Generalized Modified Gravity Models: instabilities and dynamical bounds

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with Mark Hindmarsh, JCAP

Introduction

- Cosmic acceleration

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  - ⋆ Cosmological constant

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  - ⋆ All terms compatible with this symmetry

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- It leads, in general, to a system of non-linear ODEs of fourth order in the scale factor [Mena, Santiago, Weller PRL 96 (2006)]

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- Squared Speed of propagation

$$s = B(t)/A(t)$$

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- Vector modes do not propagate [Hwang, Noh PRD 61 (2000)]

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- 2nd Eqn: $\phi - b\,\lambda^2 = R_{\rm GB}^2 = 24\beta^4 e^{4u}[u' + 1]$

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- Unviable if $b > 0$, as $0 \leq q_1 < 1$.

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- In cosmology $-\infty < \kappa < b$
- Unviable if $b \le \bar{b}(n)$, $(-\frac{1}{3} < \bar{b} \le -\frac{2}{9})$ as $\Delta = 0$

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- At $N \sim -20$ scalar modes become classically unstable, $c_0^2 = -\frac{5}{3} < 0$

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- No IC with no-ghosts modes