Dynamics of super-inflation in Loop Quantum Cosmology

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- **Superinflation in LQC**
- **Scaling solution**
- **Scalar power spectrum**
- **Number of e-folds**

Copeland, Mulryne, Nunes, Shaeri (2007)
Mulryne, Nunes (2006)

1. Loop Quantum Gravity

Theory of Gravity based on Ashtekar's variables which brings GR into the form of a gauge theory.

- Densitized triad E_i^a and $E_i^a E_i^b = q^{ab} q$
- SU(2) connection $A_a^i = \Gamma_a^i - \gamma K_a^i$

Γ_a^i - spin connection; K_a^i - extrinsic curvature; γ - Barbero-Immirzi parameter.

Quantization proceeds by using as basic variables holonomies,

$$h_e = \exp \int_e \tau_i A_a^i \dot{e}^a dt$$

in edges e, and fluxes,

$$F = \int_S \tau^i E_i^a n_a d^2 y$$

in spacial surfaces S.

2. Loop Quantum Cosmology

Focuses on minisuperspace settings with finite degrees of freedom.

Evolution of the Universe can be divided into 3 distinct phases:

- Quantum phase: $a < a_i$ and $a_i^2 = \gamma \ell_{\rm pl}^2$.

Described by a difference equation;

- Semi-classical phase: $a_i < a < a_*$.

Continuous evolution but equations modified due to non-perturbative quantization effects;

- Classical phase: $a > a_*$.

Usual continuous cosmological equations.

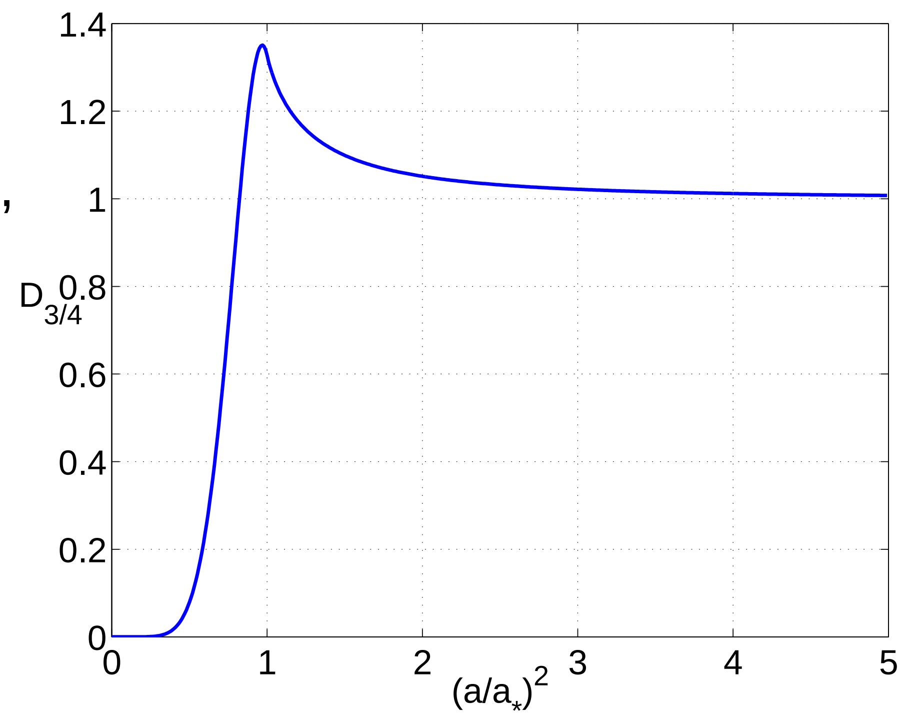
3. Inverse volume operator

Classically: $d(a) = a^{-3}$

LQC: $d_{l,j}(a) = D_l(q) a^{-3}$ where $q = \left(\frac{a}{a_*}\right)^2$

for $a \ll a_*$, $D(q) \approx D_\star a^n$, $6 < n < \infty$

for $a \gg a_*$, $D(q) \approx 1$



4. Modified semi-classical equations

1. Modified Friedmann equation is obtained from the Hamiltonian constraint $\mathcal{H} = 0$,

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{1}{a^2} = \frac{S}{3}\left(\frac{1}{2}\frac{\dot{\phi}^2}{D} + V(\phi)\right)$$

2. Modified Klein-Gordon equation is obtained from the Hamilton's equations

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\left(1 - \frac{1}{3}\frac{d\ln D}{d\ln a}\right)\dot{\phi} + D\frac{dV}{d\phi} = 0$$

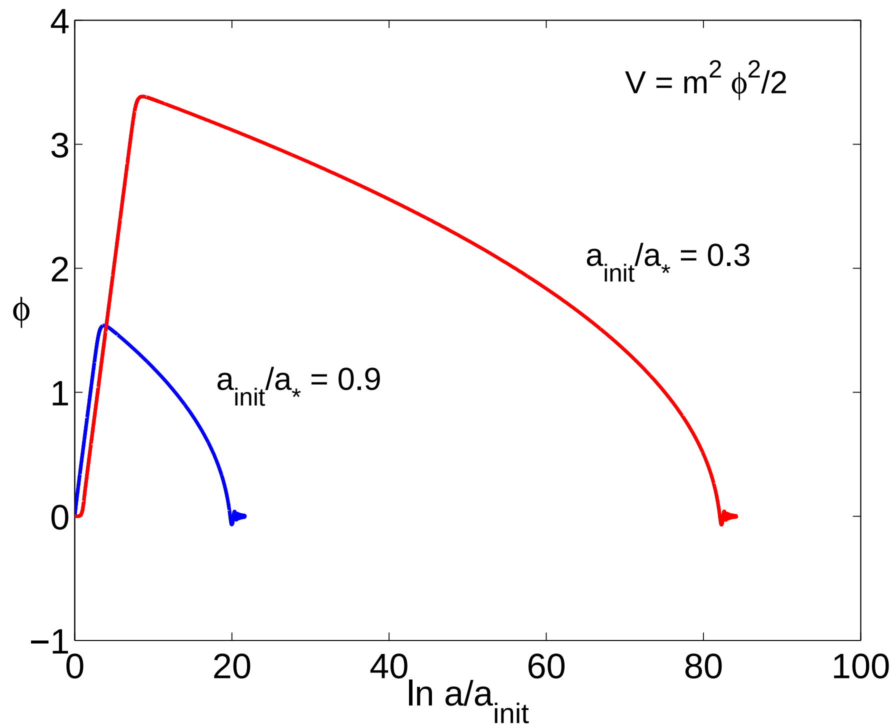
Antifrictional term when $d\ln D/d\ln a > 3$ in expanding Universe and frictional term in a contracting Universe.

3. Variation of the Hubble rate

$$\dot{H} = -\frac{S\dot{\phi}^2}{2D}\left(1 - \frac{1}{6}\frac{d\ln D}{d\ln a} + \frac{1}{6}\frac{d\ln S}{d\ln a}\right) + \frac{S}{2}\frac{d\ln S}{d\ln a}V + \frac{1}{a^2}$$

Super-inflation for $n - r = d\ln D/d\ln a - d\ln S/d\ln a > 6$.

5. Consequences for inflation (flat Universe)



Tsujikawa and Singh (2003)

1. Superinflation is brief

2. $\phi_t < 2.4\ell_{\rm pl}^{-1}$ if Hubble bound is satisfied $\Rightarrow$ not enough inflation

But can super-inflation replace standard inflation?

6. Scaling solution (Inverse volume corrections)

Scaling solution $\Leftrightarrow$ $\dot{\phi}^2/(2DV) \approx$ cnst.

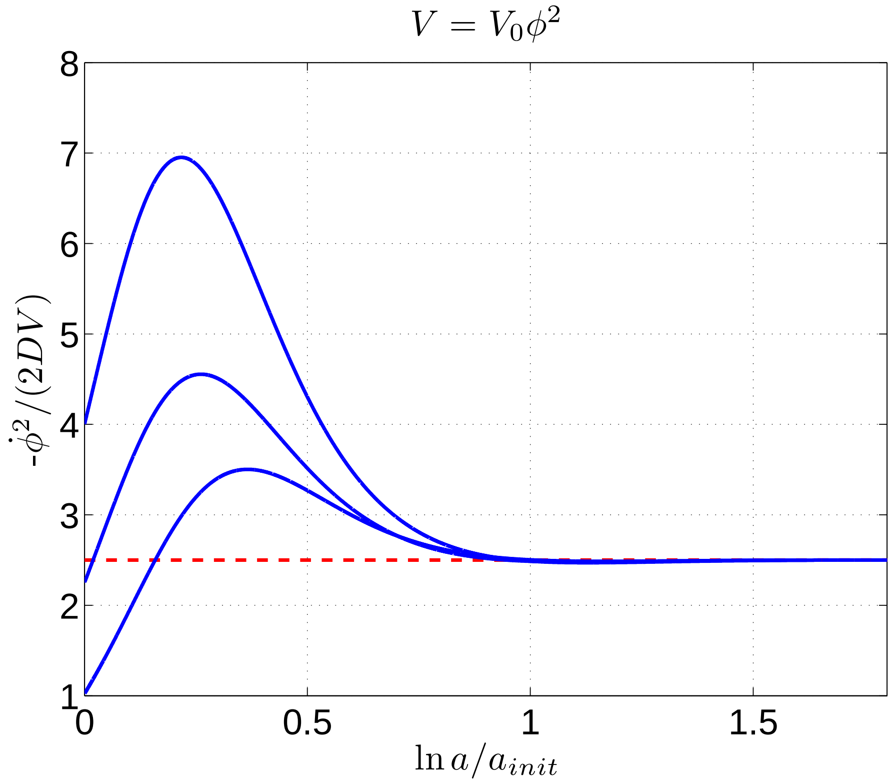
Lidsey (2004)

$$a = (-\tau)^p$$

$$p = \frac{2\alpha}{2\bar{\epsilon} - (2+r)\alpha}$$

$$\bar{\epsilon} = \frac{1}{2}\frac{D}{S}\left(\frac{V_{,\phi}}{V}\right)^2$$

$$V = V_0\,\phi^\beta$$



$\beta = 4\bar{\epsilon}/(n-r)\alpha > 0$, $\alpha = 1 - n/6$, $D \propto a^n$, $S \propto a^r$.

Scaling solution is *stable* attractor for $\bar{\epsilon} > 3\alpha^2$ or $\beta > (n-6)/n \sim \mathcal{O}(1)$.

7. Power spectrum of the perturbed field

1. Write equation of motion for the perturbed field $u = aD^{-1/2}\delta\phi$:

$$u'' + (-D\nabla^2 + m_{\rm eff}^2)u = 0$$

2. Promote u to an operator $\hat{u}$ and expand in plane waves $\hat{u} = (2\pi)^{-2/3}\int d^3k[\omega_k \hat{a}_{\rm k} + \omega_k^* \hat{a}_{-\rm k}^\dagger]e^{-i\rm k.x}$. Modes have equation of motion:

$$\omega_k'' + (Dk^2 + m_{\rm eff}^2)\omega_k = 0$$

3. Normalize modes s.t. $[\hat{a}_{\rm k}, \hat{a}_{\rm l}] = [\hat{a}_{\rm k}^\dagger, \hat{a}_{\rm l}^\dagger] = 0$ and $[\hat{a}_{\rm k}, \hat{a}_{\rm l}^\dagger] = \delta^{(3)}(\rm k - l)$ (Wronskian condition).

4. Find asymptotic value for large modes ($k \ll aH/\sqrt{D}$), when $m_{\rm eff}^2\tau^2 = {\rm cnst.}$. Power spectrum is given by:

$$\mathcal{P}_u \propto k^3\langle|\omega_k|^2\rangle \propto k^{3-2|\nu|}(-\tau)^{1-|\nu|(np+2)}$$

where $\nu = -\sqrt{1 - 4m_{\rm eff}^2\tau^2}/(2+np)$ and the spectral index is

$$\Delta n_u \equiv 3 - 2|\nu|$$

8. Fast-roll parameters and scale invariance

Near scale invariance $\Rightarrow \quad \Delta n_u = 3 - 2|\nu| \approx 0 \quad \Rightarrow \quad |\nu| = 3/2 \quad \Rightarrow$
$m_{\rm eff}^2 \tau^2 = -2, np \approx 0 \quad \Rightarrow \quad p \approx 0 \quad \Rightarrow \quad \bar{\epsilon} \gg 1 \quad \Leftrightarrow$

Steep and negative potentials.

Expand Δn_u in terms of fast-roll parameters

$$\epsilon \equiv 1/2\bar{\epsilon} = \frac{S}{D}\left(\frac{V}{V_{,\phi}}\right)^2$$

$$\eta \equiv 1 - \frac{V_{,\phi\phi}V}{V_{,\phi}^2} - \frac{1}{2}\frac{V}{V_{,\phi}}\left(\frac{D_{,\phi}}{D} - \frac{S_{,\phi}}{S}\right)$$

and admitting that $\bar{\epsilon}$ is time dependent, the spectral index gives

$$\Delta n_u \approx 4\epsilon\left[1 - \frac{n}{12}\left(1 + \frac{n}{6} - r\right) - \frac{r}{2}\right] - 4\eta$$

Scale invariance is obtained for $\epsilon \approx 0$ and $\eta \approx 0$.

9. Quadratic corrections

Using holonomies as basic variables leads to a quadratic energy density contribution in the Friedmann equation

$$H^2 = \frac{1}{3}\rho\left(1 - \frac{\rho}{2\sigma}\right)$$

with $\rho < 2\sigma$. In this work we consider

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0$$

The variation of the Hubble rate is

$$\dot{H} = -\frac{\dot{\phi}^2}{2}\left(1 - \frac{\rho}{\sigma}\right)$$

Super-inflation for $\sigma < \rho < 2\sigma$.

10. Scaling solution (quadratic corrections)

"Scaling solution" $\Leftrightarrow$ $\dot{\phi}^2/(2\sigma - V) \approx$ cnst.

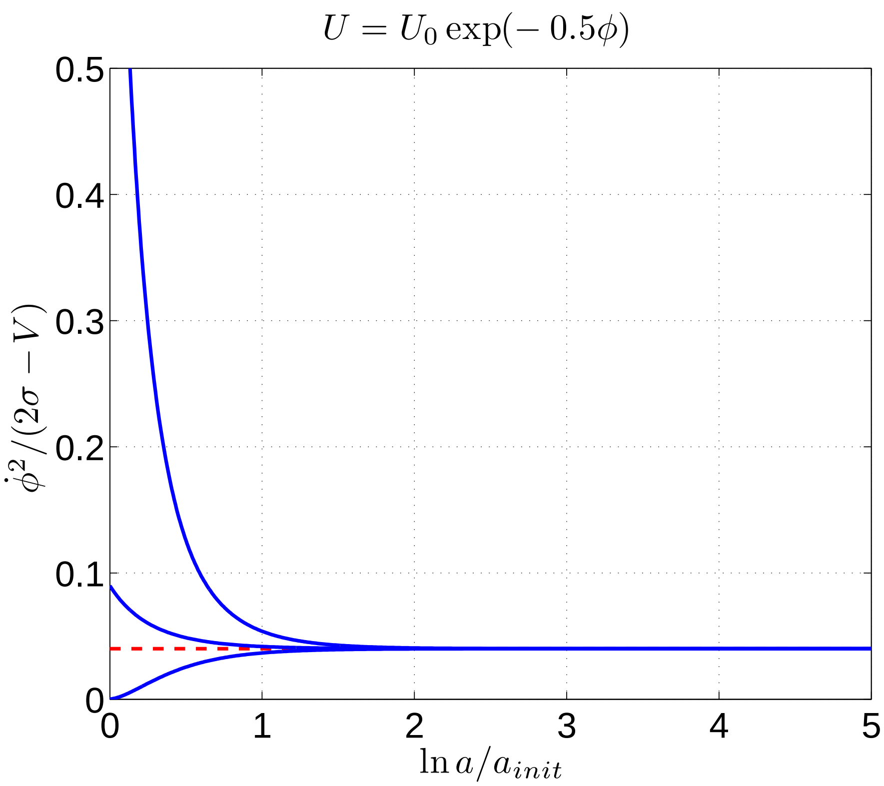
$$a = (-\tau)^p$$

$$p = -\frac{1}{\bar{\epsilon}+1}$$

$$\bar{\epsilon} = \frac{1}{2}\left(\frac{U_{,\phi}}{U}\right)^2$$

$$V = 2\sigma - U(\phi)$$

$$U = U_0\, e^{-\lambda\phi}$$



where $\lambda^2 = 2\bar{\epsilon}$.

Scaling solution is *stable* attractor for all λ or $\bar{\epsilon}$

11. Power spectrum of the perturbed field

Power spectrum is given by: $\mathcal{P}_u \propto k^3\langle|\omega_k|^2\rangle \propto k^{3-2|\nu|}(-\tau)^{1-2|\nu|}$

where $\nu = -\sqrt{1 - 4m_{\rm eff}^2\tau^2}/2$ and the spectral index is $\Delta n_u \equiv 3 - 2|\nu|$
For scaling solution

$$m_{\rm eff}^2\tau^2 = -2 + 3p(1+p)$$

Near scale invariance $\Rightarrow$ $p \approx 0$ $\Rightarrow$ $\bar{\epsilon} \gg 1$ $\Leftrightarrow$ Steep, positive potentials.

Expand Δn_u in terms of fast-roll parameters

$$\epsilon \equiv 1/2\bar{\epsilon} = \left(\frac{U}{U_{,\phi}}\right)^2 \qquad \eta \equiv 1 - \frac{V_{,\phi\phi}V}{V_{,\phi}^2}$$

and admitting that ϵ is time dependent, the spectral index gives

$$\Delta n_u \approx -4(\epsilon - \eta)$$

Scale invariance is obtained for $\epsilon \approx 0$ and $\eta \approx 0$.

12. Number of e-folds and the horizon problem

Requirement that the scale entering the horizon today exited N e-folds before the end of inflation:

$$\ln\left(\frac{a_{\rm end}H_{\rm end}}{a_N H_N}\right) = 68 - \frac{1}{2}\ln\left(\frac{M_{\rm Pl}}{H_{\rm end}}\right) - \frac{1}{3}\ln\left(\frac{\rho_{\rm end}}{\rho_{\rm reh}}\right)^{1/4}$$

1. In standard inflation: $\ln\left(\frac{a_{\rm end}H_{\rm end}}{a_N H_N}\right) \approx \ln\left(\frac{a_{\rm end}}{a_N}\right) \equiv N \approx 60$

2. In LQC with $a = (-\tau)^p$ and $p \ll 1$

$$\ln\left(\frac{a_{\rm end}H_{\rm end}}{a_N H_N}\right) = \ln\frac{\tau_N}{\tau_{\rm end}} = \ln\left(\frac{a_N}{a_{\rm end}}\right)^{1/p} = -\frac{1}{p}N$$

$$N \approx -60p$$

Number of e-folds of super-inflation required to solve the horizon problem can be of only a few.

13. Summary and questions

1. Inverse volume corrections: Scale invariance for steep negative potentials, $V = V_0\phi^\beta$;

2. Quadratic corrections: Scale invariance for steep positive potentials, $V = 2\sigma - U_0 \exp(-\lambda\phi)$;

3. Scaling solution is stable in both cases;

4. Only a few e-folds necessary to solve the horizon problem

5. What is the power spectrum of the curvature perturbation? Work in progress.