The three-point correlator and the second order Klein-Gordon equation

Cosmo07

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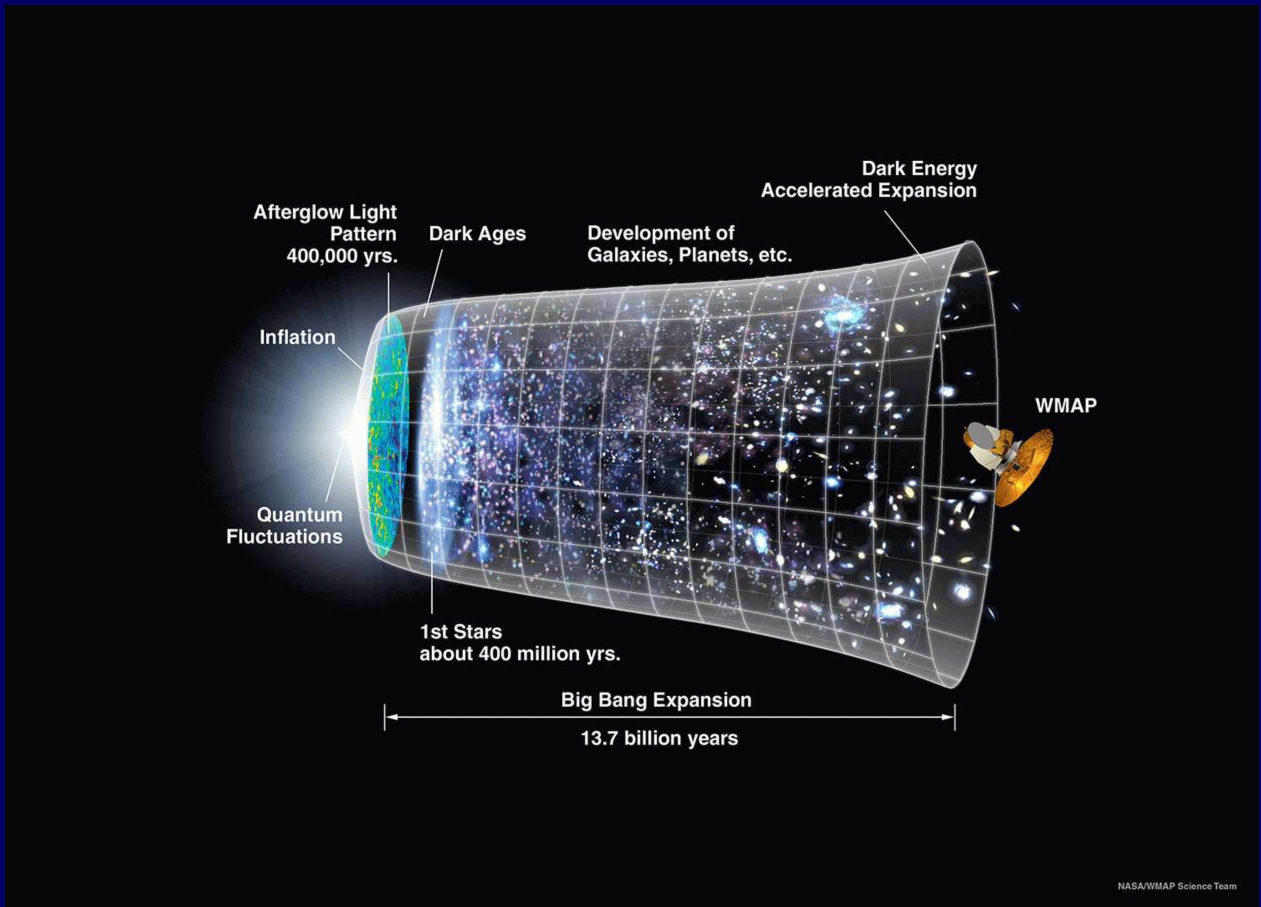
work in progress with David Lyth (Lancaster) and David Seery (QMUL)

The cosmological standard model . . .

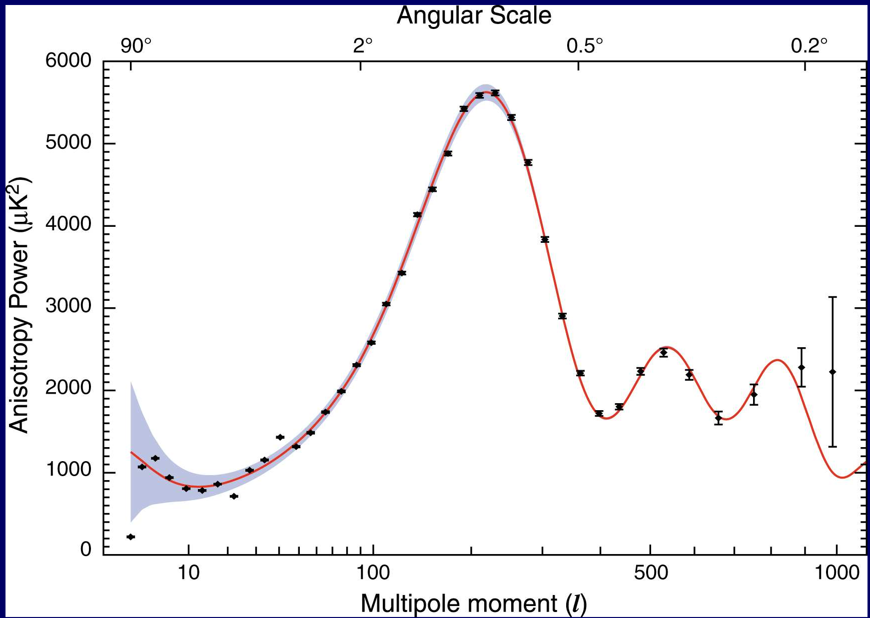
. . . goes like this:



- early period of accelerated expansion, *inflation*, primordial perturbations are sourced, dominated by scalar field φ
- then period of radiation domination
- then matter domination
- at present: accelerated expansion, due to dark energy (possibly scalar field(s) again)

Standard approach: first order

getting information from the data:



- calculate two-point correlator or power spectrum $\langle \delta\varphi_1 \delta\varphi_1 \rangle$ (using Klein-Gordon equation for field fluctuation $\delta\varphi_1$)
- translate into conserved quantity sourcing CMB anisotropies, e.g. curvature perturbation on uniform density hypersurfaces, ζ_1 (everybody's favourite . . .) $\langle \zeta_1 \zeta_1 \rangle$

- feed into Einstein equations and Boltzmann solver
- get theoretical predictions for CMB anisotropies
- compare with observations

Higher order observables

Why stop at first order?

- *until recently*: data not good enough
- *now*: amount and accuracy of data allow calculation of higher order observables
- *new data sets* on the “horizon”: 21cm anisotropy data from e.g. LOFAR complementary to CMB data

Second order theory already sufficient to calculate higher order observables consistently:

study e.g. three-point correlator

$$\langle \delta\varphi\delta\varphi\delta\varphi \rangle$$

No f_{NL}

- Could translate three-point correlator $\langle \delta\varphi\delta\varphi\delta\varphi \rangle$ into single parameter f_{NL} describing the non-gaussianity, defined as

$$\zeta = \zeta_{\mathrm{L}} + f_{\mathrm{NL}} \left(\zeta_{\mathrm{L}}^2 - \langle \zeta_{\mathrm{L}}^2 \rangle \right)$$

Komatsu and Spergel 2000

where ζ_{L}: linear (gaussian part) of perturbation

- In the following focus on $\langle \delta\varphi\delta\varphi\delta\varphi \rangle$

- See however talks by C. Hidalgo and D. Lyth (and others . . .) for insights into f_{NL} etc.

Intermezzo: standard calculation

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- Standard calculation for three-point correlator:
  using the 'Dirac' or 'interaction' picture (based *directly* on action)
  - Pioneered in single field case

    Maldacena 2002
  - extended to multi-field case

    Rigopoulos, Shellard and van Tent 2005
    Seery and Lidsey 2005
- Different approach: Heisenberg picture, using the field equations
- Why?
  - why not?
  - need field equations anyway (to calculate evolution of perturbations from $\sim$ horizon exit, particularly in the multi-field case)
  - ultimately: might be easier to implement numerically (hunch, or prejudice?)

Calculating $\langle\delta\varphi\delta\varphi\delta\varphi\rangle$

Calculate $\langle\delta\varphi\delta\varphi\delta\varphi\rangle$ using perturbation theory:

- split field into homogeneous background and perturbation:

$$\varphi(x^\mu) = \varphi_0(\eta) + \delta\varphi_1(x^i,\eta) + \frac{1}{2}\delta\varphi_2(x^i,\eta)$$

- Note:

$$\langle\delta\varphi_1\delta\varphi_1\delta\varphi_1\rangle = 0$$

$\Rightarrow$ lowest order non-zero contribution is

$$\langle\delta\varphi_1\delta\varphi_1\delta\varphi_2\rangle$$

Technical issues:

- Choose model $\Rightarrow$ simplest case: single field de Sitter
- use gauge-invariant variables: field fluctuation on uniform curvature hypersurfaces at first and second order

Malik and Wands 2003

Calculating $\langle \delta\varphi_1 \delta\varphi_1 \delta\varphi_2 \rangle$

"All" we need now: $\delta\varphi_1$ and $\delta\varphi_2$

working throughout in Fourier space, using conformal time η, we get at first order

- Klein-Gordon equation at first order (de Sitter):

$$\delta\varphi_1'' - \frac{2}{\eta}\delta\varphi_1' + k^2\delta\varphi_1 = 0$$

- get solution

$$\delta\varphi_1(\eta, k) = H\sqrt{\frac{1}{2k^3}}\,(i - k\eta)\,e^{-ik\eta}$$

  (choosing suitable initial conditions)

- alternatively: construct Klein-Gordon Green function, $G_k(\eta, \eta')$, from solutions above

Calculating $\langle \delta\varphi_1 \delta\varphi_1 \delta\varphi_2 \rangle$ continued ...

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- Second order Klein-Gordon equation

$$\delta\varphi_2{}''(\eta, k^i) + 2\mathcal{H}\delta\varphi_2{}'(\eta, k^i) + k^2\delta\varphi_2(\eta, k^i) = -\int \frac{d^3p d^3q}{(2\pi)^3}\delta(k^i - p^i - q^i) F(p^i, q^i, \eta)$$

where the interaction term $F(\eta, p^i, q^i)$ is quadratic in $\delta\varphi_1$,

$$\begin{aligned} F(\eta, p^i, q^i) &= \Bigg\{ a^2 U_{,\varphi\varphi\varphi}\delta\varphi_1(p^i)\delta\varphi_1(q^i) \\ &+16\pi G\frac{\varphi_0'}{\mathcal{H}}\left[\frac{1}{3}\left(3q^2 - p_l q^l + \frac{k^i p_i k^j q_j}{k^2}\right)\delta\varphi_1(p^i)\delta\varphi_1(q^i) \right. \\ &\left. +\mathcal{H}\left(1 + \frac{p_l q^l}{q^2} + \frac{q^2 + p_l q^l}{k^2}\right)\delta\varphi_1(p^i)'\delta\varphi_1(q^i) + \frac{p_l q^l}{q^2}\delta\varphi_1(p^i)'\delta\varphi_1(q^i)'\right]\Bigg\} \end{aligned}$$

Malik 2006

- solve via Klein-Gordon Green function

$$\delta\varphi_2(\eta, k^i) = -i\int_{-\infty}^{\eta} d\eta'\ a(\eta')^2 G_k(\eta, \eta') \int \frac{d^3p\ d^3q}{(2\pi)^6}\ (2\pi)^3\delta(k^i - p^i - q^i) F(p^i, q^i)\delta\varphi_1(p, \eta')\delta\varphi_1(q, \eta')$$

Calculating $\langle \delta\varphi_1 \delta\varphi_1 \delta\varphi_2 \rangle$ continued ...

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- After "some more algebra" finally arrive at

$$\begin{aligned}&\langle \delta\varphi(k_1^i)\delta\varphi(k_2^i)\delta\varphi(k_3^i)\rangle \\ &= -\frac{i}{6}(2\pi)^3\delta(k_1^i + k_2^i + k_3^i)\int_{-\infty(1-i\delta)}^{\eta} d\eta\ a(\eta')^2 G_{k_1}(\eta,\eta')\left[F(p,q)+F(q,p)\right] G_{k_2}(\eta,\eta') G_{k_3}(\eta,\eta') \\ &\quad + \text{c.c.} + \text{combs.}\end{aligned}$$

where 'c.c.': complex conjugate
combs.: combinations of wave vectors k_n^i

- next step: evaluate integral

- However: found a "slight" problem ...

Calculating $\langle \delta\varphi_1 \delta\varphi_1 \delta\varphi_2 \rangle$ continued . . .

. . . we get two different interaction terms $F(p, q)$ using:

- either full second order field equations to get Klein-Gordon equation (and hence $F(p, q)$)

Malik 2006

- or third order action and then constraints to get Klein-Gordon equation

Seery 2007

$\Rightarrow$ we are going to publish result as soon as we found source of discrepancy

. . .

Conclusions

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- two different approaches – Dirac picture (based on action) and Heisenberg picture (based on field equations) – both work
- we have shown that both approaches give similar results, difference being algebraic
- paper on how to calculate three-point correlator using Klein-Gordon equation to be published *very soon*!
- for details on 2nd order Klein-Gordon equation etc., see `astro-ph/0610864v4`