Dynamics of Linear Perturbations in Modified Gravity

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references: astro-ph/0611321, PRD'07

astro-ph/0607458, NJP'06, astro-ph/07.08.....

Syracuse University

f(R) Gravity

$$S = \frac{M_P^2}{2}\int dx^4\sqrt{-g}\,[R+f(R)] + \int d^4x\sqrt{-g}\mathcal{L}_m[\chi_i, g_{\mu\nu}]$$

$$\begin{cases} (1+f_R)R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}(R+f) + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)f_R = \frac{T_{\mu\nu}}{M_P^2} \\ \nabla_\mu T^{\mu\nu} = 0 \end{cases}$$

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NOT algebraic !

Background Viability

1. $f_{RR} > 0$ to have a stable high-curvature regime, to have a non-tachyonic scalar field

2. $1 + f_R > 0$ to prevent the graviton from turning into ghost

3. $f_R < 0$ negative, monotonically increasing function of R that asymptotes to zero from below

4. $|f_R^0| \leq 10^{-6}$ must be small at recent epochs to pass LGC

(Hu and Sawicki astro-ph/0705.1158)

$\Downarrow$

$$w_{eff} \simeq -1$$

(Dolgov & Kawasaki, Phys.Lett.B 573 (2003), Navarro et al. gr-qc/0611127, Sawicki and Hu astro-ph/0702278
Starobinsky astro-ph/0706.2041, Chiba, Smith, Erickcek astro-ph/0611867
Amendola et al. astro-ph/0603703-0612180, Amendola & Tsujikawa astro-ph/0705.0396)

Dynamics of Linear Perturbations in f(R) Gravity

Scalar perturbations in Conformal Newtonian gauge

$$ds^2 = -a^2(\tau)\,(1+2\Psi)\,d\tau^2 + a^2(\tau)\,(1-2\Phi)\,d\vec{x}^2$$

$$\begin{cases} T^0_0 = -\rho(1+\delta) \\ T^0_j = (\rho+P)v_j \\ T^i_j = (P+\delta P)\delta^i_j + \pi^i_j \end{cases}$$

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$$(\Psi - \Phi) = -\frac{f_{RR}}{1+f_R}\delta R$$

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New variables:

$$\begin{cases} \Phi_+ \equiv \dfrac{\Phi+\Psi}{2} \\ \chi \equiv f_{RR}\delta R \end{cases}$$

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$$(F \equiv 1 + f_R)$$

$$\begin{cases} \Phi'_+ = \dfrac{3}{2}\dfrac{a\Omega v}{HkF} - \left(1 + \dfrac{1}{2}\dfrac{F'}{F}\right)\Phi_+ + \dfrac{3}{4}\dfrac{F'}{F}\dfrac{\chi}{F} \\ \chi' = -\dfrac{2\Omega\Delta}{H^2}\dfrac{F}{F'} + \left(1 + \dfrac{F'}{F} - 2\dfrac{H'}{H}\dfrac{F}{F'}\right)\chi - 2F\Phi'_+ - 2F\left(1 + \dfrac{2}{3}\dfrac{k^2}{a^2H^2}\dfrac{F}{F'}\right)\Phi_+ \end{cases}$$

Sub-Horizon

CDM equation:

$$\delta'' + \left(1 + \frac{H'}{H}\right)\delta' + \frac{k^2}{a^2 H^2}\left(\Phi_+ - \frac{\chi}{2F}\right) = 0$$

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Einstein equations:

$$\Phi_+ \simeq \frac{3\Omega H^2}{2}\frac{a^2}{k^2}\frac{\delta}{F}$$

$$\chi \simeq \frac{k^2}{k^2 + 3a^2H^2F'/F}\frac{\Omega\delta a^2}{k^2}$$

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$$-\frac{3}{2}\underbrace{\frac{1}{F}\frac{1 + 4\frac{k^2}{a^2}\frac{f_{RR}}{F}}{1 + 3\frac{k^2}{a^2}\frac{f_{RR}}{F}}}\Omega\delta$$

time and scale dependent rescaling of Newton constant $\equiv G_{eff}$

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$$\frac{k^2}{a^2}\frac{f_{RR}}{F}$$

There is a scale associated with the extra d.o.f. :

$$\lambda_C \equiv \sqrt{\frac{f_{RR}}{F}} = \frac{1}{m_{f_R}}$$

$$-\frac{3}{2}\underbrace{\frac{1}{F}\frac{1+4\frac{k^2}{a^2}\frac{f_{RR}}{F}}{1+3\frac{k^2}{a^2}\frac{f_{RR}}{F}}}\Omega\delta$$

time and scale dependent rescaling of Newton constant $\equiv G_{eff}$

Sub-Horizon

Below this scale there is a significant departure from std GR and a scale dependence in the behavior of perturbations

- $\lambda \gg \lambda_C$

$$\chi \simeq 0, \quad G_{eff} \simeq \frac{G}{F}$$

$$\Psi \simeq \Phi$$

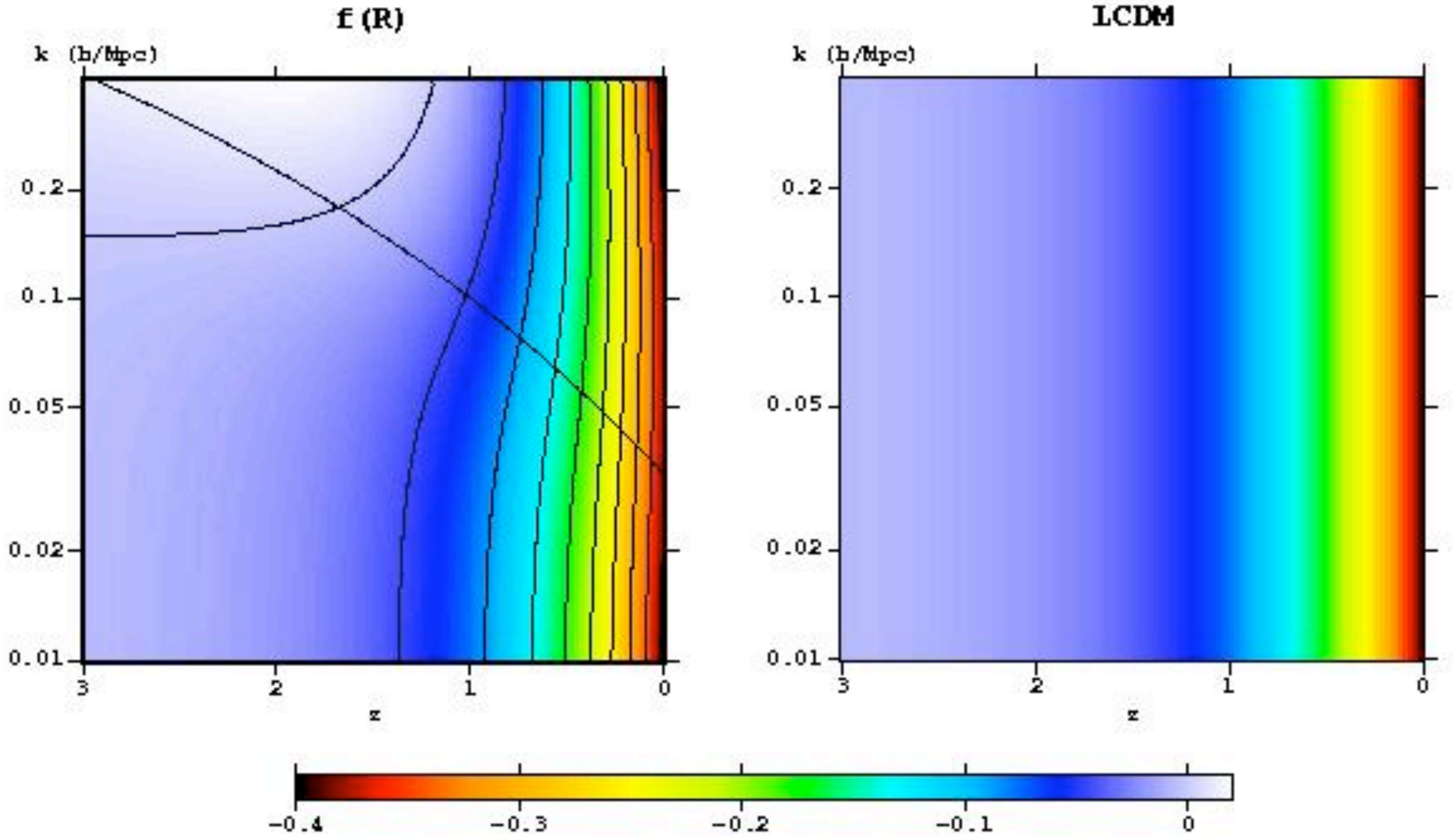
- $\lambda \ll \lambda_C$

$$\chi \simeq -\frac{2}{3} F \Phi_+, \; G_{eff} \simeq \frac{4}{3}\frac{G}{F}$$

$$\Psi \simeq 2\Phi$$

$$w_{eff} = -1$$
$$f_R^0 = -10^{-4}$$

$$\frac{d\Phi_+}{dz}$$



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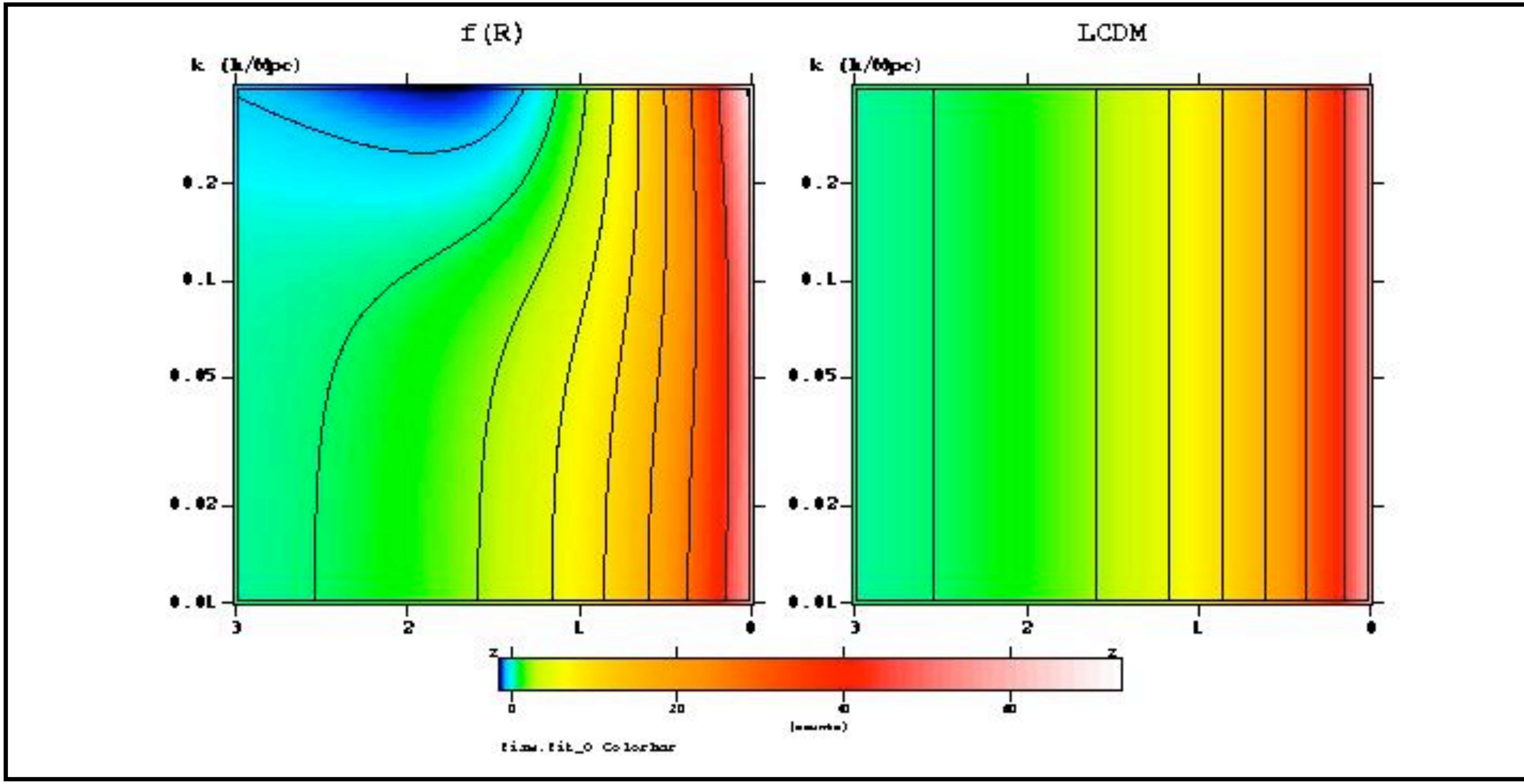
$$-\frac{d\Phi_+}{dz} \cdot \frac{\Delta(k,z)}{\Delta(k,z_i)}$$



Dynamics of Linear Perturbations in Modified Source Gravity

$$S = \int dx^4 \sqrt{-g} \left[\frac{M_P^2}{2} e^{2\psi} R + 3e^{2\psi} (\nabla\psi)^2 - U(\psi) \right] + s_m[g, \chi_i]$$

Carroll, Sawicki, Silvestri, Trodden
astro-ph/0607458, NJP'06

$$e^{2\psi} G_{\mu\nu} = \kappa^2 \left(T_{\mu\nu} + T_{\mu\nu}^{(\psi)} \right)$$

$$\psi = \psi(T) = \psi(\rho_m) \longrightarrow G_{\mu\nu} = \tilde{T}_{\mu\nu}(\rho)$$

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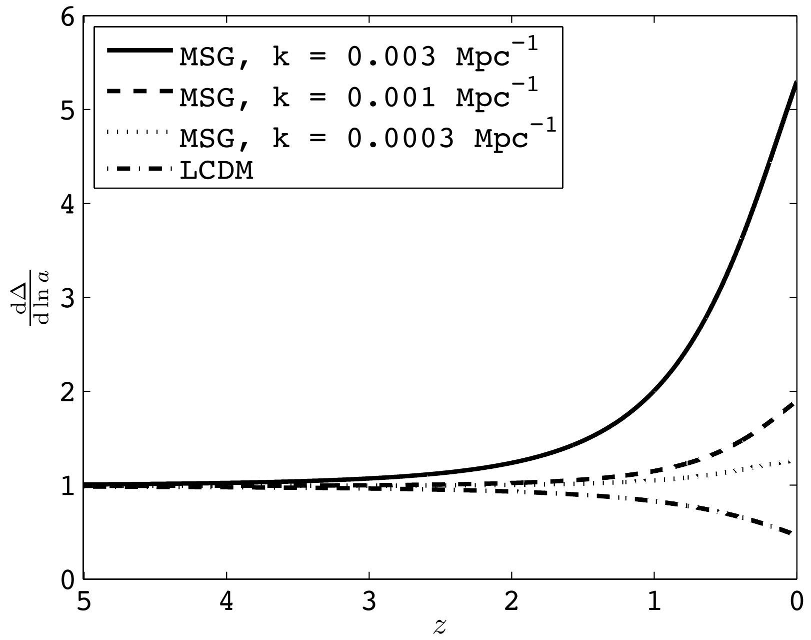
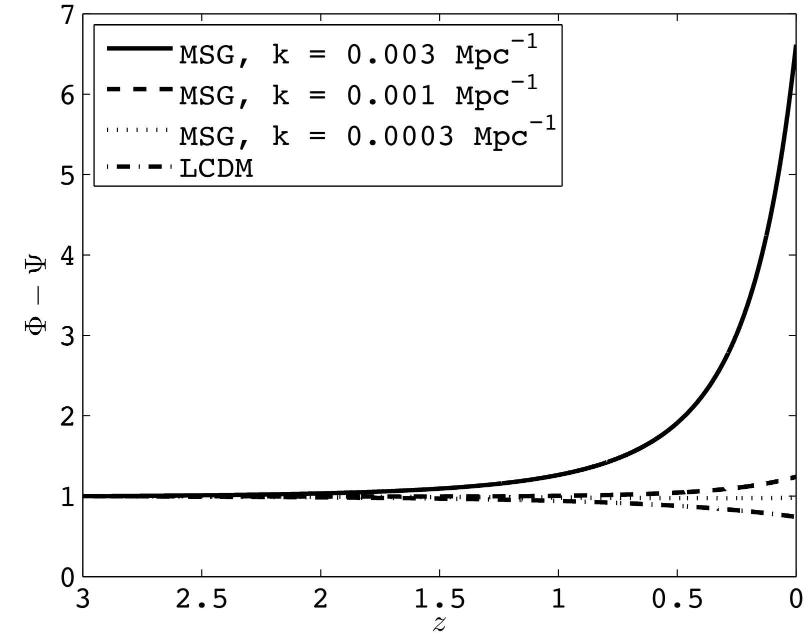
Linear perturbations

$$\Phi - \Psi = -\frac{2}{3}\frac{d\psi}{dlna}\delta$$

$$\frac{k^2}{a^2}\Psi = -\left[\frac{3\Omega e^{-2\psi}}{2}\left(1 + \frac{2}{3}\frac{d\psi}{dlna}\right) - \frac{2}{3}\frac{k^2}{a^2}\frac{d\psi}{dlna}\right]\delta$$





- scale-dependent runaway growth
- rapid structure formation drives the growth of gravitational potentials
- the ISW effect is enhanced at the lowest multipoles
- negative LSS-ISW correlation

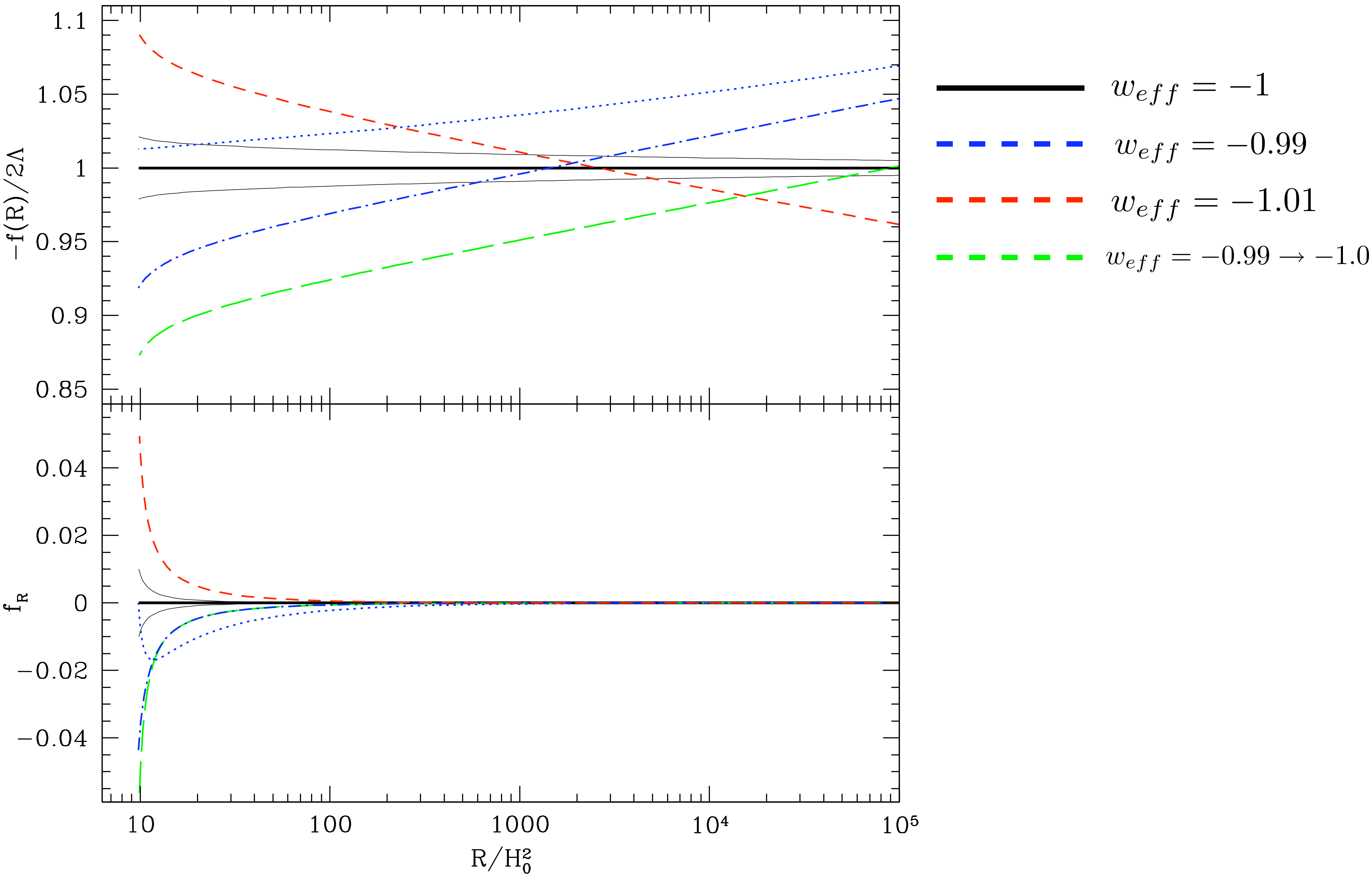
CONCLUSIONS

For f(R) and MSG models that reproduce the desired background evolution, we investigated the dynamics of linear perturbations, finding:

- transition scale related to new d.o.f. mass scale
- effective shear ⟹ slip between metric potentials Ψ and Φ
- modified, scale-dependent evolution of the metric potentials ⟹ modified ISW signal
- modified, scale-dependent evolution of matter perturbations

The ISW, its correlation with LSS and Weak Lensing might be very useful probes of modifications of gravity

THANK YOU!


$-f(R)/2\Lambda$
1.1
1.05
1
0.95
0.9
0.85
f_R
0.04
0.02
0
−0.02
−0.04
10
100
1000
10^4
10^5
R/H_0^2
$w_{eff} = -1$
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