Thank you for the invitation.

TURNAROUND

IN

CYCLIC COSMOLOGY

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OUTLINE

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References:

(1) L. Baum and P.H.F. `hep-th/0610213`. P.R.L. **98,** 071301 (2007)
(2) L. Baum and P.H.F. `hep-th/0703162`.
(3) P.H.F. `arXiv:0706.1186`.

1. Historical context.

One of the oldest questions in theoretical cosmology is whether an infinitely oscillatory universe which avoids an initial singularity can be consistently constructed. As realized by Friedmann and especially by Tolman (also LeMaitre, Einstein, De Sitter ....) one principal obstacle is the second law of thermodynamics which dictates that the entropy increases from cycle to cycle. If the cycles thereby become longer, extrapolation into the past will lead back to an initial singularity again, thus removing the motivation to consider an oscillatory universe in the first place. This led to the abandonment of the oscillatory universe by the majority of workers.

Nevertheless, an oscillatory universe is an attractive alternative to the Big Bang. One new ingredient in the cosmic make-up is the dark energy discovered only in 1998 and so it natural to ask whether this can avoid the difficulties with entropy which have dogged previous attempts.

Some work has been started to exploit the dark energy in allowing cyclicity possibly without apparently the need for inflation in Steinhardt *et al* Another new ingredient is the use of branes and a fourth spatial dimension as in Randall *et al*, Binetruy *et al* which have examined the consequences for cosmology. The Big Rip and replacement of dark energy by modified gravity have been explored in PHF and Takahashi.

If the dark energy has a super-negative equation of state, $\omega_\Lambda = p_\Lambda/\rho_\Lambda < -1$, it leads to a Big Rip (R. Caldwell) at a finite time where there exist extraordinary conditions with regard to density and causality as one approaches the Big Rip. In the present article we explore whether these exceptional physical conditions can assist in providing an infinitely-cyclic cosmology.

We shall consider the situation where if, as we approach the Big Rip, the expansion stops due to the brane contribution just short of the Big Rip and there is a turnaround at $t = t_T$ when the scale factor is deflated to a very tiny fraction (f) of itself and only one causal patch is retained, while the other $1/f^3$ patches contract independently into separate universes. The turnaround takes place an extremely short time before the Big Rip would have occurred, at a time when the universe is fractionated into many independent causal patches, see *e.g.* PHF and Takahashi (2004).

We discuss the contraction phase which occurs with a very much smaller universe than in the expansion phase and with almost vanishing entropy because it is assumed empty of dust, matter and black holes all of which were jettisoned at turnaround. A bounce at $t = \tau$ takes place a short time before a would-be Big Bang. Then, immediately after the bounce, entropy is injected by inflation (Guth) where the scale factor is enhanced by large factor and hence so is entropy. Inflation can thus be a part of the present scenario which is one distinction from the work of Steinhardt *et al.*

For cyclicity of the entropy, $S(t) = S(t+\tau)$ to be consistent with thermodynamics it is necessary that the deflationary decrease by f^3 compensate the entire entropy increase acquired during contraction and expansion including the huge increase during inflation.

A possible shortcoming of the proposal could have been the persistence of spacetime singularities in cyclic cosmologies (Borde, Guth and Vilenkin, 2003) but to our understanding for the truly cyclic universe which we here outline this problem is avoided, provided a simple constraint on the time average of the Hubble parameter is respected.

This work is presented because our discussion seems to give a plausible realization of the infinitely oscillatory universe originally saught by cosmologists on the 1920s and 1930s ignorant of dark energy

(see, however, the discussion after Eq.(172.6) of R.C. Tolman in Relativity, Thermodynamics and Cosmology. Oxford University Press (1934))

and one whose minor shortcomings can hopefully be evolved by others into a convincing scenario.

2. Expansion phase.

Let the period of the Universe be designated by τ and the bounce take place at $t = 0$ and turnaround at $t = t_T$. Thus the expansion phase is for times $0 < t < t_T$ and the contraction phase corresponds to times $t_T < t < \tau$. We employ the following Friedmann equation for the *expansion* period $0 < t < t_T$:

$$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8\pi G}{3}\left[\left(\frac{(\rho_\Lambda)_0}{a(t)^{3(\omega_\Lambda+1)}} + \frac{(\rho_m)_0}{a(t)^3} + \frac{(\rho_r)_0}{a(t)^4}\right) - \frac{\rho_{total}(t)^2}{\rho_c}\right] \quad (1)$$

where the scale factor is normalized to $a(t_0) = 1$ at the present time $t = t_0 \simeq 14Gy$.

To explain the notation, $(\rho_i)_0$ denotes the value of the density ρ_i at time $t = t_0$. The first two terms are the dark energy and total matter (dark plus luminous) satisfying

$$\Omega_\Lambda = \frac{8\pi G(\rho_\Lambda)_0}{3H_0^2} = 0.72 \tag{2}$$

and

$$\Omega_m = \frac{8\pi G(\rho_m)_0}{3H_0^2} = 0.28 \tag{3}$$

where $H_0 = \dot{a}(t_0)/a(t_0)$. The third term in the Friedmann equation is the radiation density which is now $\Omega_r = 1.3 \times 10^{-4}$.

The final term $\sim \rho_{total}(t)^2$ is derivable from a brane set-up; we use a negative sign arising from negative brane tension (a negative sign can arise also from a second timelike dimension but that gives difficulties with closed timelike paths). $\rho_{total} = \Sigma_{i=\Lambda,m,r}\rho_i$. As the turnaround is approached, the only significant terms in Eq.(1) are the first (where $\omega_\Lambda < -1$) and the last.

As the bounce is approached, the only important terms in Eq.(1) are the third and the last. (We shall later argue that the second term must be absent during contraction.) In particular, the final term of Eq. (1), $\sim \rho_{total}(t)^2$, arising from the brane set up is insignificant for almost the entire cycle but becomes dominant as one approaches $t \to t_T$ for the turnaround and again for $t \to \tau$ approaching the bounce.

3. Turnaround.

Let us assume for algebraic simplicity $\omega_\Lambda = -4/3 =$ constant. This value is already almost excluded by WMAP3 but to begin we are aiming only at consistency of infinite cyclicity. More realistic values may be discussed elsewhere. The approach to the Big Rip will follow that discussed in PHF+TT *q.v.*. With the value $\omega_\Lambda = -4/3$ we learn therefrom that the time to the Big Rip is $(t_{rip} - t_0) = 11\text{Gy}(-\omega_\Lambda - 1)^{-1} = 33\text{Gy}$ which is, within 10^{-27} second, when turnaround occurs at $t = t_T$. So if we adopt $t_0 = 14Gy$ then $t_T = t_0 + (t_{rip} - t_0) = (14 + 33)Gy = 47Gy$.

From the analysis in PHF+TT the time when a system becomes gravitationally unbound corresponds approximately to the time when the growing dark energy density matches the mean density of the bound system. For a "typical" object like the Earth (or a hydrogen atom where the mean density happens to be about the density of water $\rho_{H_2O} = 1g/cm^3$ since $10^{-24}g/(10^{-8}cm)^3 = 1g/cm^3$) water's density ρ_{H_2O} is an unlikely but practical unit for cosmic density in the oscillatory universe.

With this in mind, for the simple case of $\omega = -4/3$ we see from the Friedmann equation that the dark energy density grows proportional to the scale factor $\rho_\Lambda(t) \propto a(t)$ and so given that the dark energy at present is $\rho_\Lambda \sim 10^{-29} g/cm^3$ it follows that $\rho_\Lambda(t_{H_2O}) = \rho_{H_2O}$ when $a(t_{H_2O}) \sim 10^{29}$. We can estimate the time t_{H_2O} by taking on the RHS of the Friedmann equation only dark energy $\left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \Omega_\Lambda a^{-\beta}$ with $\beta = 3(1+\omega)$. When we specialize to $\omega = -4/3$ as illustration and require as before $\rho_\Lambda(t_{H_2O}) = \rho_{H_2O}$ then $a(t_{H_2O}) = 10^{29}$ and it follows that

$$\frac{a(t_{H_2O})}{(a(t_0) = 1)} = \left(\frac{(t_{rip} - t_0)}{(t_{rip} - t_{H_2O})} \right)^2 \tag{4}$$

so that $(t_{rip} - t_{H_2O}) = 33Gy \times 10^{-14.5} \simeq 10^{3.5} s \sim 1$ hour. [The value is sensitive to ω]

For $\omega = -4/3$, it will be useful to consider a more general critical density $\rho_c = \eta\rho_{H_2O}$, since there is nothing special about ρ_{H_2O} and to compute the time $(t_{rip} - t_\eta)$ such that $\rho_\Lambda(t_\eta) = \rho_c = \eta\rho_{H_2O}$. We then find, using $a(t_\eta) = 10^{29}\eta$, that $(t_{rip} - t_\eta) = (t_{rip} - t_0)10^{-14.5}\eta^{-1} \simeq \eta^{-1}$hours is the required result.

To discuss the turnaround analytically we keep only the first and last terms, the only significant ones, on the RHS of the Friedmann equation which becomes for the special case $\omega = 4/3$

$$\left(\frac{\dot{a}}{a}\right)^2 = \alpha_1 a - \alpha_2 a^2 \tag{5}$$

in which

$$\alpha_1 = \frac{8\pi G}{3}(\rho_\Lambda)_0 \qquad \alpha_2 = \frac{8\pi G}{3}\frac{(\rho_\Lambda)_0^2}{\rho_c} \tag{6}$$

Writing $a = z^2$ and $z = (\alpha_1/\alpha_2)^{1/2} sin\theta$ gives

$$dt = \frac{2\sqrt{\alpha_2}}{\alpha_1}\frac{d\theta}{sin^2\theta} = \frac{2\sqrt{\alpha_2}}{\alpha_1} d(-cot\theta) \tag{7}$$

Integration then gives for the scale factor

$$a(t) = \left(\frac{\alpha_1}{\alpha_2}\right) sin^2\theta = \frac{\rho_c}{(\rho_\Lambda)_0}\left[\frac{1}{1+\left(\frac{t_T - t}{C}\right)^2}\right] \quad (8)$$

where $C = -(3/2\pi G\rho_c)^{1/2}$. At turnaround $t = t_T$, $a(t_T) = [\rho_C/(\rho_\lambda)_0] = (a(t))_{max}$. At the present time $t = t_0$, $a(t_0) = 1$ and $sin^2\theta_0 = [(\rho_\Lambda)_0/\rho_C] \ll 1$, increasing during subsequent expansion to $\theta_T = \pi/4$.

A key ingredient in our cyclic model is that at turnaround $t = t_T$ (mod τ) our universe deflates dramatically with effective scale factor $a(t_T)$ shrinking before contraction to $\hat{a}(t_T) = fa(t_T)$ where $f < 10^{-28}$. This jettisoning of almost all, a fraction $(1 - f)$, of the accumulated entropy may be permitted by the exceptional causal structure of the universe. We shall see later that the parameter η at turnaround could be $\eta \sim 10^{31}$ or even larger which implies the dark energy density at turnaround of $\rho_\Lambda(t_T) > 10^{31}\rho_{H_2O}$ (Planckian density of $\rho_\Lambda \sim 10^{104}\rho_{H_2O}$ can be avoided). By the time the dark energy density reaches such values, according to the Big Rip analysis of PHF+TT even the smallest known bound systems of particles have become unbound and the constiutents causally disconnected. Possibly smaller unknown bound systems have equally become unbound and acausal.

According to this, at $t = t_T$ the universe has already fragmented into an astronomical number $(1/f^3)$ of causal patches, each of which independently contracts as a separate universe leading to an infinite multiverse. The entropy at $t = t_T$ is thus divided between these new contracting universes and our universe retains only the infinitesimal fraction f^3. Since our eternal universe has cycled an infinite number of times, the number of parallel universes is infinite.

4. Contraction phase.

The contraction phase for our universe occurs for the period $t_T < t < \tau$ (mod τ). The scale factor for the contraction phase will be denoted by $\hat{a}(t)$ while we use always the same linear time t subject to the periodicity $t + \tau \equiv t$. At the turnaround we retain a fraction f^3 of the entropy with $\hat{a}(t_T) = fa(t_T)$ and for the contraction phase the Friedmann equation is:

$$\left(\frac{\dot{\hat{a}}(t)}{\hat{a}(t)}\right)^2 = \frac{8\pi G}{3}\left[\left(\frac{(\hat{\rho}_\Lambda)_0}{\hat{a}(t)^{3(\omega_\Lambda+1)}} + \frac{(\hat{\rho}_r)_0}{\hat{a}(t)^4}\right) - \frac{\hat{\rho}_{total}(t)^2}{\rho_c}\right] \tag{9}$$

where we defined

$$\hat{\rho}_i(t) = \frac{(\rho_i)_0 f^{3(\omega_i+1)}}{\hat{a}(t)^{3(\omega_i+1)}} = \frac{(\hat{\rho}_i)_0}{\hat{a}(t)^{3(\omega_i+1)}} \tag{10}$$

In contrast to expansion we have set $\hat{\rho}_m = 0$ because our hypothesis is that the causal patch retained contains only dark energy and radiation but no matter (no black holes). This is necessary because during a contracting phase dust or matter would clump, more readily than during expansion, and interfere with cyclicity.

Perhaps more importantly, presence of dust or matter would require that our universe go in reverse through several phase transitions (recombination, QCD and electroweak to name a few) which would violate the second law of thermodynamics and be statistically impossible.

We thus require that

our universe comes back empty!

(like a milk bottle in the old days)

The contraction of our universe will proceed from one of the $1/f^3$ causal patches following the truncated Friedmann equation until the radiation term balances the brane tension term at the bounce.

5. Bounce

As an estimated time $t = \tau$ for the bounce, the contraction scale is given, using $\rho_c = \eta\rho_{H_2O}$, from Eq. (1), and using the certainty that $(\rho_a)_0 < (\rho_r)_0$ as

$$a(\tau)^4 = \left(\frac{(\rho_a)_0}{\eta\rho_{H_2O}}\right) = \left(\frac{10^{-33}}{\eta}\right) \qquad (11)$$

Now the bounce at $t = \tau$ must be before the electroweak transition at $t_{EW} = 10^{-10}s$ when $a(t_{EW}) = 10^{-15}$ and after the Planck time where $a(t_{Planck}) \sim 10^{-31}$ (Recall $a \propto T^{-1}$).

We may take as illustrative cases:

- $T_B = 10^{17} GeV, a(t_B) = 10^{-30}, \eta = 10^{87}$
- $T_B = 10^{10} GeV, a(t_B) = 10^{-23}, \eta = 10^{59}$
- $T_B = 10^{3} GeV, a(t_B) = 10^{-16}, \eta = 10^{31}$

Immediately after the bounce there is conventional inflation with enhancement $\mathrm{E} = a(\tau + \delta)/\hat{a}(\tau)$ and successful inflation requires $E > 10^{28}$. Consistency requires therefore $f < E^{-1}$ to allow for the entropy accrued during normal expansion after inflation and contraction. The fraction of entropy jettisoned from our universe at deflation at the turnaround is thus extremely close to one, being less than one and more than $(1 - 10^{-28})^3$.

6. Entropy.

Consider first the present epoch $t = t_0$. The contributions of the radiation to the entropy density s follows the relation

$$s = \frac{2\pi^2}{45} g_* T^3 \tag{12}$$

First consider only photons with $g_* = 2$. The present CMB temperature is $T = 2.73K \equiv 0.235meV \sim 1.191(mm)^{-1}$. Substitution in Eq.(12) gives a present radiation entropy density $s_\gamma(t_0) = 1.48(mm)^{-3}$. Using a volume estimate $V = (4\pi/3)R^3$ with $R = 10Gly \simeq 10^{29}mm$ gives a total radiation entropy $S_\gamma \sim 6.3 \times 10^{87}$. Including neutrinos increase g_* in Eq.(12) from $g_* = 2$ to $g_* = 3.36 = 2 + 6 \times (7/8) \times (4/11)^{4/3}$. This increases $S_\gamma = 6.3 \times 10^{87}$ to $S_{\gamma+\nu} \sim \times 10^{88}$.

The total entropy is interpretable as $exp(10^{88})$ degrees of freedom, or in information theory to a number I of qubits where $2^I = e^S$ so that $I = S/(ln2 = 0.693) \sim 10^{88}$. This is well below the holographic bound which is dictated by the area in terms of Planck units $10^{-64}mm^2$ which gives $S_{holog}(t_0) = 4\pi(10^{29}mm)^2/(10^{-32}mm)^2 \sim 10^{123}$ about 10^{35} times bigger. Some of this difference may come from supermassive black holes.

The entropy contribution from the baryons is smaller than S_γ by some ten orders of magnitude, so like that of the dark matter, is negligible. What is the entropy of the dark energy? If it is perfectly homogeneous and non-interacting it has zero for both temperature and entropy. Another viewpoint, at least for a pure cosmological constant, is that one number Λ cannot contain entropy. Finally, the 4th term in Eq.(1) corresponding to the brane term is neglible, as we have already estimated.

The conclusion is that $S_{total}(t_0) \sim 10^{88}$. Now consider the entropy at turnaround $t = t_T$ (mod τ). We have estimated that $a(t_T) = 10^{29}\eta$. The temperature T_γ of the radiation scales as $T_\gamma \propto a(t)^{-1}$ so using the entropy density of Eq.(12) a comoving 3-volume $\propto a(t)^3$ will contain the same total radiation entropy $S_\gamma(t_T) = S_\gamma(t_0)$ as at present; this is simply the usual adiabatic expansion.

The expansion from $t = 0$ (mod τ) to t_T (mod τ) is, of course, not purely adiabatic because irreversible processes take place. The first is inflation which increases entropy by $> 10^{84}$. There are phase transitions such as the electroweak transition at $t_{ew} \sim 100ps$, the QCD phase transition at $t_{QCD} \sim 100\mu s$, and recombination at $t_{rec} \sim 10^{13}s$. Further irreversible processes occur during during stellar evolution. Although the expansion of the radiation, the dominant contributor to the entropy, is adiabatic, the entropy of the matter inevitably increases with time in accord with the second law of thermodynamics.

In our model, the entropy of the matter increases between $t = 0 \pmod{\tau}$ and $t_T = 47Gy$. Setting the entropy of the dark energy to zero and the radiation as adiabatic, the matter part represented by ρ_m will cause the entropy to rise from $S(t = 0)$ to $S(t_T) = S(t = 0) + \Delta S$ where ΔS causes the contradiction plaguing the oscillatory universe in the 1920s and 1930s. The key point is that in order for entropy to be cyclic, the entropy which was enhanced by a huge factor $E^3 > 10^{84}$ at inflation must be reduced even more dramatically at some point during the cycle so that $S(t) = S(t + \tau)$ becomes possible. Since it increases during both expansion and contraction, the only logical possibility is a dramatic decrease at turnaround as accomplished by our hypothesis of one causal patch retention .

The second law or thermodynamics continues to obtain for other causally disconnected regions, each with practically vanishing entropy at turnaround, but these are permanently removed from our universe contracting instead into separate universes.

Next, we look at entropy for contraction $t_T < t < \tau \ (\bmod \ \tau)$. According to statistical mechanics one expects the entropy to increase here also, although because it is much smaller the increase must be correspondingly much smaller than during expansion and as we are assuming the universe during contraction is empty of dust until the bounce its entropy is, in any case, vanishingly small for the contraction era.

Finally, there is the issue of entropy at bounce $t = \tau \pmod{\tau}$. Immediately after the bounce inflation increases entropy by $> 10^{84}$ so cyclicity $S(t) = S(t+\tau)$ is possible providing the entropy loss at turnaround compensates the gain during inflation as well as the other entropy acquisitions. We find the counterpoise of inflation at the bounce and deflation at turnaround an appealing aspect.

It is worth a mention that there exists an argument from the conventional Friedmann equation (see Misner, Thorne and Wheeler book) that when the RHS terms all have equation of state $\omega \geq -1$ then $\ddot{a} < 0$ which disallows a bounce after which the universe starts expanding again. In the present model the brane term contributes with opposite sign, and double the magnitude, of the radiation term and $\ddot{a} > 0$.

7. Vanishing entropy of contracting universe.

The contracting universe of the cyclic model contains dark energy with zero entropy and possibly a small amount of radiation which could possess entropy. The deflation at turnaround reduces entropy from a gigantic value $O(> 10^{88})$ to an extremely low value $O(10^1)$. An unrealistic value for the dark energy equation of state $\omega = p/\rho = -4/3$ has been employed for algebraic simplicity as it makes $\rho_\Lambda \propto a$, and no attempt yet made at a realistic description of our universe. We shall now study the entropy of the contracting universe in this speculative scenario more quantitatively and now will use arbitrary $\omega = -1 - \phi$ with $\phi > 0$ so that $\rho_\Lambda \propto a^{3\phi}$.

The quantity ϕ is the most important parameter for observational discrimination between this cyclic model and a cosmological constant [1] The next test of $\phi \neq 0$ will likely come from the Planck Surveyor satellite. One wonders, therefore, how different from zero ϕ is? There is no lower bound on ϕ to make the model work except that it must be non zero. We already know $\phi < 0.1$ from the WMAP3 data. If ϕ is truly infinitesimal, the test must await improved technology. To restore optimism we shall describe an anthropic fine tuning argument that shows that extremely small ϕ is unlikely.

[1] and from the Steinhardt-Turok cyclic model.

The universe comes back empty of matter including black holes. The presence of matter during contraction causes apparently insuperable problems because accelerated structure formation will precipitate a premature bounce. Black holes, if present, will expand and proliferate with the same consequence. But the presence of radiation must also be carefully studied because although at turnaround the photon energy is infinitesimal ($E_\gamma < 10^{-200} eV$), the blue shifting during contraction leads before the bounce to production of e^+e^- pairs, undesirable because generically they will create problems with continued contraction. As we shall show there are fortunately no photons in the contracting phase of the cycle, only the truly innocuous dark energy.

The cyclic model contains one free parameter, the common density ρ_C at which the universe both turns around and bounces. Since the bounce is independent of ω we begin with it and take as bounce temperatures $T_B = 10^p$ GeV with, to be above the weak and below the Planck scales, $3 \leq p \leq 17$. This gives $\rho_C = \eta \rho_{H_2O}$ where $\eta = 10^{(\bar{1}9+4p)}$ and $\rho_{H_20} = 1g/cm^3$ is the density of water, an easily imaginable unit somewhere between the unimaginably small present mean cosmic density and the unimaginably large critical density ρ_C at turnaround and bounce.

Going now to the turnaround at time $t = t_T$ the scale factor $a(t_T)$ is given by (since $a(t_0) = 1$ and putting $\rho_0 = 10^{-29}\rho_{H_2O}$) $a(t_T)^{3\phi} = 10^{29}\eta = 10^{48+4p}$

The present radiation temperature is $(T_\gamma)_0 = 2 \times 10^{-4}$ eV, and so the radiation temperature at turnaround is

$$(T_\gamma)_T = 2 \times 10^{-4} \left(10^{(48+4p)}\right)^{-1/3\phi} \text{eV} \quad (13)$$

which is infinitesimal: putting $\phi = 0.1$, Eq.(13) gives 10^{-200} eV for p=3 and 10^{-390} eV for p=17; with $\phi = 0.01$, the photon energy is 10^{-2000} eV for p=3 and 10^{-3900} eV for p=17. In all cases, the photon wavelength is an astronomical number of orders of magnitude longer than the present Hubble length.

To evaluate the contraction entropy we need to estimate many such photons are in one causal patch at turnaround. The deflationary factor multiplying entropy at turnaround must be much less than the inverse of the inflationary increase ($> 10^{84}$) of the early universe. We take the huge number of causal patches to be $10^{90}\alpha$ where $\alpha \gg 1$ is a parameter to allow an arbitrarily larger number, and $\alpha = 1$ will give an overestimate of contraction entropy.

At turnaround the scale factor is

$$a(t_T) = \left(10^{(48+4p)}\right)^{\frac{1}{3\phi}} \qquad (14)$$

so taking the present volume as $10^{84}cm^3$ and the present radiation density as $\rho_r(t_0) = 10^{-33}g/cm^3 = 1eV/cm^3$ gives for the radiation energy in one causal patch

$$(E_r)_{patch} = \frac{1}{(100\alpha)^3}\left(10^{(48+4p)}\right)^{-\frac{1}{3\phi}} \text{ eV} \qquad (15)$$

Comparison with Eq.(13) then gives for the number of photons per causal patch

$$n_\gamma = \frac{1}{200\alpha^3} \ll 1 \qquad (16)$$

which is small even for the unrealistic case $\alpha = 1$ and essentially zero for $\alpha \gg 1$. Thus, the entropy of the contracting universe (cu) vanishes $S_{cu} = 0$ for any value of equation of state of the dark energy $\omega = p/\rho = -1 - \phi$ since Eq.(16) has no ϕ dependence.

Anthropic fine tuning argument about ϕ

The time until turnaround is given by

$$(t_T - t_0) \simeq \frac{t_0}{\phi} \tag{17}$$

so if we take, for simplicity, the origin of life to have occurred at t_0 after the most recent bounce we see from Eq. (17) that given small $\phi \ll 1$ then ϕ measures the fraction of the expansion phase taken to originate life. An anthropic argument is: it is unreasonable for the fraction ϕ, assuming it is non zero, to be extremely close to zero.

The special case $\phi = 0$ is the standard cosmological model with a cosmological constant where there is no turnaround and the future lifetime is infinite so the origin of life necessarily takes place after a vanishing fraction of the expansion lifetime. Although such an infinite expansion seems to us unaesthetic, not all colleagues share our concern.

As soon as one commits to $\phi \neq 0$, however, the anthropic type argument emerges and it is unlikely that $\phi <<< 1$. For example, if $\phi = 10^{-3}$ the length of the expansion phase is 10^4 Gy whereas life orinated after only about 10 Gy which is only 0.1% of the expansion time. If life plays a central role in our universe, as in our understanding is the spirit of the anthropic principle, such a tiny value of ϕ is strongly disfavored; one expects at least $\phi > 0.01$ so the fraction before the origin of life is $> 1.0\%$ of the total expansion time.

This encouraging argument makes it more optimistic that the next generation of observations such as the Planck Surveyor will succeed in detecting a $\phi \neq 0$.

8. Infinite past.

Theorists are comfortable with an infinite future as occurs in the standard model with a cosmological constant. In that case the universe expands exponentially forever, and other galaxies recede from ours to become invisible. Entropy gradually increases.

There seems to be less widespread acceptance of an infinite past. One reason is the old worry about entropy that it must increase and so at a finite time in the past would fall to zero. This is avoided in here. Another possible concern is provided by arguments about null geodesics into the past and whether the spacetime manifold can be past complete.

It is true that the infinite past is less familiar than the infinite future, and surely an infinite past requires a cyclic model. At each turnaround a small fraction (f) of universes will fail to cycle and we confirm that there is a high likelihood that after infinite cycles we live in a successful universe. There is no reason that an infinite past is less viable than an infinite future, although as we shall show it does require somewhat unfamiliar concepts such as the inevitability of infinite cyclicity and that the total number of universes has been infinite since time $t \longrightarrow -\infty$, suitably defined, and remains always infinite, being multiplied by a (large) finite factor at the turnaround of each of the infinite cycles.

Past null geodesics

There is a general argument about past completeness of the spacetime manifold which we address first.

We begin with the BGV no-go theorem which we shall adapt for application to this more general case, as the original no-go theorem applies to past inflation. We shall show how this no-go theorem is by-passed, as the assumptions no longer apply.

The metric is of the form

$$ds^2 = dt^2 - a(t)^2 d\mathbf{x}^2 \tag{18}$$

In this metric for a null geodesic the affine parameter λ follows the relation

$$d\lambda \propto a(t)dt \tag{19}$$

We normalize the affine parameter to the present time $t = t_0$ by choosing with $a_0 = a(t_0)$

$$d\lambda = \left[\frac{a(t)}{a_0}\right] dt \tag{20}$$

so that $d\lambda/dt = 1$ when $t = t_0$.

We multiply Eq.(19) by the Hubble parameter $H = \dot{a}/a$ where a dot denotes derivative with respect to t but now we integrate from an initial time $t_n = t_0 - n\tau$ up to $t = t_0$ to obtain with $a_n = a(t_0 - n\tau), \lambda_n = \lambda(t_0 - n\tau)$

$$\int_{\lambda_n}^{\lambda_0} H(\lambda) d\lambda = \frac{1}{a_0} \int_{a_n}^{a_0} da = nC \tag{21}$$

where in the cyclic model we have denoted the finite integral

$$\frac{1}{na_0} \int_{a_n}^{a_0} da = C \tag{22}$$

by the constant C.

The left hand side of Eq.(21) can be written as the average of the Hubble parameter

$$H_{av} \equiv \frac{1}{(\lambda_0 - \lambda_n)} \int_{t_n}^{t_0} H(\lambda) d\lambda \qquad (23)$$

over n cycles. In particular, it is important that H_{av} in Eq.(23) is independent of the integer n because of cyclicity.
Given Eq.(23), we find from Eq.(21) that

$$H_{av} = lim_{n \to \infty} \left[\frac{nC}{(\lambda_0 - \lambda_n)} \right] \qquad (24)$$

so that for $n \to \infty$, we find a backwards null geodesic $(\lambda_0 - \lambda_n) \propto n$ of infinite length and the BGV argument does not apply. Such a geodesic is exemplified by a photon propagating always at the origin $\mathbf{x} = 0$ of the spatial coordinates.

Whether or not the past incompleteness arguments apply to the competing cyclic model of Steinhardt and Turok we take no position. BGV argue that they do, but the authors disagree, so that jury is still out. But we do assert that they do not apply to the present model, as can be seen directly from our Eq. (24), where $(\lambda_0 - \lambda_n)$ necessarily becomes an infinite length past null geodesic for $n \rightarrow \infty$, given the finiteness of both H_{av} and C.

Successful and failed universes [2]

Now we turn to another issue. At each turnaround, a very large number N of new universes is spawned. Let the number of universes at time $t = t_0 - n\tau$ be Σ_n. Then the total number now is $\Sigma_0 = N^n \Sigma_n$.

This is not quite right because although almost every causal patch contains no photons and no matter, a tiny fraction $f << 1$ will contain one or more photons and hence because of pair production will fail to cycle and bounce prematurely. Similarly any other matter such as a quark or lepton in the causal patch will cause failure. This number is very small, generally $f < 1/N$ but we need to examine the failed universes to assess the probability that we may live in a successful rather than a failed universe now, after an infinite $n \to \infty$ of cycles.

[2] Suggested and solved by Dan Reichart.

Let us ignore any new universes spawned by failed universes. The number of successful universes is given after n cycles by

$$\begin{aligned} \Sigma_0^{(successful)} &= lim_{n\to\infty}[\Sigma_n(N - fN)^n] \\ &= lim_{n\to\infty}\Sigma_n[(1-f)N]^n \end{aligned} \quad (25)$$

The number of failed universes, on the other hand, is

$$\begin{aligned} \Sigma_0^{(failed)} &= lim_{n\to\infty}\Sigma_n[fN + fN(1-f)fN + fN[(1-f)N]^2 + ... + fN[(1-f)N]^{(n-1)}] \\ &= lim_{n\to\infty}\Sigma_n fN[(1-f)N]^{(n-1)}[1 - \{1/(1-f)N\}]^{-1} \\ &= lim_{n\to\infty}\Sigma_n fN[(1-f)N]^n[(1-f)N - 1] \end{aligned} \quad (26)$$

The probability for a successful universe at present is given by the ratio of Eq.(25) with the sum of Eq.(25) and Eq.(26) which gives

$$P^{(successful)} = \frac{[(1-f)N - 1]}{(N-1)} \quad (27)$$

For $N \gg 1$ and $f \ll 1$, this is approximately $P^{successful} = (1 - f)$ similarly to each single turnaround, as expected.

This is non-trivial when both subsets are infinite and if it had been that failed universes dominate instead, the model would have been untenable because our universe would be infinitely unlikely. Fortunately, this is not the case.

Total number of universes

Last but certainly not least, we study the total number of universes versus time in the past.

Suppose that $\Sigma_n < \infty$ for some finite n. Then going back another $n^{'}$ cycles we have $\Sigma_{n+n^{'}} = \Sigma_n N^{-n^{'}}$. N satisfies $N > 1$ (actually $N >> 1$) so for some $n^{'}$ the integral part of $\Sigma_{n+n^{'}} = 1$ and cyclicity fails. Therefore no finite Σ_n is permitted for any finite n. In particular, the present number of universes must be $\Sigma_0 = \infty$, as expected after an infinite number of cycles.

More subtle is the value of

$$\begin{aligned}\Sigma_\infty &= lim_{n\to\infty}(\Sigma_0 N^{-n}) \\ &= \Sigma_0[lim_{n\to\infty}(N^{-n})]\end{aligned} \tag{28}$$

which is indeterminate as the product of infinity (Σ_0) times zero. This requires some recourse to cardinality and the transfinite numbers of set theory, depending on the level of rigor demanded.

In set theory the lowest transfinite is $\aleph_0$ (Aleph-zero) and the simplest assumption is that the number of universes is always $\aleph_0$, the cardinality of the primes, the integers or the rational numbers. When $\aleph_0$ is multiplied by a finite number N, it remains $\aleph_0$. This holds for any finite n in Eq.(28) and so extends back an arbitrarily long time in the past. For the infinite past, one cannot really say anything from Eq.(28).

The process is not time-reversal invariant and the global entropy of all universes increases with time consistent with the second law of thermodynamics. Considering only our universe, however, the entropy as well as the density and temperature are cyclic and never infinite. This is as near to infinite cyclicity as seems possible consistent with statistical laws. The old problem confronting Tolman is avoided by removing entropy to an unobservable exterior region; one may say in hindsight that the problem lay in considering only one universe.

Summary.

The standard cosmology based on a Big Bang augmented by an inflationary era is impressively consistent with the detailed data from WMAP3 when dark energy, most conservatively a cosmological constant, is included. Our objections to this standard model are more aesthetic than motivated directly by observations. The first objection is the nature of the initial singularity and the initial conditions. A second objection, not of concern to all colleagues, is that the predicted fate of the universe is an infinitely long expansion.

We have outlined here a cyclic cosmology resting on phantom dark energy where these objections are ameliorated: the classical density and temperature never become infinite and future expansion is truncated. Also, our proposal of deflation naturally leads to a multiverse picture, somewhat reminiscent of that predicted in eternal inflation, though here the proliferation of universes must be infinite and originates at the opposite end of a cyclic cosmology, at its maximum rather than at its minimum size.

We have shown that the entropy of the contracting universe is not only very small but actually vanishing so the entropy problem is solved more completely than originally envisioned. It offers an explanation of why the entropy pre inflation is zero.

We have argued that an infinite past time is consistent and that the present cyclic model is an exemplar. The presence of patches which fail to cycle is not a problem as after an infinite number of cycles the probability of being in a successful universe as we find ourselves is practically one. Also, it is mandatory that the total number of universes is infinite, equal to the constant $\aleph_0$, for times arbitrarily far into the past. This idea is unfamiliar but appears to us to be an inevitable concomitant of an infinite past.

We present this cyclic universe proposal mainly in the hope that it will stimulate an improved and more consistent formulation by others.

Thank you for your attention