The origin of primordial non-gaussianity

David H. Lyth

Particle Theory and Cosmology Group

Physics Department

Lancaster University

Introduction

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- Powerful discriminator between models for origin of ζ
  - Like n, n' and r
- Only upper bounds at present
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- This talk mostly about fundamental theory
  - Little about models
    - Less about observation

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- Constant value $\zeta(\mathbf{x})$ at $T \sim 10$ keV provides the initial condition for adiabatic perturbations.

The correlators

Spectrum $\mathcal{P}_\zeta$, bispectrum[†] $f_{\rm NL}$, trispectrum[††] $\tau_{\rm NL}$:

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\begin{aligned}
\langle \zeta_{\mathbf{k}} \zeta_{\mathbf{k}'} \rangle &= (2\pi)^3 \delta(\mathbf{k}+\mathbf{k}') K_1 \mathcal{P}_\zeta \\
\frac{5}{3} \langle \zeta_{\mathbf{k}} \zeta_{\mathbf{k}'} \zeta_{\mathbf{k}''} \rangle &= (2\pi)^3 \delta(\mathbf{k}+\mathbf{k}'+\mathbf{k}'') K_2 \mathcal{P}_\zeta^2 f_{\rm NL} \\
\langle \zeta_{\mathbf{k}} \zeta_{\mathbf{k}'} \zeta_{\mathbf{k}''} \zeta_{\mathbf{k}'''} \rangle_{\rm c} &= (2\pi)^3 \delta(\mathbf{k}+\mathbf{k}'+\mathbf{k}''+\mathbf{k}''') K_3 \mathcal{P}_\zeta^3 \tau_{\rm NL}
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where the kinematic factors depend on the wave-vectors:

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\begin{aligned}
K_1 &\equiv 2\pi^2 / k^3 \\
K_2 &\equiv K_1(k) K_1(k') + \text{ 5perms} \\
K_3 &\equiv K_2 K_1(|\mathbf{k}+\mathbf{k}''|) + 23\text{perms}
\end{aligned}
$$

† Komatsu/Spergel 2000; Maldacena 2003 †† Boubekeur/DHL 2005

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$$\begin{aligned}\zeta(\mathbf{x},t) &= N(\phi_i(\mathbf{x}),\rho(t)) - N(\phi_i,\rho(t)) \\ &= N_i(t)\delta\phi_i(\mathbf{x}) + \frac{1}{2}N_{ij}(t)\delta\phi_i(\mathbf{x})\delta\phi_j(\mathbf{x}) + \cdots\end{aligned}$$

DHL, Malik & Sasaki 05; DHL & Rodriguez 05 (non-perturbative)

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TREE LEVEL SPECTRUM

$$\langle \zeta_{\mathbf{k}} \zeta_{\mathbf{k}'} \rangle_{\text{tree}} = N'^2 \langle \varphi_{\mathbf{k}} \varphi_{\mathbf{k}'} \rangle = (2\pi)^3 N'^2 \frac{2\pi^2}{k^3} \left(\frac{H_*}{2\pi} \right)^2 \delta(\mathbf{k} + \mathbf{k}')$$

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&= \frac{1}{2}\frac{N'^2N''}{(2\pi)^3}\int\langle\varphi_{\mathbf{k}_1}\varphi_{\mathbf{q}}\rangle\langle\varphi_{\mathbf{k}_2}\varphi_{\mathbf{k}_3-\mathbf{q}}\rangle d^3q + \text{ 5 perms.} \\
&= \frac{(2\pi)^3}{2}\frac{N'^2N''}{(2\pi)^3}\left(\frac{H_*}{2\pi}\right)^4 K_2\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) \quad \text{(-4)}
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\end{aligned}$$

Gives $(3/5)f_{\text{NL}}^{\text{tree}} = \frac{1}{2}\frac{N''}{N'^2}$

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LOOP CONTRIBUTION TO SPECTRUM

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Infrared divergent, use box size L then

$$\mathcal{P}_\zeta{}^{\rm loop} = N''^2\left(\tfrac{H_*}{2\pi}\right)^4 \ln(kL),$$ negligible if $\ln(kL)\sim 1$

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Box size matters in principle, maybe in practice too

Unsurprising: correlators give outcome of measurement by *typical* observer in the box

ζ from non-gaussian perturbation φ

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Extra contribution $$\boxed{\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle = N'^3\langle \varphi_{\mathbf{k}_1}\varphi_{\mathbf{k}_2}\varphi_{\mathbf{k}_3}\rangle}$$

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(i) Calculate action to at least third order in φ

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Assume single-field slow-roll inflation

Just gravitational interaction, at tree-level Seery/Lidsey 2005

$$\boxed{\langle \varphi_{\mathbf{k}_1}\varphi_{\mathbf{k}_2}\varphi_{\mathbf{k}_3}\rangle = -(2\pi)^3\left(\frac{H_*}{2\pi}\right)^4 \frac{\pi^4(k_1^3+k_2^3+k_3^3)}{k_1^3k_2^3k_3^3}\sqrt{2\epsilon}\, f\, \delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)}$$

with $2 < f(k_1,k_2,k_3) < 11/3$ and $\sqrt{2\epsilon} = -M_{\rm P}V'/V$

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$f_{\rm NL}$ from $\frac{1}{2} N''(\delta\phi)^2$ *and* non-gaussianity of $\delta\phi$

$$\boxed{\frac{3}{5} f_{\rm NL} = -\frac{1}{4}\left[n - 1 + (r/8) y(k_1, k_2, k_3)\right] \sim \pm 10^{-2}}$$

$0 < y < 5/6$ Maldacena 2003; Seery/Lidsey 2005

The curvaton model

Mollerach 90; Linde/Mukhanov 96; DHL/Wands 01; Moroi/Takahashi 01

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- Curvaton model is not an epicycle!
  - Candidate sRHν discovered serendipitously (Hamaguchi/Murayama/Yanagida 01)

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Prediction of curvaton model

$$\zeta = \frac{\partial N}{\partial \sigma}\delta\sigma + \frac{1}{2}\frac{\partial^2 N}{\partial \sigma^2}\delta\sigma^2$$

- Allow evolution of curvaton $\sigma_{\rm os}(\sigma_*)$
  - At decay $\rho_\sigma/\rho \equiv \Omega_\sigma$

$$\boxed{\frac{3}{5}f_{\rm NL} = \frac{3}{4\Omega_\sigma}\left(1 + \frac{\sigma_{\rm os}\sigma''_{\rm os}}{(\sigma'_{\rm os})^2}\right) - 1 - \frac{1}{2}\Omega_\sigma}$$

- Instead of δN, can use cosmological perturbation theory

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- need to know unperturbed ϕ_* within the chosen box

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- Observers; go for $|f_{\rm NL}| < 1$ (using 21 cm anisotropy)?
  - go for n' too, but forget r if $r < 10^{-2}$