DM in the Constrained MSSM - A Bayesian approach

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and

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hep-ph/0602028 $\longrightarrow$ JHEP06, hep-ph/0611173$\longrightarrow$ JHEP07, arXiv:0705.2012 and arXiv:0707.0622

SuperBayes package, superbayes.org

Outline

- the Constrained MSSM (CMSSM)
- limitations of fixed-grid scans
- Bayesian Analysis of the CMSSM
- fits of observables
- mean quality of fit and the CMSSM
- direct detection of dark matter
- indirect detection of dark matter
- summary

Constrained MSSM ...aka mSUGRA

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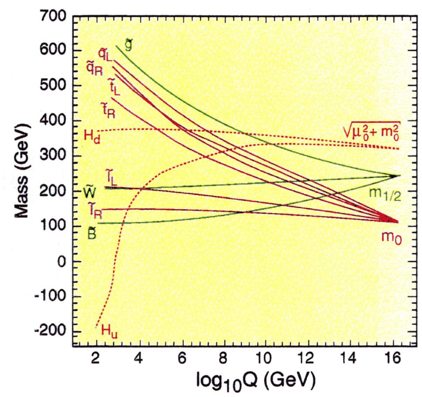
At $M_{\rm GUT} \simeq 2 \times 10^{16}$ GeV:

- gauginos $M_1 = M_2 = m_{\widetilde{g}} = m_{1/2}$ (c.f. MSSM)
- scalars $m^2_{\widetilde{q}_i} = m^2_{\widetilde{l}_i} = m^2_{H_b} = m^2_{H_t} = m^2_0$
- 3–linear soft terms $A_b = A_t = A_0$



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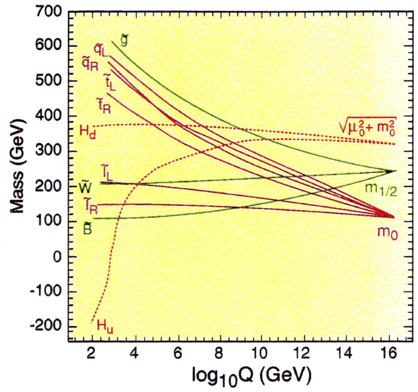

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- radiative EWSB

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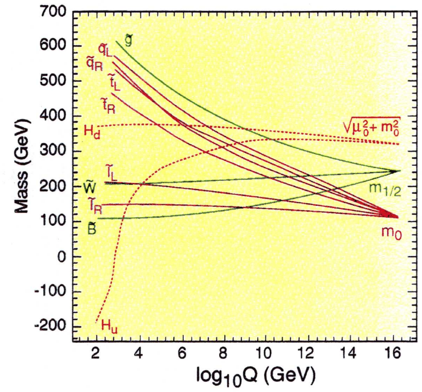

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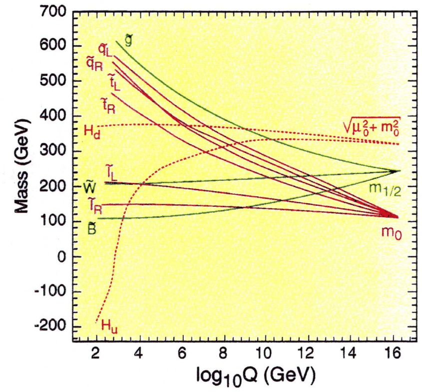

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- mass spectra at m_Z: run RGEs, 2–loop for g.c. and Y.c, 1-loop for masses
- some important quantities $(\mu, m_A, \ldots)$ very sensitive to procedure of computing EWSB & minimizing V_H

we use SoftSusy and FeynHiggs

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"usual" fixed-grid scans

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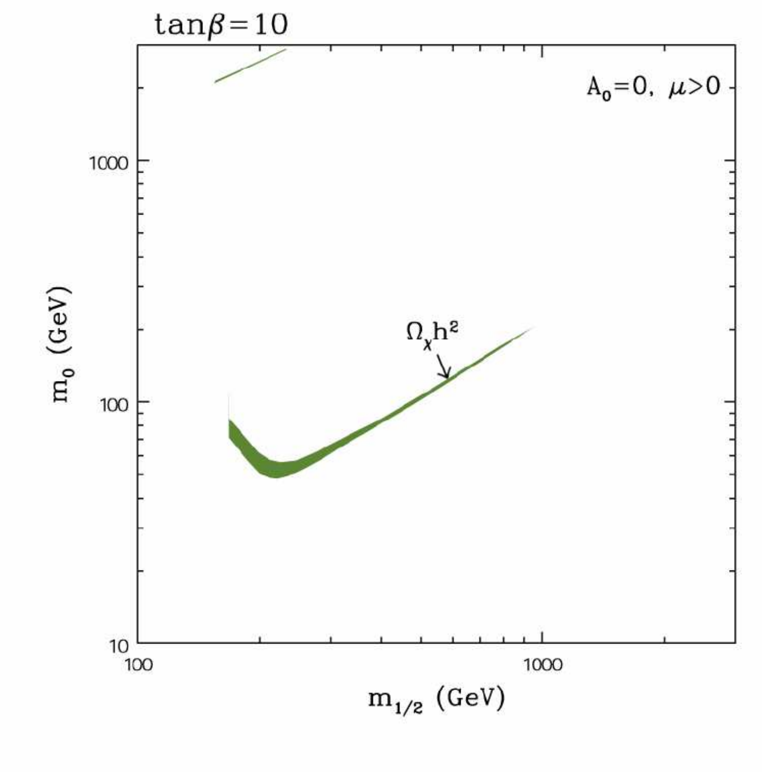

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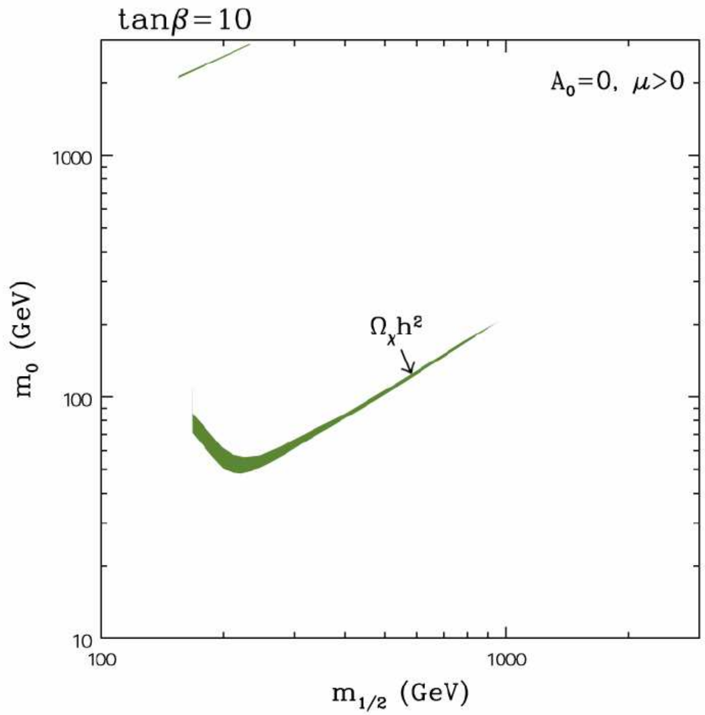

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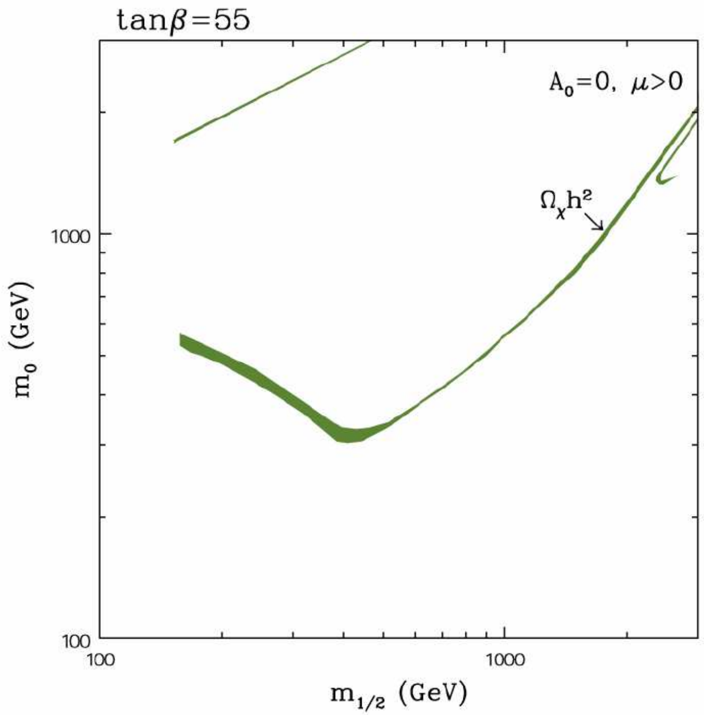

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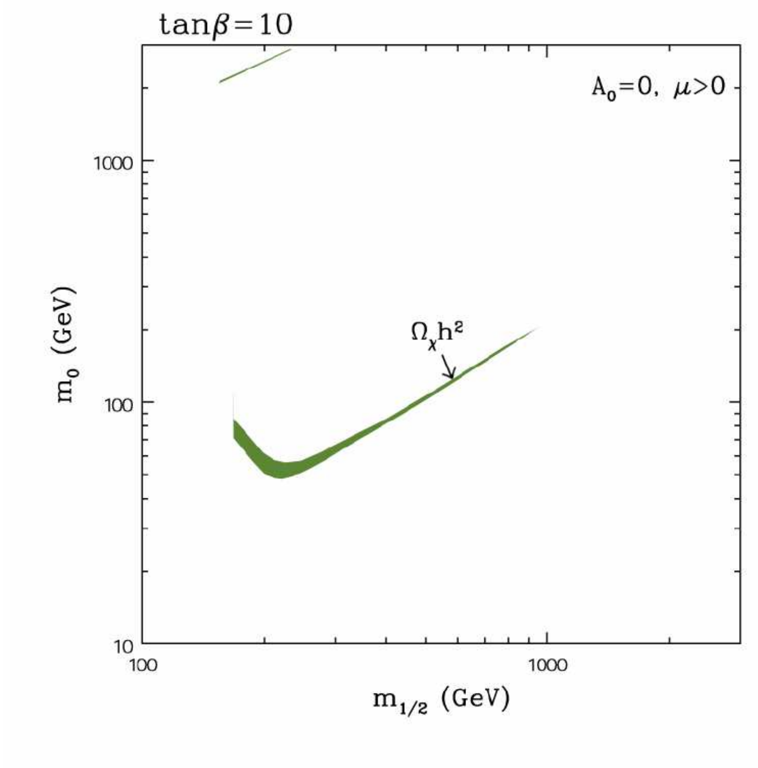

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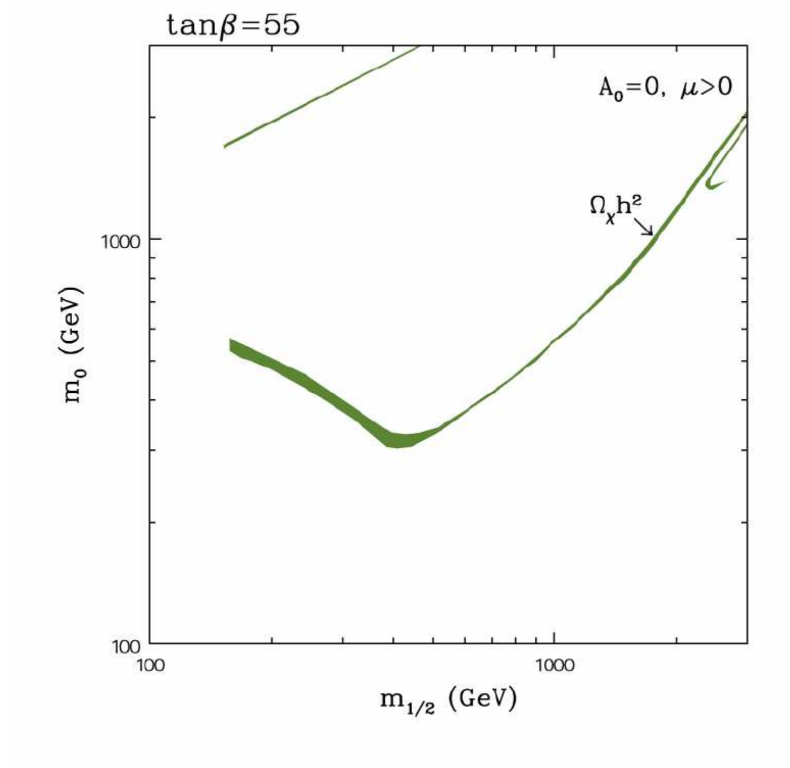

- fixed-grid scans, assuming rigid $\mathbf{1}\boldsymbol{\sigma}$ or $\mathbf{2}\boldsymbol{\sigma}$ experimental ranges
- green: consistent with WMAP-3yr (at $\mathbf{2}\boldsymbol{\sigma}$)
- all the rest excluded by LEP, $\mathbf{BR}(\bar{B} \to X_s\gamma)$, $\Omega_\chi h^2$, EWSB, charged LSP,...

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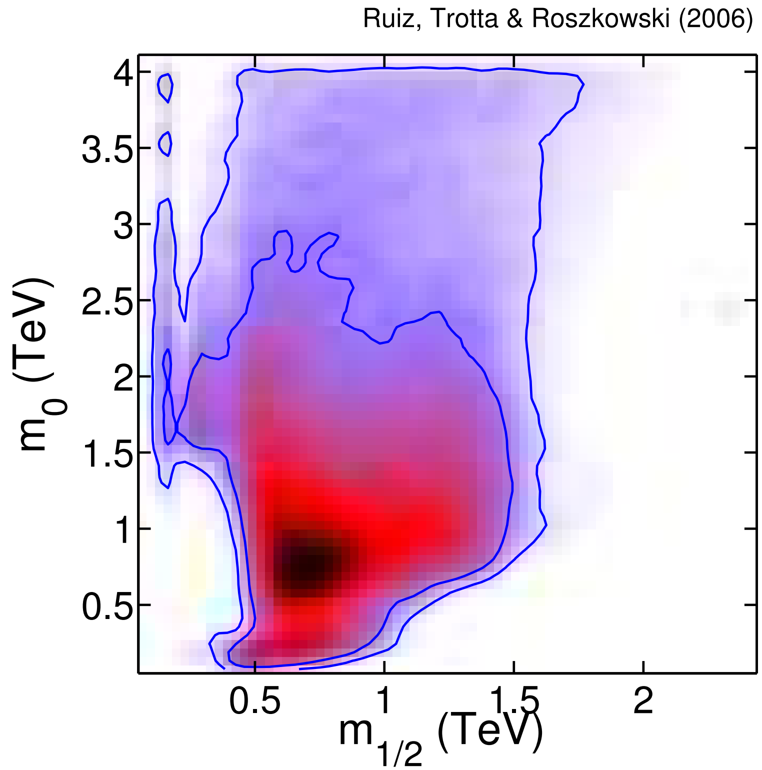

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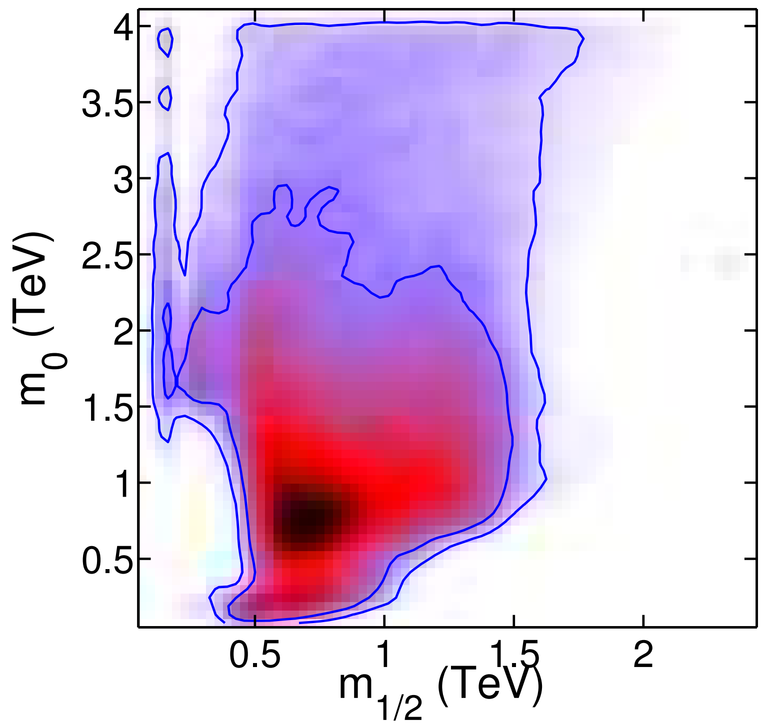
Ruiz, Trotta & Roszkowski (2006)



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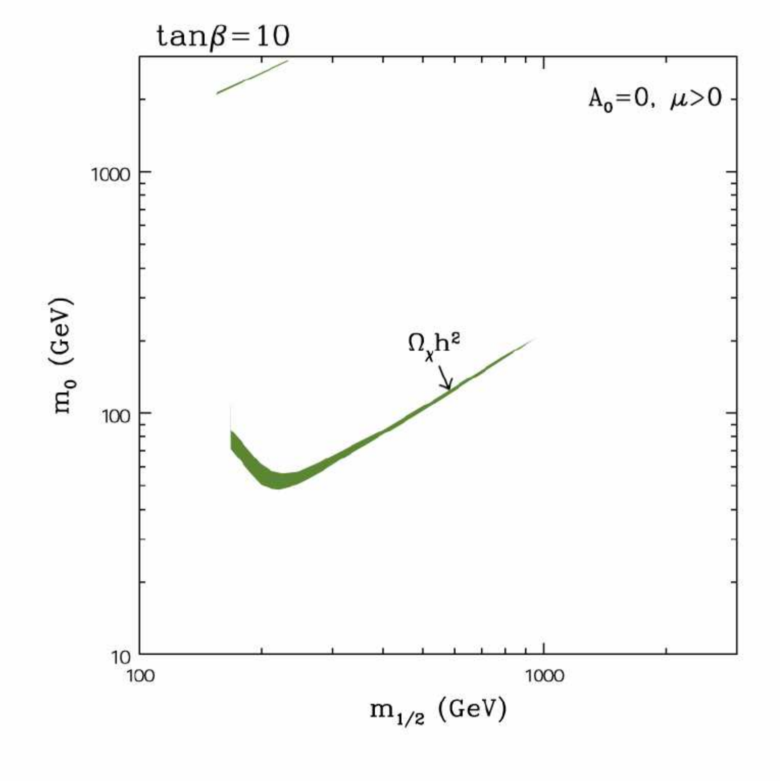

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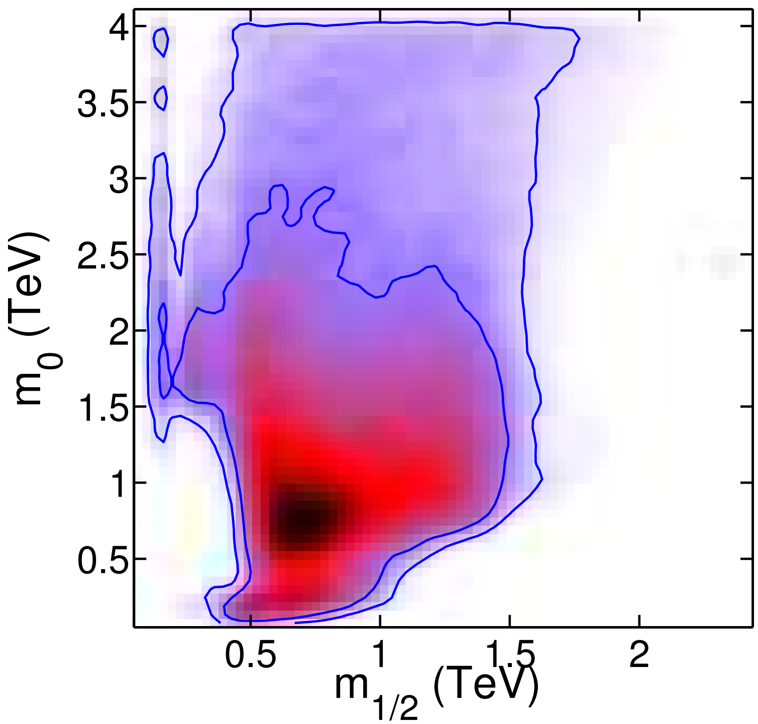

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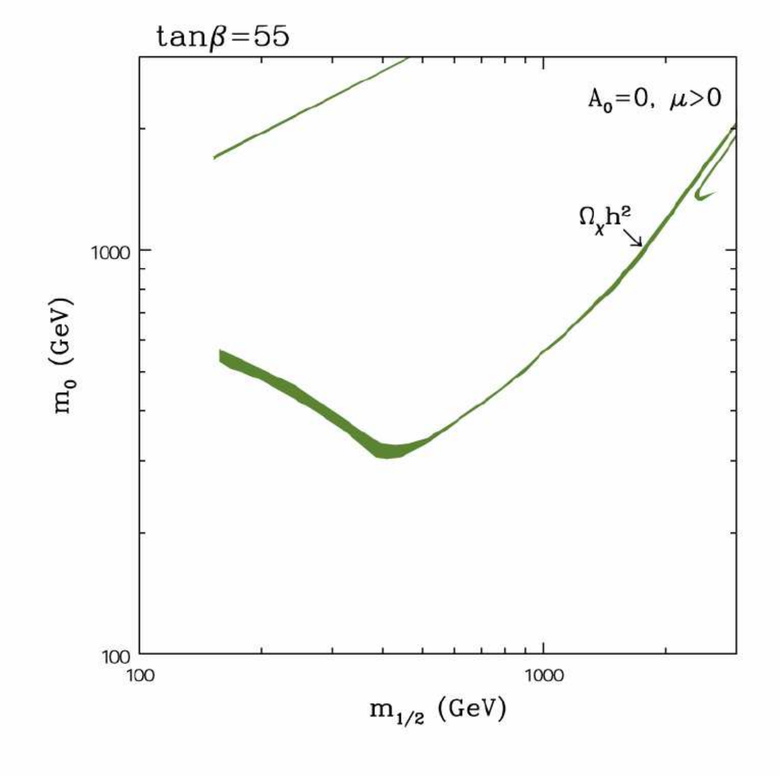

Note: In both an outdated SM value of $\mathbf{BR}(\bar{B} \to X_s\gamma)$ used. See below.

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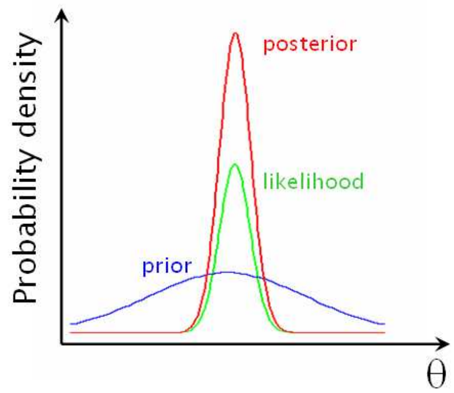

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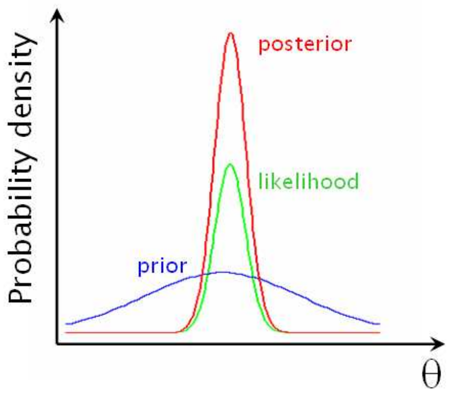

- Bayes' theorem: posterior pdf

$$p(\theta, \psi | d) = \frac{p(d|\xi)\pi(\theta,\psi)}{p(d)}$$

- $p(d|\xi)$: likelihood
- $\pi(\theta, \psi)$: prior pdf
- $p(d)$: evidence (normalization factor)

$$\text{posterior} = \frac{\text{likelihood} \times \text{prior}}{\text{normalization factor}}$$

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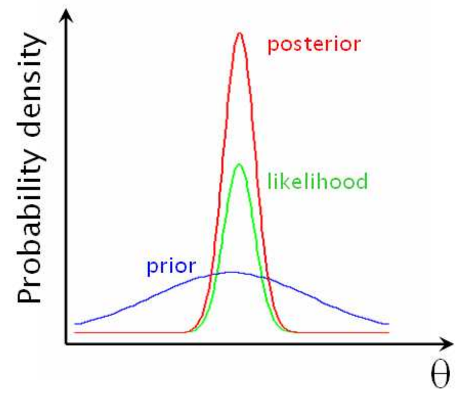

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- usually marginalize over SM (nuisance) parameters $\psi \Rightarrow p(\theta|d)$

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Bayesian Analysis of the CMSSM

- $\theta = (m_0, m_{1/2}, A_0, \tan\beta)$: CMSSM parameters
- $\psi = (M_t, m_b(m_b)^{\overline{MS}}, \alpha_{\rm em}(M_Z)^{\overline{MS}}, \alpha_s^{\overline{MS}})$: SM (nuisance) parameters
- priors – assume flat distributions and ranges as:

flat priors: CMSSM parameters
$50\,\text{GeV} < m_0 < 4\,\text{TeV}$
$50\,\text{GeV} < m_{1/2} < 4\,\text{TeV}$
$\lvert A_0 \rvert < 7\,\text{TeV}$
$2 < \tan\beta < 62$

flat priors: SM (nuisance) parameters
$160\,\text{GeV} < M_t < 190\,\text{GeV}$
$4\,\text{GeV} < m_b(m_b)^{\overline{MS}} < 5\,\text{GeV}$
$0.10 < \alpha_s^{\overline{MS}} < 0.13$
$127.5 < 1/\alpha_{\rm em}(M_Z)^{\overline{MS}} < 128.5$

- vary all 8 (CMSSM+SM) parameters simultaneously, scan with MCMC
- include all relevant theoretical and experimental errors

Experimental Measurements

(assume Gaussian distributions)

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SM (nuisance) parameter	Mean μ	Error σ (expt)
M_t	171.4 GeV	2.1 GeV
$m_b(m_b)^{\overline{MS}}$	4.20 GeV	0.07 GeV
$\alpha_s^{\overline{MS}}$	0.1176	0.002
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new $M_W = 80.413 \pm 0.048$ GeV (Jan 07, not yet included)
new $M_t = 170.9 \pm 1.8$ GeV (Mar 07, not yet included)
$\mathrm{BR}(\bar{B} \to X_s\gamma) \times 10^4$:
new SM: 3.15 ± 0.23 (Misiak & Steinhauser, Sept 06) used here

Derived observable	Mean μ	Errors σ (expt)	τ (th)
M_W	80.392 GeV	29 MeV	15 MeV
$\sin^2\theta_{\rm eff}$	0.23153	16×10^{-5}	15×10^{-5}
$\delta a_\mu^{\rm SUSY} \times 10^{10}$	28	8.1	1
$\mathrm{BR}(\bar{B} \to X_s\gamma) \times 10^4$	3.55	0.26	0.21
ΔM_{B_s}	17.33	0.12	4.8
$\Omega_\chi h^2$	0.119	0.009	$0.1\,\Omega_\chi h^2$

take as precisely known: $M_Z = 91.1876(21)$ GeV, $G_F = 1.16637(1) \times 10^{-5}$ GeV^{-2}

Experimental Limits

Derived observable	upper/lower limit	Constraints $\xi_{\rm lim}$	τ (theor.)
$\mathrm{BR}(B_s \to \mu^+\mu^-)$	UL	1.5×10^{-7}	14%
m_h	LL	114.4 GeV (91.0 GeV)	3 GeV
$\zeta_h^2 \equiv g_{ZZh}^2 / g_{ZZH_{\rm SM}}^2$	UL	$f(m_h)$	3%
m_χ	LL	50 GeV	5%
$m_{\chi_1^\pm}$	LL	103.5 GeV (92.4 GeV)	5%
$m_{\tilde{e}_R}$	LL	100 GeV (73 GeV)	5%
$m_{\tilde{\mu}_R}$	LL	95 GeV (73 GeV)	5%
$m_{\tilde{\tau}_1}$	LL	87 GeV (73 GeV)	5%
$m_{\tilde{\nu}}$	LL	94 GeV (43 GeV)	5%
$m_{\tilde{t}_1}$	LL	95 GeV (65 GeV)	5%
$m_{\tilde{b}_1}$	LL	95 GeV (59 GeV)	5%
$m_{\tilde{q}}$	LL	318 GeV	5%
$m_{\tilde{g}}$	LL	233 GeV	5%
(σ_p^{SI}	UL	WIMP mass dependent	$\sim 100\%$)

Note: DM direct detection σ_p^{SI} not applied due to astroph'l uncertainties (eg, local DM density)

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- for several uncorrelated observables (assumed Gaussian):

$$\mathcal{L} = \exp\left[-\sum_i \frac{\chi_i^2}{2}\right]$$

Probability maps of the CMSSM

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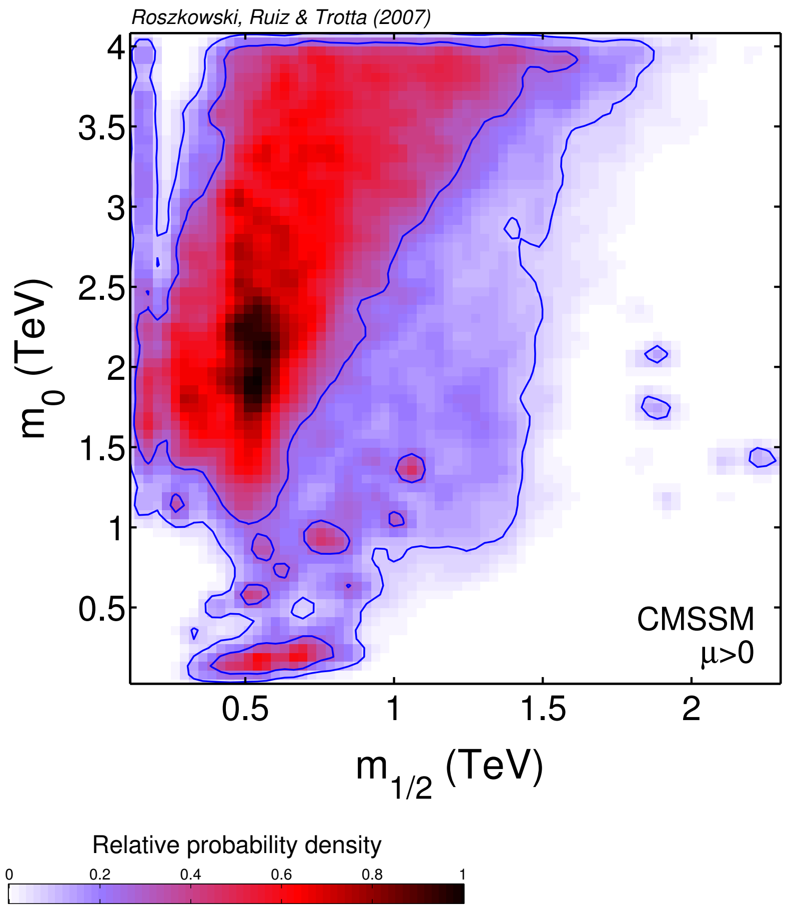
arXiv:0705.2012



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- Bayesian analysis
- relative probability density fn
- flat priors
- 68% total prob. – inner contours
- 95% total prob. – outer contours
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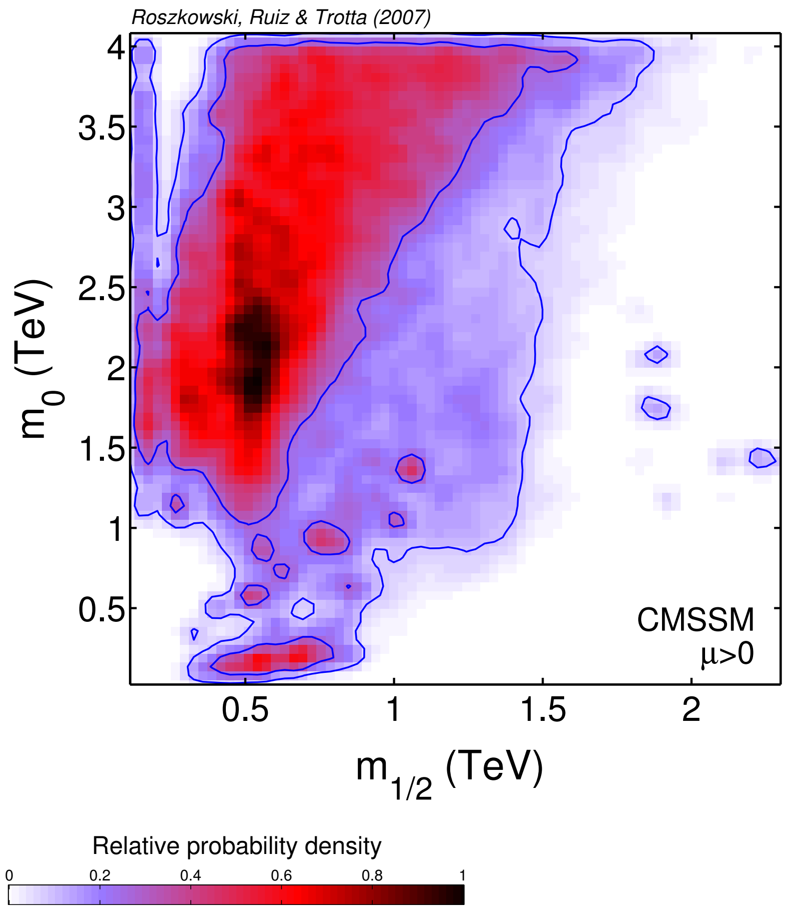

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similar study by Allanach+Lester(+Weber) (but no mean qof),
see also, Ellis et al (EHOW, χ^2 approach, no MCMC, they fix SM parameters!)

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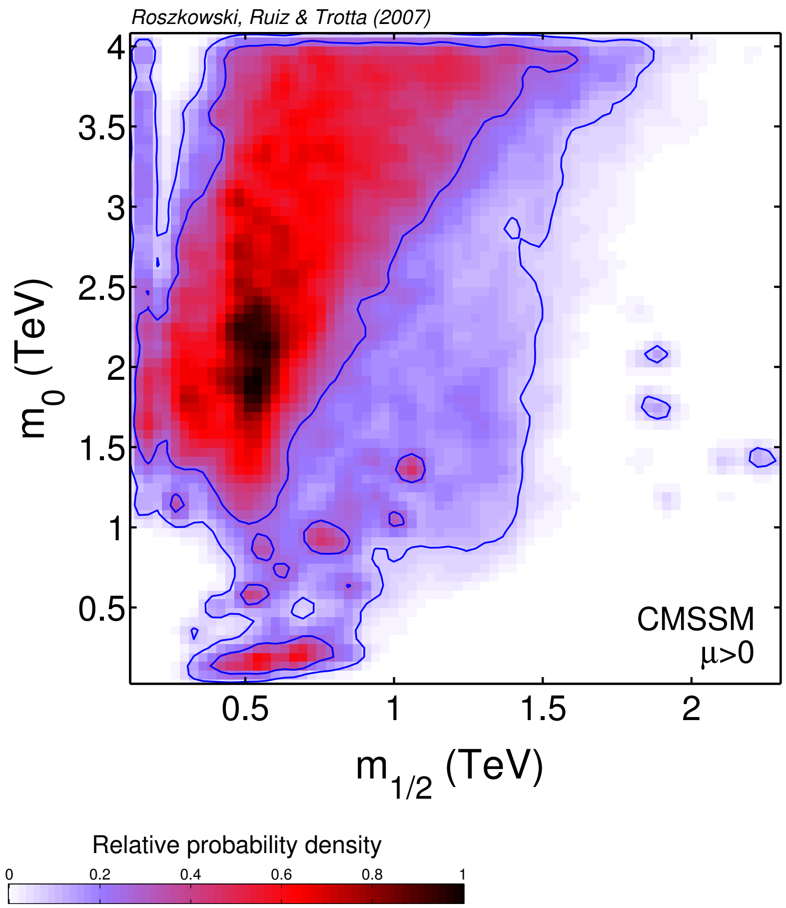

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unlike others (except for A+L), we vary also SM parameters

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- other ideas: traces of WIMP annihilation in dwarf galaxies, in rich clusters, etc

  more speculative

Dark matter detection: σ_p^{SI}

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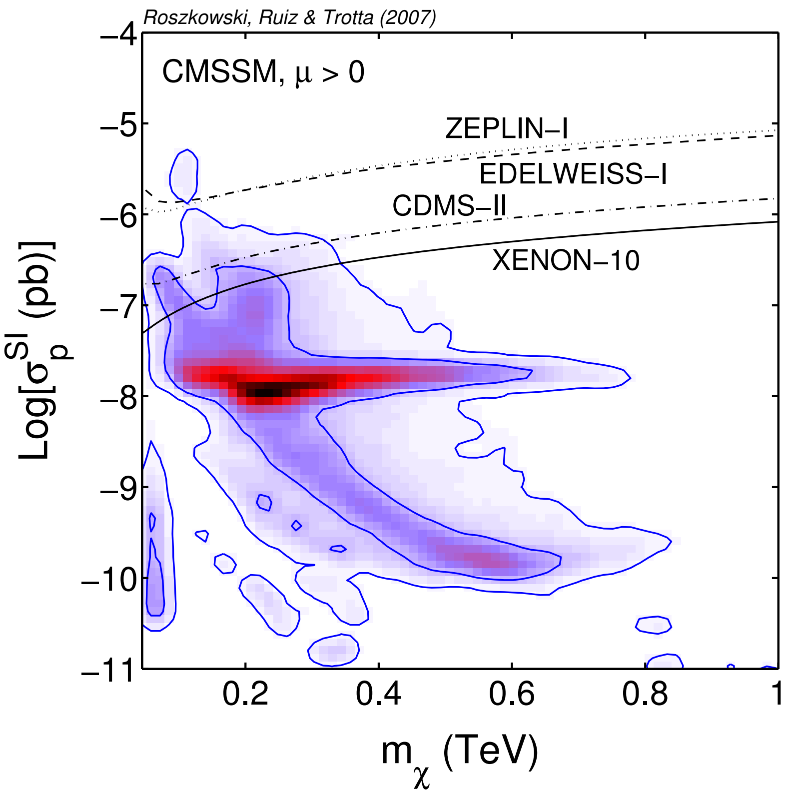
MCMC+Bayesian analysis



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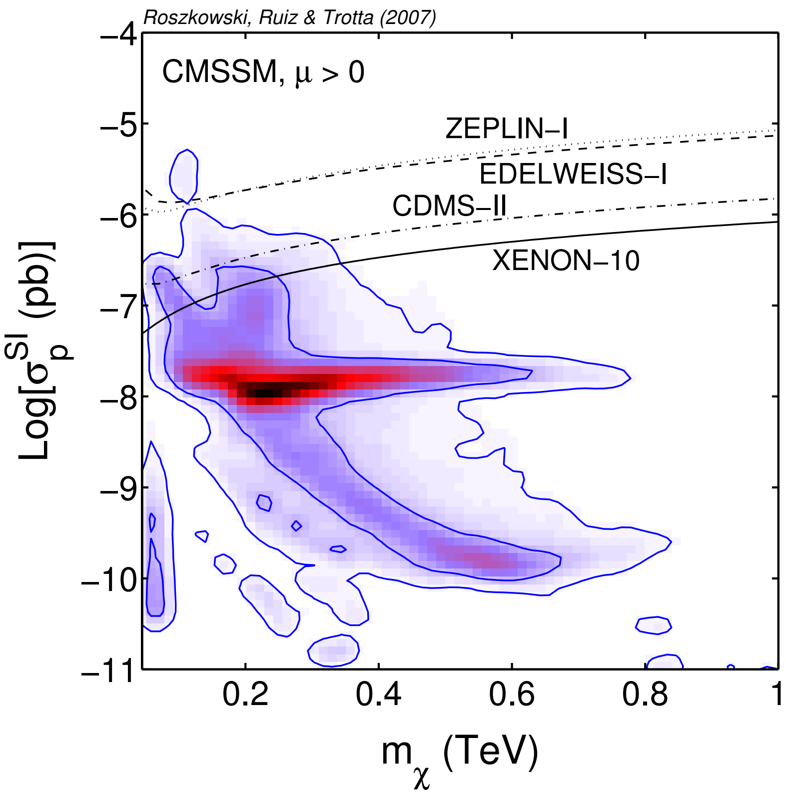
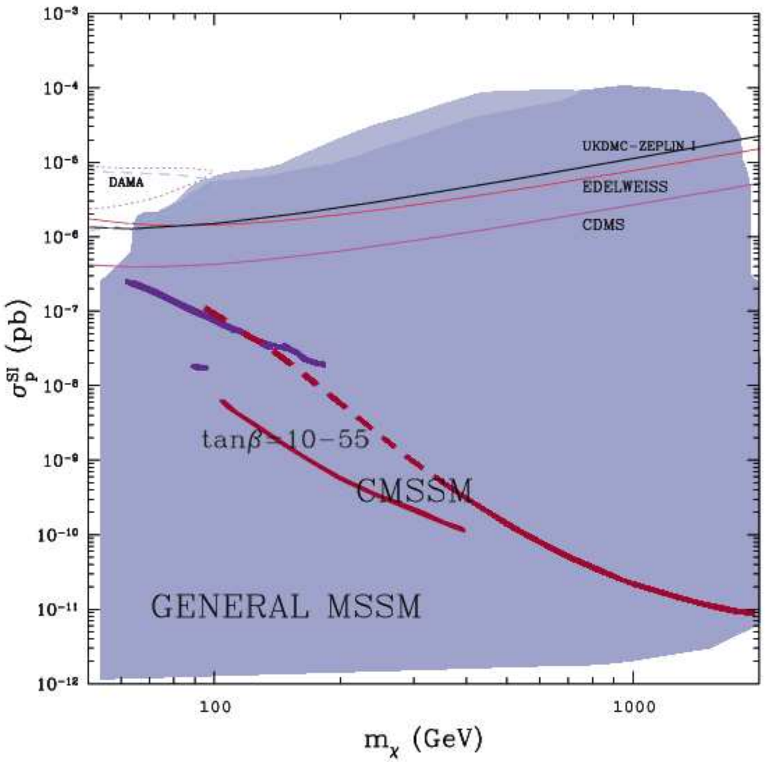
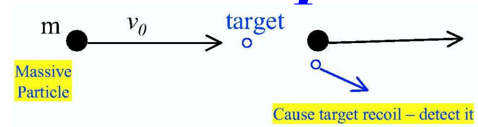
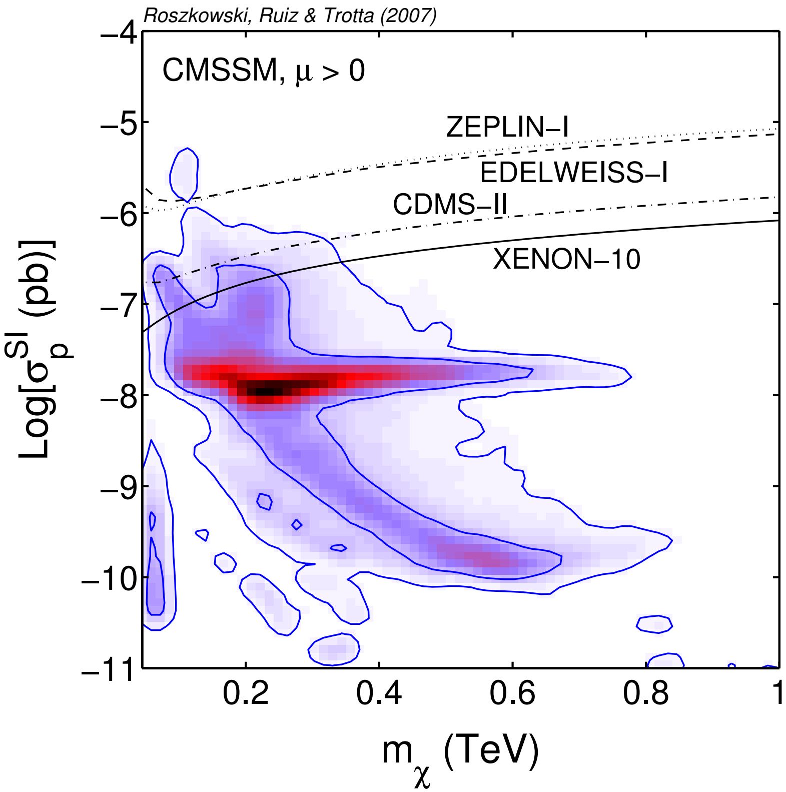


compare: fixed grid scan



Prospects for direct detection: σ_p^{SI}





Bayesian analysis, flat priors (MCMC)

internal (external): $\mathbf{68\%}$ ($\mathbf{95\%}$) region

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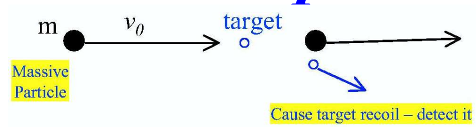

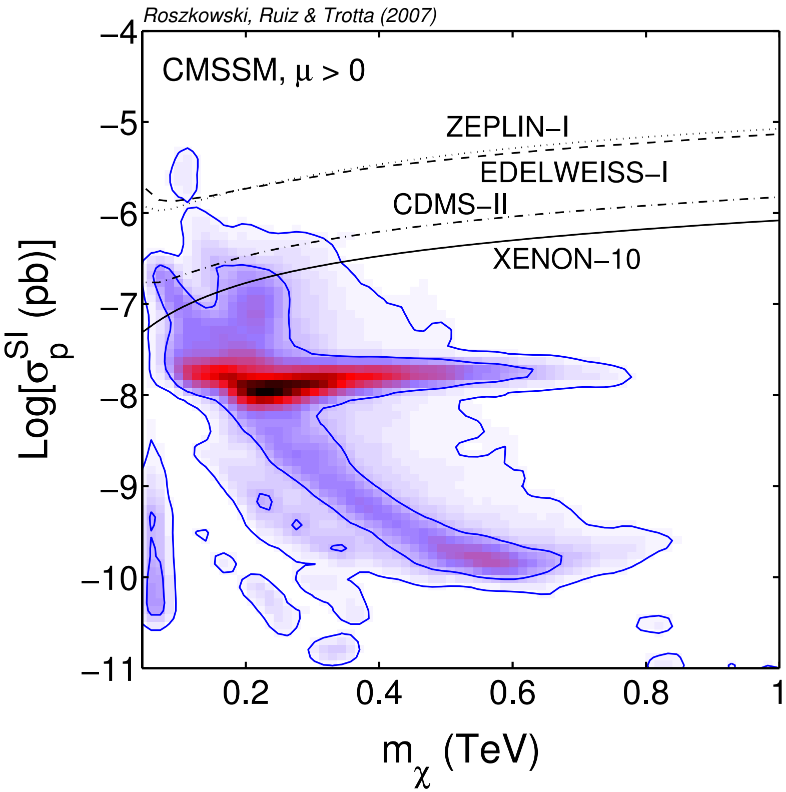

Bayesian analysis, flat priors (MCMC)

XENON-10 (June 07):
new limit $\sigma_p^{SI} \lesssim 10^{-7}$ pb:

also CDMS–II (?)

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(large $m_0 \gg m_{1/2}$), outside of the LHC reach

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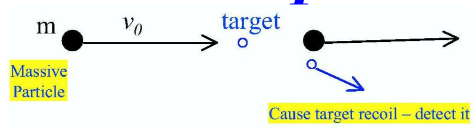

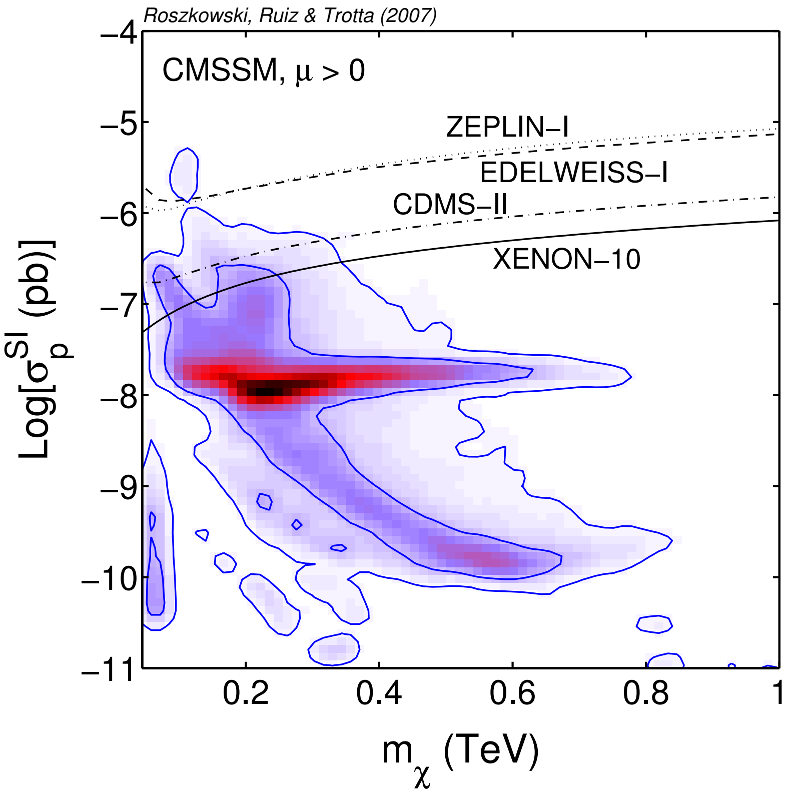

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most probable range: 10^{-7} pb $\lesssim \sigma_p^{SI} \lesssim 10^{-10}$ pb

partly outside of the LHC reach ($m_\chi \lesssim 400$ GeV)

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...not a settled matter

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$$\rho_{DM}(r) = \rho_c / \left(\frac{r}{a}\right)^{\gamma} \left[1 + \left(\frac{r}{a}\right)^{\alpha}\right]^{(\beta-\gamma)/\alpha}$$

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some most popular models:

halo model	a (kpc)	r_0 (kpc)	(α, β, γ)	small r: $\propto r^{-\gamma}$	large r: $\propto$
isothermal cored	3.5	8.5	$(2, 2, 0)$	flat	r^{-2}
NFW	20.0	8.0	$(1, 3, 1)$	r^{-1}	r^{-3}
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Many open questions: clumps??, central cusp??, spherical or tri–axial??,. . .

Gamma Rays from the Galactic Center

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- in the GC: ρ_{DM} is likely to be larger
- WIMP pair annihilation $\chi\chi \longrightarrow \mathrm{SMparticles} \propto \rho_\chi^2$ will be enhanced
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- separate particle physics and astrophysics inputs; define:

$$J(\psi) = \frac{1}{8.5\,\mathrm{kpc}} \left(\frac{1}{0.3\,\mathrm{GeV/cm^3}}\right)^2 \int_{\mathrm{l.o.s.}} dl\, \rho_\chi^2(r(l,\psi))$$

and

$$\bar{J}(\Delta\Omega) = (1/\Delta\Omega) \int_{\Delta\Omega} J(\psi) d\Omega$$

$\Delta\Omega$ - finite angular resolution of a GR detector

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- diff'l flux from the cone $\Delta\Omega$

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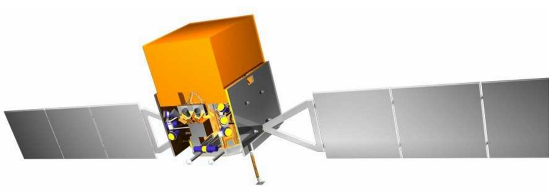
- all-sky survey
- effective energy range 20 MeV to 300 GeV, very good energy resolution
- angular resolution $\Delta\Omega\simeq 10^{-5}\mathrm{sr}$ (or $\sim 0.15\ \mathrm{deg}$ for $E_\gamma > 10$ GeV)



GRs from the GC in the CMSSM

use GLAST parameters

Bayesian posterior probability maps

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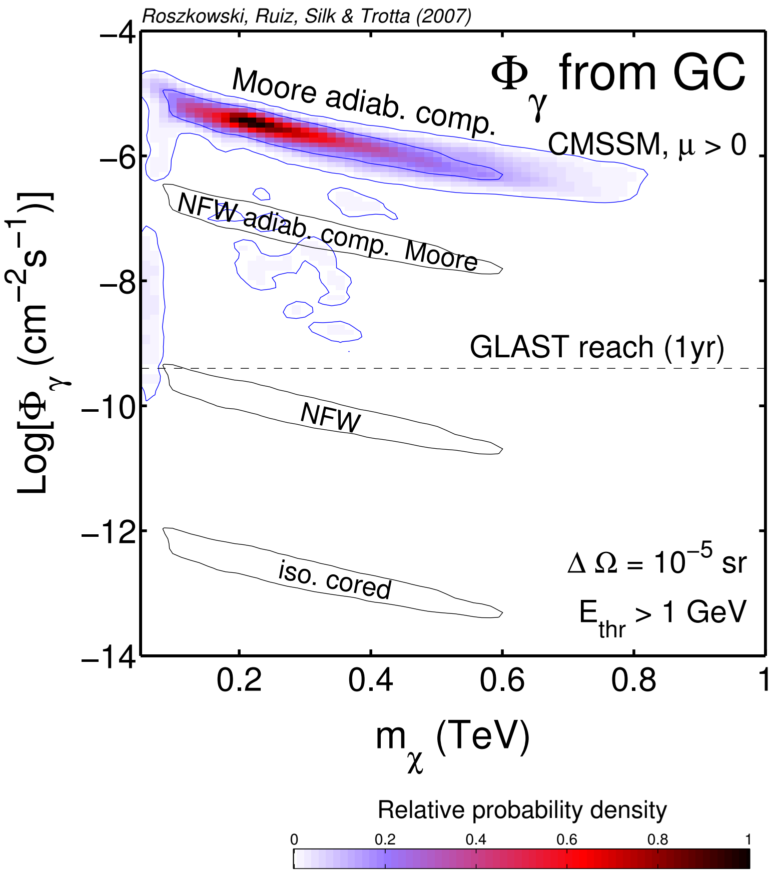

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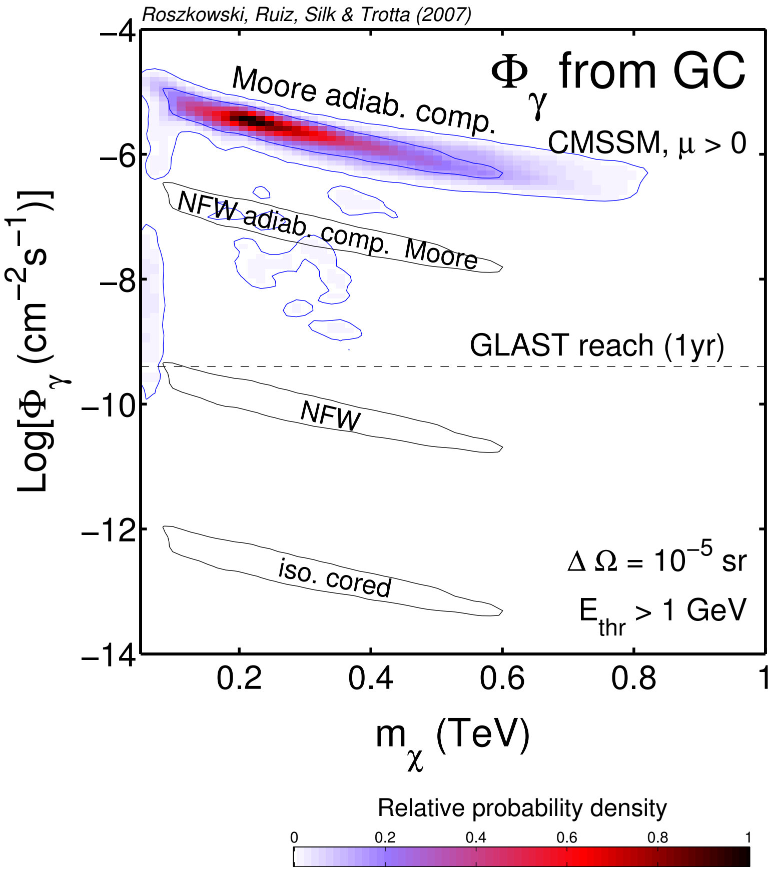
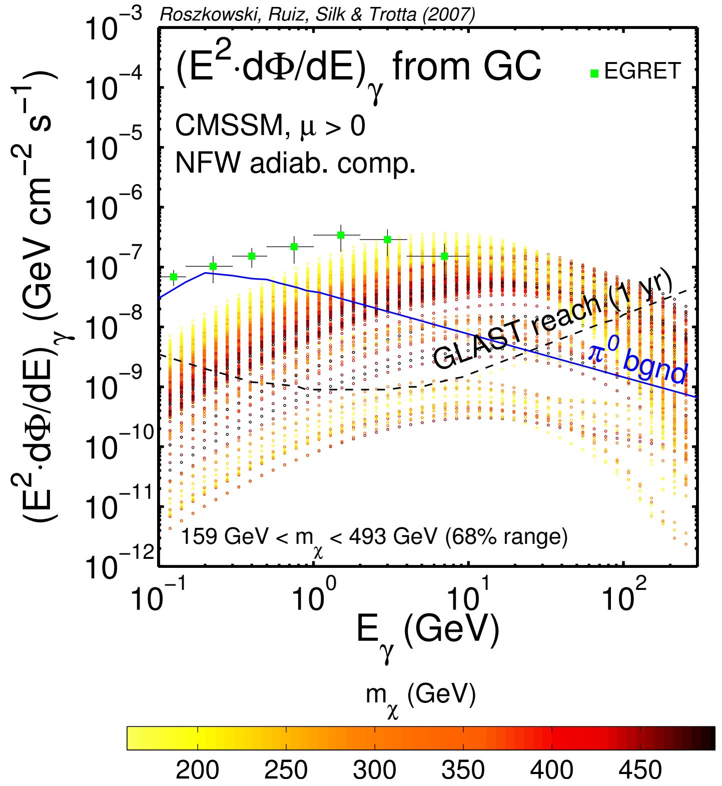


Bayesian posterior probability maps

γ-ray energy spectrum: example



159 GeV $< m_\chi <$ 493 GeV (68% range)

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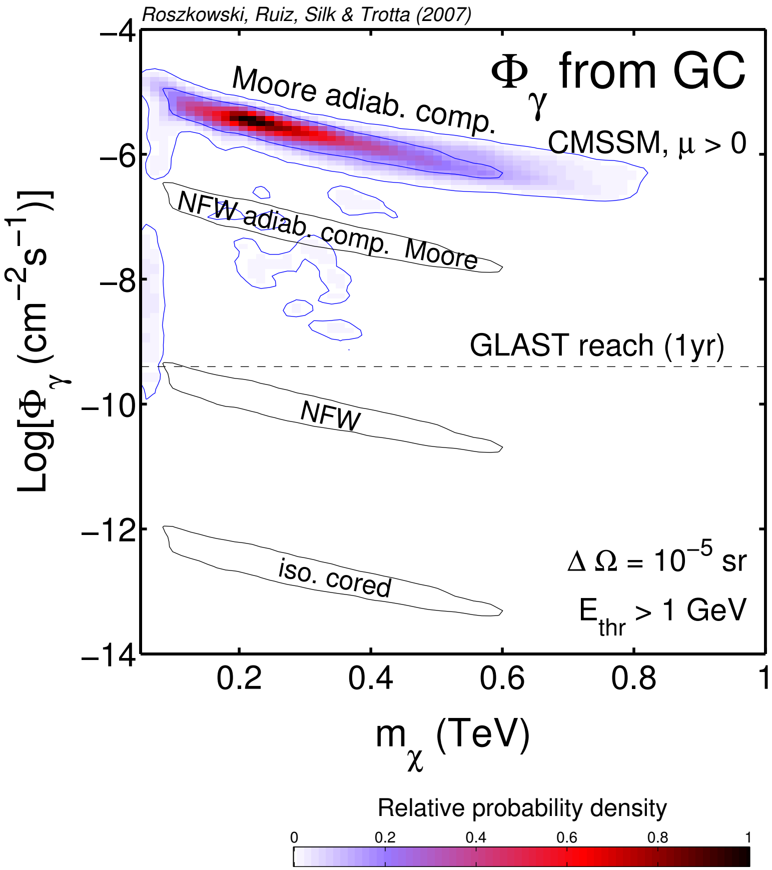

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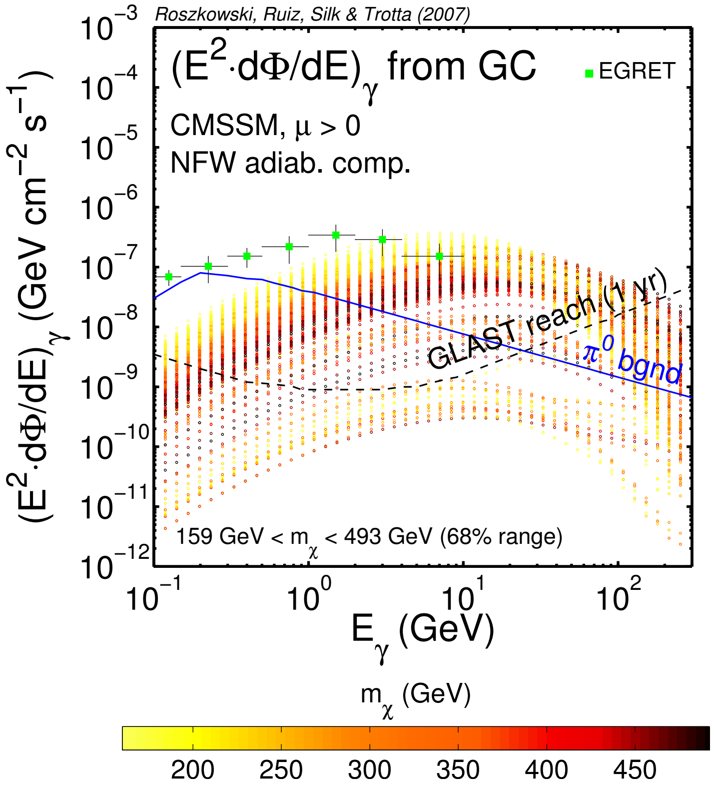

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GLAST prospects critically depend on how cuspy is the GC

- if more cuspy than NFW: all 95% CMSSM range will be explored (at 95% CL)
- even if signal detected: much uncertainty in determining m_χ

Positron flux and PAMELA

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- E_{e^+} from from DM annihilations
- propagate in interstellar magnetic field

$$K(\epsilon) = 2.1 \times 10^{28} \epsilon^{0.6} \mathrm{cm}^2 \, \mathrm{sec}^{-1}$$

$$\epsilon = E_{e^+} / 1 \, \mathrm{GeV}$$

- much less halo model dependence
- loose energy via inverse Compton scattering

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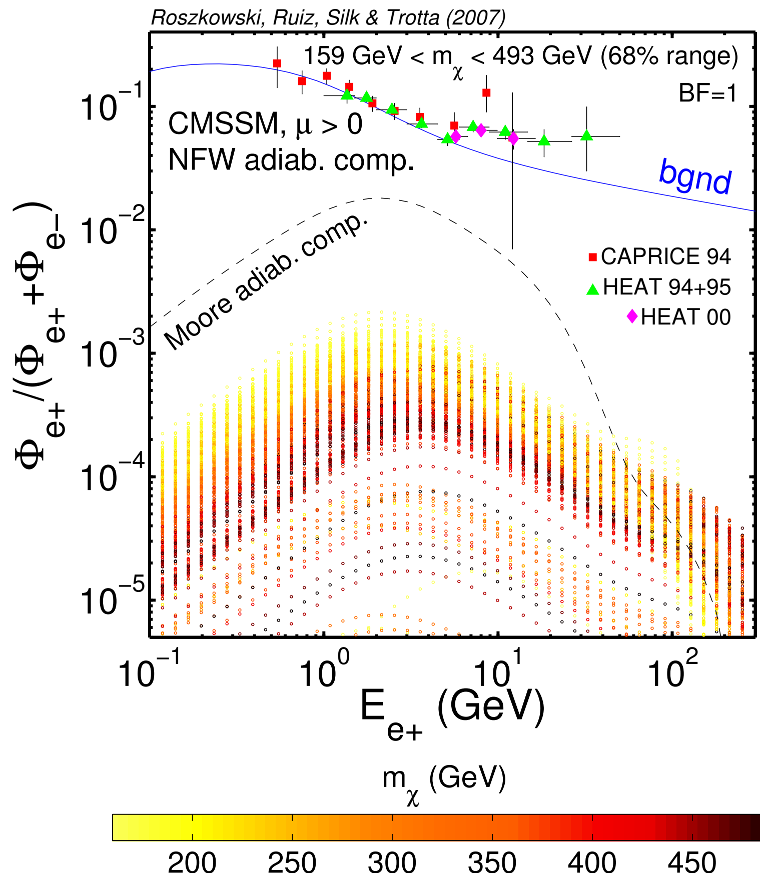

(scales linearly with boost factor)

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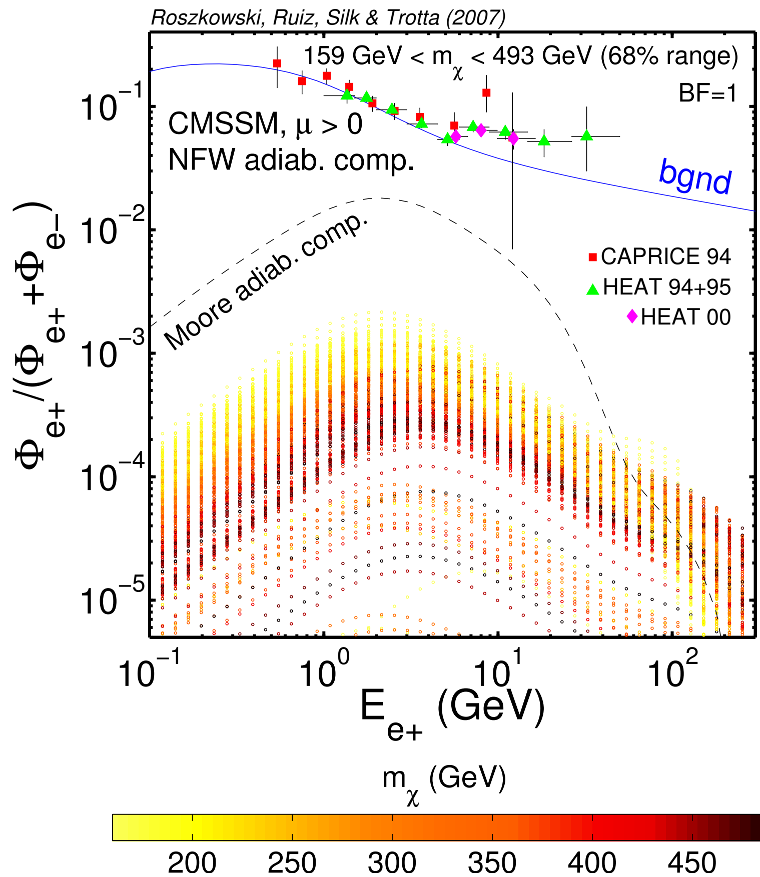

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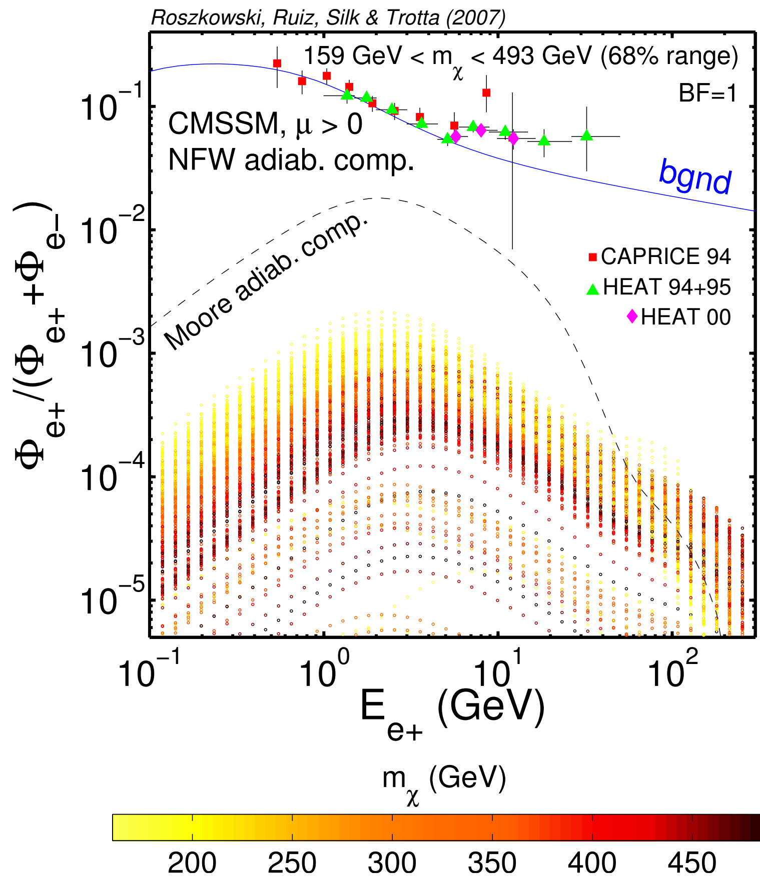

⇒ prospects for PAMELA rather poor

(...unless large boost factor)

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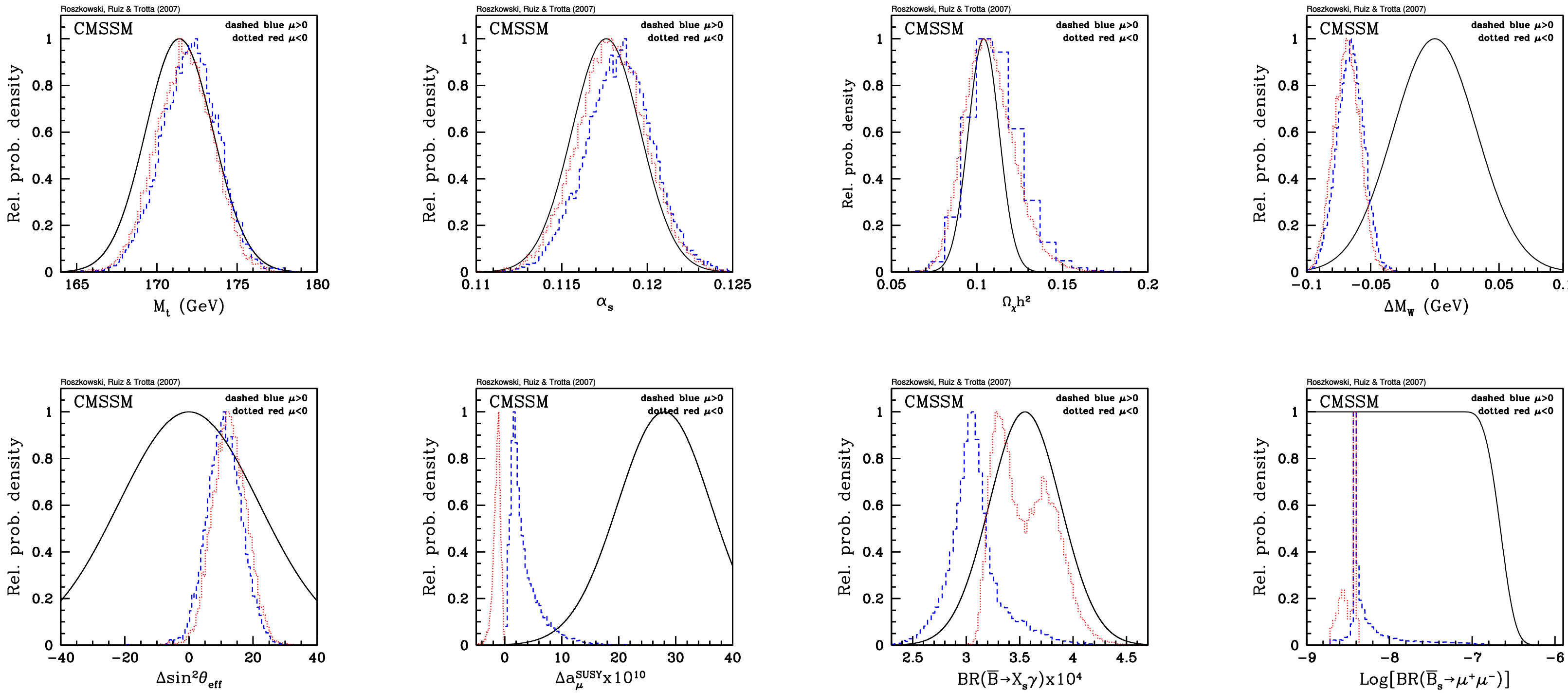
- positrons from DM: signal unlikely at PAMELA (unless large boost factor)

Backup

Fits of Observables



- good fits: M_t, α_s, $\Omega_\chi h^2$, $\mathrm{BR}(\bar{B} \to X_s \gamma)$ (for $\mu < 0$!)
- not so good: M_W, $\sin^2\theta_{\mathrm{eff}}$, $\mathrm{BR}(\bar{B} \to X_s \gamma)$ (for $\mu > 0$!)
- bad: $\Delta a_\mu^{\mathrm{SUSY}}$ (for both signs of μ!)

The Likelihood

incorporates information about the observational data

- the mapping $\xi(m)$ comes with uncertainties
- experimental uncertainty σ_i
- theoretical uncertainty τ_i
- introduce "exact" mapping $\hat{\xi}(\theta, \chi)$
- the likelihood:

$$p(d|\xi) = \int p(d|\hat{\xi}) p(\hat{\xi}|\xi) d^m \hat{\xi}$$

where

$$p(\hat{\xi}|\xi) = \frac{1}{(2\pi)^{m/2}|C|^{1/2}} \exp\left(-\frac{1}{2}(\xi - \hat{\xi}) C^{-1} (\xi - \hat{\xi})^T\right)$$

C: $m \times m$ covariance matrix
if uncorrelated: $C = \mathrm{diag}\left(\tau_1^2, \ldots, \tau_m^2\right)$

$$p(d|\hat{\xi}) = \frac{1}{(2\pi)^{m/2}|D|^{1/2}} \exp\left(-\frac{1}{2}(d - \hat{\xi}) D^{-1} (d - \hat{\xi})^T\right)$$

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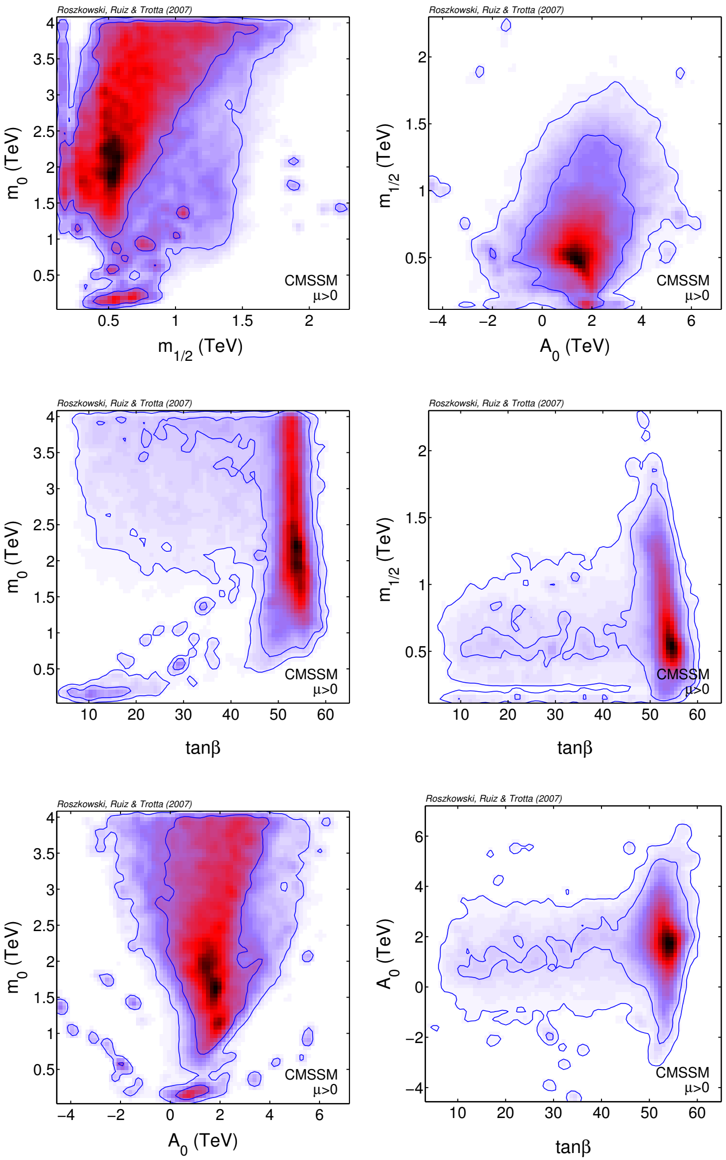
- total error for each observable: $s_i = \sqrt{\sigma_i^2 + \tau_i^2}$

Probability maps of the CMSSM

$\mu > 0$:



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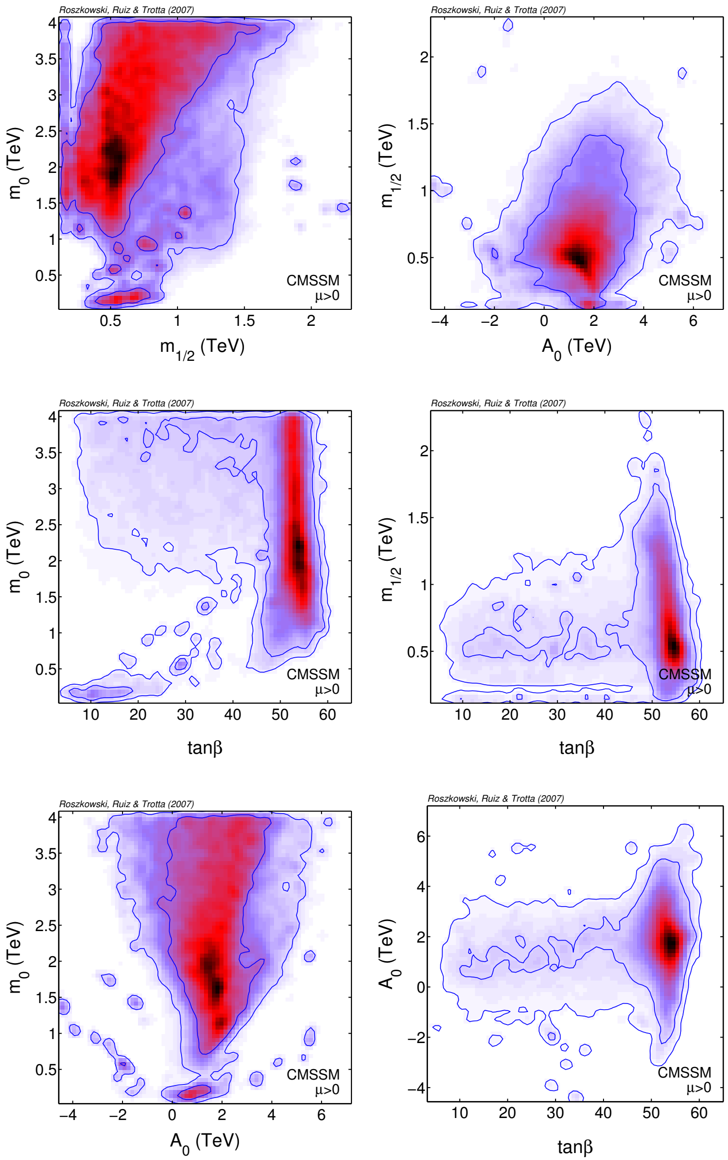
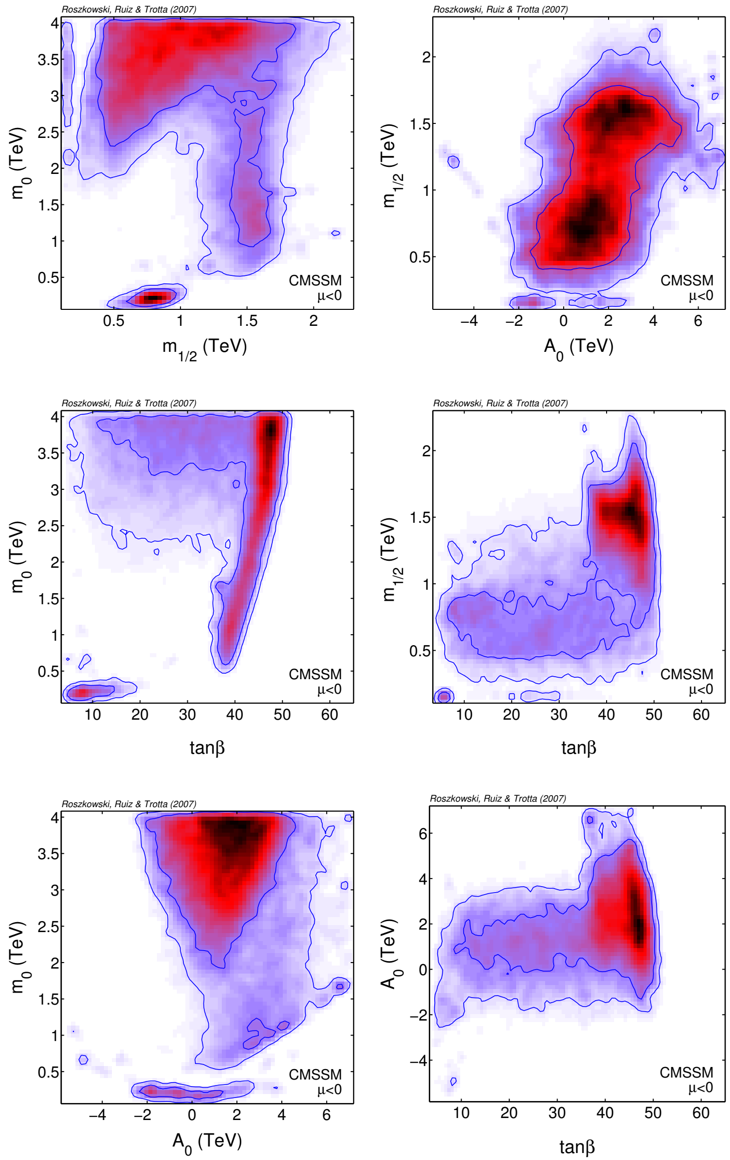


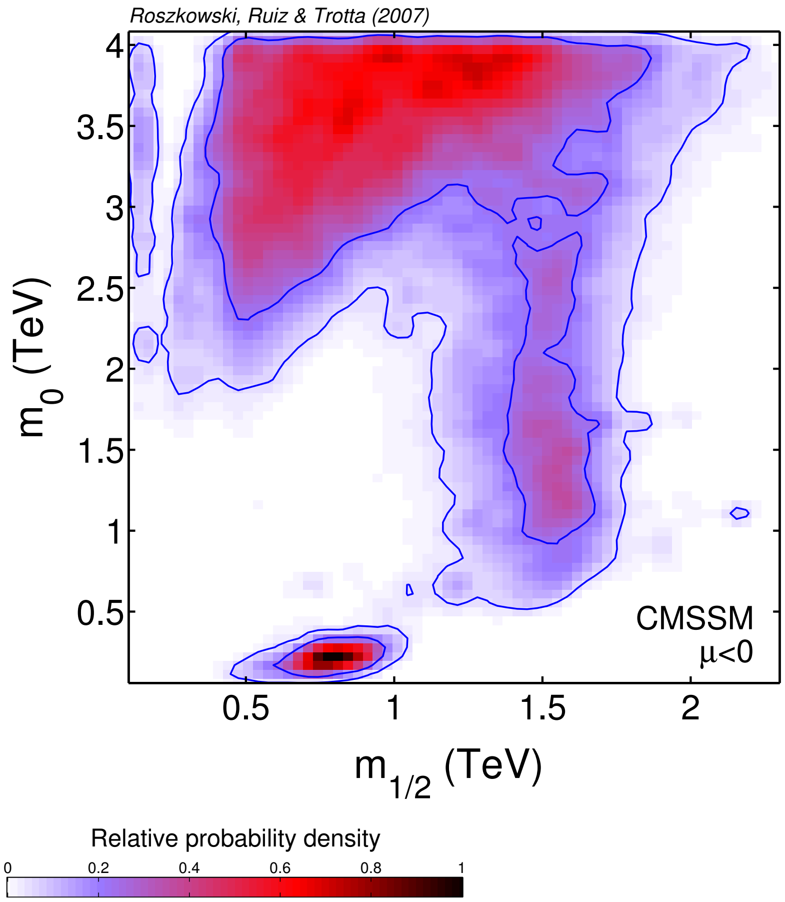
$\mu < 0$:



Posterior pdf vs. mean qof: $\mu < 0$

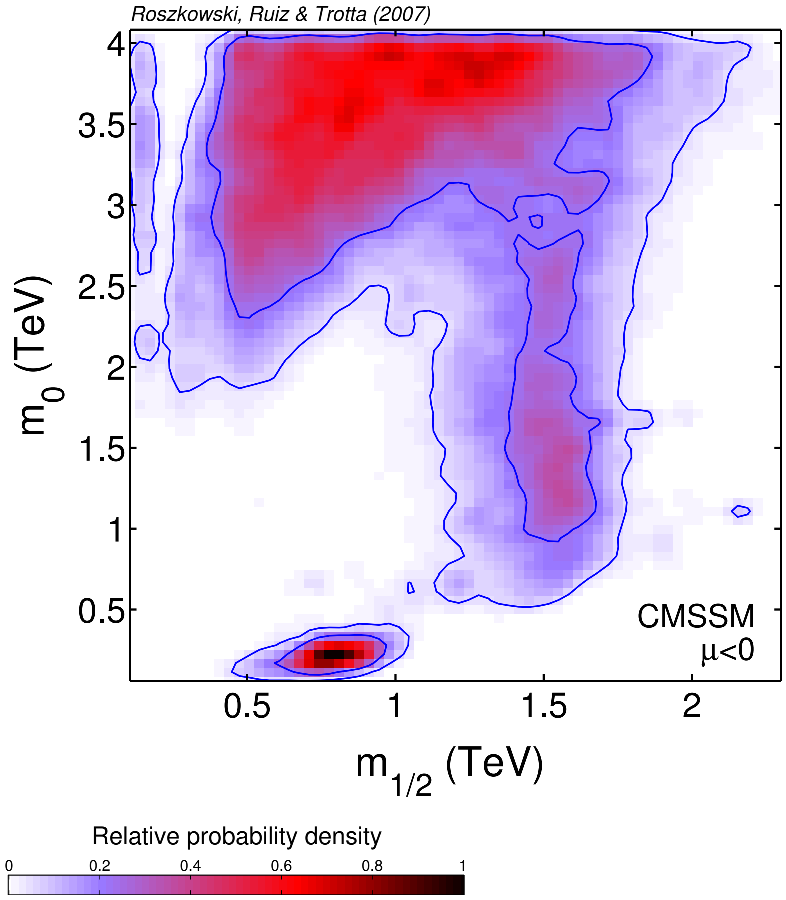
Posterior pdf vs. mean qof: $\mu < 0$

posterior pdf

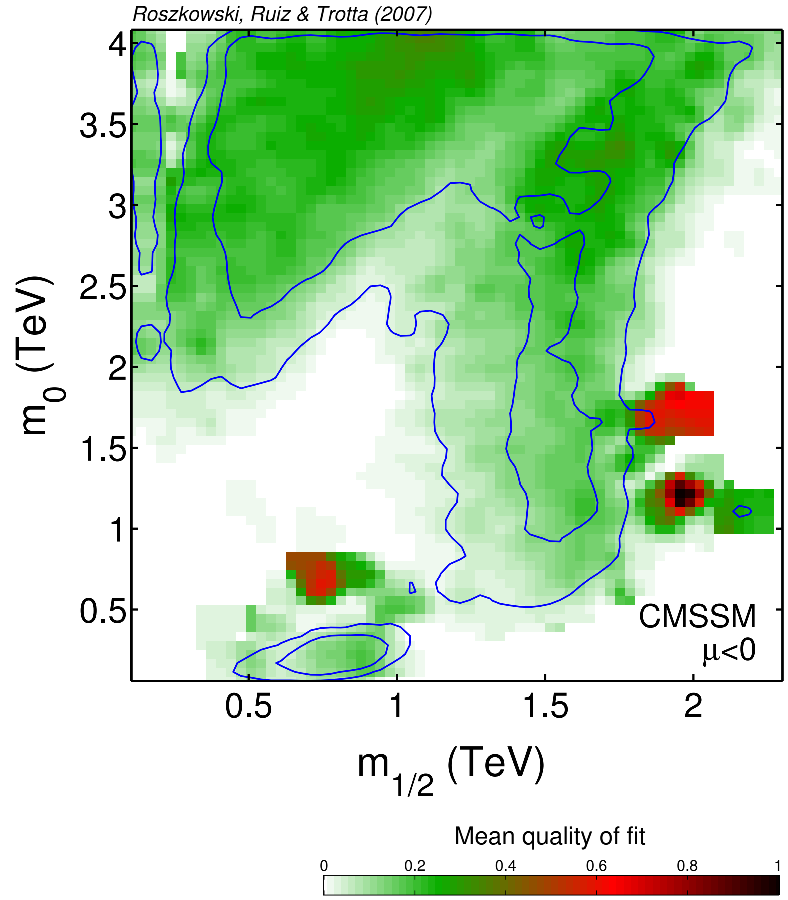


Posterior pdf vs. mean qof: $\mu < 0$

posterior pdf



mean qof



$\mu < 0$ generally poorer fit to the data