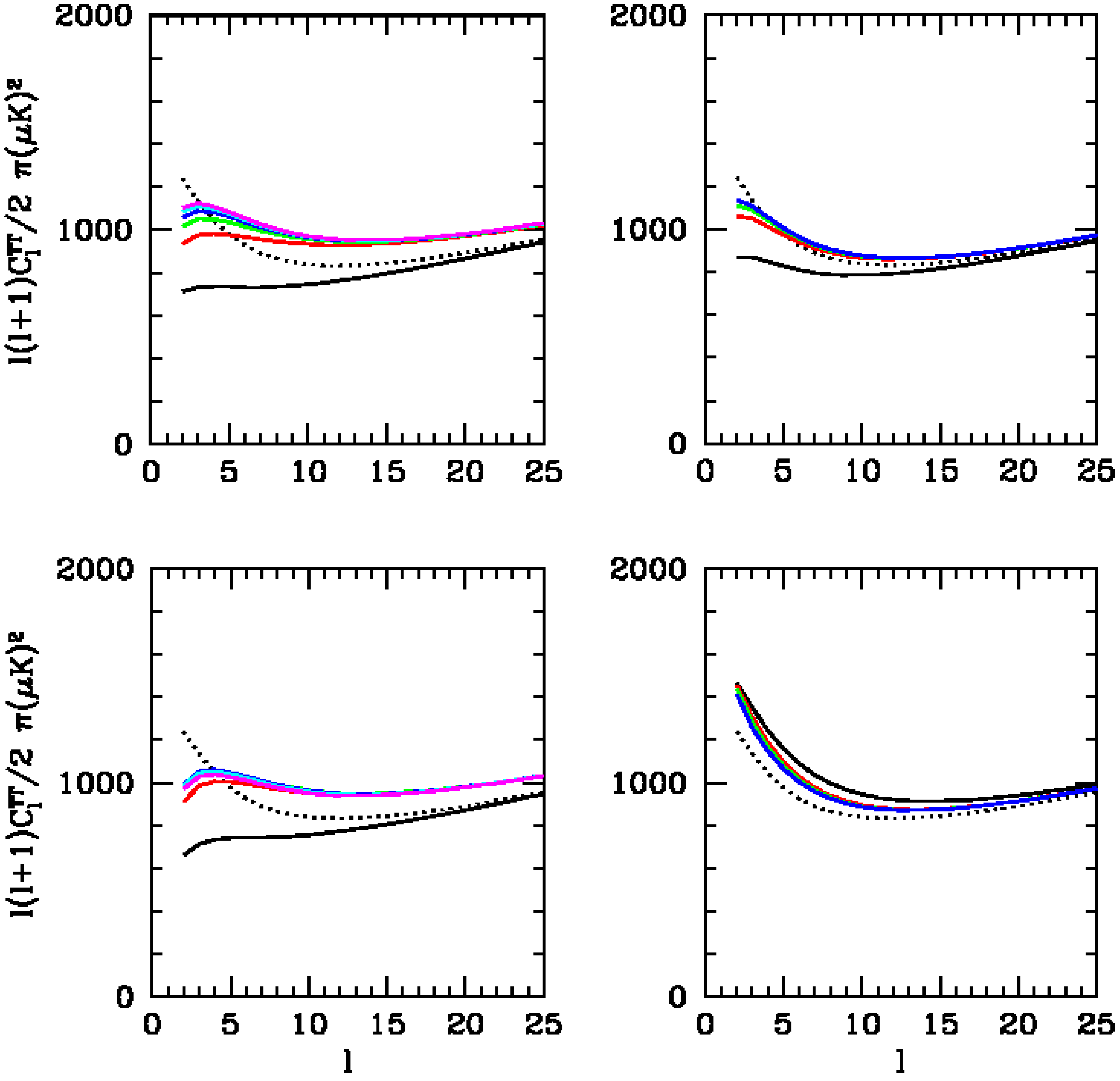
LONGITUDINAL
(SCALAR)

$$c_V^2 = \frac{\mu}{\rho + P}$$

TRANSVERSE
(VECTOR)

NB : $\dfrac{\delta P}{\delta \rho} = \dfrac{dP}{d\rho}$

ADIABATIC



W = -1/3

W = -2/3

NON-ADIABATIC
MODEL

(LEWIS & WELLER,
BEAN & DORE)

ADIABATIC
ELASTIC MODEL

NB : NO PERTS
→ **MEANINGLESS**
RESULTS

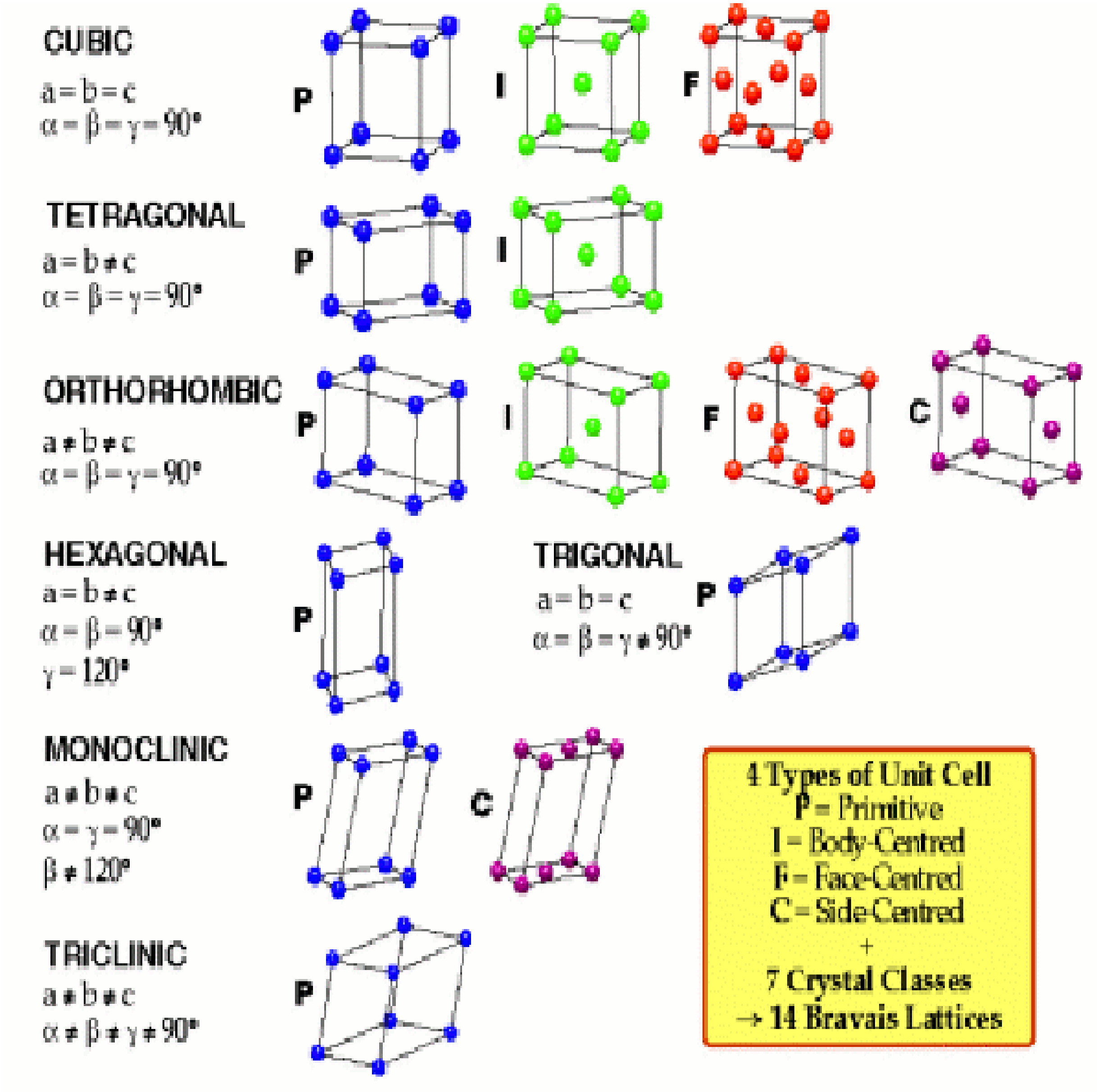
POINT SYMMETRIES

POSSIBLE SYMMETRIES OF ELASTICITY TENSOR ARE CLASSIFIED BY THE BRAVAIS LATTICES

eg FROM LANDAU & LIFSCHITZ

<u>NON- ZERO MODULI</u>

TRICLINIC	18
MONOCLINIC	12
ORTHORHOMIBIC	9
TETRAGONAL	6
RHOMBOHEDRAL	6
HEXAGONAL	5
CUBIC	3
ISOTROPIC	2



CASE STUDY : CUBIC SYMMETRY

PRESSURE ISOTROPIC :

$P^{\mu\nu} = P\gamma^{\mu\nu}$ → FRW BACKGROUND

ELASTICITY TENSOR :

BULK MODULUS +
2 SHEAR MODULI

$$E^{11} = E^{22} = E^{33} = \beta + \frac{4}{3}\mu_{\mathrm{L}} + P$$

$$E^{12} = E^{13} = E^{23} = \beta - \frac{2}{3}\mu_{\mathrm{L}} - P$$

$$E^{44} = E^{55} = E^{66} = P + \mu_{\mathrm{T}}$$

STABILITY :

$$\frac{\mu_{\mathrm{L}}}{\rho}, \frac{\mu_{\mathrm{T}}}{\rho} > \frac{1}{6}$$

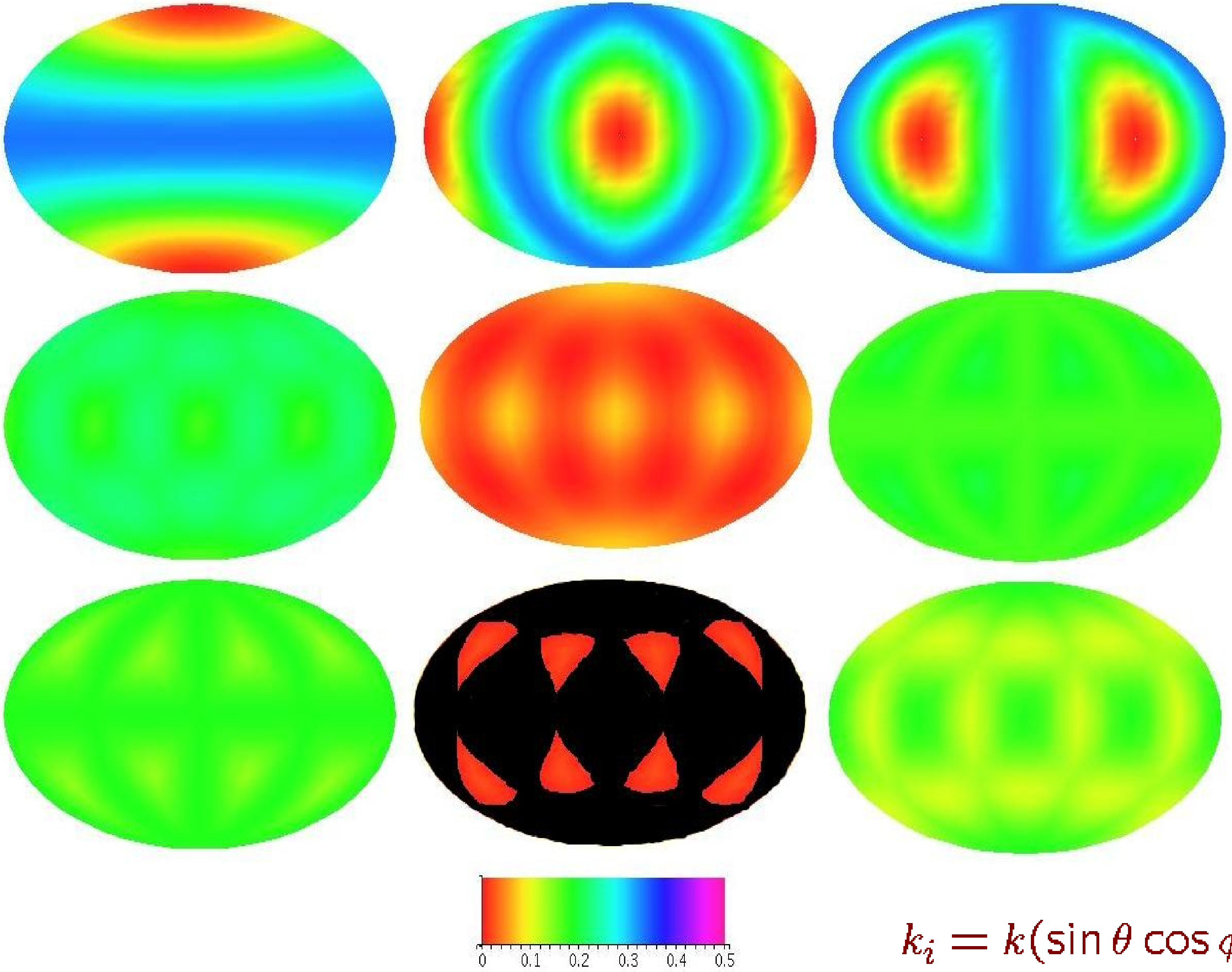
WHERE

1 = xx, 2 = yy, 3 = zz
4 = xy, 5 =yz, 6 = zx

SHEAR MODULI HAVE BEEN COMPUTED FOR THE PRIMITIVE LATTICES AND A NUMBER OF COMPOUND LATTICES

(BATTYE, CHACHOUA & MOSS 2005)

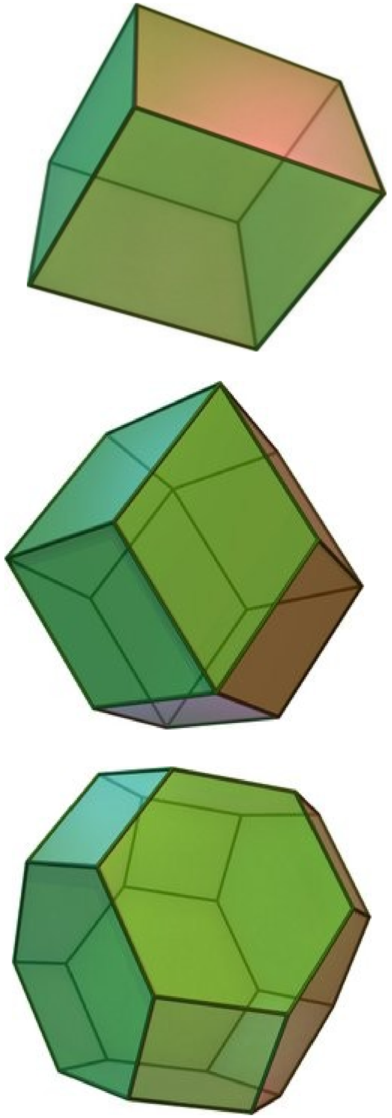
VARIABLE SOUND SPEEDS : $c^2(k,\theta,\phi)$



SIMPLE
CUBE

FCC

BCC



$$k_i = k(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$

ANISOTROPY FROM ADIABATIC PERTS

- ie. FROM INFLATION

INITIAL CONDITIONS

THOSE USED
FOR INFLATION →

$$h_{ij} = 6\left(\hat{k}_i\hat{k}_j - \frac{1}{3}\delta_{ij}\right)k^{-3/2}$$

$$\dot{h}_{ij} = 0 = \xi^i = \dot{\xi}^i = 0$$

POWER SERIES SOLUTION W = - 2/3

$$\boxed{\begin{aligned} A &= \frac{1}{3}\left(\frac{1}{4} - \frac{\mu_\mathsf{T}}{\rho}\right) \\ B &= -\frac{1}{2\rho}\left(\mu_\mathsf{T} - \mu_\mathsf{L}\right) \end{aligned}}$$

Δμ

$$\xi^i = \frac{\tau^4 k^{3/2}}{12}\begin{pmatrix} A & B\hat{k}_x\hat{k}_y & B\hat{k}_x\hat{k}_z \\ B\hat{k}_x\hat{k}_y & A & B\hat{k}_y\hat{k}_z \\ B\hat{k}_x\hat{k}_z & B\hat{k}_y\hat{k}_z & A \end{pmatrix}\begin{pmatrix} \hat{k}_x \\ \hat{k}_y \\ \hat{k}_z \end{pmatrix} + \mathcal{O}(\tau^5)$$

"WOULD- BE SCALAR MODE"

$$\xi^S = \tau^4 k^{3/2}\left[\frac{A}{12} - \frac{B}{6}\left(\hat{k}_x^2\hat{k}_y^2 + \hat{k}_y^2\hat{k}_z^2 + \hat{k}_z^2\hat{k}_x^2\right)\right] + \mathcal{O}(\tau^5)$$

CUBIC
SYMMETRY

COMPUTATION OF CORRELATION MATRIX

$$
\begin{aligned}
C_{\ell m \ell' m'} &= \langle a_{\ell m} a^*_{\ell' m'} \rangle \\
&= (4\pi)^2 \int dk\, k^2 P_{\mathrm{i}}(k) |\Delta_{\ell m \ell' m'}(k)|^2
\end{aligned}
$$

WHERE

$$|\Delta_{\ell m \ell' m'}(k)|^2 = (-i)^{\ell-\ell'} \int d\Omega Y_{\ell m}(\hat{k}) Y^*_{\ell' m'}(\hat{k}) \boxed{\Delta_\ell(k\hat{k}) \Delta_{\ell'}(k\hat{k})}$$

INDEP OF DIRECTION

& $$\Delta_\ell(k\hat{k}) = \int_0^{\eta_0} d\eta\, S(k\hat{k}, \eta) j_\ell\,(k(\eta_0 - \eta))$$

STANDARD DECOMPOSITION :

$$\frac{\Delta T}{T}(\theta, \phi) = \sum a_{\ell m} Y_{\ell m}(\theta, \phi)$$

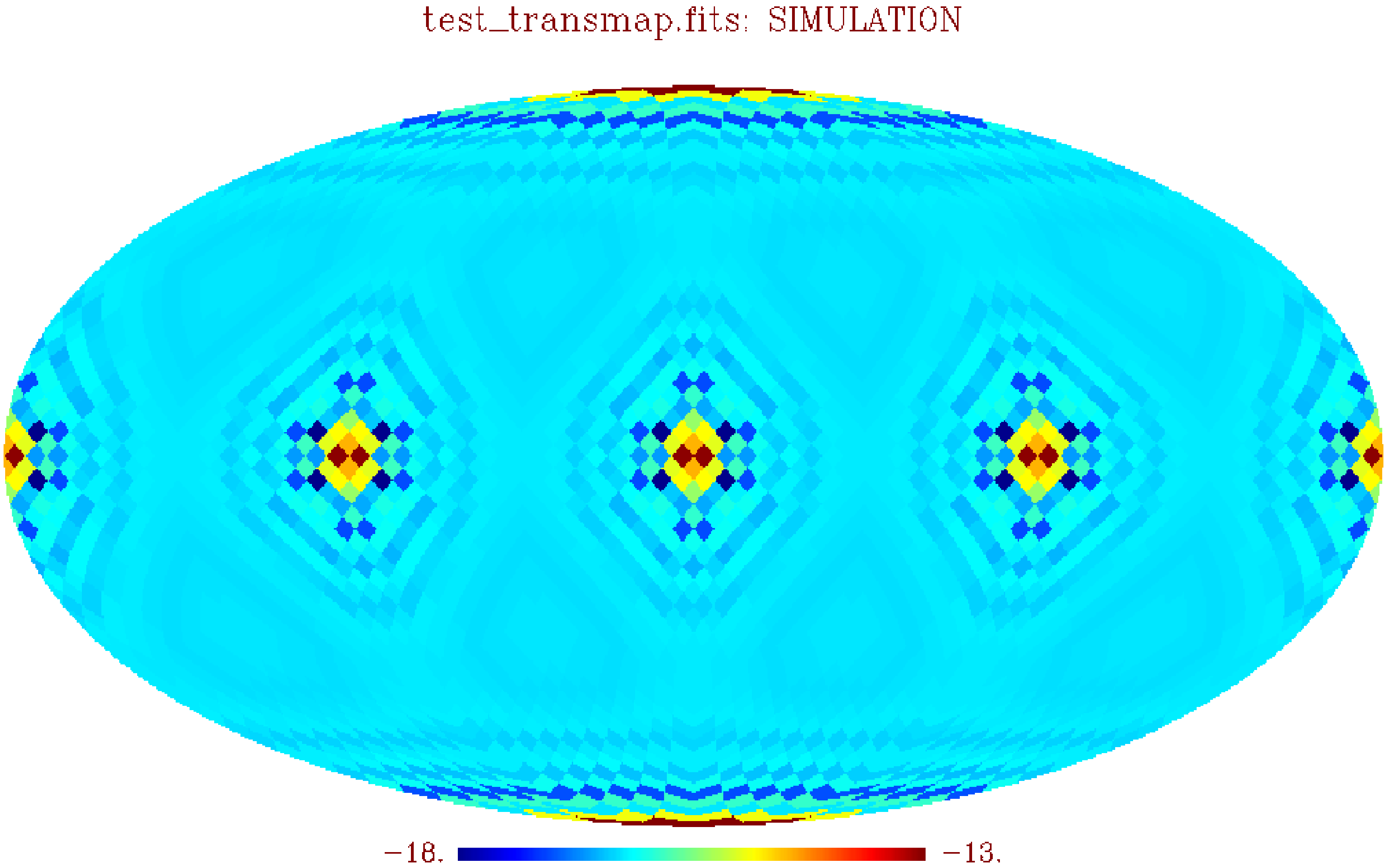
IF $S(k\hat{k}, \eta) = S(k, \eta)$

$$\Downarrow$$

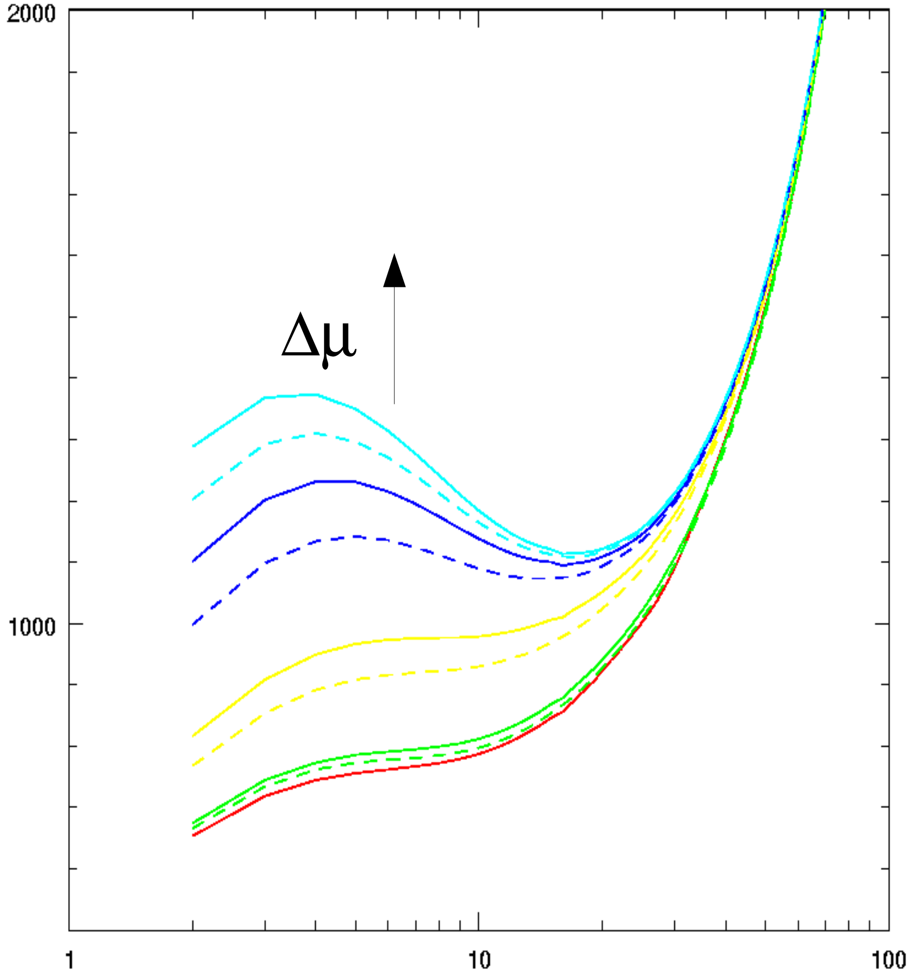
$$C_{\ell m \ell' m'} = \delta_{\ell \ell'} \delta_{m m'} C_\ell$$

$$a_{\ell m} \sim N(0, C_\ell)$$

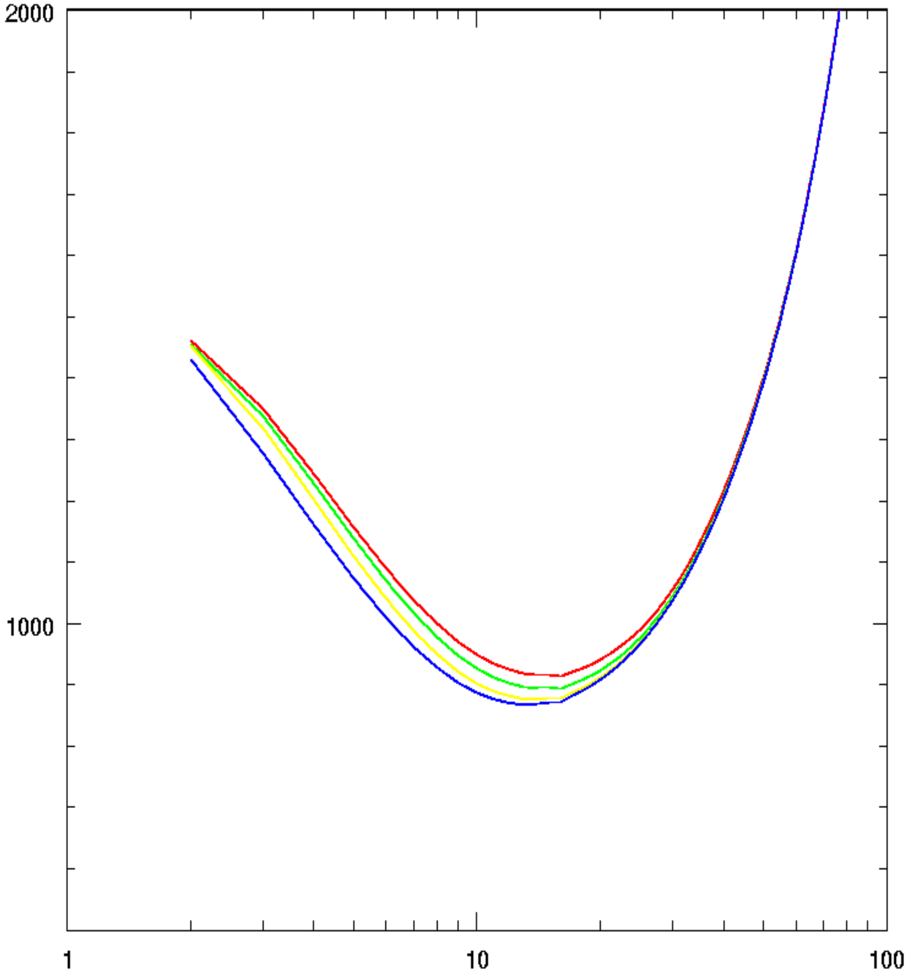
TRANSFER FUNCTION FOR l=2 AND k=0.005 Mpc^{-1}



PRELIMINARY RESULTS : 1 - C_{lmlm}



W = -1/3



W = -2/3

PRELIMINARY RESULTS : 2

$$C_{2m2m'} = \begin{array}{c} \begin{array}{ccccc} -2 & -1 & 0 & 1 & 2 \end{array} \\ \begin{pmatrix} 0.99 & 0.00 & 0.00 & 0.00 & -0.07 \\ 0.00 & 1.05 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.92 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 1.05 & 0.00 \\ -0.07 & 0.00 & 0.00 & 0.00 & 0.99 \end{pmatrix} \end{array}$$

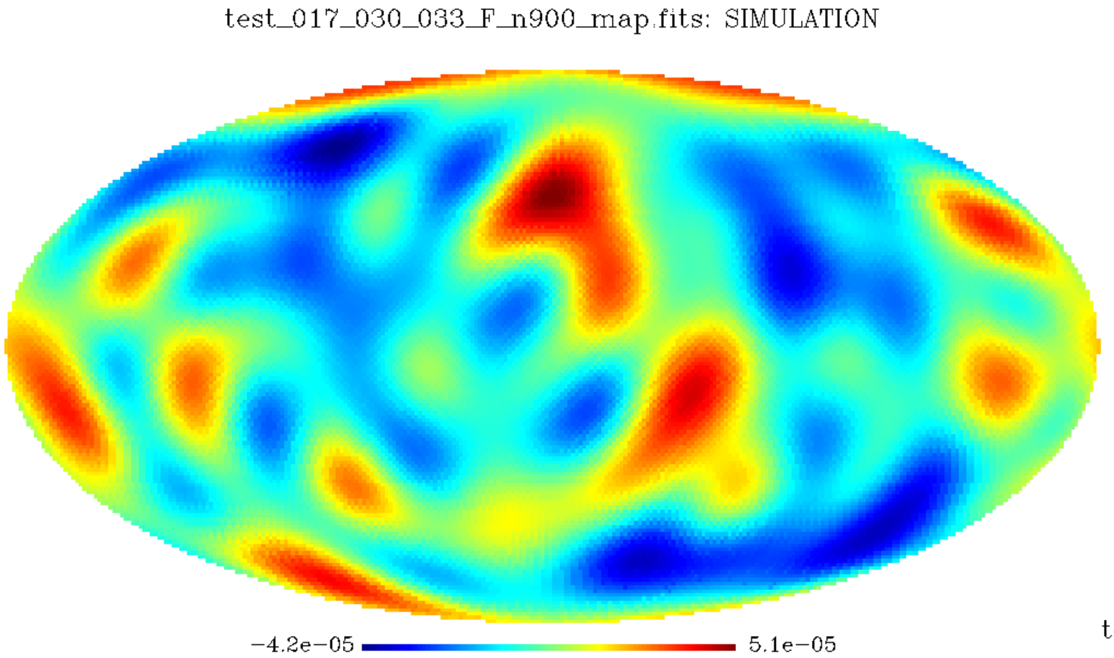
$$C_{4m4m'} = \begin{array}{c} \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 0.94 & 0.00 & 0.00 & 0.00 & -0.03 \\ 0.00 & 0.99 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 1.02 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 1.04 & 0.00 \\ -0.03 & 0.00 & 0.00 & 0.00 & 0.97 \end{pmatrix} \end{array}$$

PRELIMINARY RESULTS : 3

$$C_{\ell 0 \ell' 0} = \begin{array}{c} \begin{array}{ccccccc} l=2 & l=3 & l=4 & l=5 & l=6 & l=7 & l=8 \end{array} \\ \begin{pmatrix} 0.90 & 0.00 & -0.05 & 0.00 & -0.05 & 0.00 & -0.01 \\ 0.00 & 1.01 & 0.00 & -0.06 & 0.00 & -0.04 & 0.00 \\ -0.05 & 0.00 & 1.03 & 0.00 & -0.05 & 0.00 & -0.05 \\ 0.00 & -0.06 & 0.00 & 1.04 & 0.00 & -0.05 & 0.00 \\ -0.05 & 0.00 & -0.05 & 0.00 & 1.02 & 0.00 & -0.04 \\ 0.00 & -0.04 & 0.00 & -0.05 & 0.00 & 1.01 & 0.00 \\ -0.01 & 0.00 & -0.05 & 0.00 & -0.04 & 0.00 & 1.00 \end{pmatrix} \end{array}$$

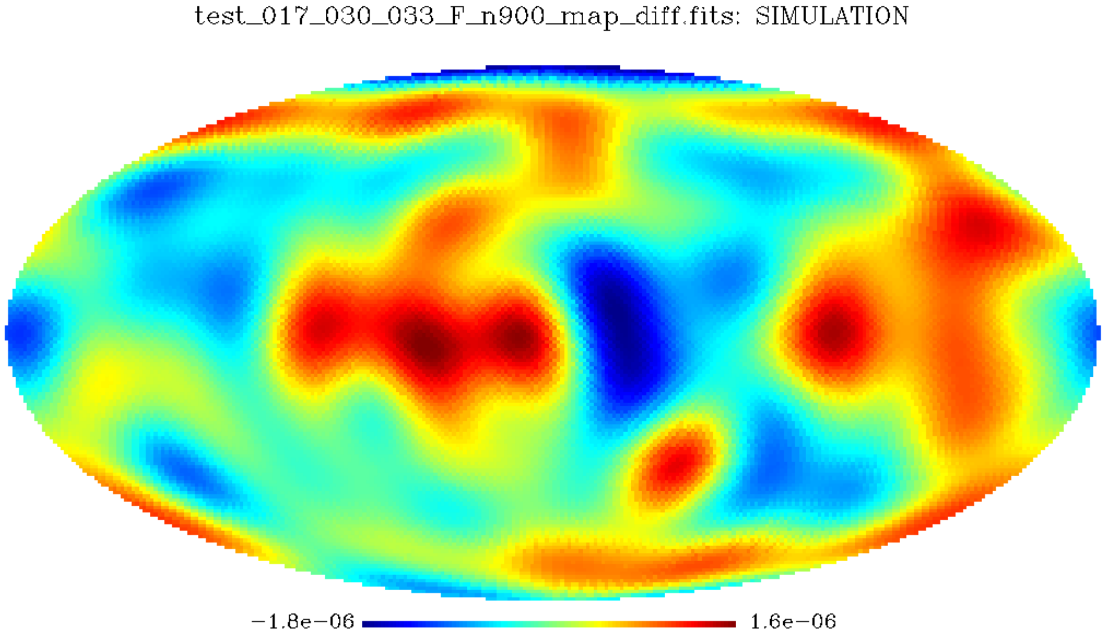
CORRELATIONS ~ 5% POSSIBLE FOR W = -1/3 MUCH LESS FOR W = -2/3

PRELIMINARY RESULTS : 4



ALL l=2-10

SUBTRACT MAP WITH
DIAGONAL COVARIANCE
MATRIX



CONCLUSIONS

- ELASTIC DARK ENERGY IS A PARAMETERIZATION OF DARK ENERGY
- MODELS CAN BE STABLE IF THERE IS SUFFICIENT RIGIDITY
- INTERESTING TO LINK CMB ALIGNMENTS AND DARK ENERGY
- ANISOTROPIC DARK ENERGY IS A NATURAL EXTENSION
- WE HAVE INVESTIGATED THE SIMPLE CASE OF CUBIC SYMMETRY
- AND HAVE COMPUTED THE CORRELATION MATRIX
- OFF DIAGONAL TERMS <10% + FEW μK IN THE MAP
- NB ONE IS NOT RESTRICTED TO CUBIC SYMMETRY

PLAN OF TALK

LINK BETWEEN DARK ENERGY AND LARGE SCALE CMB ?

COSMOLOGICAL PERTURBATIONS

- SCALAR- VECTOR- TENSOR SPLIT

ELASTIC DARK ENERGY

Battye & Moss PRD76 023005 (2007)

- BASIC IDEA
- ISOTROPIC ELASTICITY
- SOUNDS SPEEDS & STABILITY

ANISOTROPIC PERTURBATIONS

Battye & Moss PRD74 041301 (2006)

- ANISOTROPIC ELASTCITY
- DIRECTIONAL SOUND SPEEDS
- ANISOTROPY FROM ADIABATIC INITIAL CONDITIONS

CORRELATED MODES ON LARGE SCALES
IN THE CASE OF CUBIC SYMMETRY

Battye & Moss
In preparation

SCALAR- VECTOR- TENSOR SPLIT

ENERGY-MOMENTUM TENSOR →

$$\begin{aligned}\delta T_{\mu\nu} &= (\delta\rho + \delta P)u_\mu u_\nu - \delta P g_{\mu\nu} \\ &+(\rho + P)(V_\mu u_\nu + V_\nu u_\mu) \\ &+\Pi_{\mu\nu}\end{aligned}$$

SCALAR

VECTOR

VELOCITY :

$$V^i = V^{\mathsf{S}}\hat{k}_i + V^{\mathsf{V1}}\hat{l}_i + V^{\mathsf{V2}}\hat{m}_i$$

ANISOTROPIC STRESS :

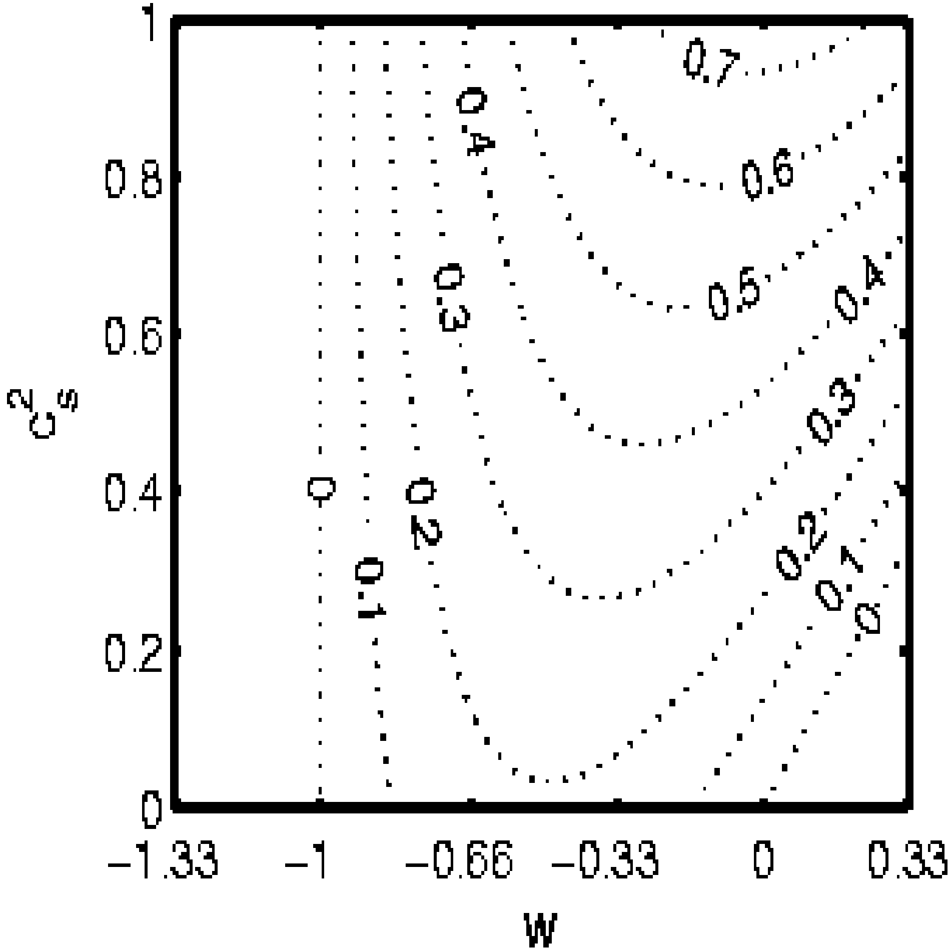
$$\Pi_{ij} = \left(\hat{k}_i\hat{k}_j - \frac{1}{3}\delta_{ij}\right)\Pi^{\mathsf{S}} + \Pi^{V}_{ij} + \Pi^{\mathsf{T}}_{ij}$$

SCALAR

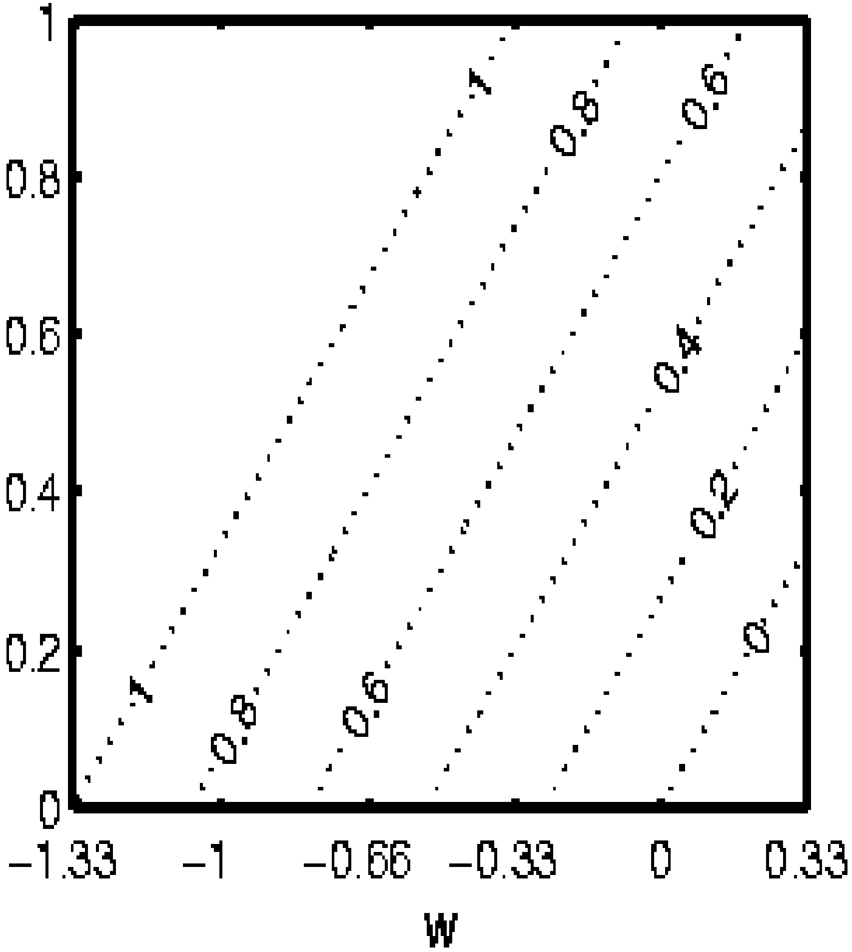
VECTOR

TENSOR

DARK ENERGY PHENOMENOLOGY



μ/ρ VARYING



C_V^2 VARYING

EQUATIONS OF MOTION

SCALAR :

$$\ddot{\xi}^S + (1-3w)\frac{\dot{a}}{a}\dot{\xi}^S + c_S^2 k^2 \left[\xi^S + \frac{1}{2k}h\right] + 3k(c_S^2 - w)\eta = 0$$

VECTOR :

$$\ddot{\xi}^V + (1-3w)\frac{\dot{a}}{a}\dot{\xi}^V + c_V^2 k^2 \left(\xi^V - \frac{1}{k}H^V\right) = 0$$

WHERE

$$h_{ij} = h\hat{k}_i\hat{k}_j + 6\eta\left(\hat{k}_i\hat{k}_j - \frac{1}{3}\delta_{ij}\right) + \hat{k}_i H_j^V + \hat{k}_j H_i^V + H_{ij}^T$$

SCALAR SECTOR →

$$\begin{aligned} \dot{\delta}_s &= -(1+w)\left(\theta_s + \frac{1}{2}\dot{h}\right) \\ \dot{\theta}_s &= -(1-3w)\frac{\dot{a}}{a}\theta_s + \frac{w}{1+w}k^2\delta_s \\ &\quad -k^2(c_S^2 - w)\left[-\delta_s + 3(1+w)\eta\right] \end{aligned}$$

COMPARISON TO OTHER MODELS

EQUATIONS OF MOTION FOR A GENERAL FLUID →

$$\dot{\delta} = -(1+w)\left(\theta + \frac{1}{2}\dot{h}\right) - 3wH\Gamma$$

ENTROPY PERT

$$\dot{\theta} = -(1-3w)H\theta + \frac{k^2 w}{1+w}\left(\delta + \Gamma - \frac{2}{3}\Pi\right)$$

$$w\Gamma = \left(\frac{\delta P}{\delta\rho} - \frac{dP}{d\rho}\right)\delta^{\rm rest}$$

ANISOTROPIC STRESS

NON- ADIABATIC (SCALAR FIELD)

(Hu ; Weller & Lewis; Bean & Dore)

$$w\Gamma = (c_s^2 - w)\left[\delta + 3H(1+w)\frac{\theta}{k^2}\right]$$

$$\Pi = 0$$

$$c_s^2 = \frac{\delta P}{\delta\rho}$$

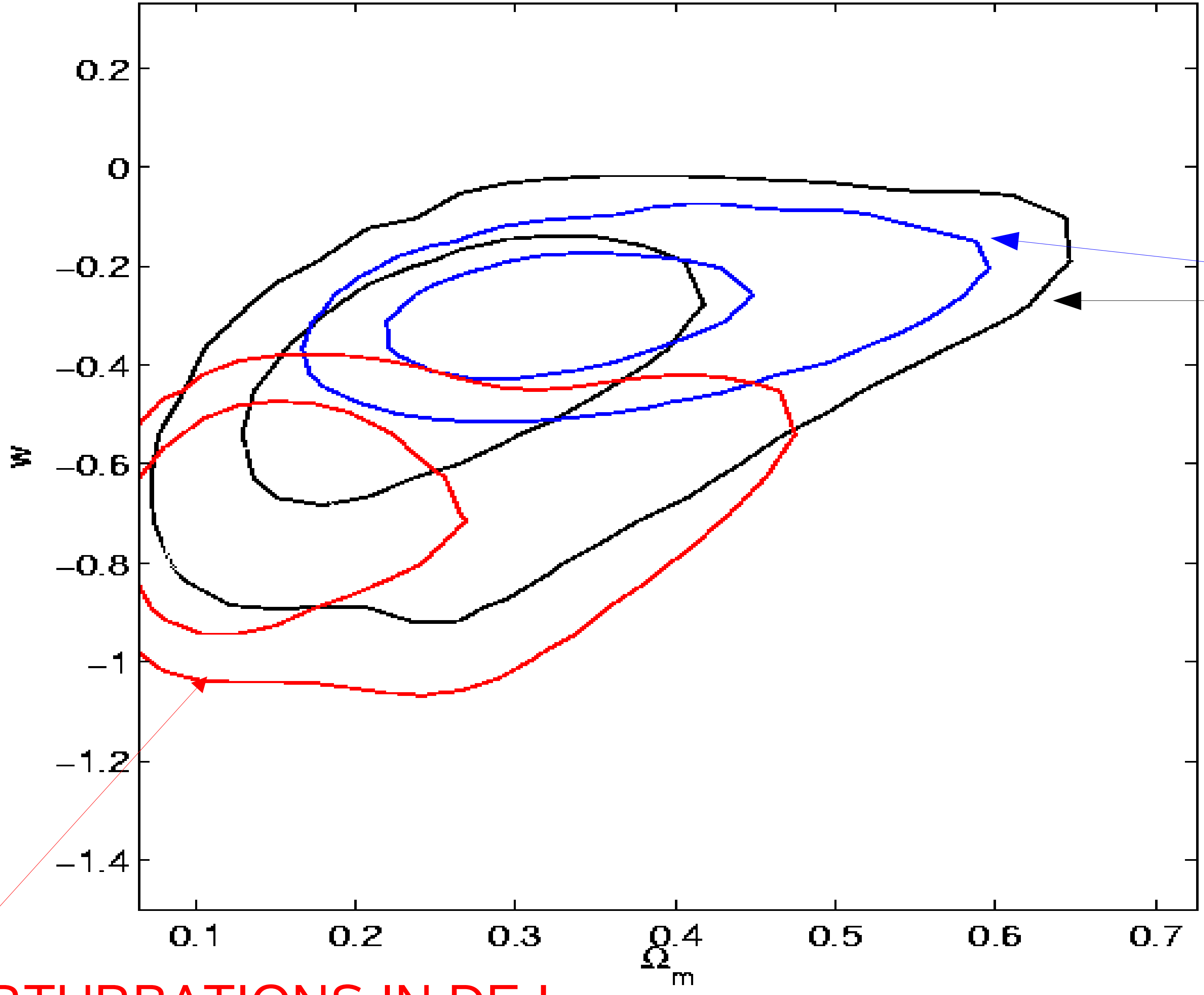
ADIABATIC (ELASTIC)

(Bucher & Spergel; Battye, Bucher & Spergel)

$$w\Gamma = 0$$

$$\Pi = \frac{3}{2w}(c_s^2 - w)\left[-\delta + 3(1+w)(\eta - \eta_I)\right]$$

IMPORTANCE OF PERTURBATIONS



TWO
MODELS
INCLUDING
PERTS

NO PERTURBATIONS IN DE !

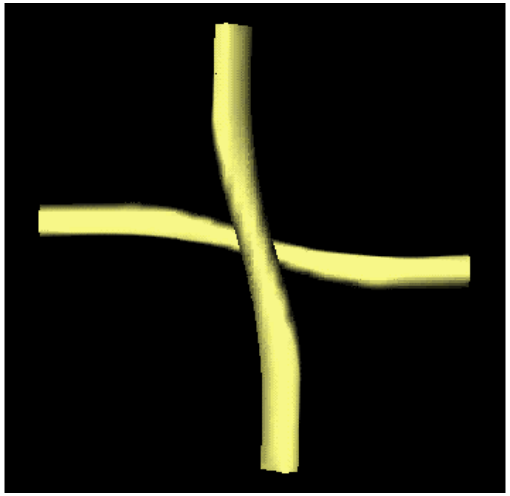
SIMPLE DIMENSIONAL SCALING

(VILENKIN 1984, KIBBLE 1986)

STRINGS

$$\rho_{\mathrm{str}} = \frac{\mu L}{V} \propto \frac{1}{a^2}$$

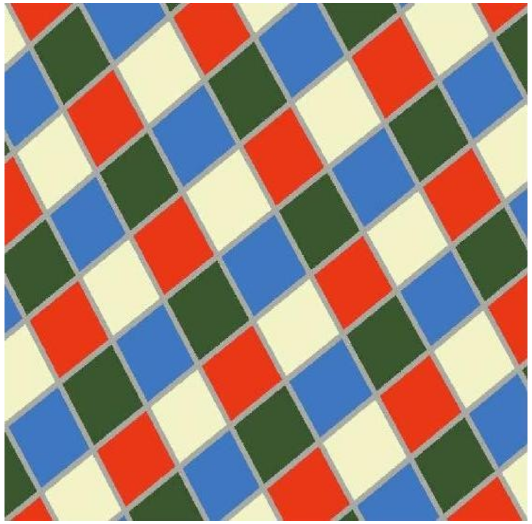
w=- 1/3



DOMAIN WALLS

$$\rho_{\mathrm{dw}} = \frac{\sigma A}{V} \propto \frac{1}{a}$$

w=- 2/3



CMB ANISOTROPIES

$$\frac{\Delta T}{T} = \frac{\Delta T}{T}\Big|_{\mathrm{ISW}} + \frac{\Delta T}{T}\Big|_{LS}$$

ANISOTROPIC

ISOTROPIC
(COMPUTE
USING CAMB)

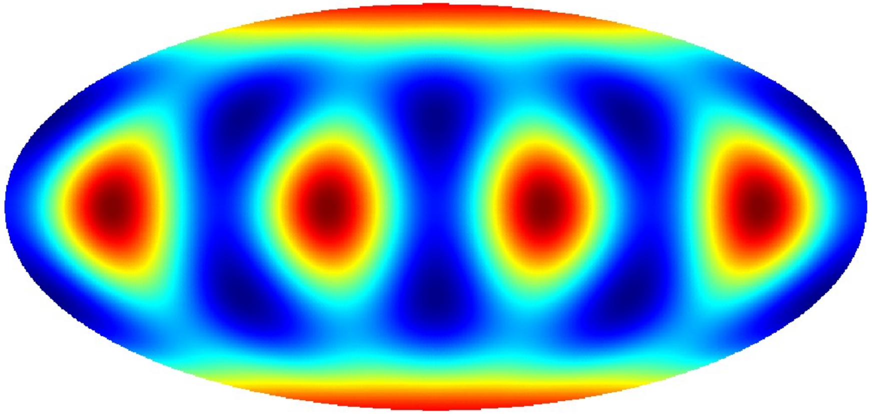
$$\frac{\Delta T}{T}\Big|_{\mathrm{ISW}}(\hat{n}) = \sum_{i,\ell} b_\ell^{(i)} X_\ell^{(i)}(R\hat{n})$$

SYMMETRY ADAPTED
SPHERICAL HARMONICS (SASH)
eg BETHE 1947

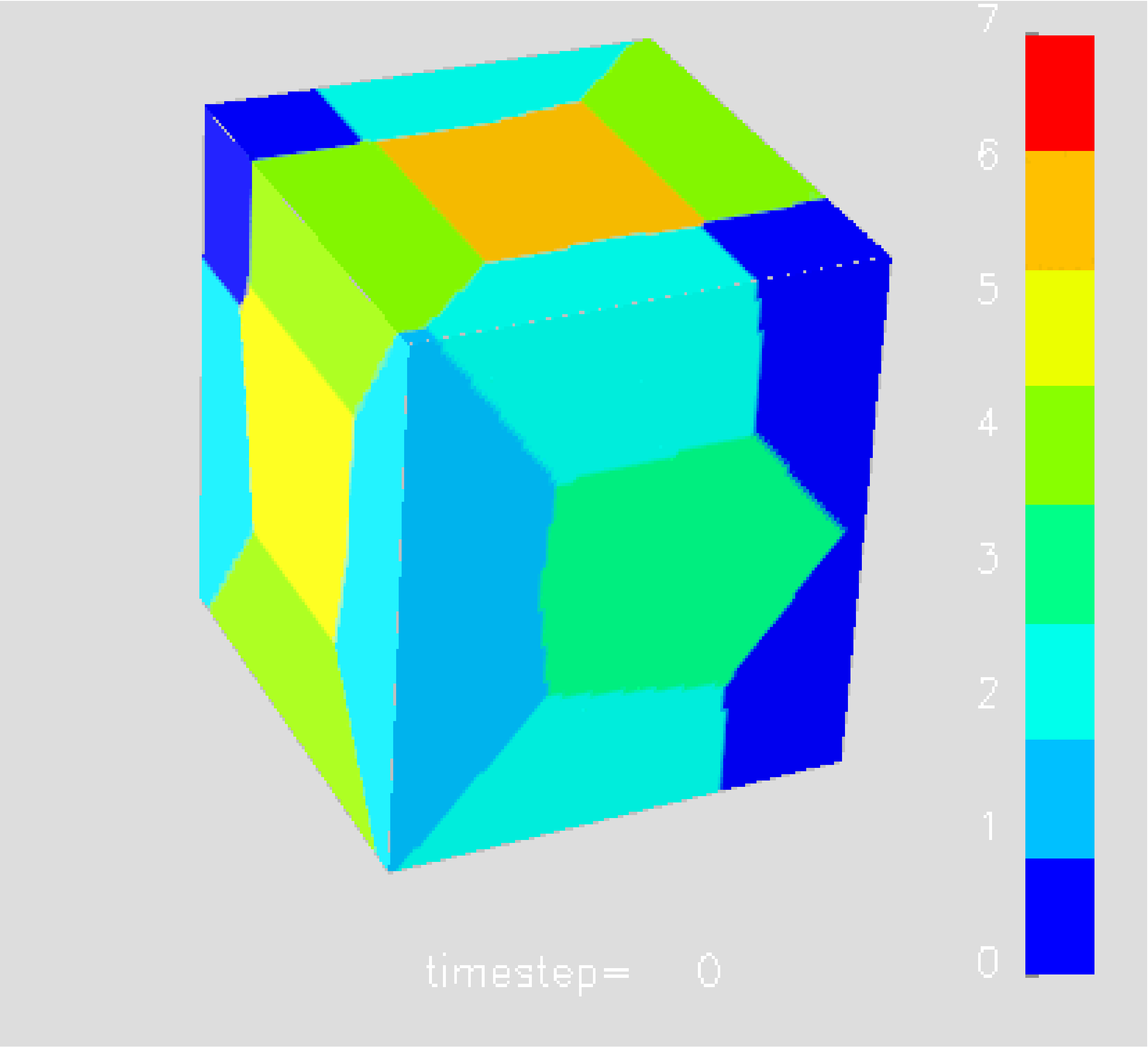
EXAMPLE OF SASH : l =4

$$X_4(\hat{n}) = AY_{4,0}(\hat{n}) + B\left(Y_{4,4}(\hat{n}) + Y_{4,-4}(\hat{n})\right)$$

$$Y_{\ell m}(R\hat{n}) = \sum D_{\ell m m'}(R) Y_{\ell m}(\hat{n})$$



DOMAIN WALL DOMINATED UNIVERSE



$$\rho_{\mathrm{dw}} = \frac{\sigma A}{V} \propto \frac{1}{a}$$

$\rightarrow$ W = -2/3

DIMENSIONS :

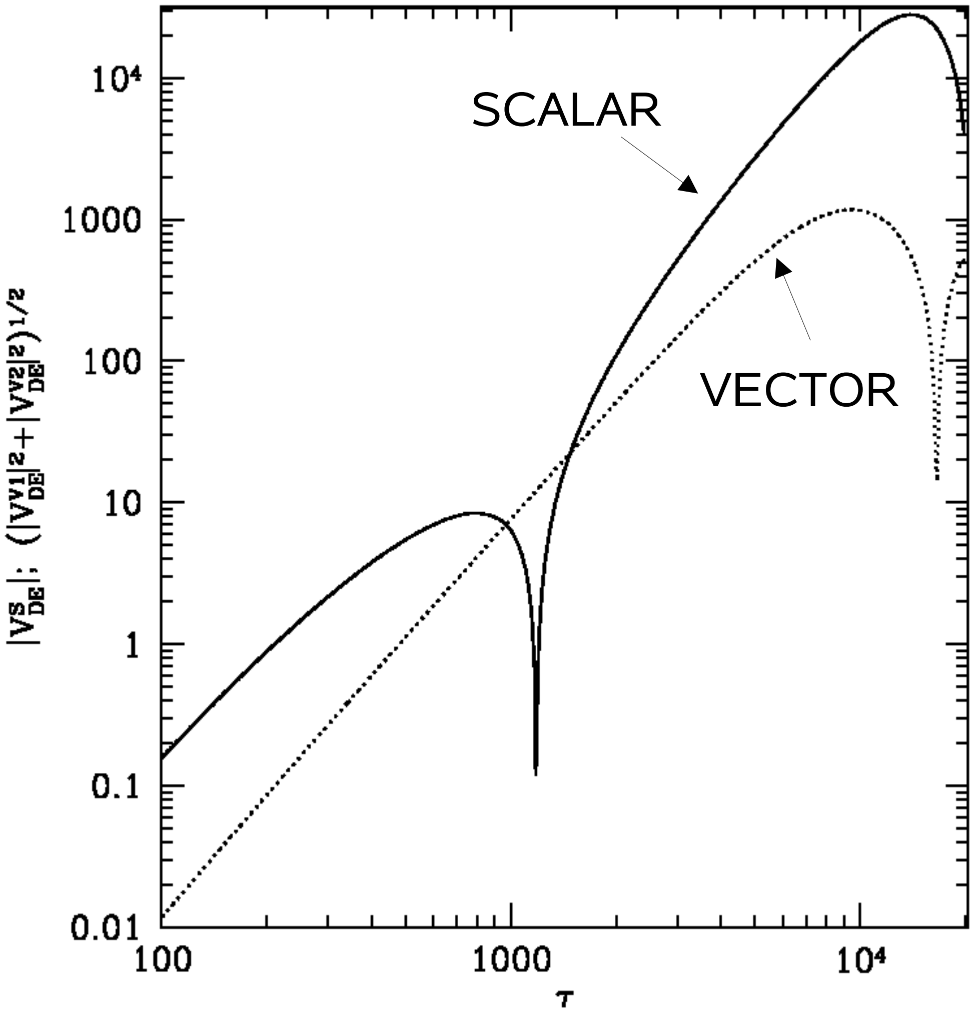
$\eta \sim 100\mathrm{keV}$

$L_{\mathrm{dw}} \sim 30\mathrm{pc}$

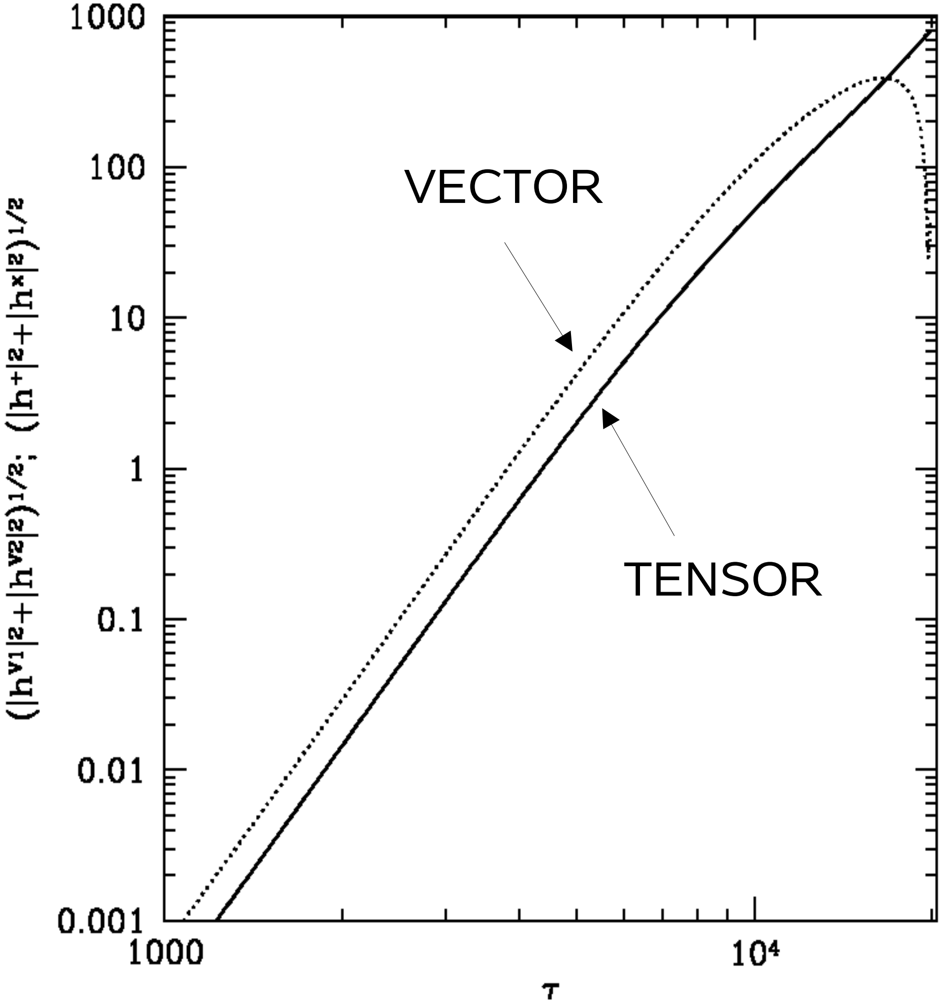
$\sigma \sim 10^{-9}\mathrm{gm}^{-2}$

TIME EVOLUTION :

$\theta = \pi/2$, $\varphi = \pi/8$, $\Delta\mu/\rho=0.01$, w=-2/3, k=0.001Mpc^{-1}

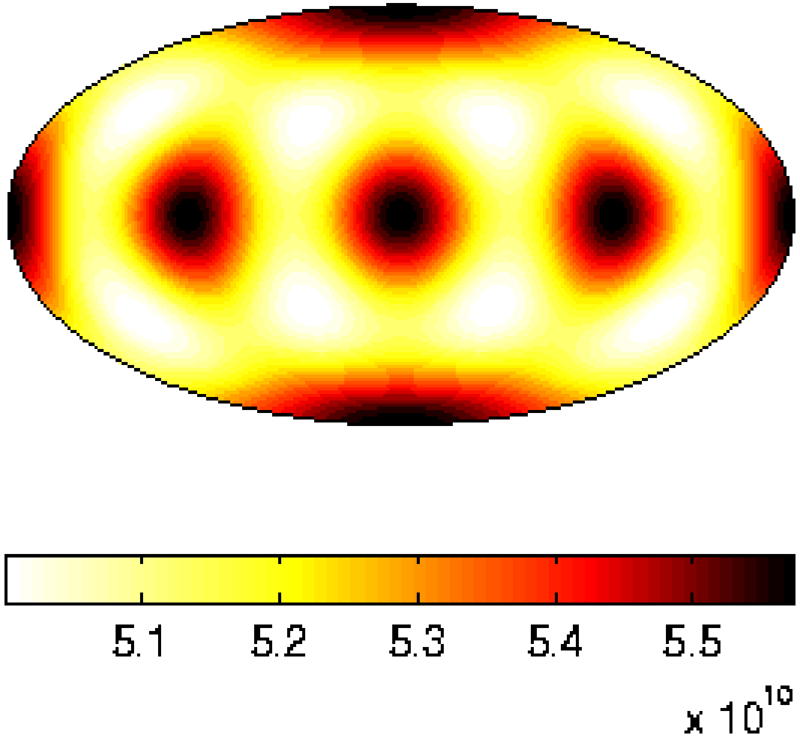


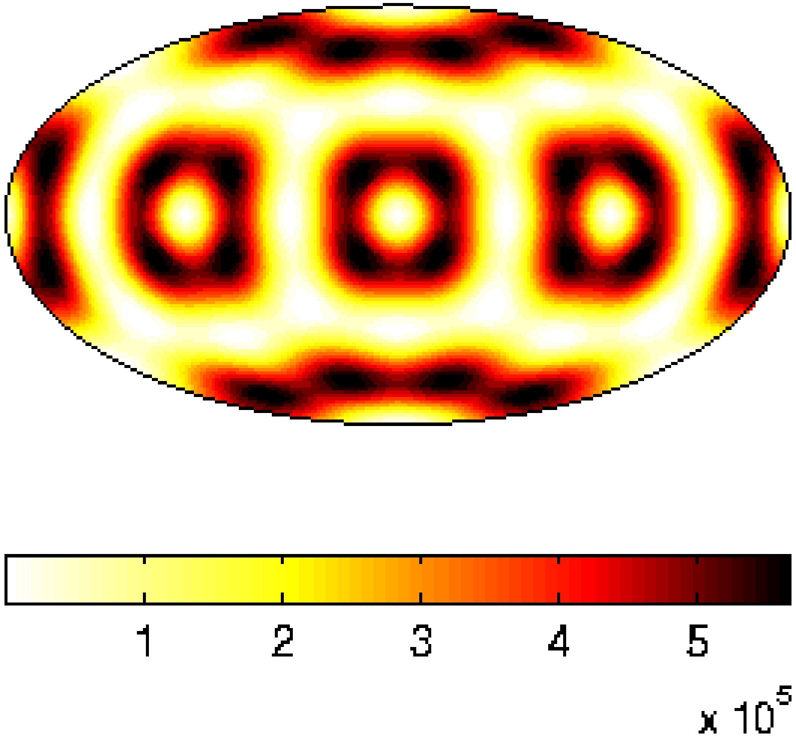
VELOCITY



METRIC PERTS

SPATIAL DISTRIBUTION : $k=0.001\mathrm{Mpc}^{-1}$





$$P(k) = |\delta_\mathrm{T}|^2$$

WHERE

$$\delta_\mathrm{T} = \Omega_\mathrm{DE}\delta_\mathrm{DE} + \Omega_\mathrm{c}\delta_\mathrm{c} + \Omega_\mathrm{r}\delta_\mathrm{r}$$

$$\delta_\mathrm{DE} = (1+w)\left(k\xi_\mathrm{DE} + \frac{1}{2}h\right)$$