Superluminal signals in k-essence cosmology

Sergei Winitzki

Ludwig-Maximilians University, Munich, Germany

Cosmological scenarios with k-essence are invoked in order to explain the observed late-time acceleration of the universe. These scenarios avoid the need for fine-tuned initial conditions (the "coincidence problem") because of the attractor-like dynamics of the k-essence field in the presence of another matter component. We obtain a complete classification of Lagrangians admitting stable attractor-like cosmological solutions ("trackers"), among all Lagrangians of the form $\mathcal{L} = K(\phi)F(X)$. We show that all k-essence scenarios with these Lagrangians necessarily involve an epoch where perturbations of k-essence propagate faster than light. In the context of k-essence cosmology, the superluminal epoch does not lead to causality violations.

J. U. Kang, V. Vanchurin, and S. W. (2007)

COSMO 2007, University of Sussex, August 2007

Cosmology with k-essence field

k-essence $=$ scalar field with noncanonical kinetic terms, e.g.

$$\mathcal{L} = K(\phi)F(X), \qquad X \equiv \frac{1}{2}\partial_\mu\phi\partial^\mu\phi$$

Matter $+$ k-essence field $=$ attractor behavior if $K(\phi) = \phi^{-2}$

Armendáriz-Picón, Mukhanov, Steinhard 2000

- Tracking behavior during radiation domination

— alleviates the coincidence problem

- Speed of sound must exceed 1 at some time

Bonvin, Caprini, Durrer 2006

Superluminal propagation of perturbations $=$ trouble?

Main results

- Which Lagrangians $K(\phi)F(X)$ provide attractor behavior? (after exhaustive study, have list)

- Which Lagrangians provide viable cosmological scenarios? (only $K(\phi) \to \phi^{-2}$ at $\phi \to \infty$)

- Is there a superluminal epoch? (yes)

- Is there a causality violation? (no)

Scenario of k-essence cosmology

Effective scalar field theory with unconventional kinetic terms:

$$\mathcal{L} = K_1(\phi)X + K_2(\phi)X^2 + \dots$$

$$X = \frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}$$

Armendáriz-Picón, Damour, Mukhanov 1999

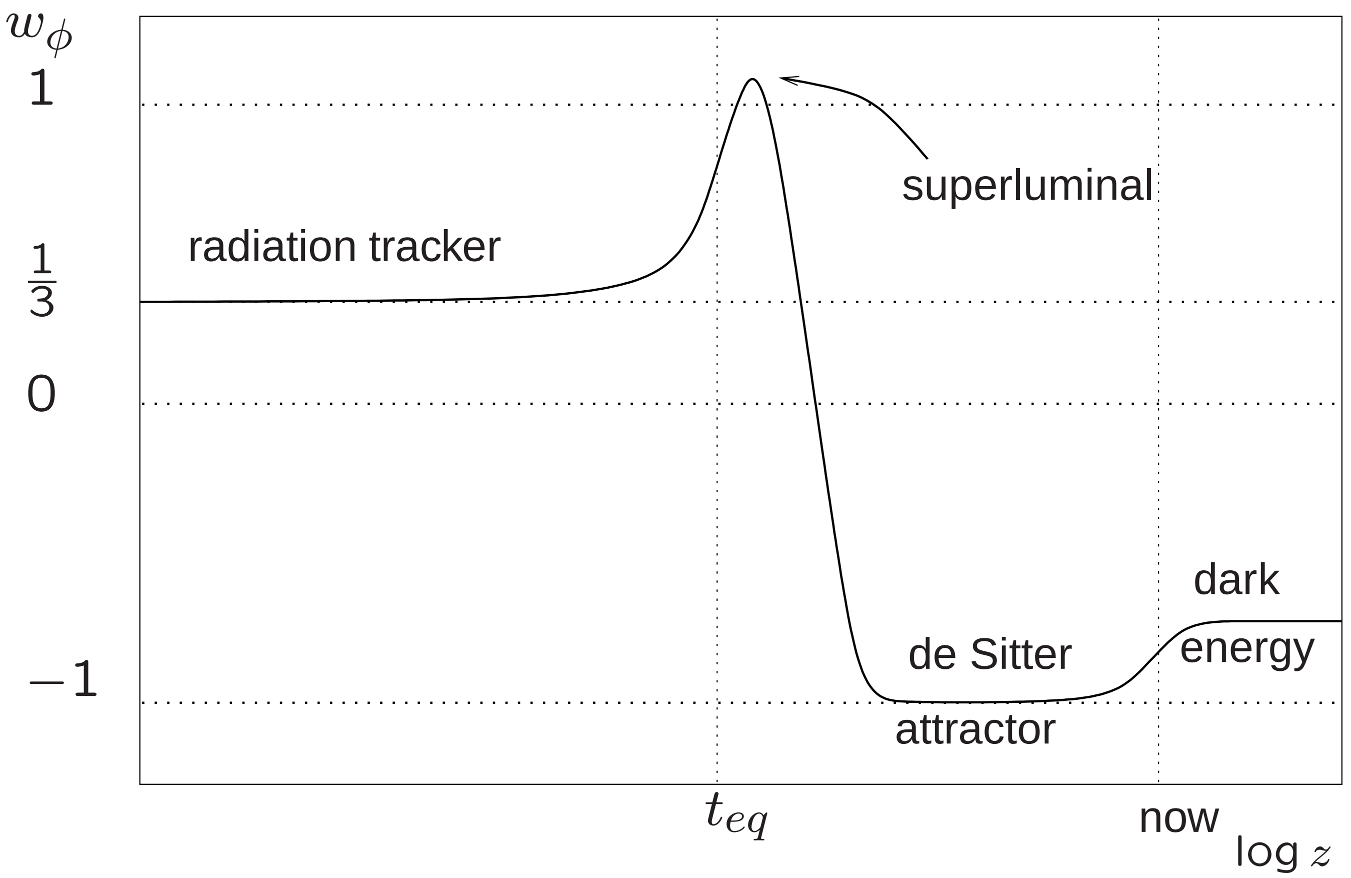
More generally: $\mathcal{L} = K(\phi)F(X)$; minimal coupling to gravity

Homogeneous cosmology: $\phi = \phi(t)$, $a = a(t)$

Consider k-essence with a dominant matter component
($w_m =$ const)

Scenario of k-essence cosmology

Evolution of the k-essence equation of state w_ϕ:



Evolution of k-essence and matter

Friedmann equation:

$$\frac{\dot{a}}{a} = H = \sqrt{\frac{8\pi G}{3}\left(\varepsilon_\phi + \varepsilon_m\right)}$$

Field equation: $\dot{\varepsilon}_\phi = -3H\varepsilon_\phi\left(1 + w_\phi\right)$, $w_\phi = w_\phi(\dot{\phi})$

Matter equation: $\dot{\varepsilon}_m = -3H\varepsilon_m\left(1 + w_m\right),\ w_m =$ const

Consider $\dot{\phi}$, H, ε_m as functions of ϕ instead of time:

$\Rightarrow$ 2 nonlinear ODEs
$$\begin{cases} \dfrac{d\dot{\phi}}{d\phi} = A(\phi, \dot{\phi}, \varepsilon_m) \\ \dfrac{d\varepsilon_m}{d\phi} = B(\phi, \dot{\phi}, \varepsilon_m) \end{cases}$$

Stable attractors

At late times ($\phi \to \infty$), we require that

$$\left.\begin{array}{c} \dfrac{\varepsilon_m}{\varepsilon_\phi + \varepsilon_m} \approx \text{const} \\ w_\phi \approx \text{const} \end{array}\right\} \quad \text{are stable attractors}$$

as long as $w_m = \text{const}$.

Goals:

- Find Lagrangians admitting such solutions
- Suitable sequence of attractors (radiation/dust/dark energy)?

Initial requirements on Lagrangians

- k-essence field ϕ is not a ghost and not tachyonic
- $\dot{\phi} > 0$ and $\phi \to \infty$ as $t \to \infty$

Lagrangians $\mathcal{L} = K(\phi)F(X)$ satisfy the requirements if:

- $F(X)$ is a monotonically growing function of X
- $F(0) \leq 0$

Results

...300 equations après... [Kang, Vanchurin, S.W. 2007]

Obtained 12 possible types of stable attractors, e.g.

$$\mathcal{L} = \frac{1+K_0(\phi)}{\phi^2}F(X), \quad K_0(\phi) \to 0 \quad \text{at} \quad \phi \to \infty$$

$$\mathcal{L} = \frac{K_0(\phi)}{\phi^\alpha}F(X), \quad F(X) \propto X^n \quad \text{at} \quad X \to 0, \quad n \geq 2$$

$$K_0(\phi) \to \infty \quad \text{at} \quad \phi \to \infty, \alpha = \frac{4n}{(2n-1)(1+w_m)}$$

and 10 other cases...

Have cases where $\dot{\phi} \to 0$ or $\dot{\phi} \to$ const $\neq 0$

- Only the first type of Lagrangian has *all* needed attractors
- Its properties are known from previous work

Superluminal signal propagation

[Bonvin, Caprini, Durrer 2006]

In k-essence scenarios with Lagrangians $\mathcal{L} = \phi^{-2} F(X)$:

- must have an epoch where $w_\phi > 1$ [calculation]
- if $w_\phi > 1$ then also have an epoch where $c_s^2 > 1$ [calculation]

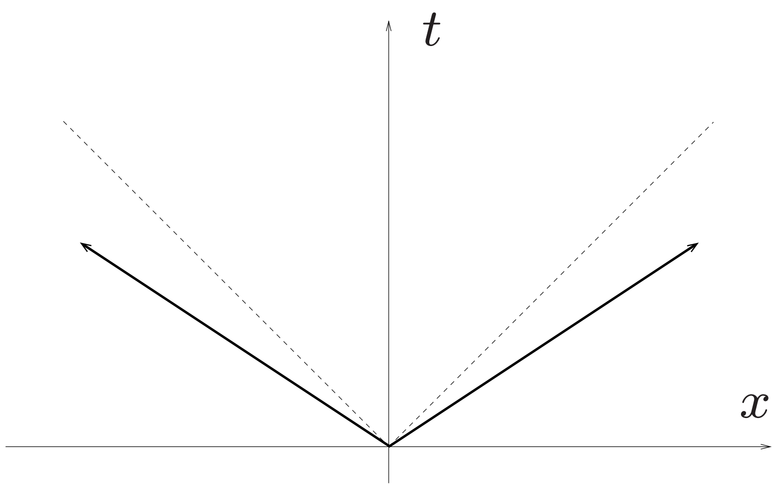
- Can send signals faster than light...

...uh oh...

- ...but only in *one* reference frame

Superluminal signals

Perturbations propagate with sound speed $c_s > 1$



In the cosmological reference frame, $\phi = \phi(t)$, *all* signals will propagate in the positive t direction

$\Rightarrow$ No causality violations on cosmological background

Conclusions

Classification of attractors in models of k-essence:

- with Lagrangian $\mathcal{L} = K(\phi)F(X)$: done

Lagrangians must be of the form $\mathcal{L} \approx \phi^{-2}F(X)$ at large ϕ

A superluminal epoch is necessarily present

No causality violations in cosmological context

- with more general Lagrangians (DBI, ...): open

Existence of an epoch with $w_\phi > 1$, $w'_\phi > 1$

Let $v \equiv \dot{\phi}$, Lagrangian for k-essence $\mathcal{L} = K(\phi)Q(v)$

Tracker solutions require $v \to$ const, $r \equiv \varepsilon_\phi / \left(\varepsilon_\phi + \varepsilon_m\right) \to$ const

During radiation/dust eras: $v \approx v_r$, $r \approx 0.01$, and $v \approx v_d$, $r \approx 1$; also have $v_d < v_r$ because $Q(v)$ is monotonic

Tracking condition [derived]: $r = 6\pi G Q'^2/(vQ' - Q) \equiv F(v)$

Since $F(v_r) < F(v_d)$, must have $F'(v_1) < 0$, then $w_\phi(v_1) > 1$

Since $w_\phi(v_1) > 1$ and $w_\phi(v_d) < 0$, for some v_* we have $w'_\phi(v_*) > 0, w_\phi(v_*) > 1$

Existence of a superluminal epoch $c_s > 1$

At the value v_* we have $w'_\phi(v_*) > 0$ and $w_\phi(v_*) > 1$

Calculation gives

$$w'_\phi(v) = \frac{\left(1 + w_\phi\right)\left(c_s^2 - w_\phi\right)}{v c_s^2} > 0$$

Hence $c_s^2(v_*) > w_\phi(v_*) > 1$

- There exist an epoch with $c_s^2 > 1$ during dust domination