A graviton propagator for inflation

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based on work done with Tomislav Prokopec

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$\longrightarrow$Two loop graviton backreaction

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- de Sitter space is ’nondynamical’
- It is not possible to construct a physical viable, de Sitter invariant propagator for the MMC scalar and the graviton

These issues are resolved if the background is quasi de Sitter (H depends mildly on time)

Quasi de Sitter space

$$H \equiv \frac{\dot{a}}{a} = H_0 - H_0^2 \epsilon t + \mathcal{O}(\epsilon^2), \qquad \epsilon \equiv -\frac{\dot{H}}{H^2} \ll 1, \qquad \dot{\epsilon} = \mathcal{O}(\epsilon^2).$$

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In conformal time $a d\eta = dt$

$$g_{\mu\nu} = a^2 \eta_{\mu\nu}, \qquad \eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1), \qquad a = -\frac{1}{\eta H}(1+\epsilon) + \mathcal{O}(\epsilon^2)$$

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we define the distance function

$$y(x;\tilde{x}) = \frac{1}{\eta\tilde{\eta}}\left(-(|\eta - \tilde{\eta}| - \imath\varepsilon)^2 + ||\vec{x} - \vec{\tilde{x}}||^2\right)$$

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The nonsingular part of the KG equation becomes

$$\left[y(4-y)\Big(\frac{d}{dy}\Big)^2 - \Big(D(y-2)+(D-2+2w)y\epsilon\Big)\frac{d}{dy} - (1+2\epsilon)\frac{\mu^2}{H^2}\right]\imath\Xi(y) = 0.$$

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Thus we have an hypergeometric equation if $m^2 = \mathcal{O}(\epsilon)$.

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$$\boxed{\begin{aligned} \imath\Delta(x;\tilde{x}) = & \frac{(a\tilde{a})^{-\epsilon(D/2-1)}}{(1+\epsilon)^{D-2}} \frac{H_0^{D-2}}{(4\pi)^{D/2}} \frac{\Gamma[\frac{D-1}{2}+\nu_D]\Gamma[\frac{D-1}{2}-\nu_D]}{\Gamma[\frac{D}{2}]} \\ & \times {}_2F_1\Big[\frac{D-1}{2}+\nu_D, \frac{D-1}{2}-\nu_D; \frac{D}{2}; 1-\frac{y}{4}\Big], \\ \nu_D^2 = & \Big(\frac{D-1}{2}\Big)^2 - \frac{m^2}{H_0^2} + \frac{\epsilon}{2}(D-1)(D-2), \qquad m^2 = \mathcal{O}(\epsilon). \end{aligned}}$$

Constructing the graviton propagator

We consider the action

$$S_{\rm EH} = \frac{1}{\kappa}\int \sqrt{-\hat{g}}\Big(\hat{R} - (D-2)\Lambda\Big) + S_M ,$$

where

$$S_M = \int \sqrt{-\hat{g}}\Big(- \frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi - V(\phi)\Big)$$

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With a gauge fixing term

$$-\frac{1}{2}a^{D+4}(\nabla_\mu\tilde{\psi}^{\mu\alpha})(\nabla_\nu\tilde{\psi}^\nu{}_\alpha), \qquad \tilde{\psi}_{\mu\nu} \equiv \psi_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\psi^\alpha{}_\alpha.$$

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We obtain the following action

$$\int a^D \eta^{\alpha\mu}\eta^{\beta\nu}\psi_{\alpha\beta}\left(\Box_S\left(\frac{1}{4}\delta^\rho_\mu\delta^\sigma_\nu - \frac{1}{8}g_{\mu\nu}g^{\rho\sigma}\right) - \frac{D-2}{2}(H^2+\dot{H})\delta^0_\mu\delta^\rho_\nu\delta^\sigma_0\right)\psi_{\rho\sigma},$$

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This gives the following propagator equation:

$$\mathcal{D}_{\mu\nu}{}^{\rho\sigma}[\imath_{\rho\sigma}\Delta^{\alpha\beta}](x;\tilde{x}) = \imath\delta^{(\alpha}_\mu\delta^{\beta)}_\nu\delta^D(x-\tilde{x}).$$

with

$$\mathcal{D}_{\mu\nu}{}^{\rho\sigma} = a^D\left[\square_S\left(\frac{1}{2}\delta^{(\rho}_\mu\delta^{\sigma)}_\nu - \frac{1}{4}\eta_{\mu\nu}\eta^{\rho\sigma}\right) - (D-2)(H^2+\dot{H})\delta^0_{(\mu}\delta^{(\rho}_{\nu)}\delta^{\sigma)}_0\right],$$

Constructing the graviton propagator

The propagator in any FLRW spacetime is given by

$$\boxed{\begin{aligned}&\left[{}_{\rho\sigma}\Delta^{\alpha\beta}\right] = \\ &\left(2\bar{\delta}_{\rho}^{(\alpha}\bar{\delta}_{\sigma}^{\beta)} - \frac{2}{D-3}\bar{\eta}_{\rho\sigma}\bar{\eta}^{\alpha\beta}\right)\Delta_0 + 4\delta^0_{(\rho}\bar{\delta}^{(\alpha}_{\sigma)}\delta^{\beta)}_0\Delta_1 \\ &+\left[\frac{2}{(D-2)(D-3)}(\eta_{\rho\sigma} + (D-2)\delta^0_\sigma\delta^0_\rho)(\eta^{\alpha\beta} + (D-2)\delta^\beta_0\delta^\alpha_0)\right]\Delta_2,\end{aligned}}$$

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$$a^D\Big(\Box_S - n(D-n-1)H^2(1-\epsilon)\Big)\Delta_n = \imath\delta^D(x-\tilde{x}).$$

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The solution to this equation in QdS was just constructed.

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- The propagator allows for a study of quantum corrections to tensor cosmological perturbations.
- To do the same for scalar perturbations, the mixing between gravitational and matter degrees of freedom needs to be taken into account.