Conditions for the cosmological viability of f(R) dark energy models

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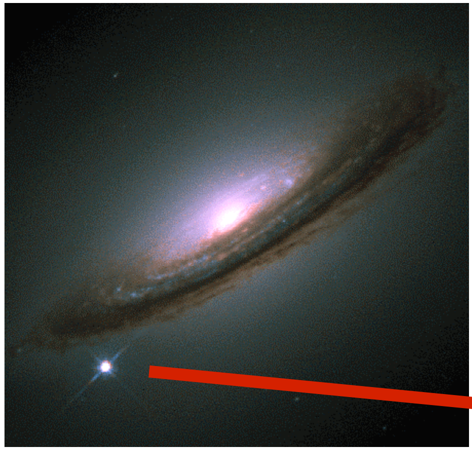
Dark energy

Equation of state

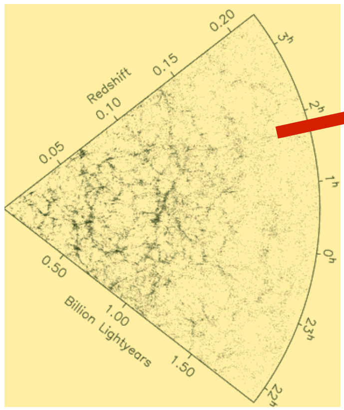
$\Omega_{DE} \approx 0.7$ ← $w_{DE} \approx -1$

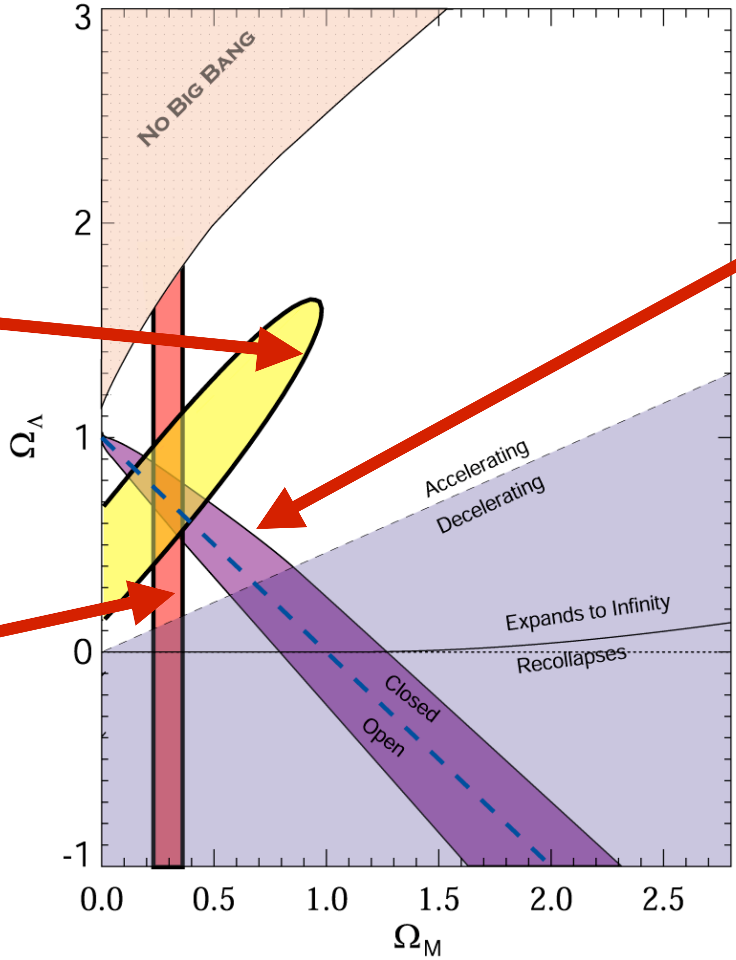
$\Omega_{DM} \approx 0.25$ ← $w_{DM} \approx 0$

SNe Ia

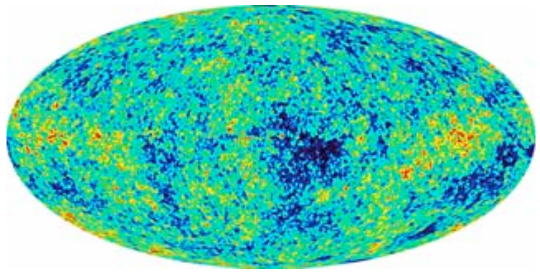


LSS





CMB



The current universe is accelerating!

What is the origin of dark energy?

There are two approaches to dark energy.

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

 **-equations**

(i) Changing gravity

f(R) gravity models,
Scalar-tensor models,
DGP braneworld,
…..

(ii) Changing matter

Quintessence,
K-essence,
Tachyon,
Chaplygin gas,
…..

Generally these models give rise to a very light mass:

$$m_\phi \approx H_0 \approx 10^{-33} eV$$

‘Changing gravity’ models

f(R) gravity, scalar-tensor gravity, DGP braneworld models,..

Dark energy may originate from some modifications from Einstein gravity.

The simplest model: **f(R) gravity**

$$S = \int d^4x\sqrt{-g}\left[f(R)/2 \ + L_m\right]$$

ΛCDM **model:** $f(R) = R - 2\Lambda$

Starobinsky’s inflation model: $f(R) = R + \alpha R^2$

→ **Used for early universe inflation**

f(R) modified gravity models can be used for dark energy ?

Example of f(R) dark energy models

M. Turner

I want to explain dark energy without using scalar fields…

$$f(R) = R - \frac{\mu^{2(n+1)}}{R^n}$$

Capoziello, Carloni and Troisi (2003)
Carroll, Duvvuri, Troden and Turner (2003)

It is possible to have a late-time acceleration as the second term becomes important as R decreases.

In the small R region we have

$$a \propto t^q, \quad q = \frac{(2n+1)(n+1)}{(n+2)} \qquad w_{\rm DE} = -1 + \frac{2(n+2)}{3(2n+1)(n+1)}$$

When $n = 1$ we have $q = 2$ and $w_{\rm DE} = -2/3$.

However this model does not have a standard matter era prior to the late-time acceleration.

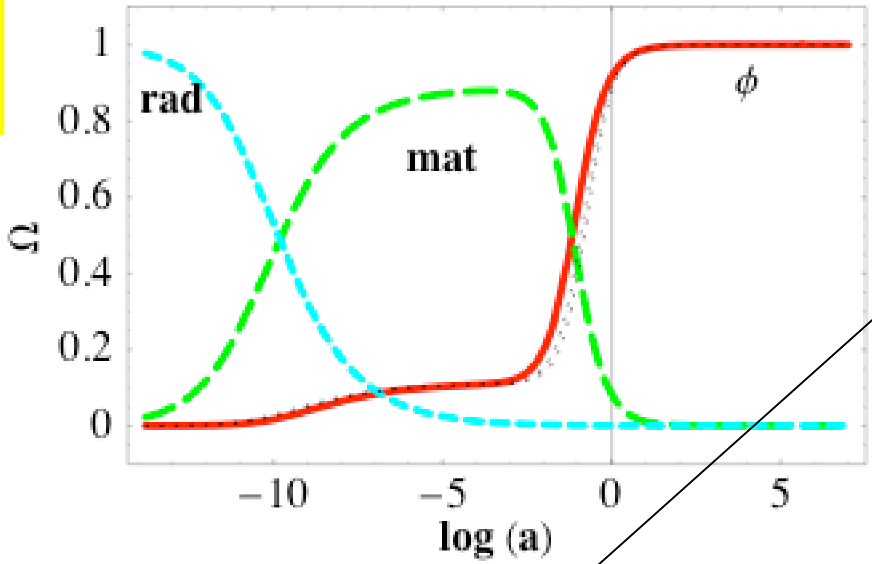
L. Amendola, D. Polarski and S.T., PRL98, 131302 (2007)

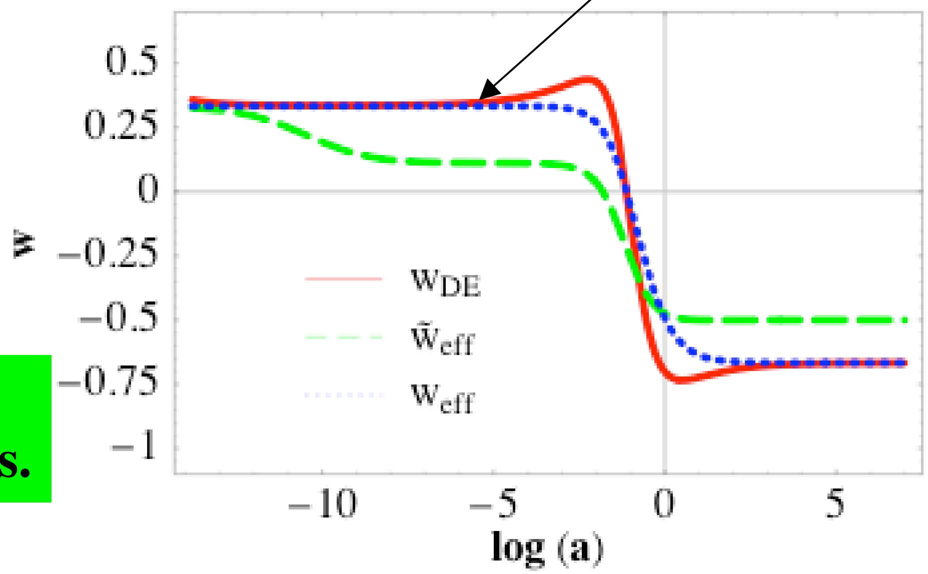
$$f(R) = R - \frac{\mu^{2(n+1)}}{R^n}$$

Dark energy is strongly coupled to dark matter in this model.

The absence of a standard matter epoch.

Incompatible with observations.





$w_{eff} = 1/3$

The scale factor evolves as

$a \propto t^{1/2}$

What are general conditions for the cosmological viability of f(R) dark energy models?

L. Amendola, D. Polarski, R. Ganouji and S.T., PRD75, 083504 (2007)

General analysis

$$S = \int d^4x\sqrt{-g}\left[f(R)/2\kappa^2 + L_m + L_{rad}\right]$$

$$\kappa^2 = 8\pi G$$

In the FRW background we have

$$3FH^2 = \kappa^2(\rho_{\rm m} + \rho_{\rm rad}) + \frac{1}{2}(FR - f) - 3H\dot{F}\,,$$

$$-2F\dot{H} = \kappa^2\left(\rho_{\rm m} + \frac{4}{3}\rho_{\rm rad}\right) + \ddot{F} - H\dot{F}\,,$$

$$F \equiv \frac{{\rm d}f}{{\rm d}R}\,.$$

We wish to carry out general analysis without specifying the form of f(R).

Autonomous equations

See the dark energy review
E. Copeland, M. Sami and S.T. (2006)

We introduce the following variables:

$$x_1 = -\frac{\dot{F}}{HF}, \quad x_2 = -\frac{f}{6FH^2}, \quad x_3 = \frac{R}{6H^2} = \frac{\dot{H}}{H^2} + 2, \quad x_4 = \frac{\kappa^2 \rho_{\rm rad}}{3FH^2}.$$

Then we obtain $\Omega_{\rm m} \equiv \frac{\kappa^2 \rho_{\rm m}}{3FH^2} = 1 - x_1 - x_2 - x_3 - x_4$ **and**

$$\frac{{\rm d}x_1}{{\rm d}N} = -1 - x_3 - 3x_2 + x_1^2 - x_1 x_3 + x_4 \,,$$

$$\frac{{\rm d}x_2}{{\rm d}N} = \frac{x_1 x_3}{m} - x_2(2x_3 - 4 - x_1) \,,$$

$$\frac{{\rm d}x_3}{{\rm d}N} = -\frac{x_1 x_3}{m} - 2x_3(x_3 - 2) \,,$$

$$\frac{{\rm d}x_4}{{\rm d}N} = -2x_3 x_4 + x_1 x_4 \,,$$

where $N = \ln(a)$ **,**

$$m(r) = \frac{R f_{,RR}}{f_{,R}}$$ **and** $$r = -\frac{R f_{,R}}{f} = \frac{x_3}{x_2}$$

The above equations are closed.

ΛCDM **model:** $f(R) = R - 2\Lambda$ ⟹ $m(r) = \frac{Rf_{,RR}}{f_{,R}} = 0$

The parameter m(r) characterizes the deviation from the ΛCDM model.

The cosmological dynamics is well understood by the geometrical approach in the (r, m) plane.

(i) Matter point: P_M

P_M : $(r, m) \approx (-1, 0)$ $w_{eff} = -\frac{m}{1+m}$

The existence of the viable saddle matter epoch requires

$m(r) > 0,$ $-1 < m'(r) < 0$ **at** $r \approx -1$

(ii) De-sitter point P_A

P_A : $r = -2$ $w_{eff} = -1$

For the stability of the de-Sitter point, we require $0 < m(r = -2) < 1$

Viable trajectories

and S.T. (arXiv:0705.396)

(i) ΛCDM

(ii) $f(R) = (R^b - \Lambda)^c$

$bc \approx 1$

(iii) $f(R) = R - \alpha R^n$

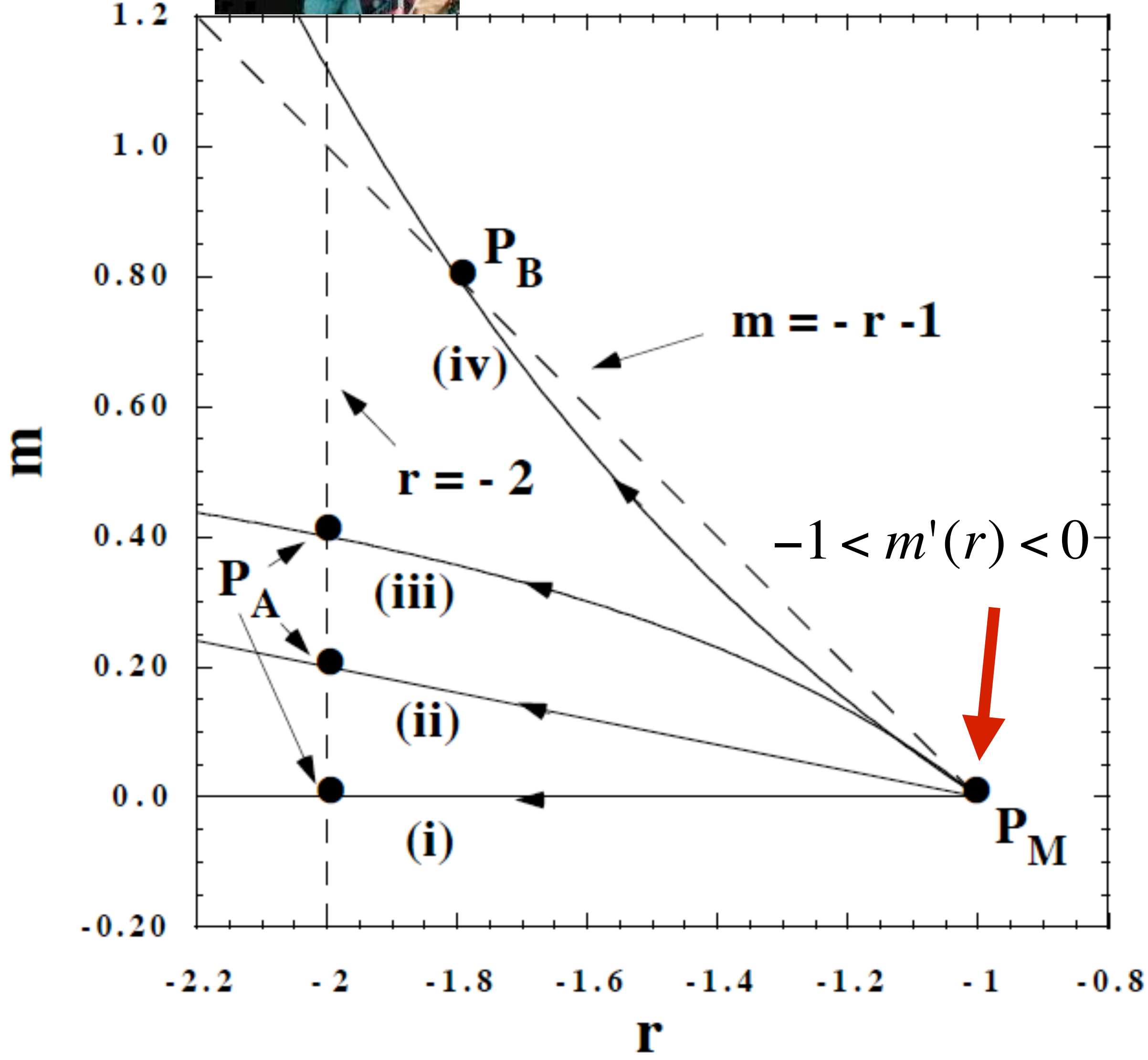
$\alpha > 0, \quad 0 < n < 1$

(iv)

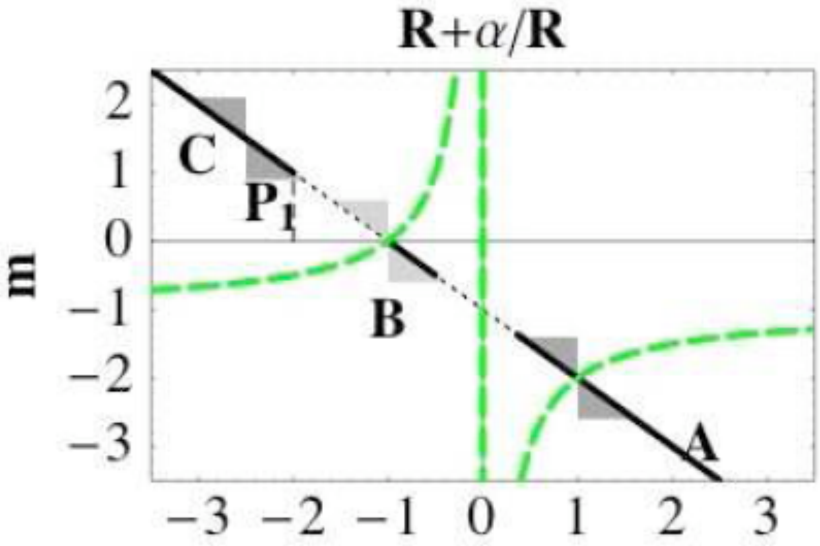
$m(r) = -C(r+1)(r^2 + ar + b)$

Constant m model:

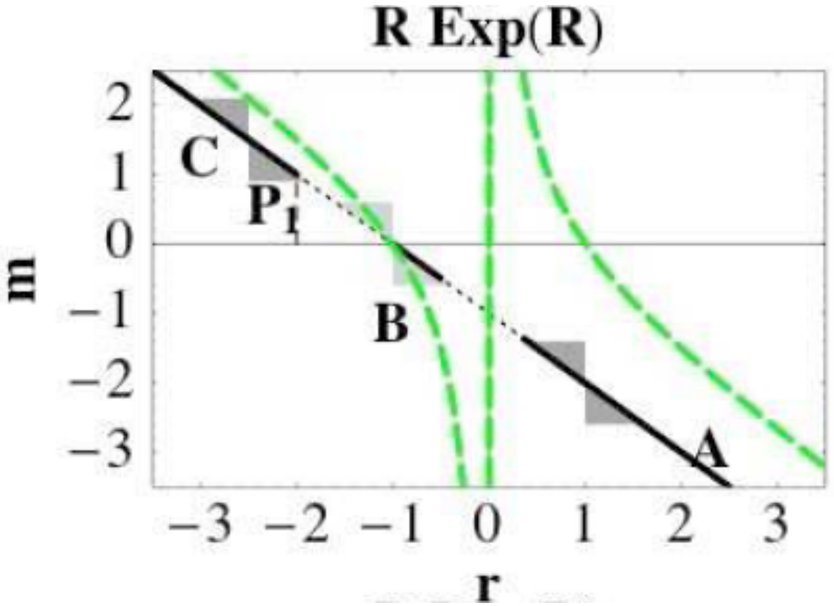
$$f(R) = R^{1+m} - \Lambda$$



Examples of non-viable models



$$m = -\frac{1+r}{r}$$



$$m = -r + \frac{1}{r}$$

Are there cosmologically viable models satisfying local gravity constraints ?

Effective gravitational constant in f(R) models is

$$G_{eff} = G[1 + e^{-M_\phi l}]$$

$$M_\phi^{\ 2} = V_{,\phi\phi} \approx \frac{1}{3 f_{,RR}}$$

l **: length scale at which gravity experiments are carried out.**

To satisfy LGC, we require $M_\phi l >> 1$

$$\Rightarrow \quad m(R_S) << \left(\frac{l}{H_0^{\ -1}}\right)^2 \frac{\rho_S}{\rho_0}$$

R $_s$ **is the curvature on the local structure.**

In the case of Cavendish-type experiment ($\rho_s = 10^{17} \rho_0$ **) , we have**

$$m(R_s) << 10^{-43}$$

The quantity m(R) needs to be very small in the high-curvature region: $R >> R_0 \approx H_0^{\ 2}$

Models that satisfy local gravity constraints

Hu and Sawicki: $$f(R) = R - \lambda R_0 \frac{(R/R_0)^{2n}}{(R/R_0)^{2n}+1}$$

Starobinsky: $$f(R) = R - \lambda R_0 \left[1 - \left(1 + R^2/R_0^{\ 2}\right)^{-n}\right]$$

$n, \lambda > 0$

$R_0 \approx H_0^{\ 2}$

Hu

Starobinsky

$f(R=0)=0$ ➡ **Cosmological constant disappears in a flat space.**

$R >> R_0$ ➡ $f(R) \approx R - \lambda R_0$ **and** $m(r) = C(-r-1)^{2n+1}$

➡ **The LGC is** $(\rho_s/\rho_0)^{2(n+1)} >> (H_0^{\ -1}/l)^2$

➡ $n \geq 2$ **is sufficient in most of experiments.**

These models also satisfy the criterion of cosmological viability and can evolve from the matter point P_M to the de-Sitter point P_A

Another viable model was proposed Appleby and Battye (next talk!).

Conclusions

1. **We derived conditions for the cosmological viability of f(R) dark energy models. This is useful to exclude some of the models, e.g.,** $f(R) = R - \mu^{2(n+1)}/R^n$
2. **It is also possible to construct cosmologically viable models that satisfy local gravity constraints. These behave as**

$$m(r) = C(-r-1)^{2n+1} \quad (n \geq 2) \quad \textbf{for} \quad R >> R_0$$

→ **Now I am considering observational constraints on these models.**