Classical approximation to quantum cosmological correlation functions

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arXiv:0707.0842

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Introduction

Motivation: cosmological perturbations

- Precision of theoretical calculations is increasing
- Important to know errors from assumptions

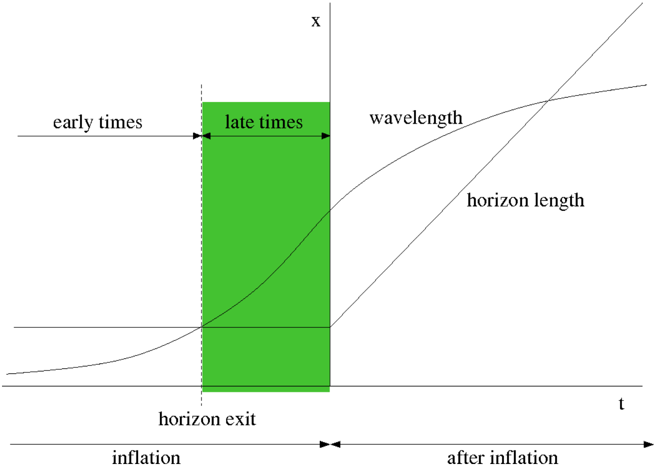
Goal: estimate size of quantum corrections after horizon exit

- Quantum effects are neglected after horizon exit
  - stochastic approach

    G.I. Rigopoulos, E.P.S. Shellard and B.J.W. van Tent, *Phys. Rev.* **D73** (2006) 083521, 083522
  - δN formalism

    D.H. Lyth and Y. Rodriguez, *Phys. Rev. Lett.* **95**, 121302 (2005)
- Estimate errors from this assumption by considering toy model

Introduction



Introduction

Toy model

- ϕ^3 theory:

$$\mathcal{L}[\phi] = -\sqrt{-g}\left(\frac{1}{2}\partial_\mu\phi(x)\partial^\mu\phi(x) + \frac{1}{3!}\lambda\phi^3(x) + \frac{1}{2}\delta_m\phi^2\right)$$

- de Sitter background

$$ds^2 = -dt^2 + a(t)^2 d\mathbf{x}^2 = a^2(\tau)(-d\tau^2 + d\mathbf{x}^2)$$

  - conformal time: $\tau = -\int_t^\infty dt'/a(t')$, runs from $-\infty$ to 0
  - scale factor: $a(t) = \exp(Ht)$, or $a(\tau) = -1/H\tau$, with Hubble scale H
  - horizon exit: $|k\tau| = 1$

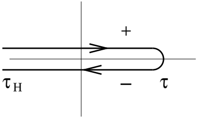
- We consider equal time correlation functions

$$\langle\phi(\tau, \mathbf{x}_1)\ldots\phi(\tau, \mathbf{x}_n)\rangle$$

Quantum theory

Closed Time Path formalism

In nonequilibrium quantum field theory, one calculates the expectation value $\langle Q(\tau)\rangle$ by using the



Closed Time Path formalism

Time integration along Schwinger-Keldysh contour.

$$\langle 0|\, U^\dagger(\tau,\tau_H)\, Q(\tau)\, U(\tau,\tau_H)\, |0\rangle$$

where $U(\tau_1,\tau_2) = T\exp\left[-i\int_{\tau_1}^{\tau_2} d\tau'\; H(\tau')\right]$ is the time evolution operator

S. Weinberg, *Phys. Rev.* **D72** (2005) 043514, **D74** (2006) 023508

Quantum theory

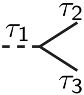
Closed Time Path formalism

Feynman rules in the Keldysh basis

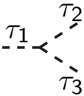
$$\begin{pmatrix} \phi^{(1)} \\ \phi^{(2)} \end{pmatrix} = \begin{pmatrix} (\phi^+ + \phi^-)/2 \\ \phi^+ - \phi^- \end{pmatrix}$$

$$= F(k, \tau_1, \tau_2)$$

$$= -iG^R(k, \tau_1, \tau_2)$$



$$= -i\lambda\, a^4(\tau_1)\delta(\tau_1 - \tau_2)\delta(\tau_1 - \tau_3)$$



$$= -i\frac{\lambda}{4}\, a^4(\tau_1)\delta(\tau_1 - \tau_2)\delta(\tau_1 - \tau_3)$$

Quantum theory

Example: one loop correction to two point function

Two point function

$$\int d^3x \, e^{i\mathbf{k}\cdot\mathbf{x}} \langle \phi(\tau, \mathbf{x}) \phi(\tau, \mathbf{0}) \rangle$$

Tree level:

$$F(k, \tau, \tau)$$

One loop level:

Quantum theory

Example: one loop correction to two point function

Result:

$$\frac{\lambda^2}{36(2\pi)^2 k^3}\Bigg\{\frac{7}{9\epsilon}+\frac{392}{27}-\frac{7}{3}\gamma-\frac{17}{18}\pi^2-\frac{4}{3}\ln 2-4\zeta(3)-\ln\frac{2\mu}{H}+\frac{4}{9}\ln(-k\tau_H)+$$
$$\left(\frac{2}{\epsilon}+15-\frac{17}{3}\gamma-\frac{2}{3}\pi^2-\frac{8}{3}\ln 2-3\ln\frac{2\mu}{H}+\frac{8}{3}\ln(-k\tau_H)\right)\ln\frac{\tau}{\tau_H}+$$
$$\left(\frac{2}{\epsilon}+\frac{22}{3}-2\gamma-2\ln 2+4\ln(-k\tau_H)\right)\ln^2\frac{\tau}{\tau_H}+\frac{8}{3}\ln^3\frac{\tau}{\tau_H}+\mathcal{O}(\frac{\tau}{\tau_H})+\mathcal{O}(\epsilon)\Bigg\}$$

with the infrared regulator

$$\epsilon=\frac{m^2}{3H^2}$$

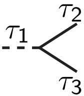
Classical theory

It can be shown that the diagrams in the classical theory form a subset of the Feynman diagrams of the quantum theory

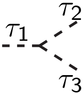
Quantum Feynman rules

$$= F(k, \tau_1, \tau_2)$$

$$= -iG^R(k, \tau_1, \tau_2)$$



$$= -i\lambda\, a^4(\tau_1)\delta(\tau_1 - \tau_2)\delta(\tau_1 - \tau_3)$$



$$= -i\frac{\lambda}{4}\, a^4(\tau_1)\delta(\tau_1 - \tau_2)\delta(\tau_1 - \tau_3)$$

Example

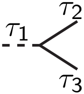
Classical theory

It can be shown that the diagrams in the classical theory form a subset of the Feynman diagrams of the quantum theory

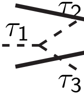
Classical graphical rules

$= F_{\rm cl}(k, \tau_1, \tau_2)$

$= -iG^R(k, \tau_1, \tau_2)$



$= -i\lambda\, a^4(\tau_1)\delta(\tau_1 - \tau_2)\delta(\tau_1 - \tau_3)$



~~$= -i\frac{\lambda}{4}\, a^4(\tau_1)\delta(\tau_1 - \tau_2)\delta(\tau_1 - \tau_3)$~~

Example

G. Aarts and J. Smit, *Nucl.Phys.* **B511** (1998) 451-478

Classical theory

We obtain the 'best' classical approximation for

$$F_{\rm cl}(k,\tau_1,\tau_2) = F(k,\tau_1,\tau_2)$$

- With this choice for $F_{\rm cl}$, the classical theory is ultraviolet divergent
- This can be treated by introducing an ultraviolet cutoff Λ, which should be chosen larger than H.

Late times

Late time behaviour (after horizon exit, $|k\tau| \ll 1$)

- Analysis by counting powers of τ
  - $\tau^{n>0}$: decay quicky after horizon exit
  - **τ^0: powers of $\ln \tau$ $\rightarrow$ growing late time contributions**
  - $\tau^{n<0}$: do not occur
- Separately for small ($|p\tau| \ll 1$) and large ($|p\tau| \gtrsim 1$) internal momenta p

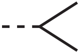
Late times

Small internal momenta

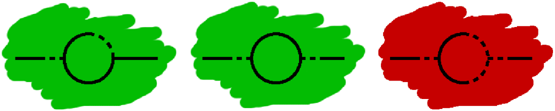
The two point functions can be expanded in $k\tau$ or $p\tau$ (similar to gradient expansion)

Diagrams with at least one  vertex

- Proportional to $\tau^{n>0} \rightarrow$ **no** late time contributions!

Diagrams with only  vertices

- Proportional to $\tau^0 \rightarrow$ **late time contributions!**
- These are exactly the diagrams of the classical theory

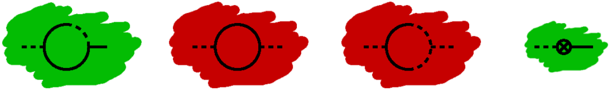
Example: 

Late times

Large internal momenta

1PI diagrams with one external dashed line lead to late time contributions

Example

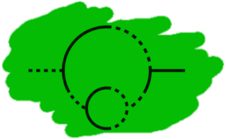


- At one loop these late time contributions can be reproduced by the classical theory, if the ultraviolet cutoff $\Lambda > H$
- These contributions could be missed by existing classical methods (stochastic approach, δN formalism)

Late times

Quantum corrections

In diagrams with 2 or more loops, there can be late time contributions from diagrams containing a  vertex.

Example: 

These diagrams do not occur in the classical theory; they are **quantum corrections**

Conclusions

- Errors from quantum corrections are of order λ^4 (two loops)
- Errors from using a cutoff $\leq H$ are of order λ^2 (one loop)

For late times, correlation functions have the form:

$$\mathcal{C}_{\lambda,k}\Big\{ f^{(0)}_{\mathrm{cl},<} + \lambda^2 \left(f^{(1)}_{\mathrm{cl},<} + f^{(1)}_{\mathrm{cl},>} \right) + \lambda^4 \left(f^{(2)}_{\mathrm{cl},<} + f^{(2)}_{\mathrm{cl},>} + f^{(2)}_{\mathrm{q}} \right) + \lambda^6 \left(f^{(3)}_{\mathrm{cl},<} + f^{(3)}_{\mathrm{cl},>} + f^{(3)}_{\mathrm{q}} \right) + \dots \Big\}$$

Free two point functions

$$F(k,\tau_1,\tau_2) = \frac{H^2}{2k^3}[(1+k^2\tau_1\tau_2)\cos k(\tau_1-\tau_2)+ k(\tau_1-\tau_2)\sin k(\tau_1-\tau_2)]$$
$$= \frac{H^2}{2k^3}[1+\mathcal{O}(k^2\tau_i^2)]$$
$$G^R(k,\tau_1,\tau_2) = \theta(\tau_1-\tau_2)\frac{H^2}{k^3}[(1+k^2\tau_1\tau_2)\sin k(\tau_1-\tau_2)- k(\tau_1-\tau_2)\cos k(\tau_1-\tau_2)]$$
$$= \theta(\tau_1-\tau_2)\,\frac{H^2}{3k^3}\,[k^3(\tau_1^3-\tau_2^3)+\mathcal{O}(k^5\tau_i^5)]$$

Comparing contributions from small and large internal momenta

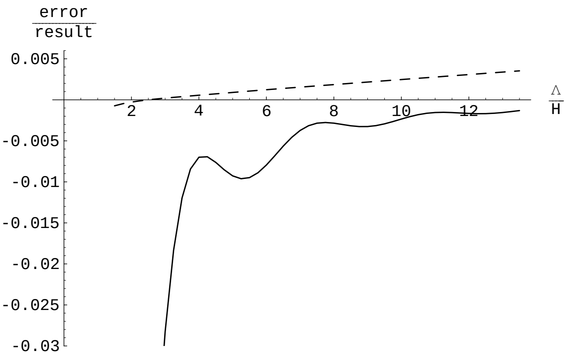
Using mass as infrared regulator, and comoving split in momenta

- small: $\propto \frac{\lambda^2}{\epsilon} \ln^2 \frac{\tau}{\tau_H}$ $\quad (\epsilon = m^2/3H^2)$
- large: $\propto \lambda^2 \ln^3 \frac{\tau}{\tau_H}$

Using finite space as infrared regulator, and physical split in momenta

- small: $\propto \lambda^2 \ln^3 \frac{\tau}{\tau_H}$
- large: $\propto \lambda^2 \ln^2 \frac{\tau}{\tau_H}$

Errors



$$k\tau_H = -0.4,\ k\tau = -0.03,\ 2\mu = H$$