Braneworlds

Kazuya Koyama
ICG, University of Portsmouth

general relativity + particle physics: expanding 4D universe model

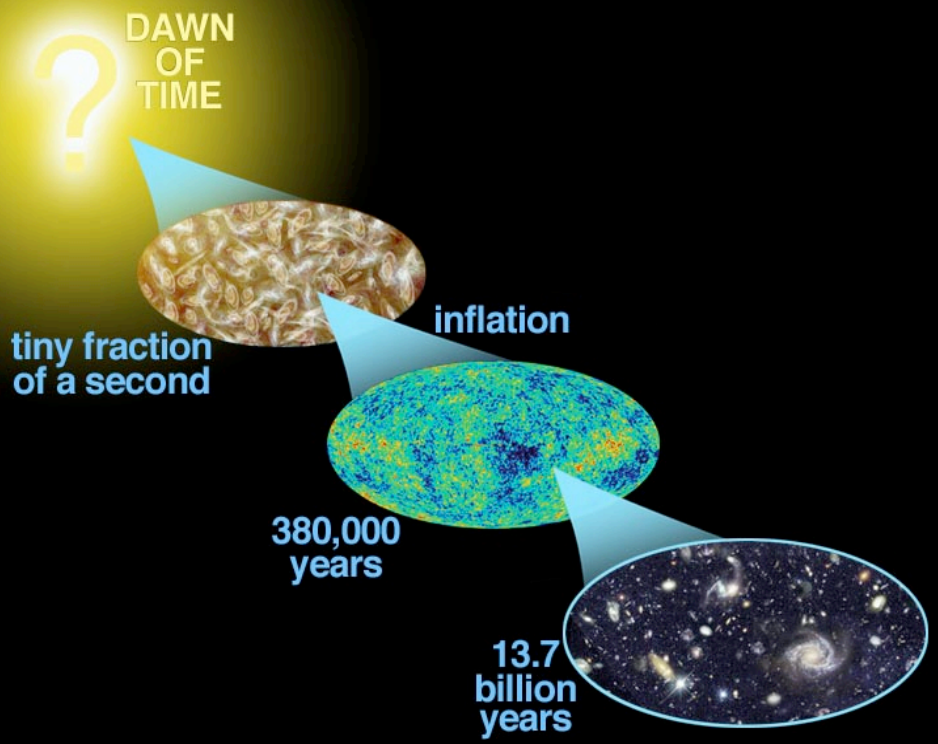
- but

  dark matter?

  dark energy?

- and

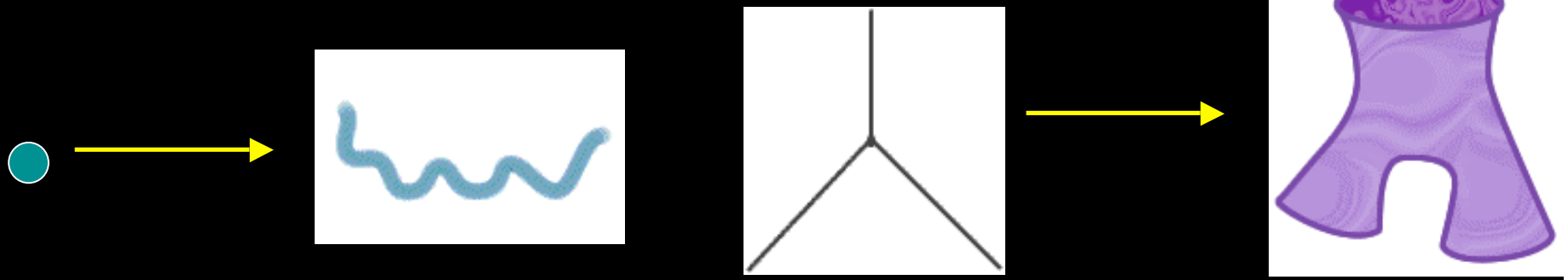
  very early times?



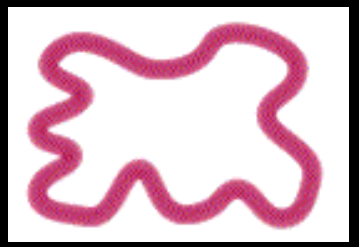
need quantum gravity + unified theory

String theory

- removes the infinities of QFT



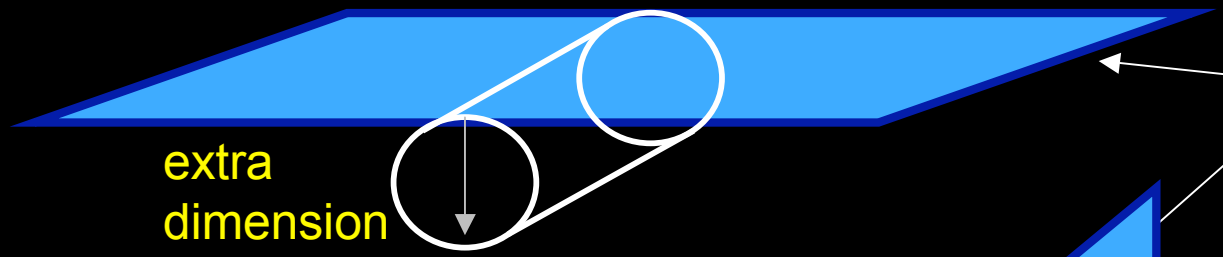
- and includes the graviton



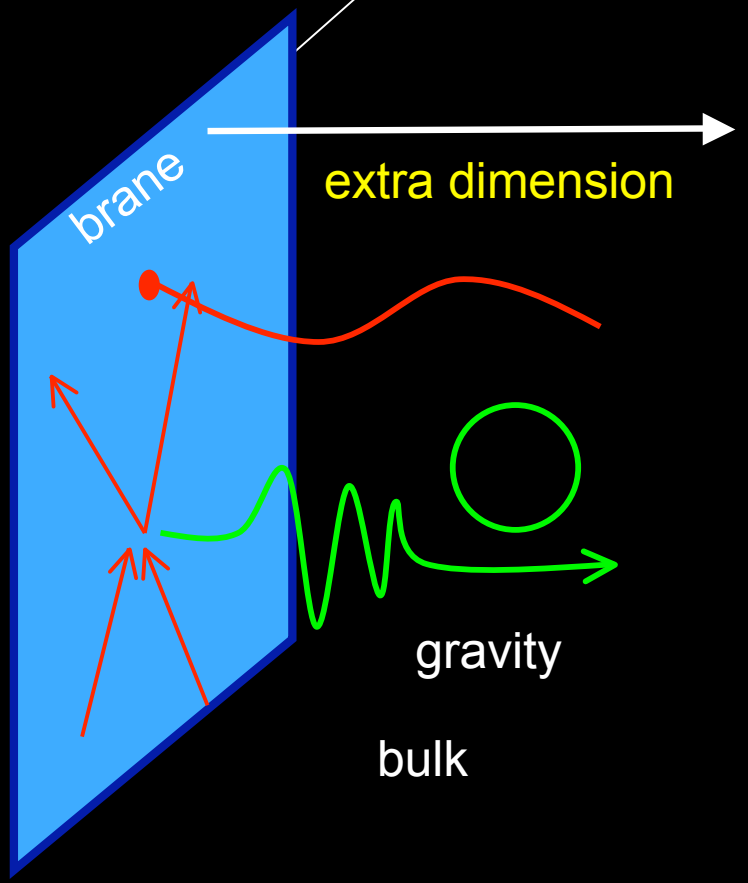
- but the price is – **9 space dimensions**

  Why don't we see the extra-dimensions?

- Kaluza-Klein
  - Internal extra dimension is too small to be seen



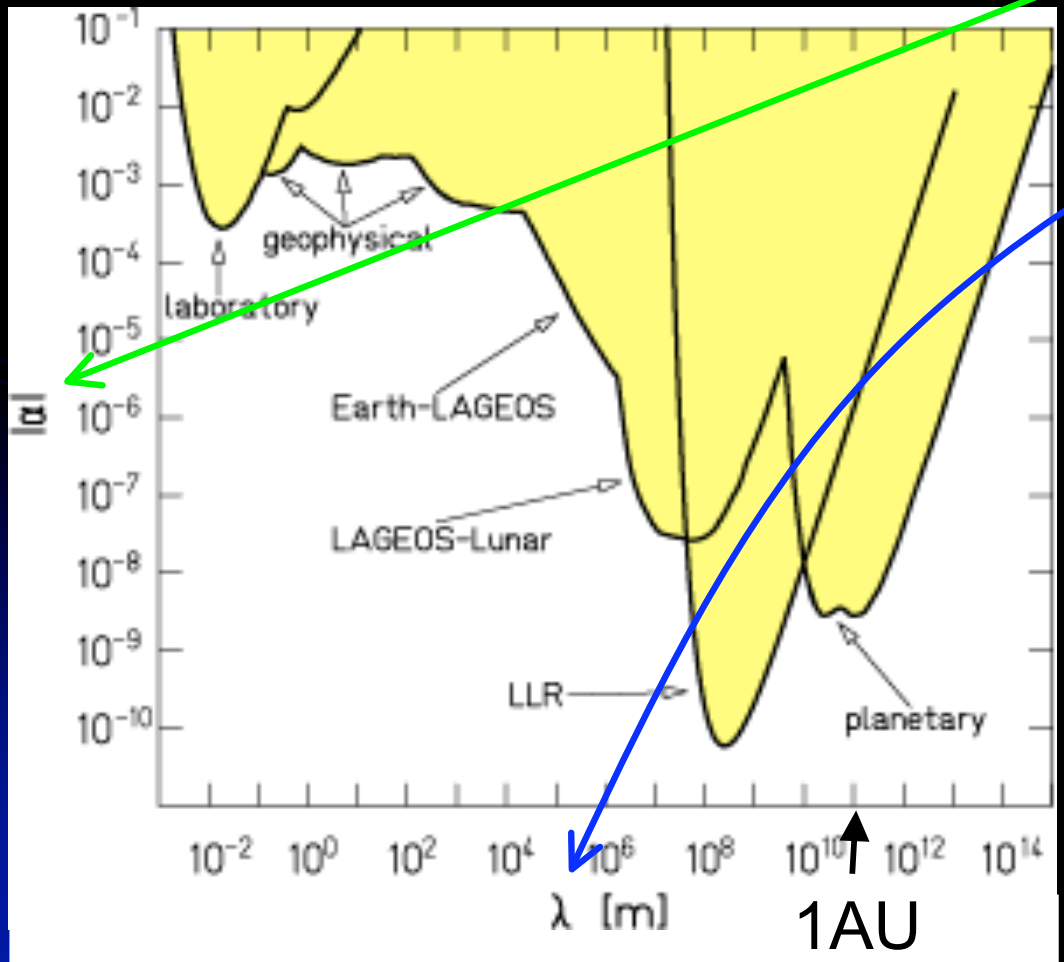
- Braneworld
  - Matter fields are restricted to lower dimensional brane
  - External bulk felt only through gravity



Extra-dimension changes law of gravity

Current constraints on deviations from Newton's law

$$V(r) = G\frac{m_1 m_2}{r}\left[1+\alpha\exp\left(-\frac{r}{\lambda}\right)\right]$$



4D gravity

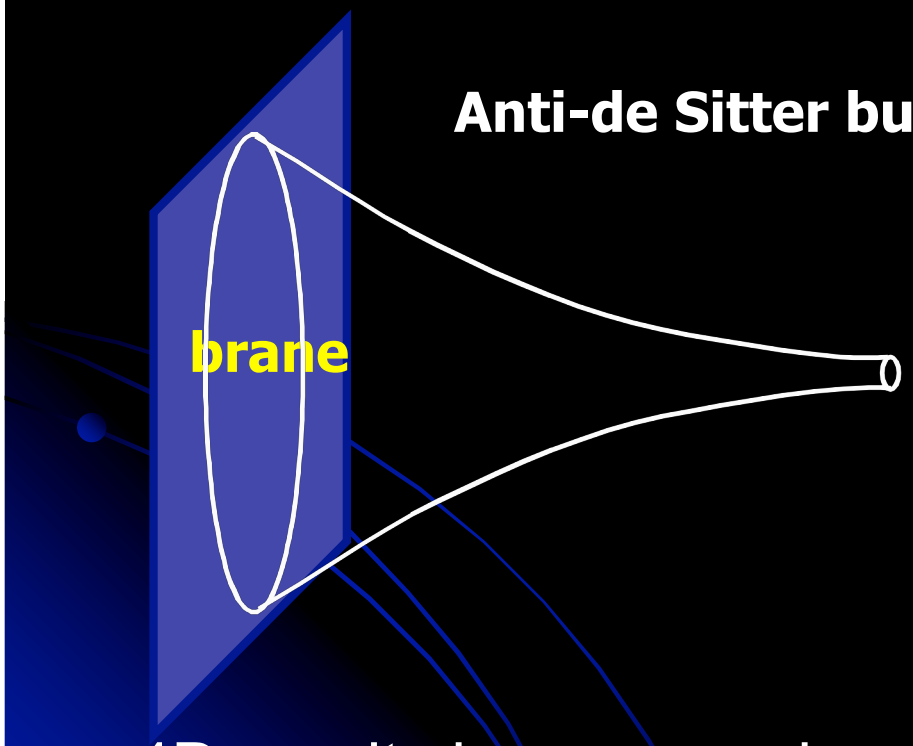
$0.1\ \text{mm} < \lambda < 10^{14}\ \text{m}$

(Adelberger et.al. hep-th/0307284)

How to recover 4D gravity?

Localization of gravity (Randall-Sundrum)

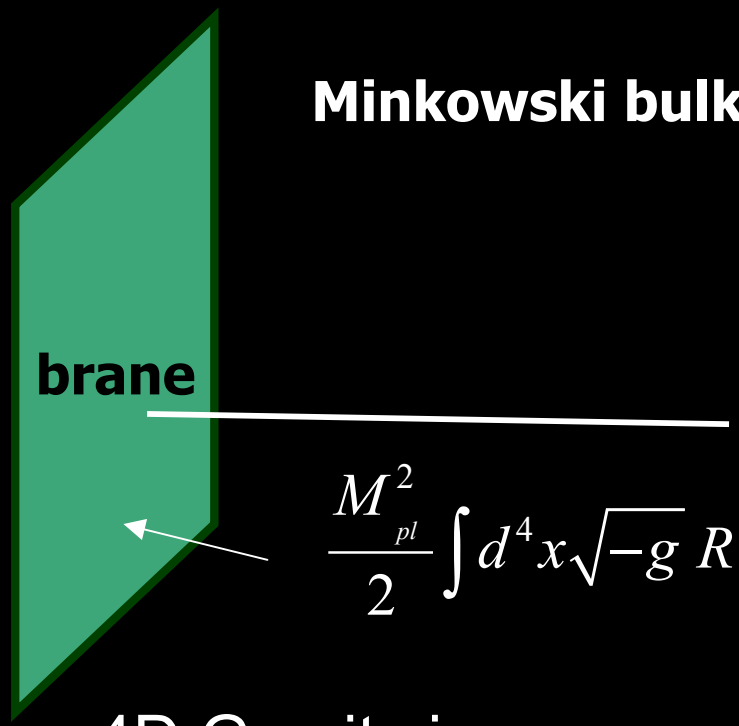
Anti-de Sitter bulk



4D gravity is recovered on large scales $L > 0.1\ \mathrm{mm}$

Brane induced gravity (Dvali-Gabadadze-Porrati)

Minkowski bulk



4D Gravity is recovered on small scales $L < H_0^{-1}$

Brane world model 1

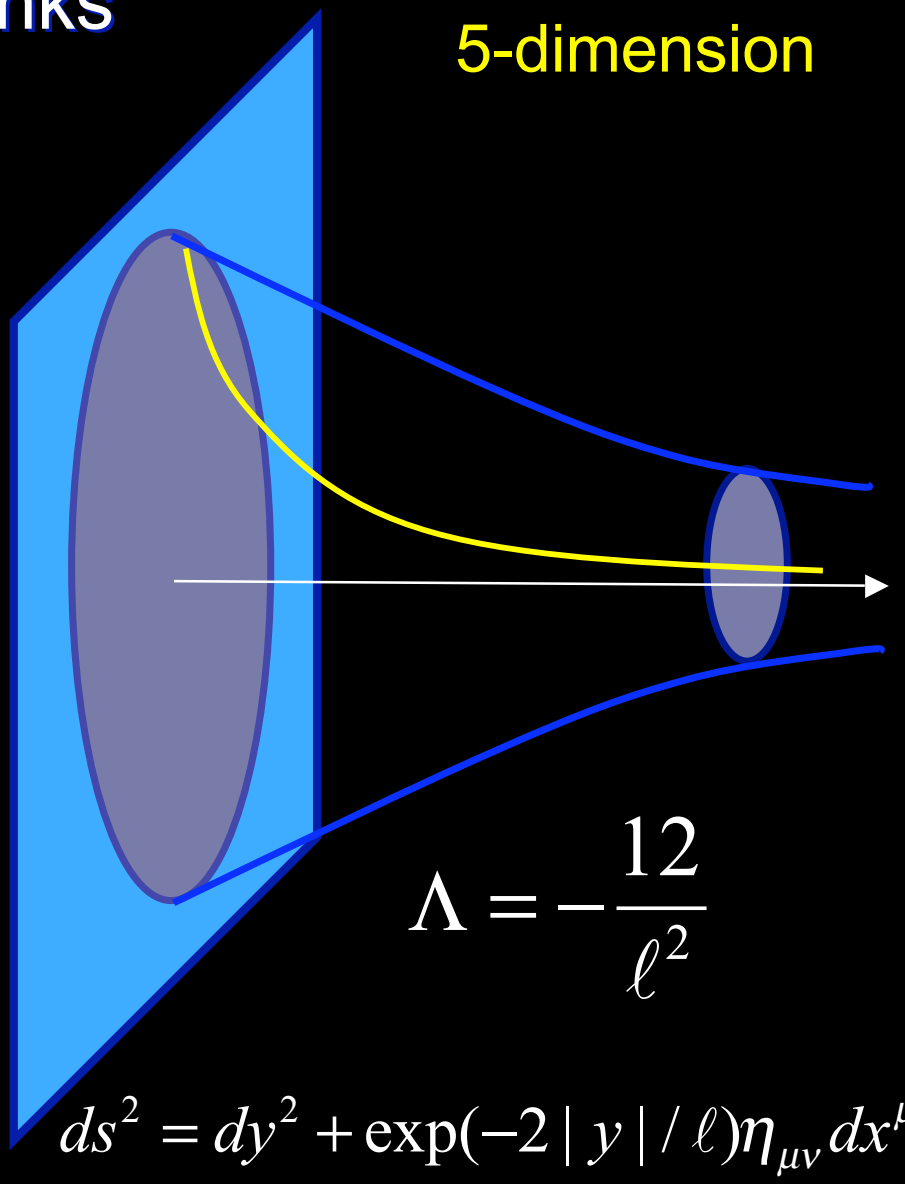
-Localization of gravity-

- Warped geometry (Randall and Sundrum '99)
  - Extra-dimension shrinks toward infinity
  - curvature scale ℓ

4D GR is recovered on large scales

$$r > \ell$$

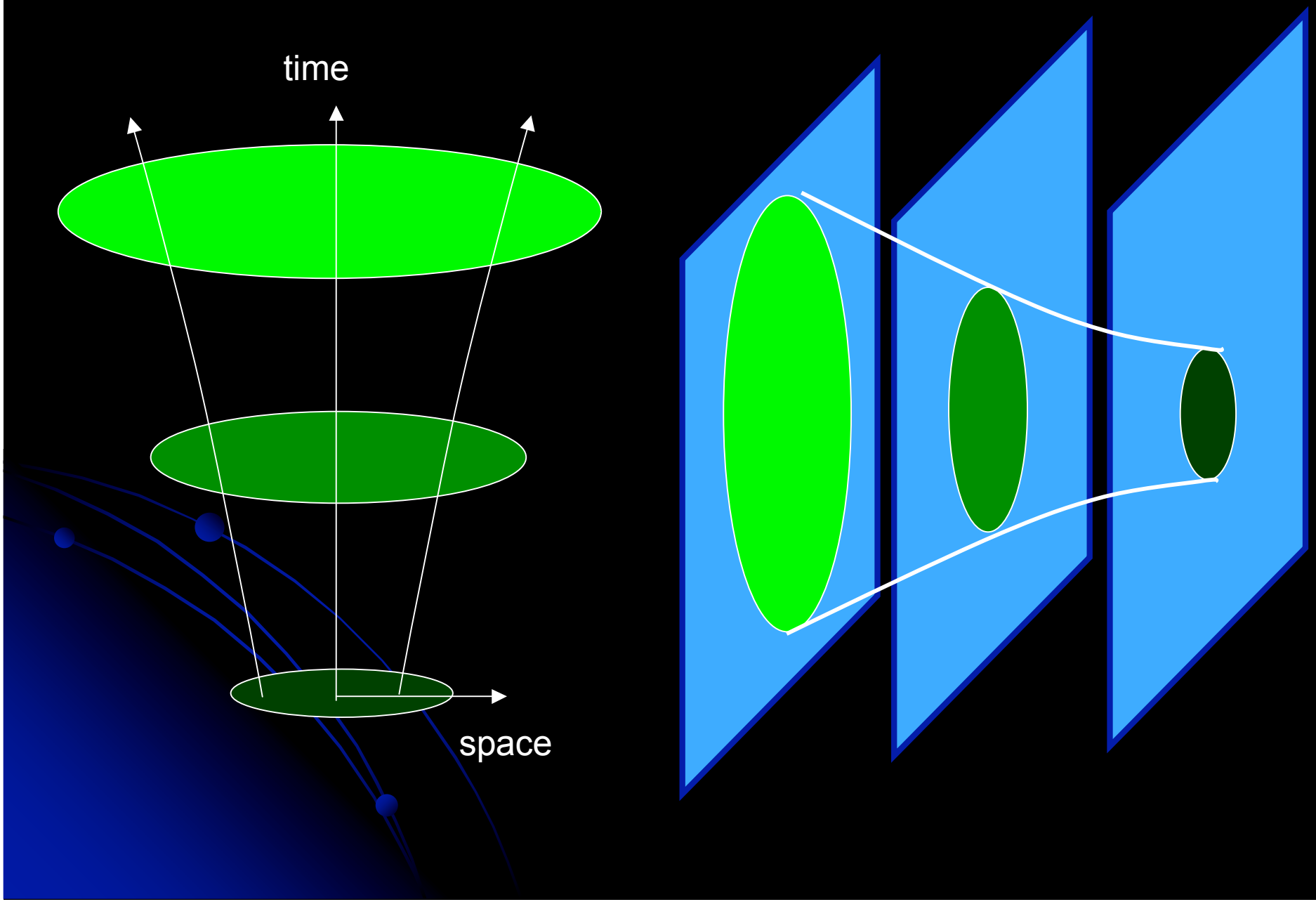
$$\Rightarrow \quad \ell < 0.1\text{mm}$$



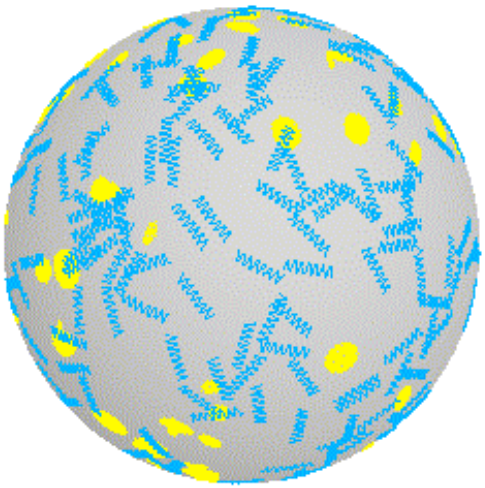
$$\Lambda = -\frac{12}{\ell^2}$$

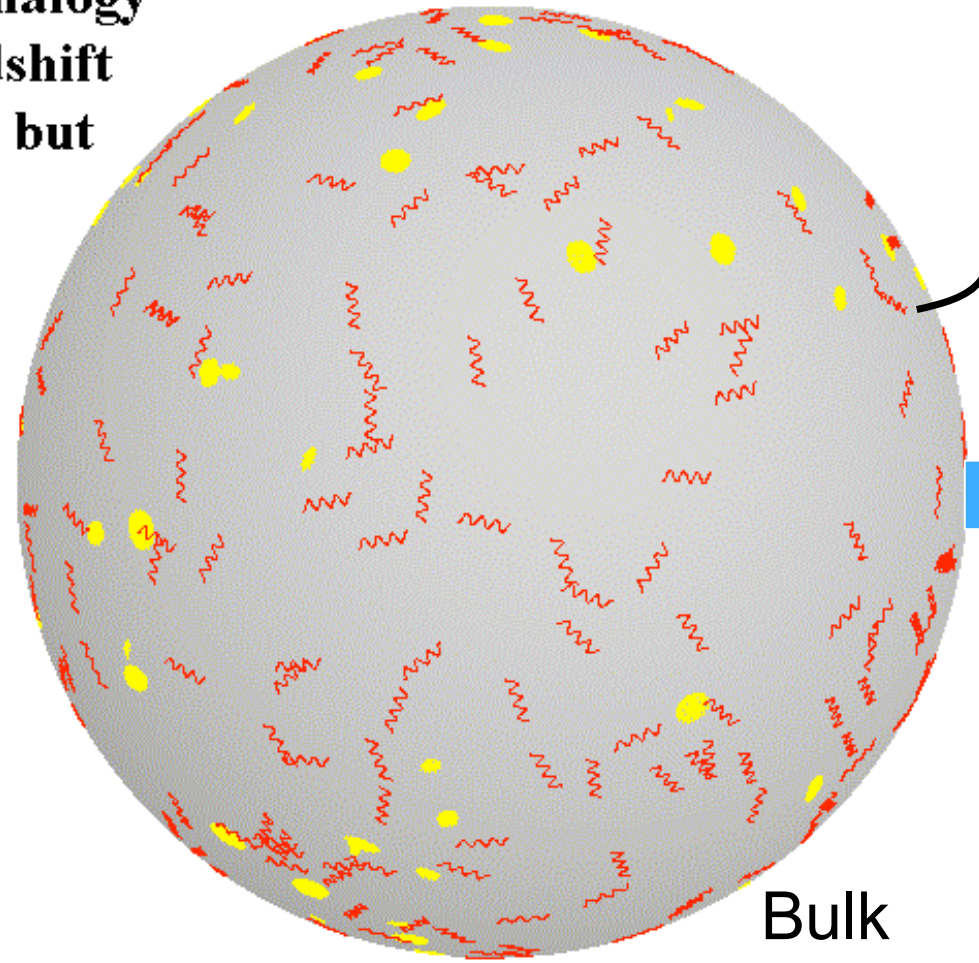
$$ds^2 = dy^2 + \exp(-2|y|/\ell)\eta_{\mu\nu}dx^\mu dx^\nu$$

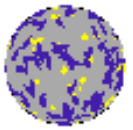
Cosmic expansion = movement of the brane



Expanding Balloon Analogy
Photons move and redshift
Galaxies spread apart but stay the same size





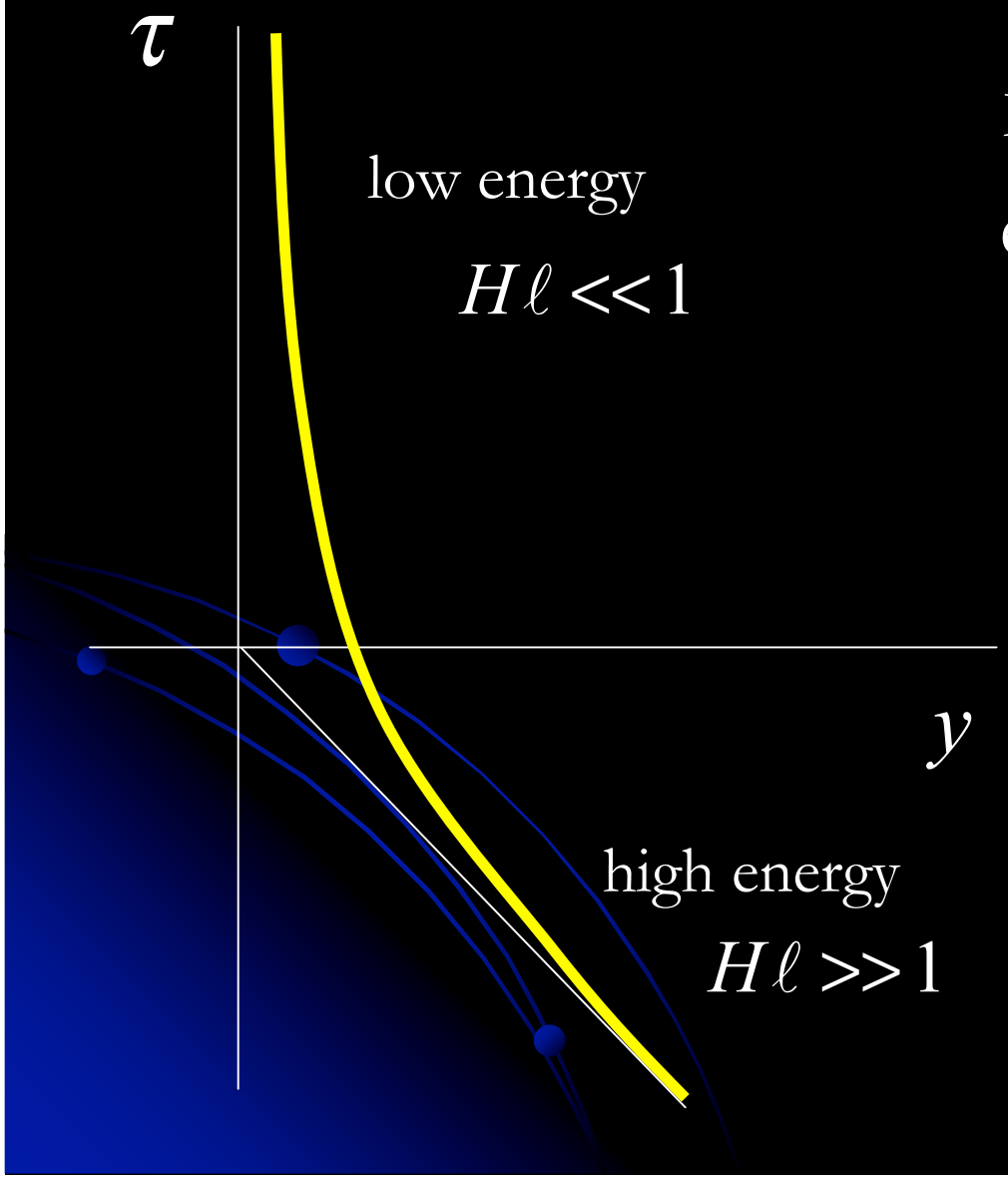


Bulk

(extra-dimensions)

- Brane is moving in static coordinate

$$ds^2 = dy^2 + \exp(-2y/\ell)(-d\tau^2 + \delta_{ij}dx^i dx^j)$$



Motion of the brane

$$\exp(-y_B(\tau)/\ell) = a(t), \quad \tau = T(t)$$

$$\begin{cases} \left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{\rho}{3M_{pl}^2} + \frac{\rho^2}{36M_5^6} \\ M_{pl}^2 = M_5^3 \ell \\ \dot{T} = \frac{1}{a(t)}\sqrt{1 + \left(\frac{H}{\ell}\right)^2} \end{cases}$$

t cosmic time

Deviations from 4D GR in the early universe

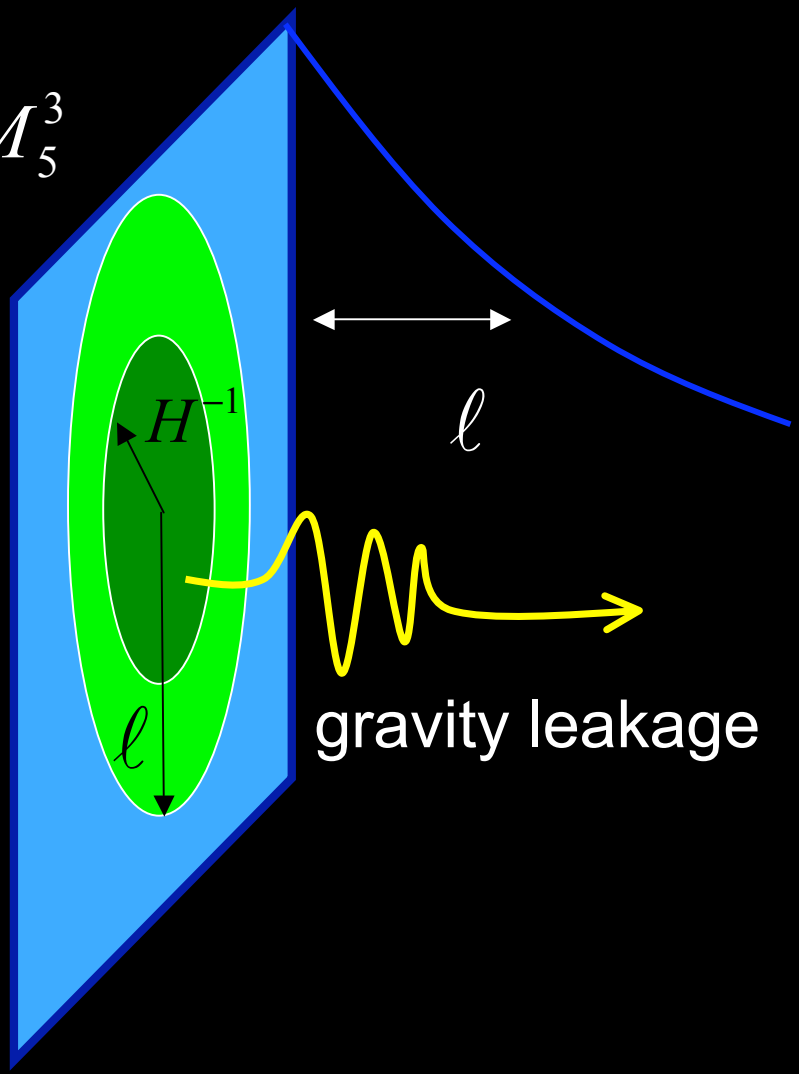
Two high energy effects

- Modified Friedmann equation

$$H^2 = \frac{\rho}{3M_{pl}^2} + \frac{\rho^2}{36M_5^6}, \quad M_{pl}^2 = \ell M_5^3$$

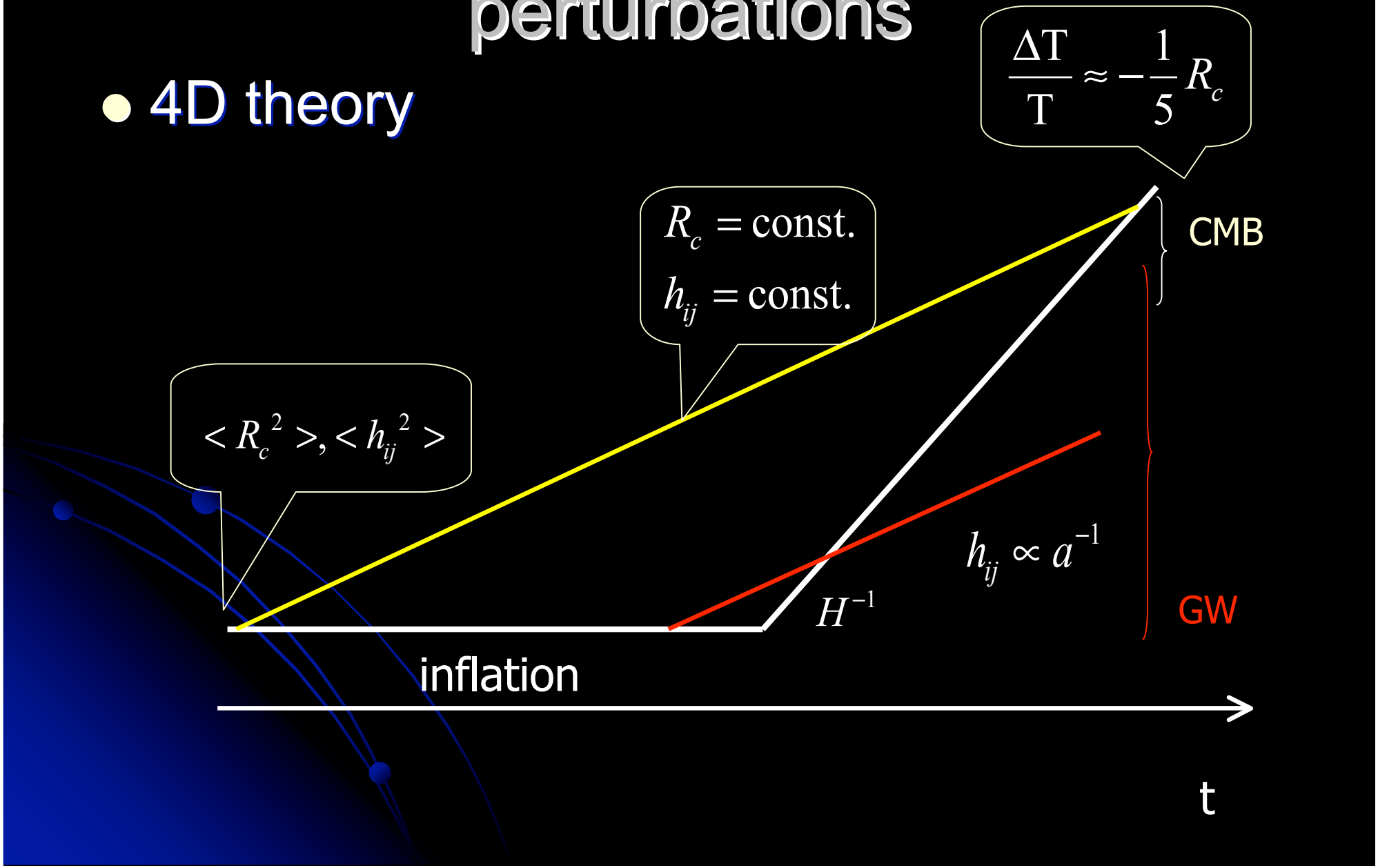
for $H\ell \gg 1$ ➡ $H \propto \rho$

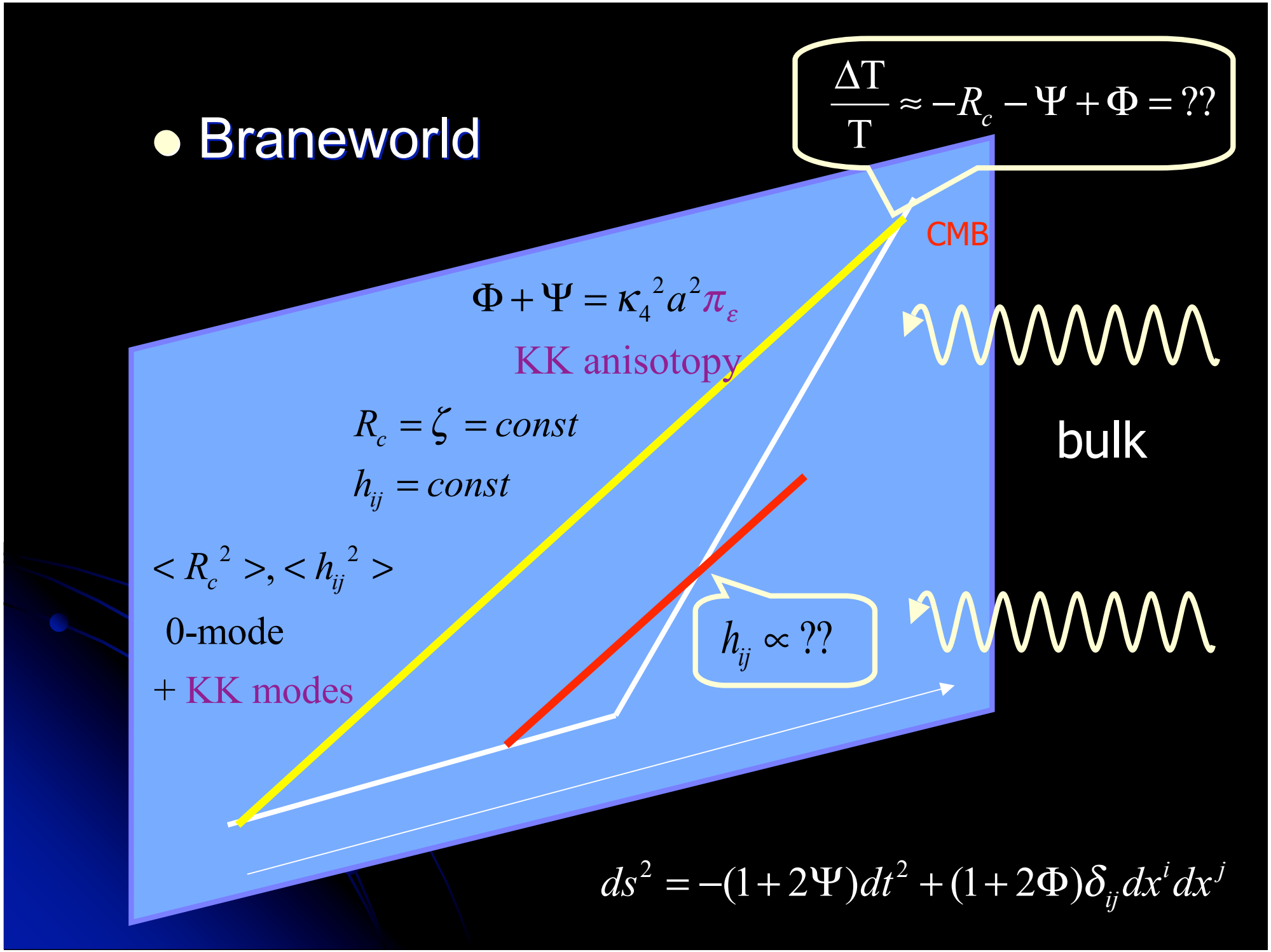
- Gravity leakage
  gravity becomes 5D

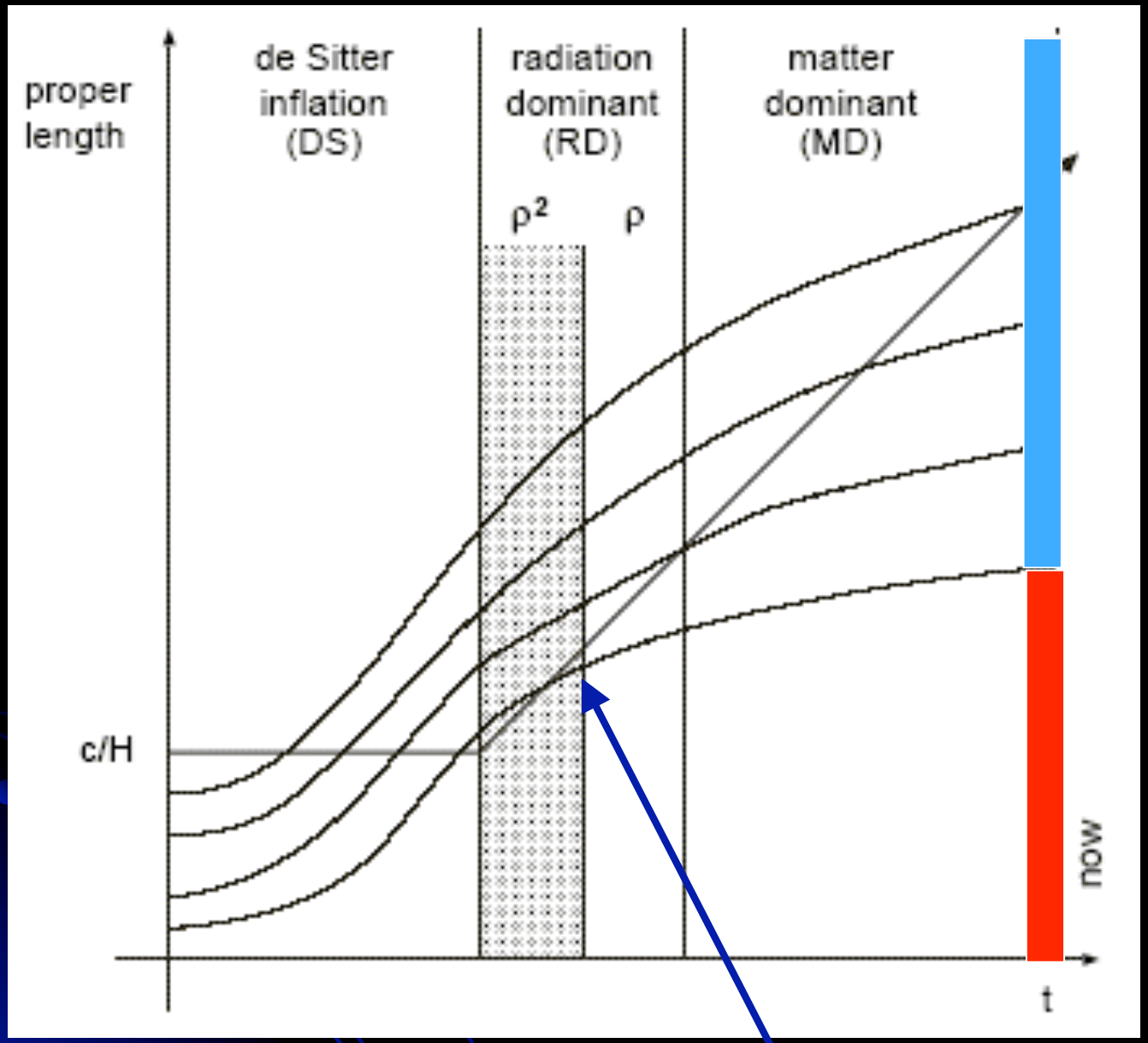


Evolution of cosmological perturbations

- 4D theory




• Braneworld
$\frac{\Delta T}{T} \approx -R_c - \Psi + \Phi = ??$
CMB
$\Phi + \Psi = \kappa_4^2 a^2 \pi_\varepsilon$
KK anisotopy
$R_c = \zeta = const$
$h_{ij} = const$
$< R_c^2 >, < h_{ij}^2 >$
0-mode
+ KK modes
bulk
$h_{ij} \propto ??$
$ds^2 = -(1+2\Psi)dt^2 + (1+2\Phi)\delta_{ij}dx^i dx^j$



4D

$f_{crit} = 2\times10^{-4}\,Hz$

$k^{-1} = 10\,AU$

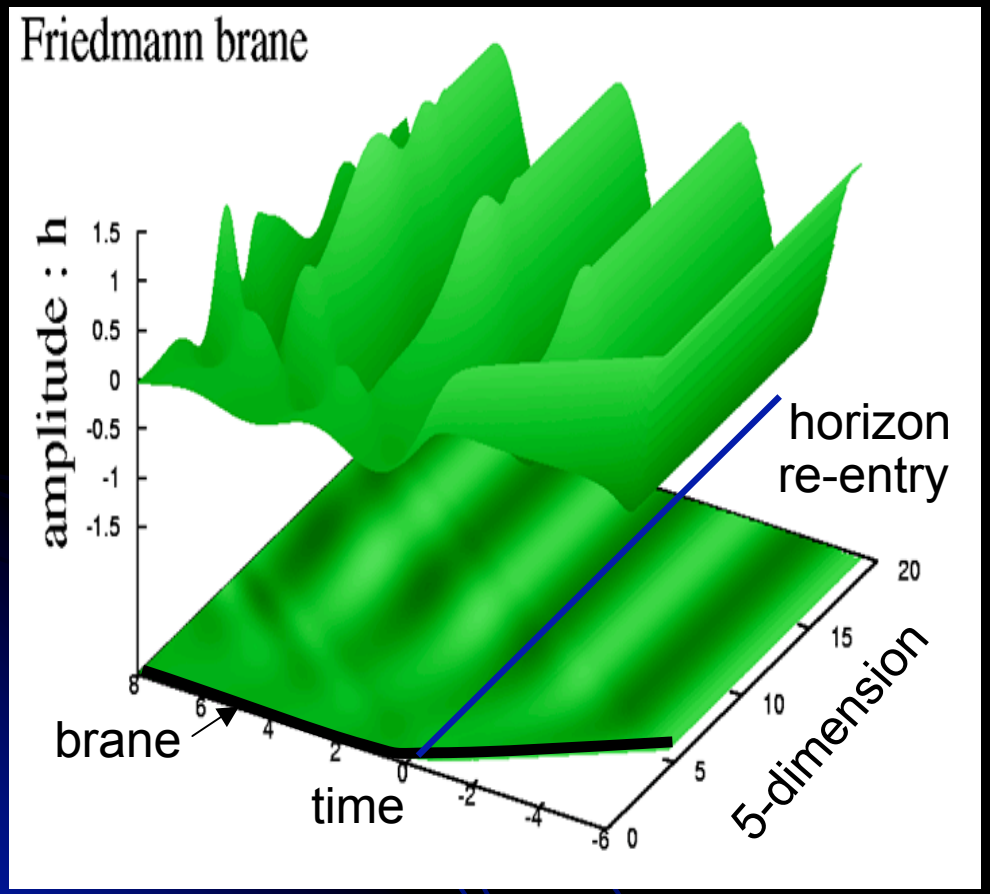
$(\ell = 1mm)$

5D

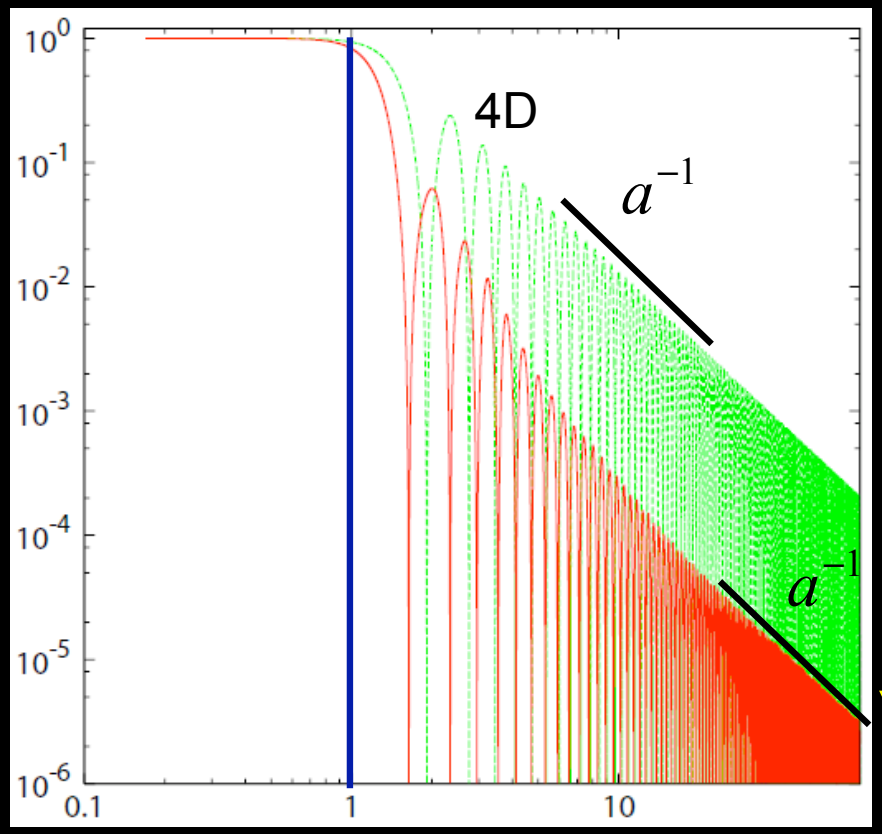
$H^{-1} = \ell$

- Tensor perturbations (Hiramatsu, KK, Taruya '03, '04)

  evolution of a mode that enters horizon at $H\ell \gg 1$



Evolution on the brane



time

Resulting spectrum (Hiramatsu, '05)

- radiation dominated universe

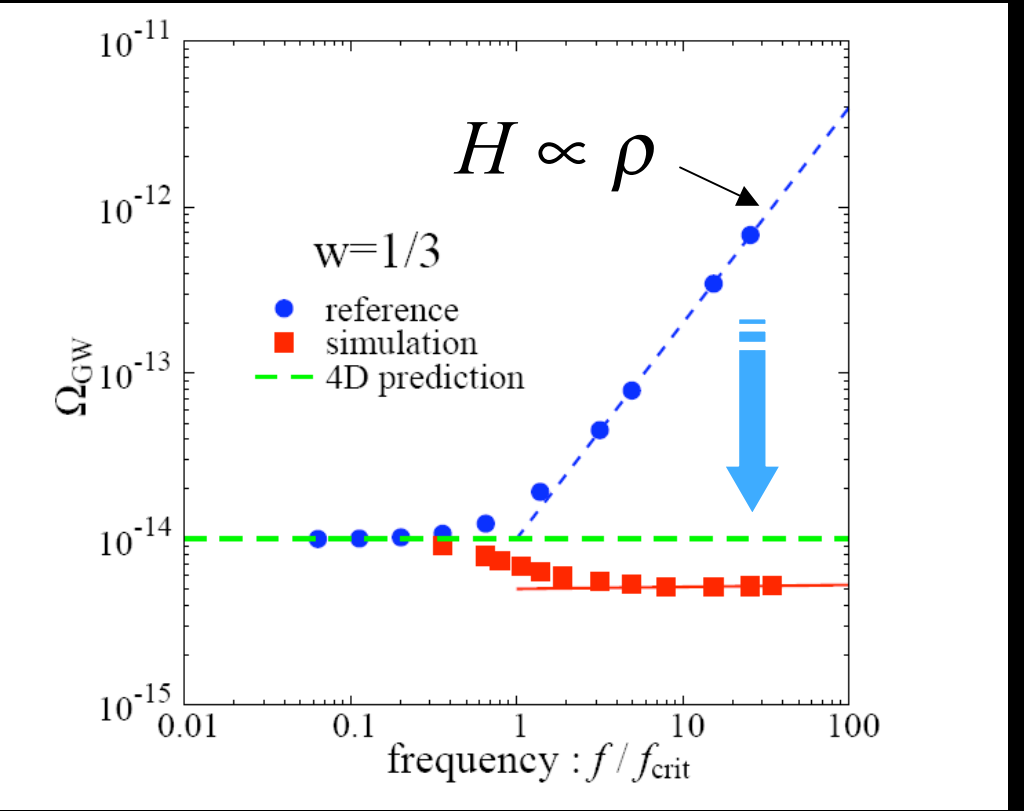
  effect from modified Friedmann equation is exactly canceled by effect from gravity leakage

- Spectrum depends on w

$$\Omega_{5D} \propto f^{\frac{2(3w-1)}{3(3w+2)}} \qquad (f > f_{\rm crit})$$

cf. $\Omega_{4D} \propto f^{\frac{2(3w-1)}{3w+1}}$

(Seahra, Kobayashi & Tanaka)



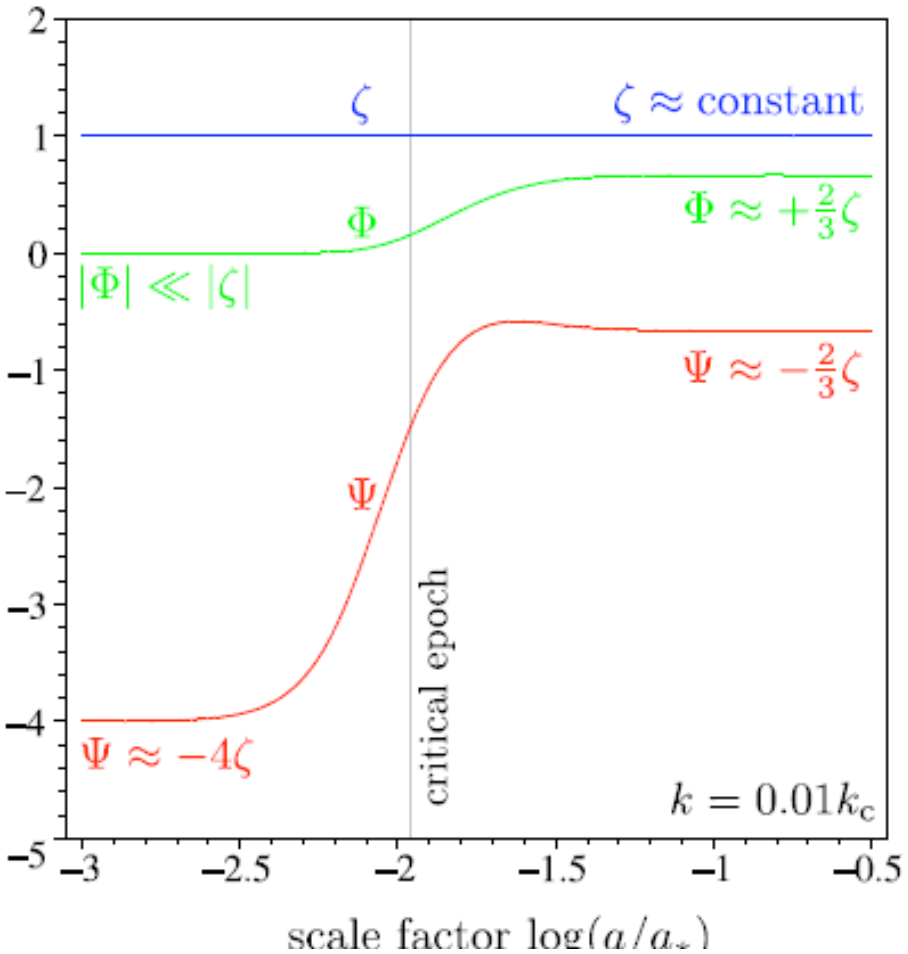
Scalar perturbations

- Metric perturbations on large scales

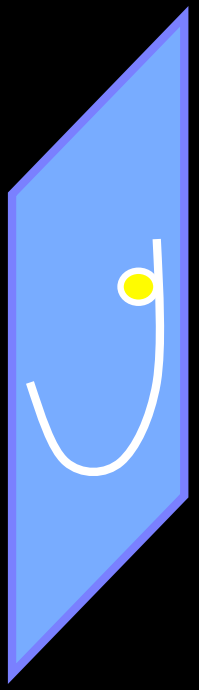
At high energies, GR solution $\Phi = -\Psi$ is largely modified

However, at late times,
GR solution is recovered

No modification to
CMB from the evolution
after inflation



Primordial fluctuations



- Modified Friedmann equation

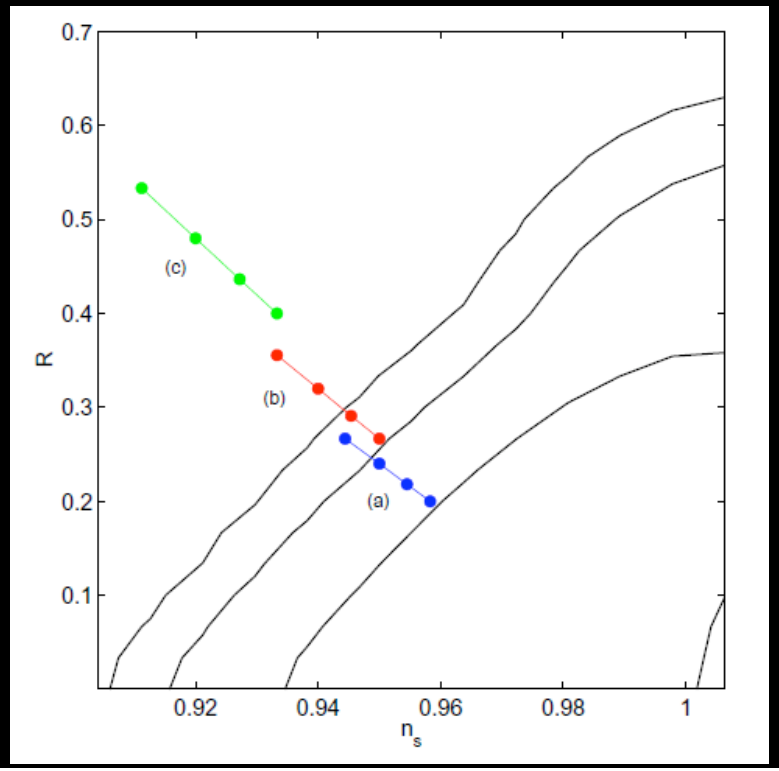
$$H^2 = \frac{\rho}{3M_{pl}^2} + \frac{\rho^2}{36M_5^6}, \quad M_{pl}^2 = \ell M_5^3$$

slow-roll becomes easy

closer to scale invariant

tensor-scalar ratio is suppressed

(for a given potential)

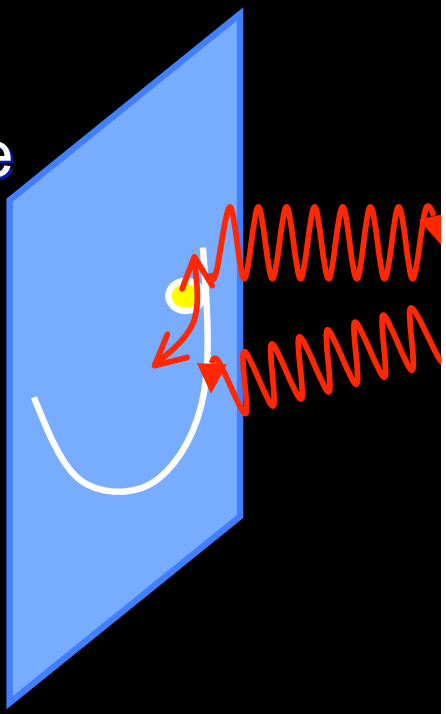


- Gravity leakage

inflaton perturbations are strongly coupled to the 5D perturbations

the classical behaviour is affected dramatically

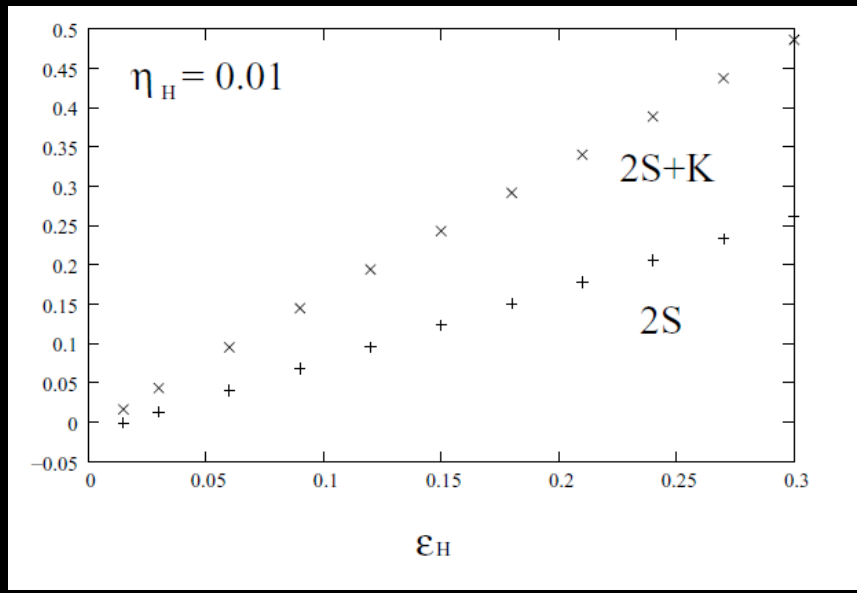
However, particle production is suppressed by slow-roll parameter



$$P_\varsigma = \left(\frac{H}{\dot{\phi}}\right)^2 \left(\frac{H}{2\pi}\right)^2 \times \left[1 + 2S(\varepsilon,\eta) + K(\varepsilon)\right]$$

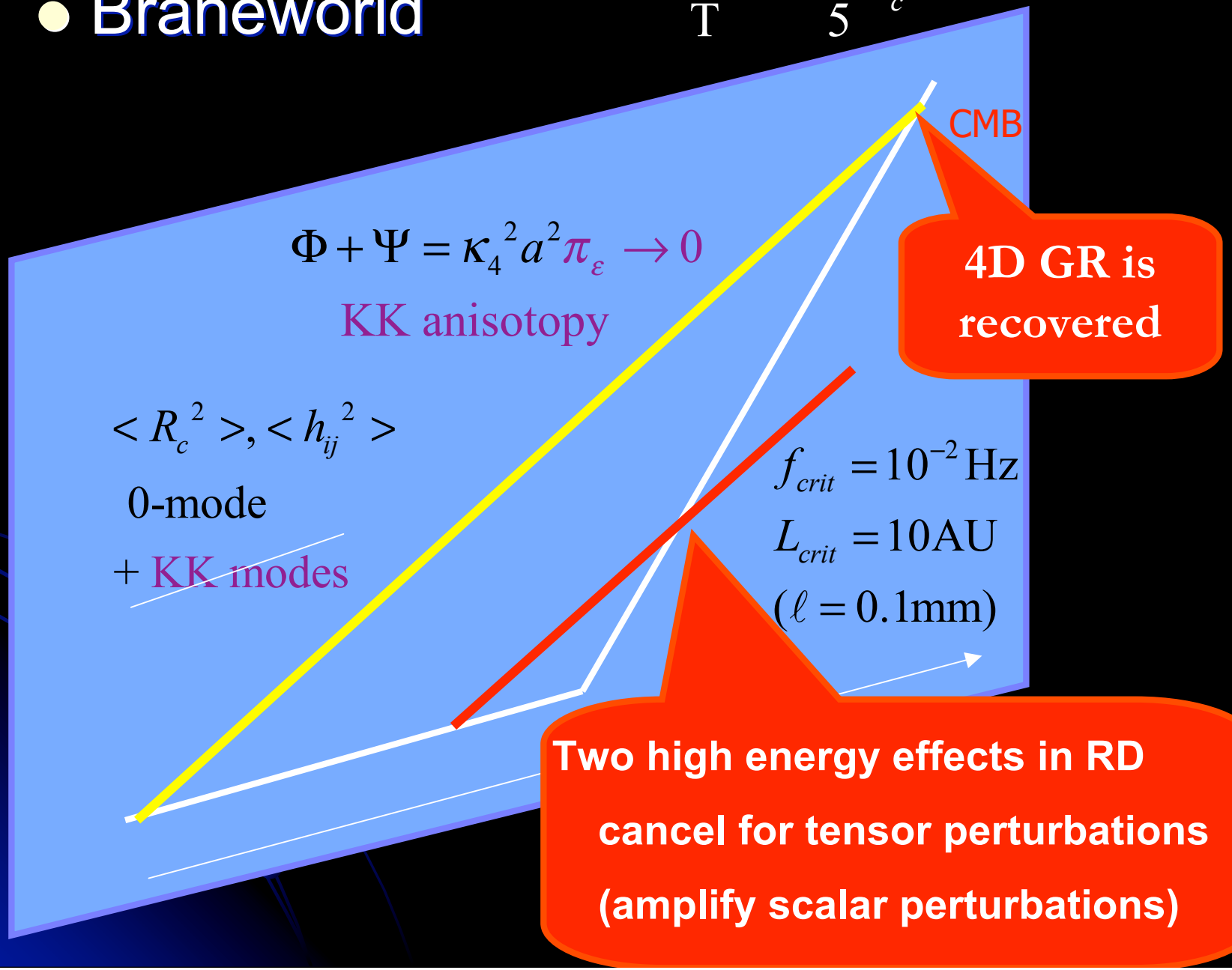
Stewart-Lyth corrections

5D corrections to vacuum



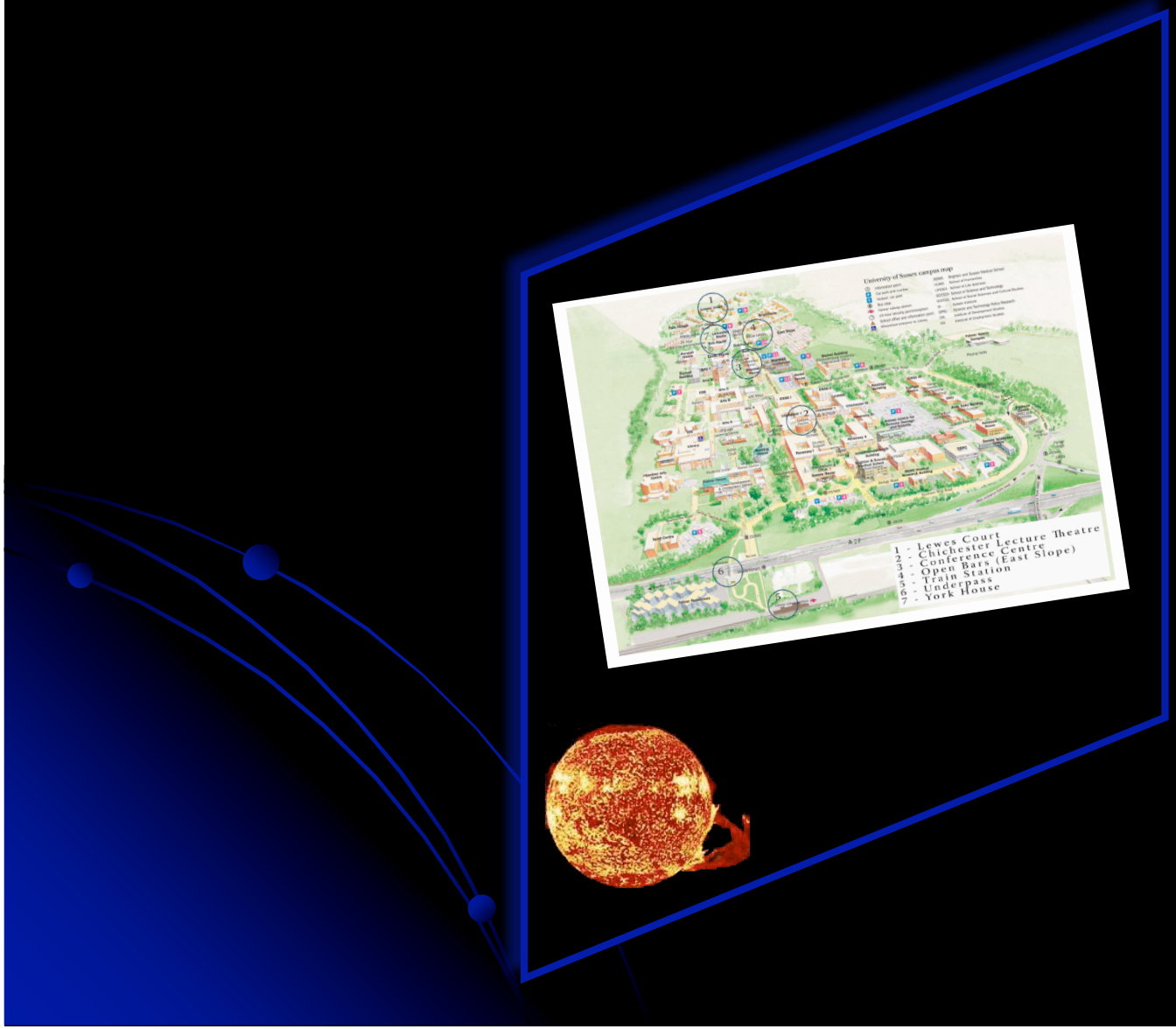
- Braneworld

$$\frac{\Delta T}{T} \approx -\frac{1}{5} R_c$$



Brane world model 2

-induced gravity-



DGP model

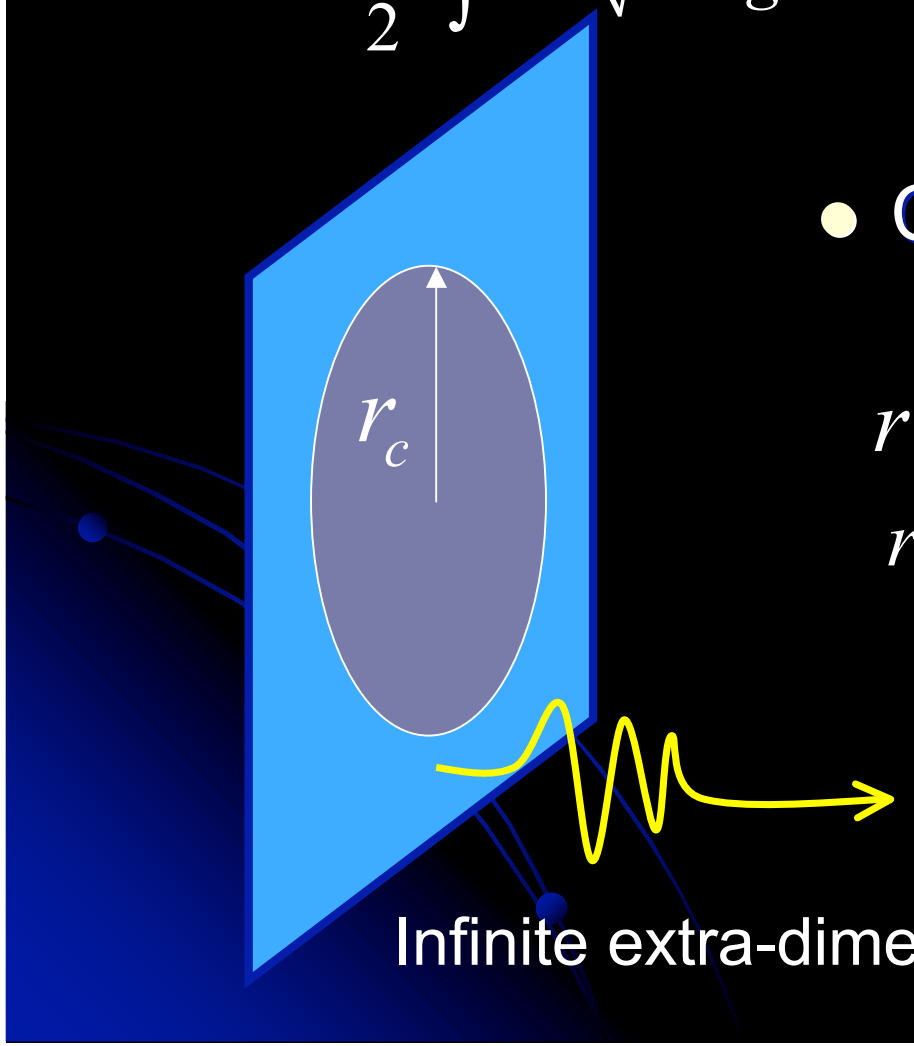
(Dvali, Gabadadze,Porrati, '00)

$$S=\frac{M_5^3}{2}\int d^5x\sqrt{-{}^{(5)}g}\,{}^{(5)}R+\frac{M_{pl}^2}{2}\int d^4x\sqrt{-g}\,R+\int d^4x\,L_m$$

- Crossover scale $r_c \equiv \dfrac{M_{pl}^2}{2M_5^3}$

$r < r_c$ 4D Newtonian gravity

$r > r_c$ 5D Newtonian gravity



Cosmology of DGP

$$\frac{M_{pl}^2}{2}\int d^4x\sqrt{-g}\,R + \int d^4x\,L_m + \frac{M_{pl}^3}{4r_c}\int d^5x\sqrt{-{}^{(5)}g}\,{}^{(5)}R$$

- Friedmann equation (Deffayet)

$$\frac{H}{r_c} = H^2 - \frac{8\pi G}{3}\rho$$

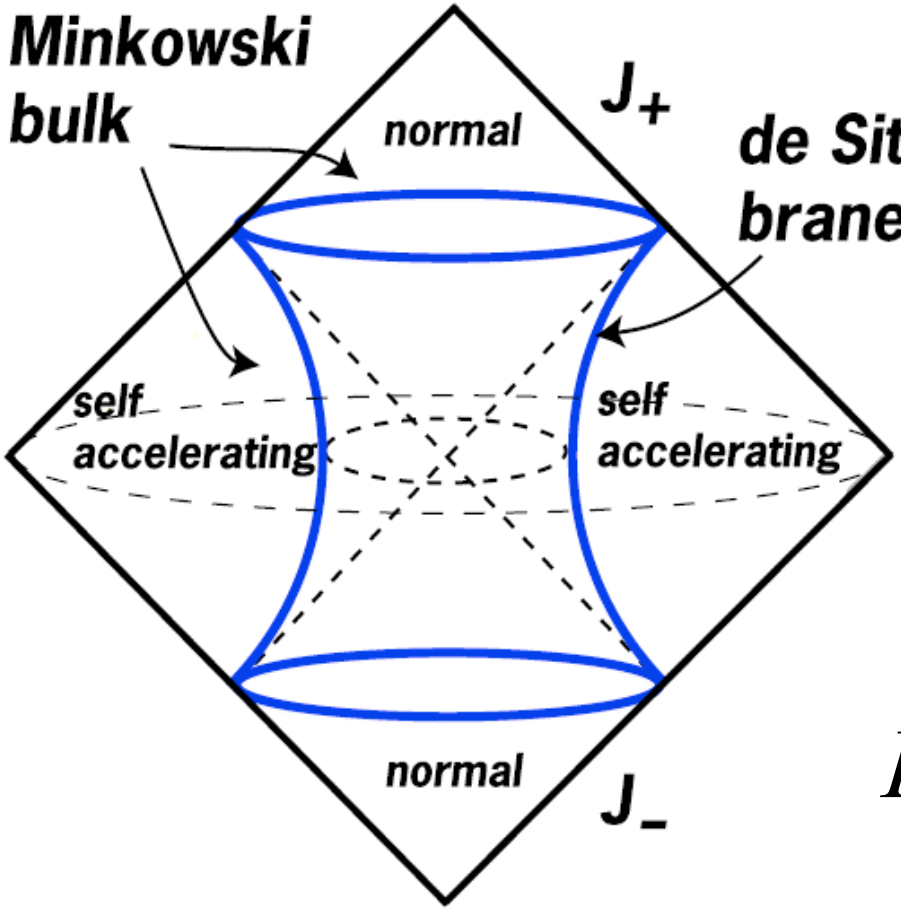
early times $Hr_c \to \infty$ 4D Friedmann

late times $\rho \to 0$ $\quad H \to \frac{1}{r_c}$

As simple as LCDM model

and as fine-tuned as LCDM $r_c \sim H_0^{-1}$

4D de Sitter =hyperboloid in 5D Minkowski



K^{μ}_{ν} :Extrinsic curvature

$$G^{\mu}{}_{\nu} - \kappa_4^{-2} T^{\mu}{}_{\nu} = \frac{1}{r_c}(K^{\mu}{}_{\nu} - K\delta^{\mu}{}_{\nu})$$

Embedding of 4D spacetime in 5D

Gravity on small scales

(Lue et.al, KK and R.Maartens)

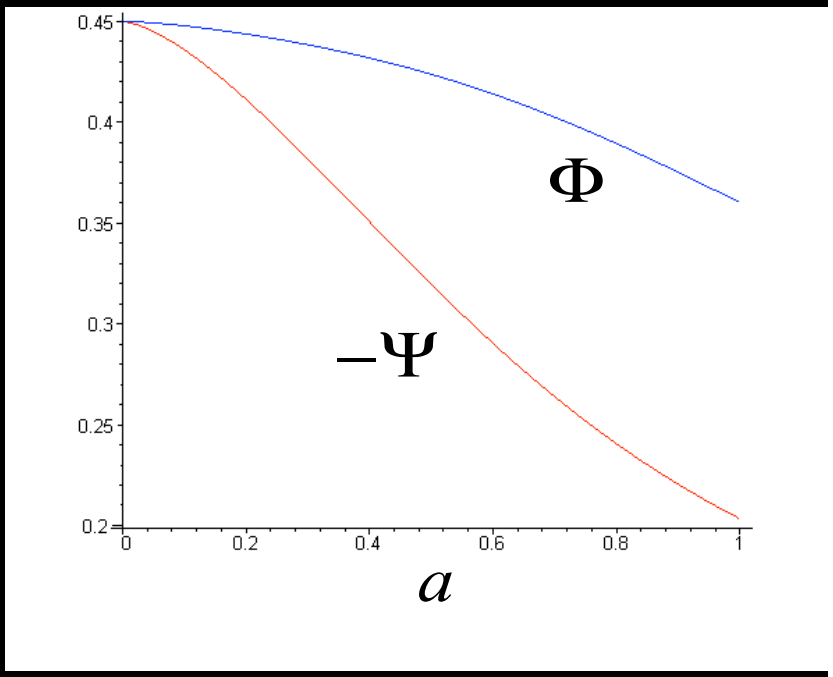
- Linear metric perturbations under horizon scales

$$ds^2 = -(1+2\Psi)dt^2 + a(t)^2(1+2\Phi)d\vec{x}^2$$

$$\frac{k^2}{a^2}\Phi = 4\pi G\left(1-\frac{1}{3\beta}\right)\rho\delta,$$

$$\frac{k^2}{a^2}\Psi = -4\pi G\left(1+\frac{1}{3\beta}\right)\rho\delta,$$

$$\beta = 1-2r_c H\left(1+\frac{\dot{H}}{3H^2}\right)$$



Scalar tensor theory with Brans-Dick parameter $\omega = \frac{3}{2}(\beta - 1)$

Consistent with local experiments?

- Small scales gravity is not described by GR

  $r < r_c$:4D Newtonian but not 4D GR!

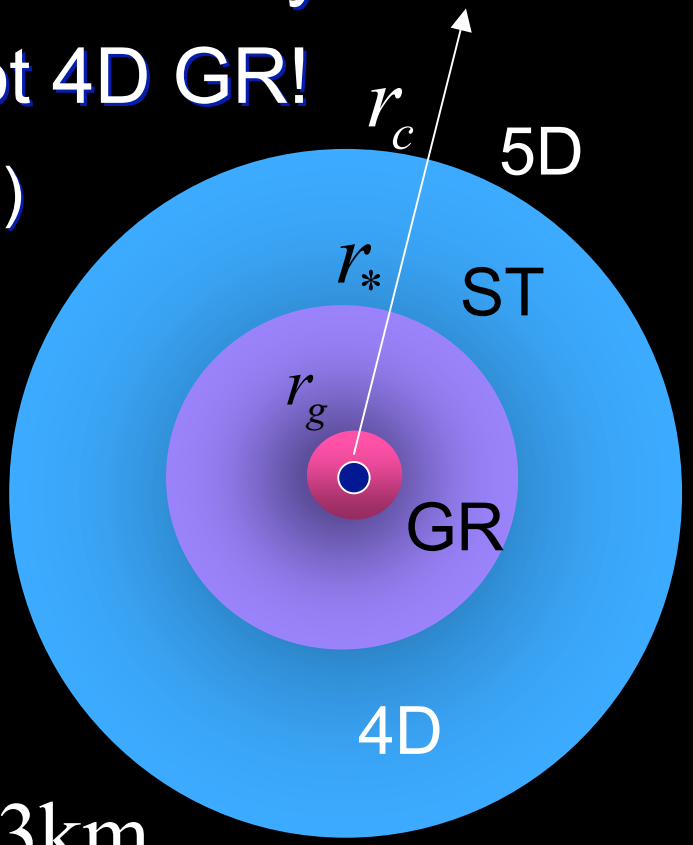
  (Scalar-Tensor theory $\omega = O(1)$)

- Non-linear shielding

  theory becomes GR at

$$r < r_* = \left(r_g r_c^2\right)^{\frac{1}{3}}$$

(Deffayet et.al.)

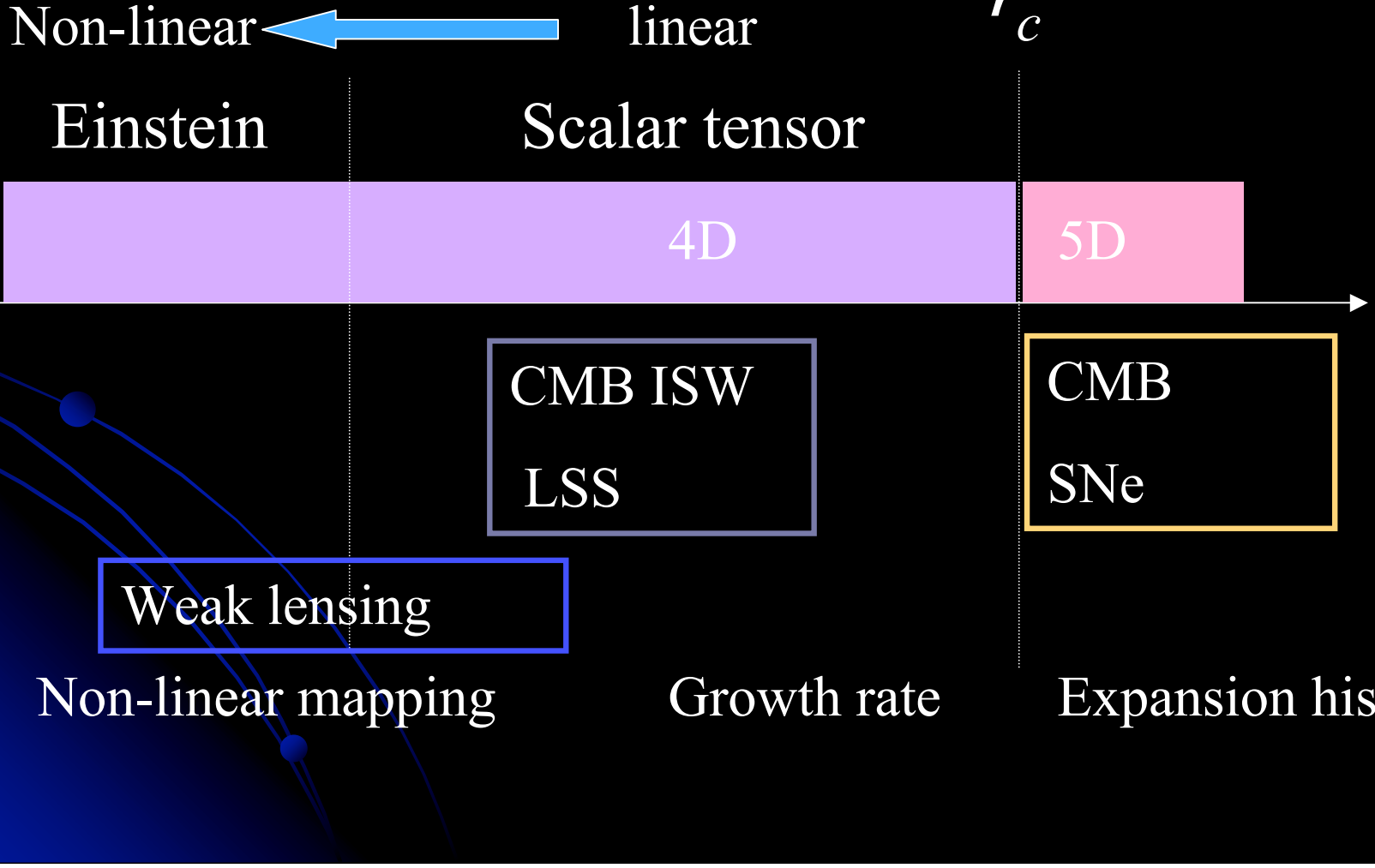


solar-system $r_g = 2GM_\odot \sim 3\mathrm{km}$

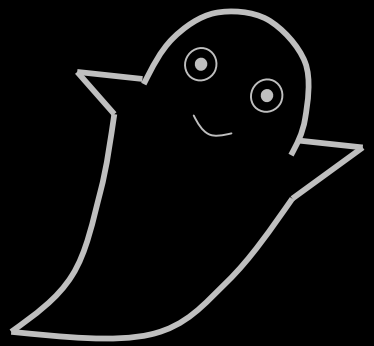
constraints can be evaded if $r_c > H_0^{-1} \sim 10^{28}\,\mathrm{cm}$

Cosmology as a probe of DGP gravity

$$r < r_* = \left(r_g r_c^2\right)^{\frac{1}{3}}$$



Caveat - a ghost



- Negative BD parameter

$$\omega = \frac{3}{2}(\beta - 1) \quad \beta = 1 - 2 r_c H \left(1 + \frac{\dot{H}}{3H^2} \right)$$

In Einstein frame, kinetic term for the scalar $\propto -\frac{3}{2}\beta$

if $\beta < 0$ the scalar becomes a ghost

(Luty, Porrati, Rattazzi; Nicolis, Rattazzi)

- Fatality of the DGP model?

  Validity of the linearized analysis?

  Fate of the ghost instability?

Conclusion

- 5D models - RS model and DGP model
  - A new picture of our universe
    - 4D Expanding universe = moving 4D brane
    - The way how 4D brane is embedded in high-dim bulk affects intrinsic curvature inside 4D brane

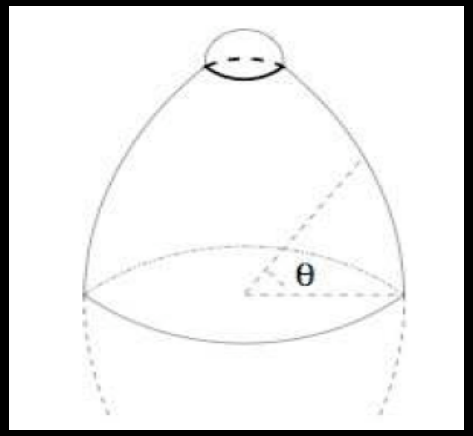
      ➡ alternative to dark energy
  - Inherently higher dimensional effects
    - Gravity leakage

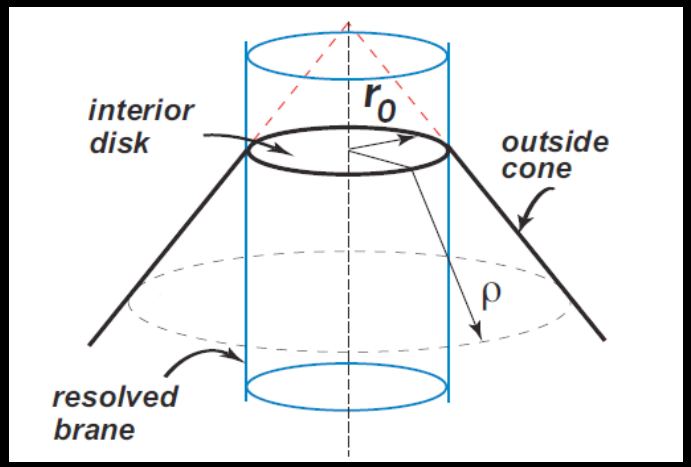
      now well understood for cosmological perturbations

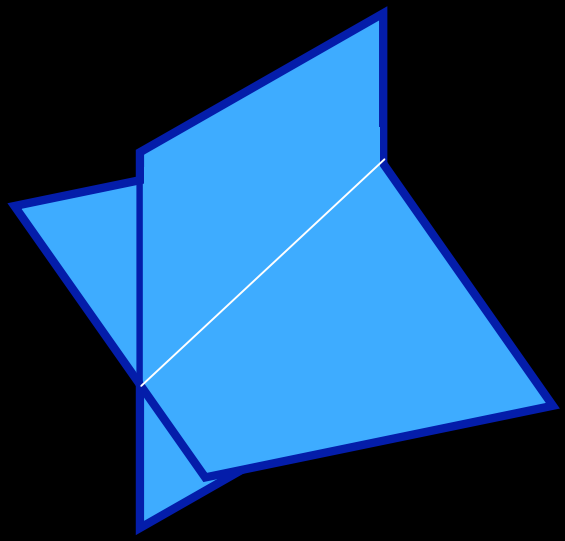
      non-linear dynamics is still less known

      (BH solutions, non-linear dynamics in DGP)

- Beyond 5D toy models
  - 6D models -regularizations







Compact

Non-compact + induced

intersecting

  - String theory