Renormalisation Group Running of Gravitational Couplings

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motivation

- **physics of classical gravity**

  Einstein's theory $G_N = 6.67 \times 10^{-11} \frac{m^3}{\mathrm{kg}\, s^2}$

  $G_N \approx$ const. on length scales between $\sim 10^{-2} - 10^{28}\,\mathrm{cm}$

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  Planck length $\ell_{\mathrm{Pl}} = \left(\frac{\hbar G_N}{c^3}\right)^{1/2} \approx 10^{-33}\,\mathrm{cm}$

  expect strong corrections to G_N at scales $\ll \ell_{\mathrm{Pl}}$.

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- **renormalisation group**

  scale-dependence of gravitational couplings $G_N \to G_N(\mu)$

  non-trivial RG fixed point at short distances?

renormalisation group scaling

- **RG scaling of gravitational coupling**

  dimensionless coupling $g(\mu) = Z_N(\mu)^{-1} \cdot G_N \cdot \mu^{D-2}$

  anomalous dimension $\eta_N = -\frac{\mathrm{d}\ln Z_N}{\mathrm{d}\ln\mu}$

  RG running $\frac{\mathrm{d}g}{\mathrm{d}\ln\mu} = (D - 2 + \eta_N)\, g$

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- **fixed points**

  Gaussian: $g = 0$ **dominates GR**

  non-Gaussian: $\eta_N = 2 - D$ **may dominate deep UV (deep IR)**

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  UV fixed point implies **weakly coupled gravity** at high energies

$$\mu \to \infty : \qquad G(\mu) \to g_* \mu^{2-D} \ll G_N$$

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- **fixed points**

  Gaussian: $g = 0$ **dominates GR**

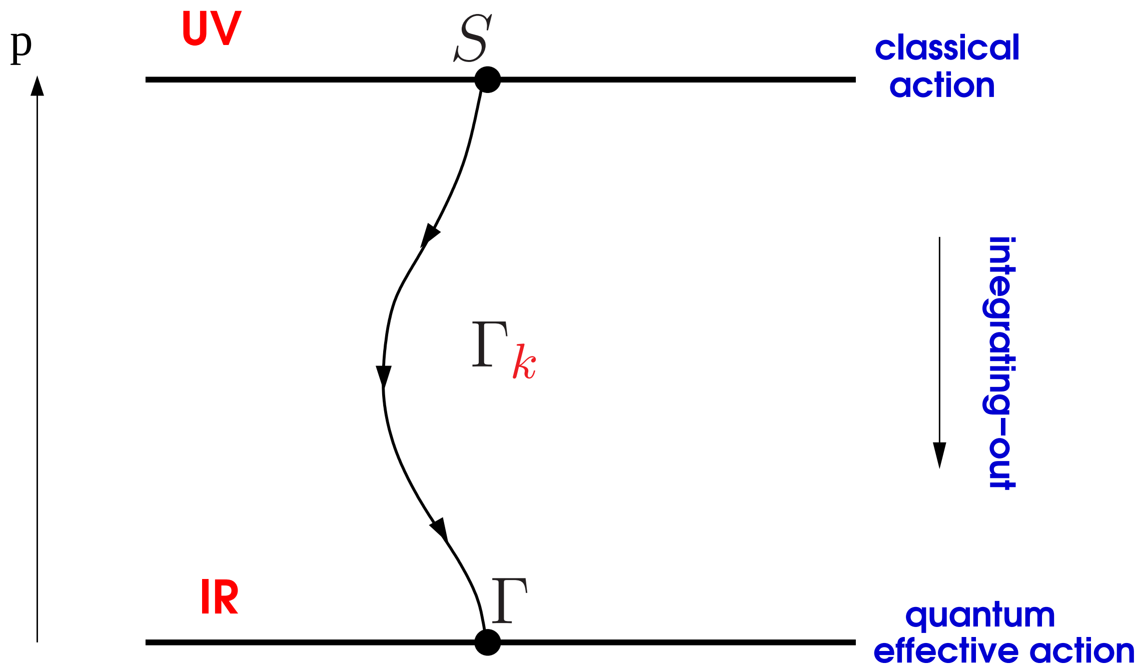
  non-Gaussian: $\eta_N = 2 - D$ **may dominate deep UV (deep IR)**

  IR fixed point implies **strongly coupled gravity** at low energies

$$\mu \to 0 : \qquad G(\mu) \to g_* \mu^{2-D} \gg G_N$$

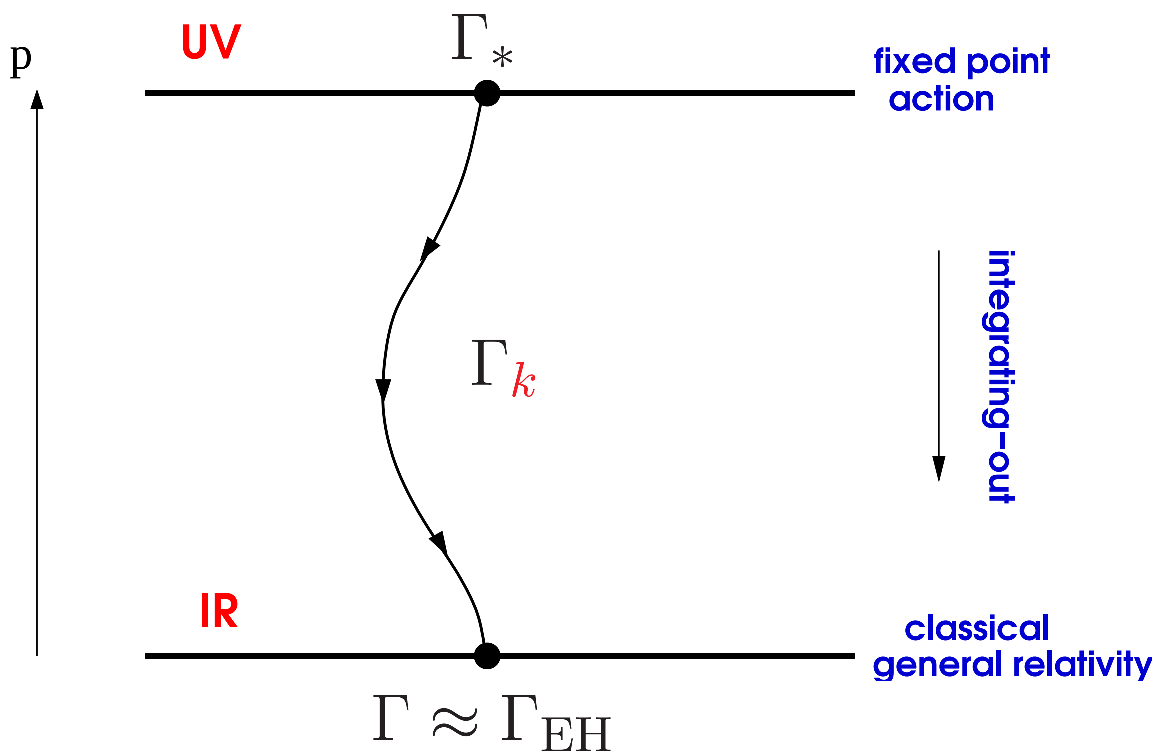
Wilson's renormalisation group

- **integrating-out momentum degrees of freedom**



Wilson's renormalisation group

- **for quantum gravity**



Wilson's renormalisation group

- **effective action**

$$\Gamma_k = \frac{1}{16\pi G_k} \int \sqrt{g}\,(\Lambda_k + R + \cdots) + S_{\text{matter},k} + S_{\text{gf},k} + S_{\text{ghosts},k}$$

up to now: $\sqrt{g},\ \sqrt{g}R,\ \sqrt{g}R^2, \cdots,\ \sqrt{g}R^8$, **matter fields**

Reuter (1996), Souma (1999), Lauscher, Reuter (2001), Reuter, Saueressig (2001),
DL (2003), Percacci, Perini (2003), Bonnano, Reuter (2004), Bonanno (2005),
Lauscher, Reuter (2005), Percacci (2005), Fischer, DL (2006)

Codello, Percacci (2006), Codello, Percacci, Rahmede (2007)

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- **wilsonian RG flow**

$$k \frac{\mathrm{d}}{\mathrm{d}k} \Gamma_k[g_{\mu\nu}] = \frac{1}{2} \,\mathrm{Tr} \, \left[\left(\Gamma_k^{(2)}[g_{\mu\nu}] + R_k \right)^{-1} k \frac{\mathrm{d}R_k}{\mathrm{d}k} \right]$$

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- **running couplings**

  projection of $k\partial_k\Gamma_k$ onto $\sqrt{g},\ \sqrt{g}R,\ \sqrt{g}R^2, \cdots$

Einstein-Hilbert theory

$$\beta_g = (D - 2 + \eta)\, g \qquad \eta = \frac{g\, b_1(\lambda)}{1 + g\, b_2(\lambda)}$$

$$\beta_\lambda = (-2 + \eta)\lambda + g(a_1 - \eta\, a_2) \qquad \lambda_k = \Lambda_k / k^2$$

$$a_1 = \frac{D(D-1)(D+2)}{2(1-2\lambda)} + \frac{D(D+2)}{1-2\alpha\lambda} - 2D(D+2)$$

$$a_2 = \frac{D(D-1)}{2(1-2\lambda)} + \frac{D}{1-2\alpha\lambda}$$

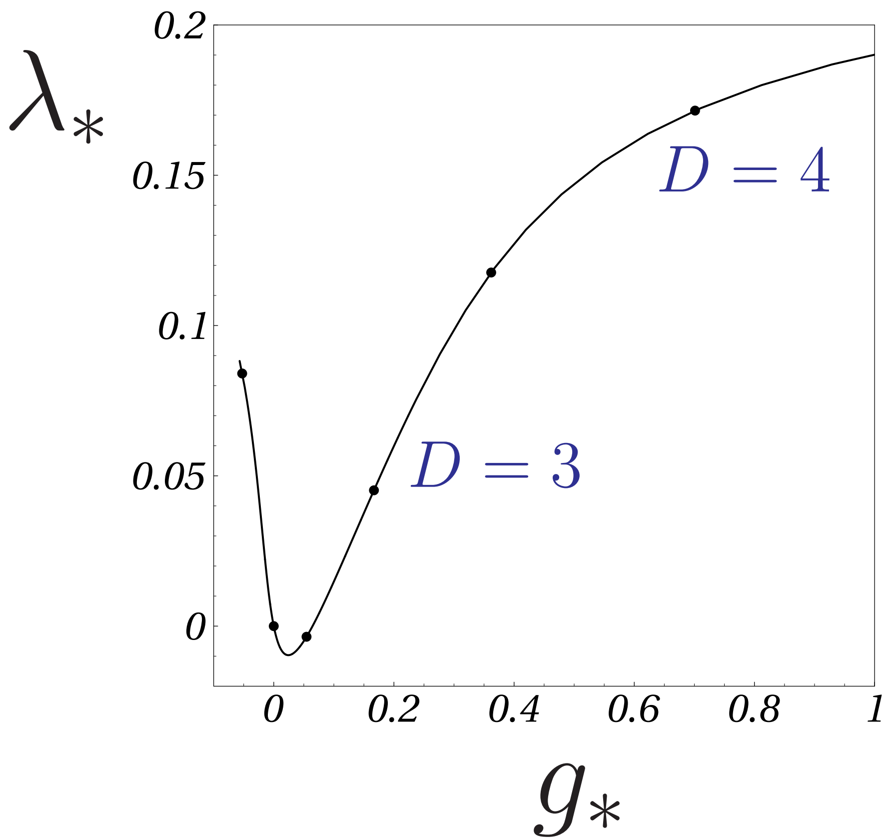
$$\begin{aligned} b_1 = &-\frac{1}{3}(1+\frac{2}{D})(D^3+6D+12) \\ &-\frac{(D+2)(D^3-4D^2+7D-8)}{(D-1)(1-2\lambda)^2} + \frac{D(D+2)(D^3-2D^2-11D-12)}{12(D-1)(1-2\lambda)} \\ &-\frac{2(D+2)(\alpha D^2-2\alpha D-D-1)}{D(1-2\alpha\lambda)^2} + \frac{(D+2)(D^2-6)}{6(1-2\alpha\lambda)} \end{aligned}$$

$$\begin{aligned} b_2 = &-\frac{D^3-4D^2+7D-8}{(D-1)(1-2\lambda)^2} + \frac{(D+2)(D^3-2D^2-11D-12)}{12(D-1)(1-2\lambda)} \\ &-\frac{2(\alpha D^2-2\alpha D-D-1)}{D(1-2\alpha\lambda)^2} + \frac{(D+2)(D^2-6)}{6D(1-2\alpha\lambda)} \end{aligned}$$

(DL ′03)

UV fixed point

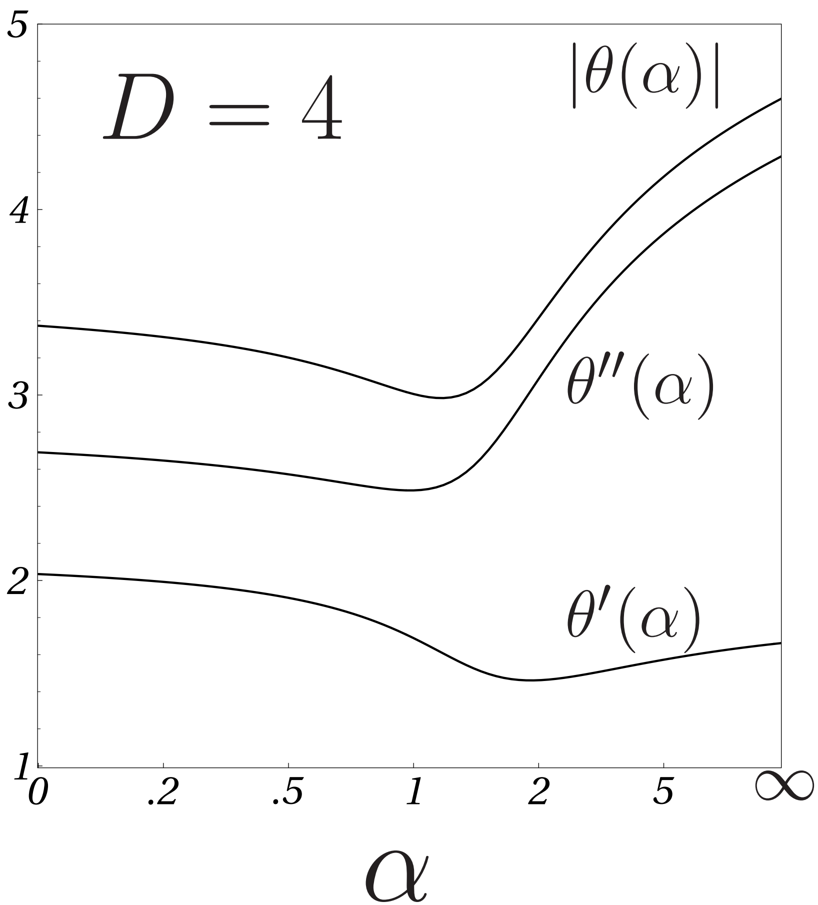
- **continuity**



- **continuous link with perturbative fixed point in $D = 2 + \epsilon$ dimensions**
- **real fixed point unique for any dimension**

universality

- **gauge fixing (in)dependence**

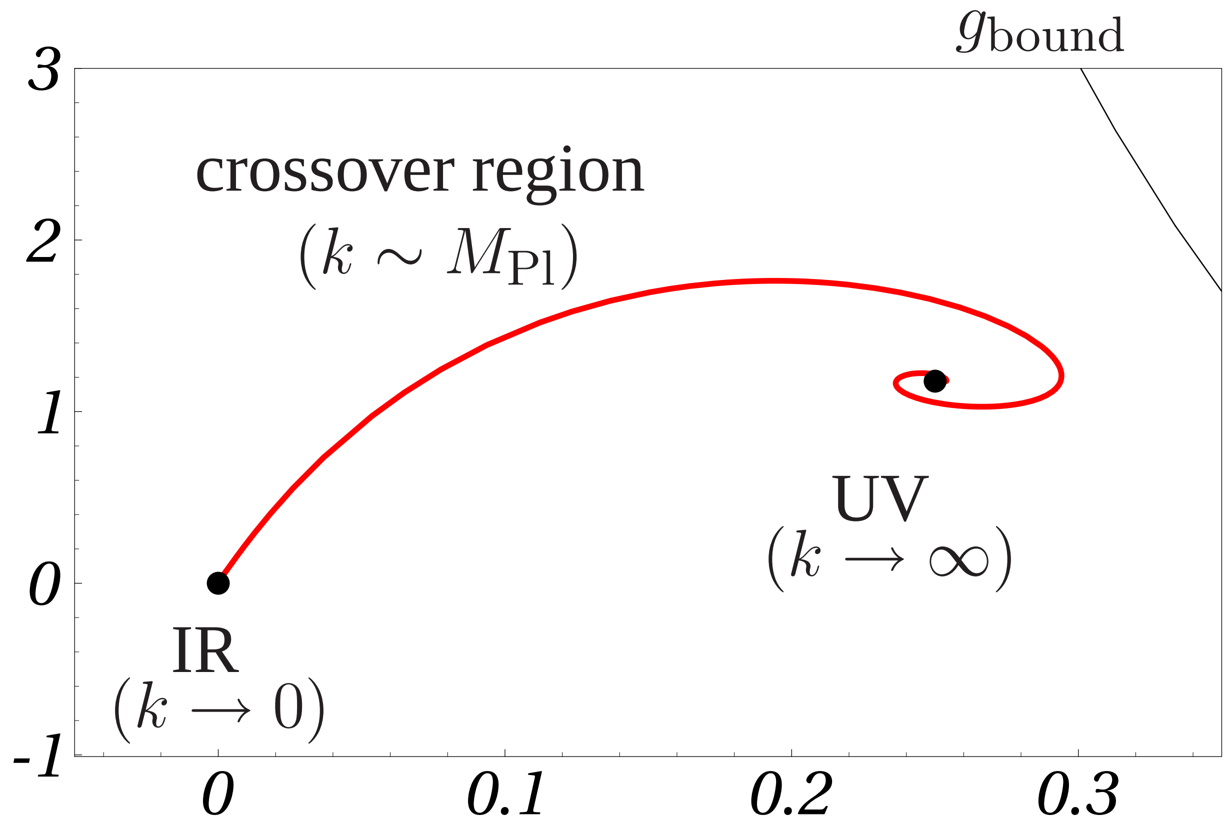


- **universal eigenvalues at criticality $\theta = \theta' + i\theta''$**
- **Landau-de Witt gauge is RG fixed point** (DL, Pawlowski '98)
- **consistent for all $\alpha \in [0, 1]$**
- **large-α behaviour correct**
- **θ consistent with Regge lattice simulations** (Hamber '00)

phase diagram

- **separatrix in four dimensions** (DL ’03)

$$g = G_k\, k^2$$



$$\lambda = \Lambda_k / k^2$$

phase diagram

- **phase diagram in four dimensions** (Fischer, DL ’07)

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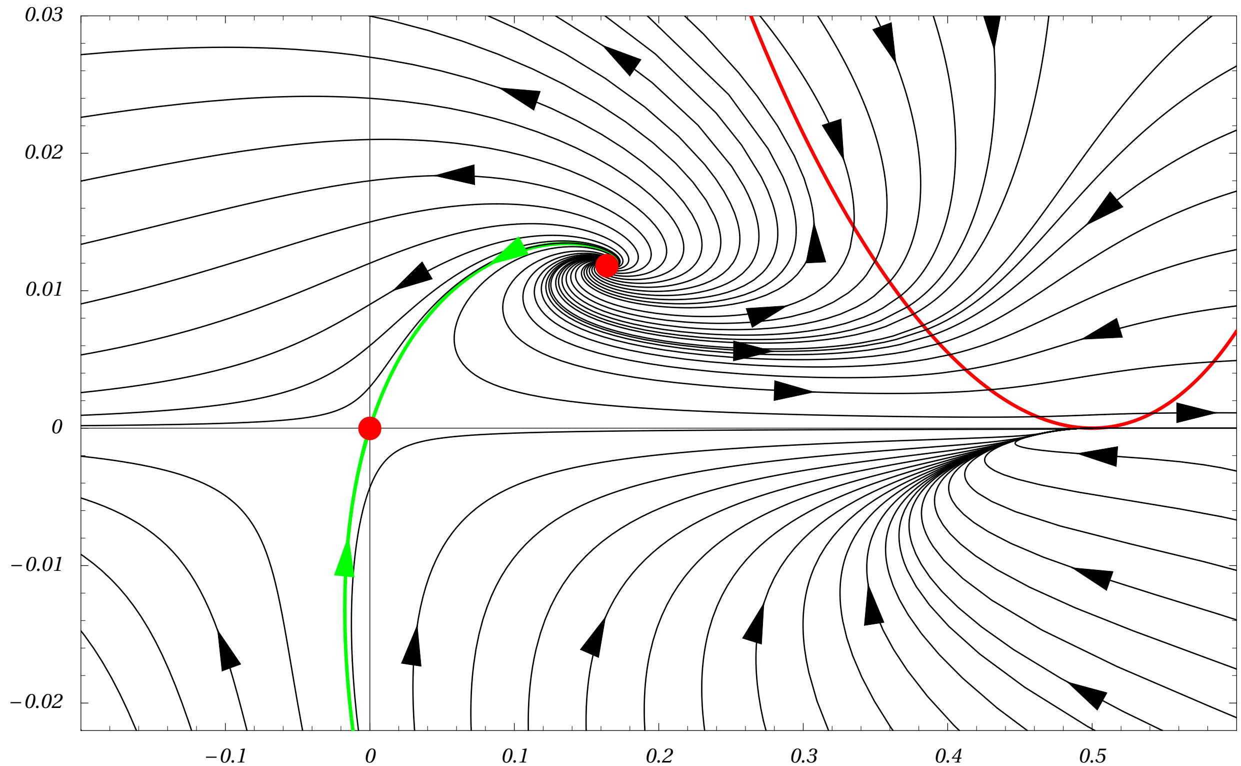
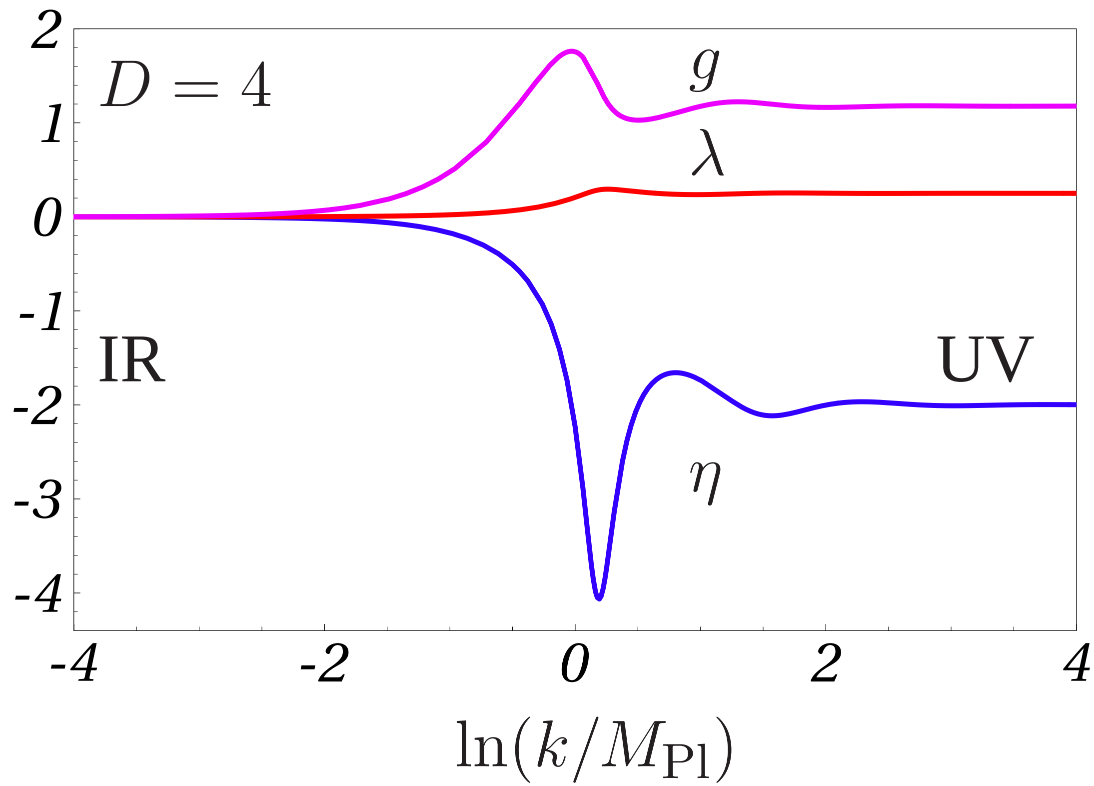


$$\lambda = \Lambda_k / k^2$$

flow trajectories

- **cross-over behaviour**

  integrated flow with $\sqrt{g}R$, $\sqrt{g}$



higher dimensions

with Peter Fischer (U Aachen)
PLB (2006) hep-th/0602203, hep-th/0606135

- **D-dimensional Einstein Hilbert theory**

  unique UV fixed point

  cutoff independence

  gauge-fixing independence

  analytical result

$$\lambda_* = \frac{D^2 - D - 4 - \sqrt{2D(D^2 - D - 4)}}{2(D-4)(D+1)}$$

$$g_* = \Gamma(\tfrac{D}{2} + 2)(4\pi)^{D/2-1} \frac{(\sqrt{D^2 - D - 4} - \sqrt{2D})^2}{2(D-4)^2(D+1)^2}$$

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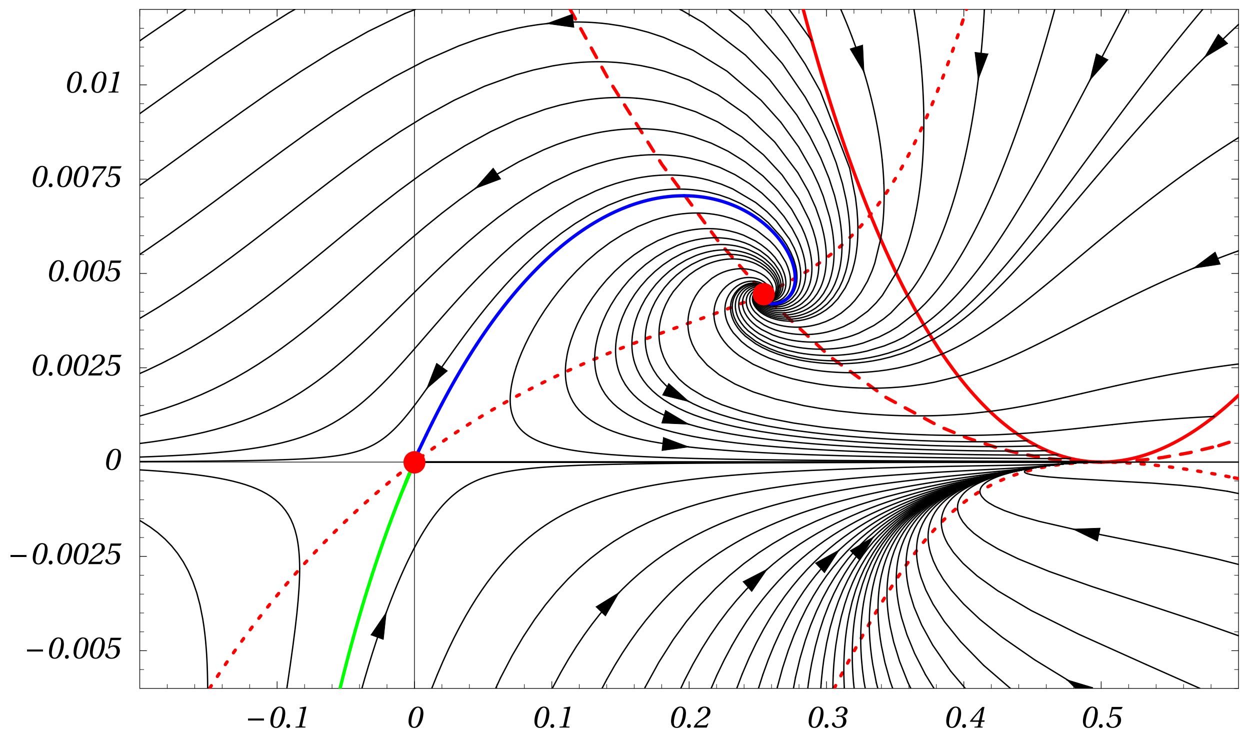
- **phase diagram in six dimensions**

$$g = G_k\, k^4$$



$$\lambda = \Lambda_k / k^2$$

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  running of couplings is important

  Wilson's method very powerful, many applications including gravity

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  growing evidence for non-trivial UV fixed point

  various RG studies, different approximations

Reuter (1996), Souma (1999)
Lauscher, Reuter (2001), Reuter, Saueressig (2001)
Forgacs, Niedermayer (2002), Niedermayer (2002)
DL (2003), Percacci, Perini (2003)
Bonnano, Reuter (2004), Percacci (2004)
Bonanno (2005), Lauscher, Reuter (2005)
Percacci (2005), Fischer, DL (2006)
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  UV fixed point persists DL (2003), Fischer, DL (2006)

  strengthens four-dimensional result

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- **outlook: weak gravity signatures**

  cosmology, astrophysics

  high energy colliders DL, Plehn (2007), Hewett, Rizzo (2007)