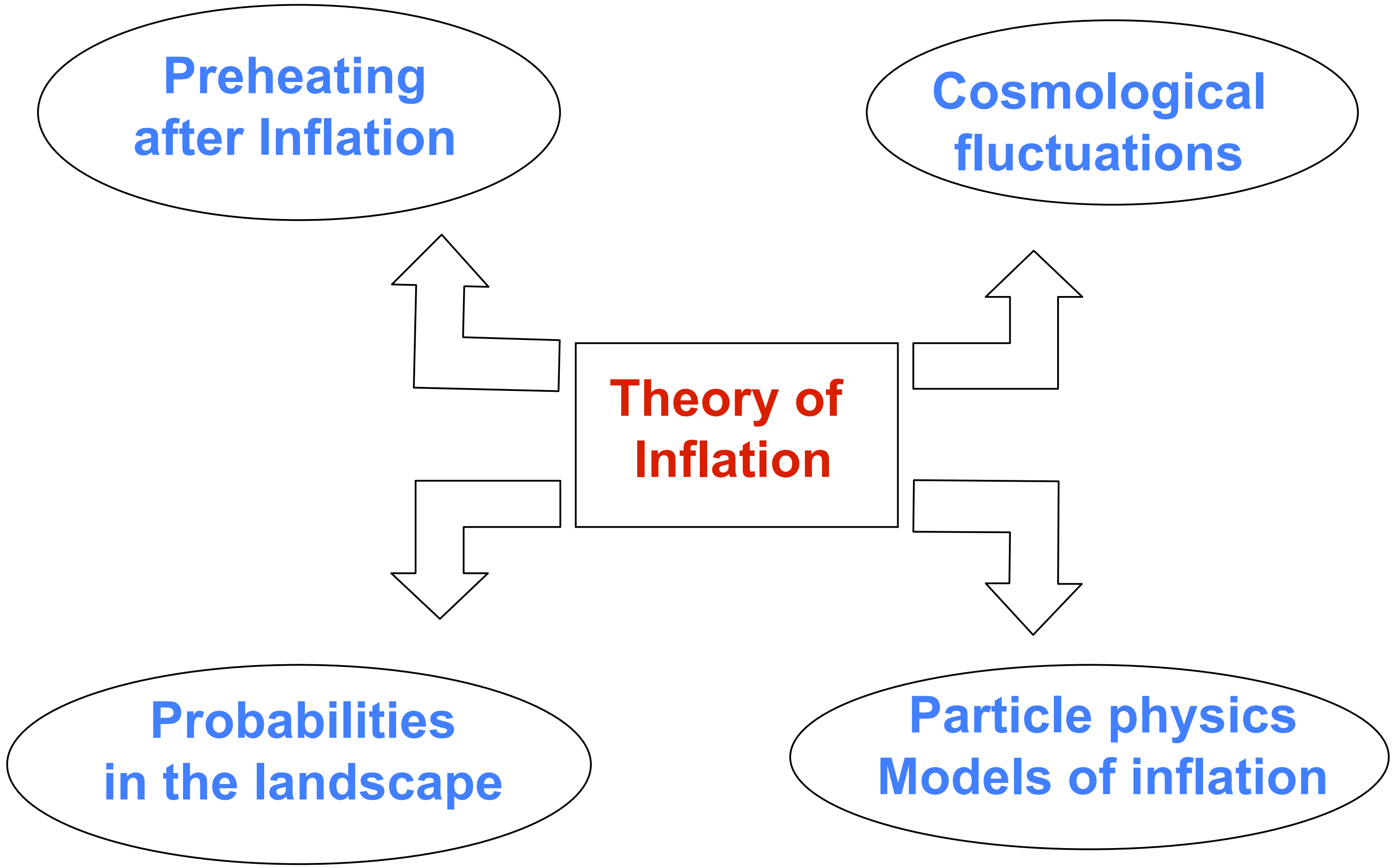

Preheating
after Inflation
Cosmological
fluctuations
Theory of
Inflation
Probabilities
in the landscape
Particle physics
Models of inflation

Making and Probing Inflation

Family Portrait of Inflation

Inflation in String Theory

How small can be r

Gravity Waves from Preheating

Lev Kofman

CITA ICAT Canadian Institute for Theoretical Astrophysics L'institut canadien d'astrophysique theorique

Cosmo07 Sussex 21 August 2007

Concise History of the Early Universe

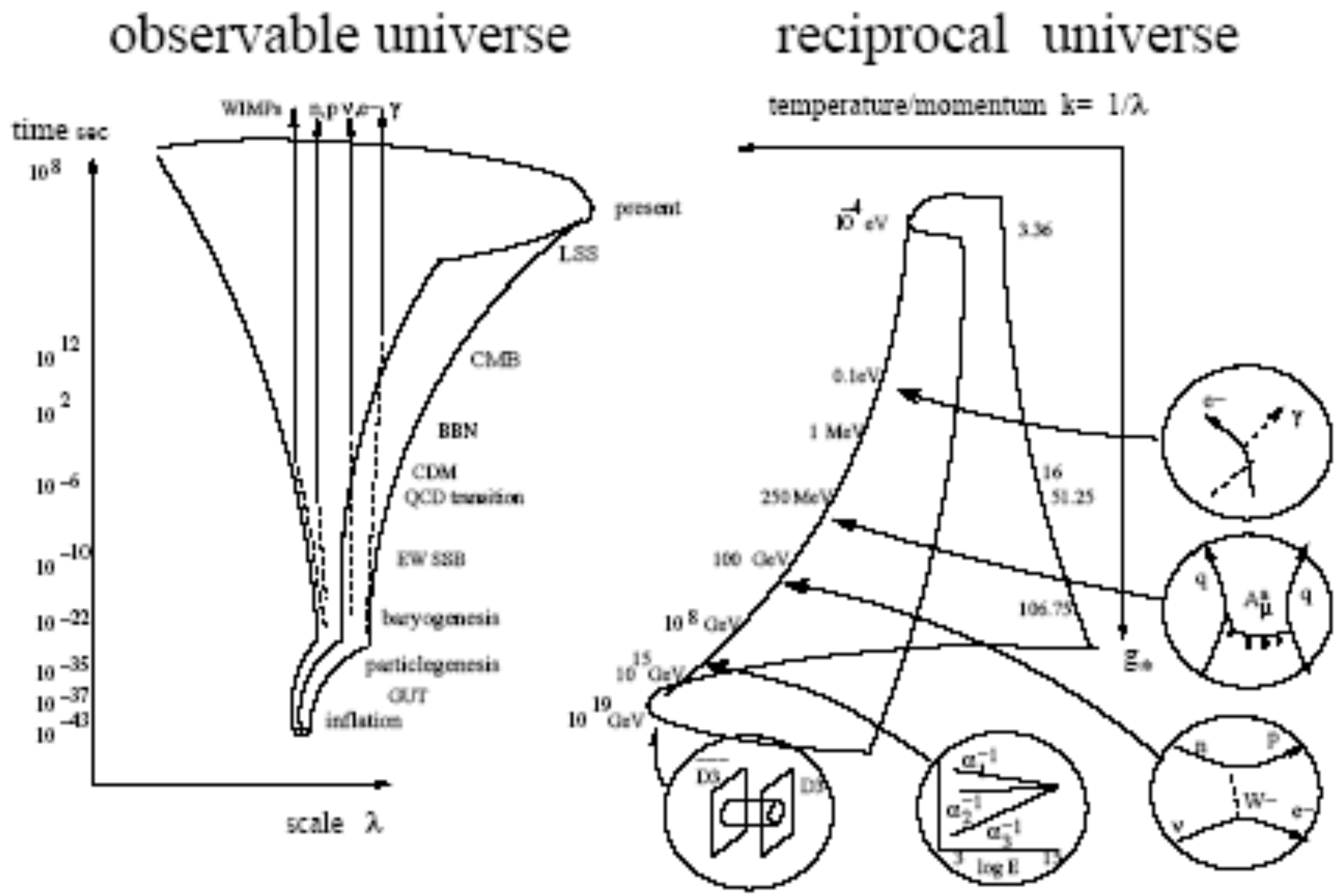
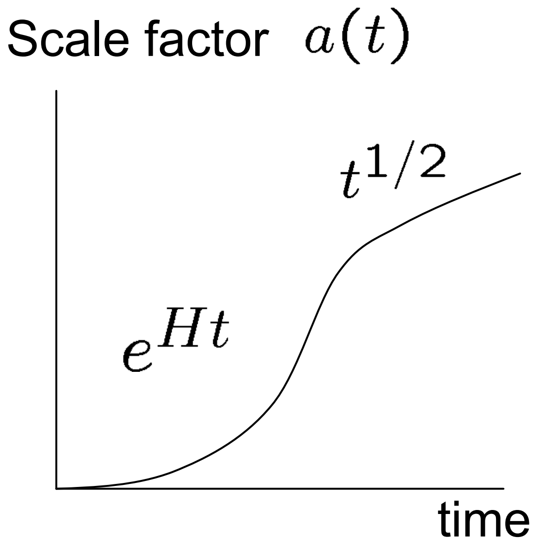


Fig. 1. Left: composition of the expanding universe is changing with time as it cools down. Right: reciprocal universe it terms of physical momenta of particles expands backwards in time to open particle physics at higher and higher temperatures. Icons illustrate physics at different energies.

Early Universe Inflation



Equation of State $t \leq 10^{-35}$ sec

$$p \approx -\epsilon$$

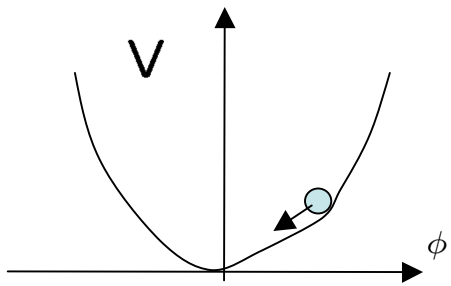
Inflation $a(t) \approx e^{Ht}$

Realization of Inflation

Scalar field

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0$$

$$p = \frac{1}{2}\dot{\phi}^2 - V$$
$$\epsilon = \frac{1}{2}\dot{\phi}^2 + V$$

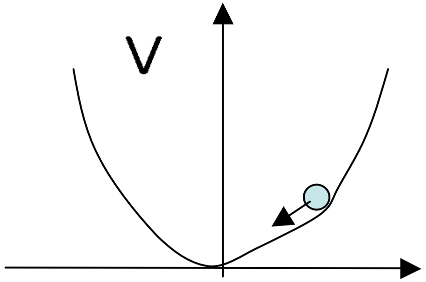


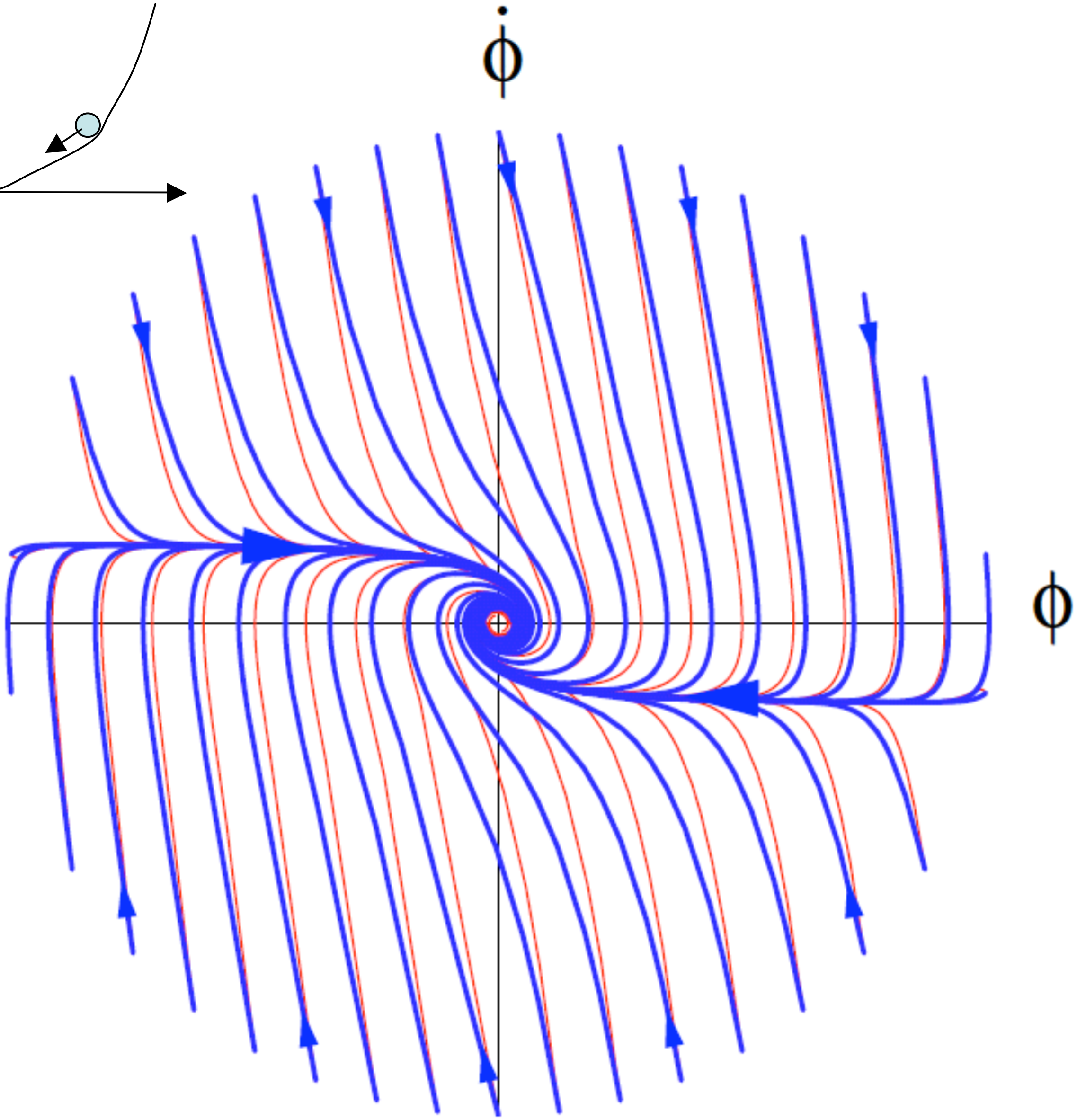
slow roll $\dot{\phi}^2 \ll V$

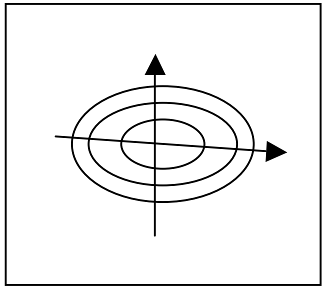
choise of $V(\phi)$

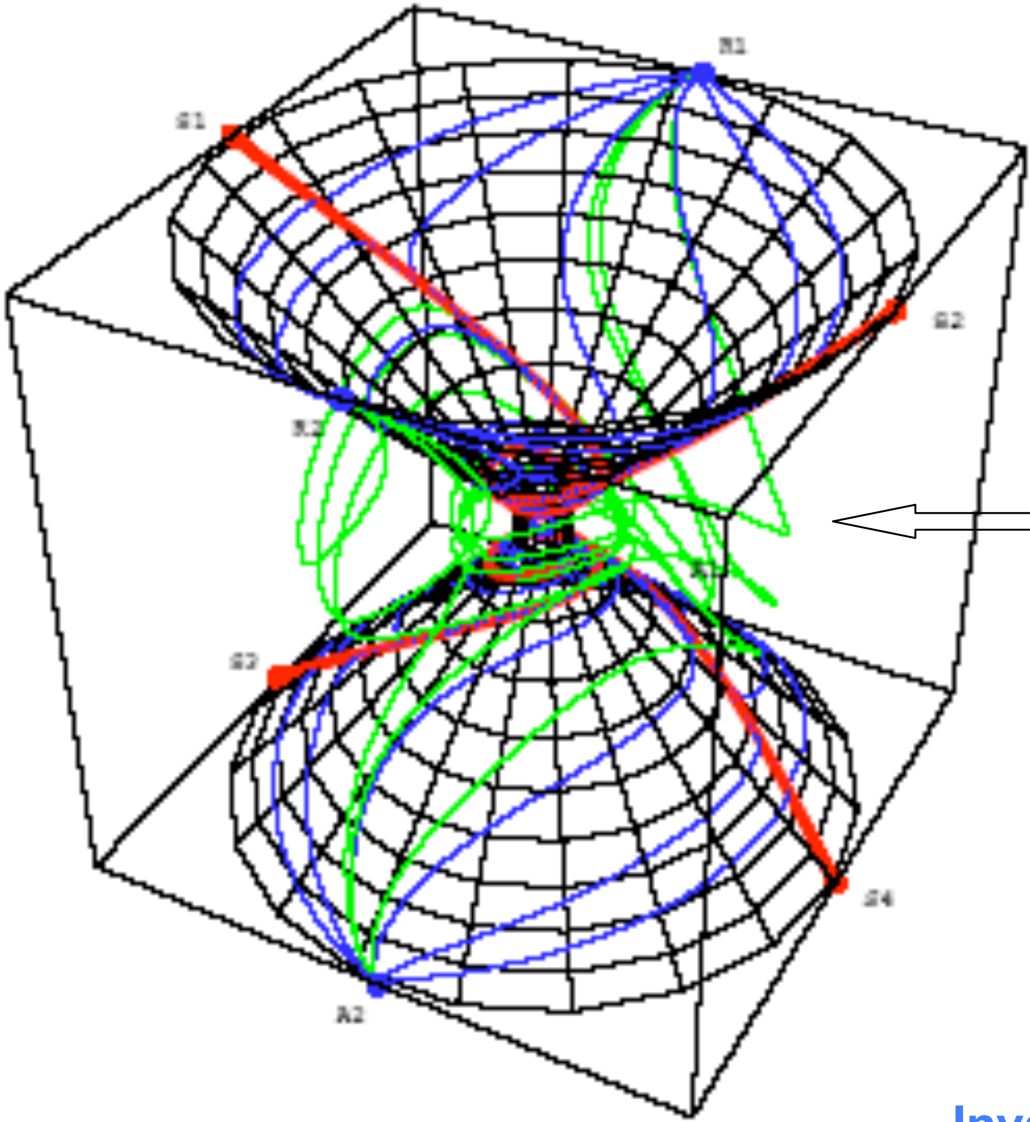
Family phase portrait of inflation

$$\ddot{\phi} + 3H\phi + V_{,\phi} = 0$$
$$3H^2 = \frac{8\pi}{M_p^2}\left(\dot{\phi}^2/2 + V\right)$$

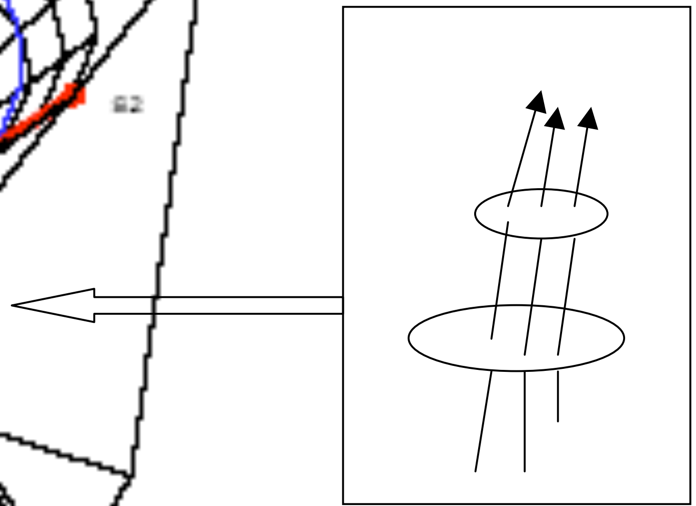








$d\mu$ is invariant



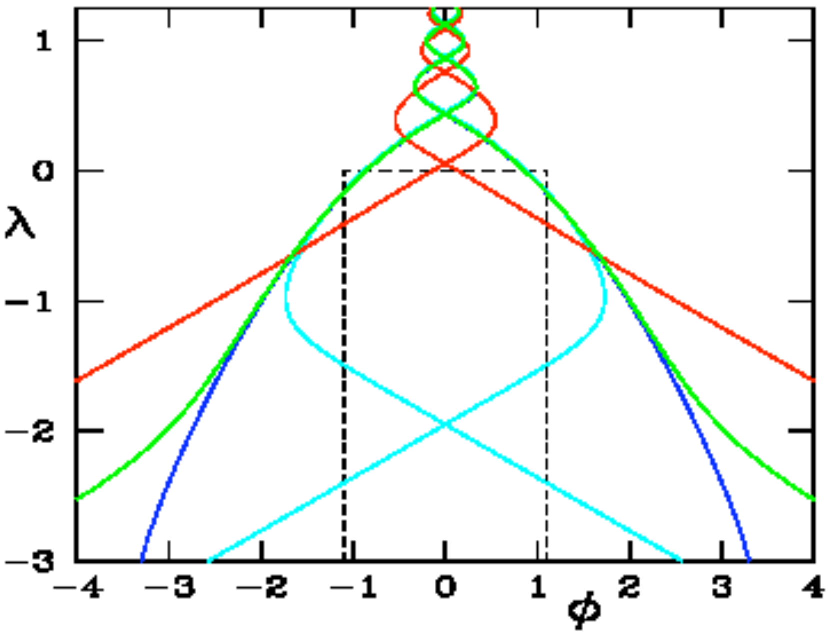
Invariant measure

$$d\mu = -a^2 \frac{\dot{\phi}}{H} d\dot{\phi} \wedge da + a^2(\dot{\phi} + V_{,\phi}/H) d\phi \wedge da + d\dot{\phi} \wedge d\phi$$

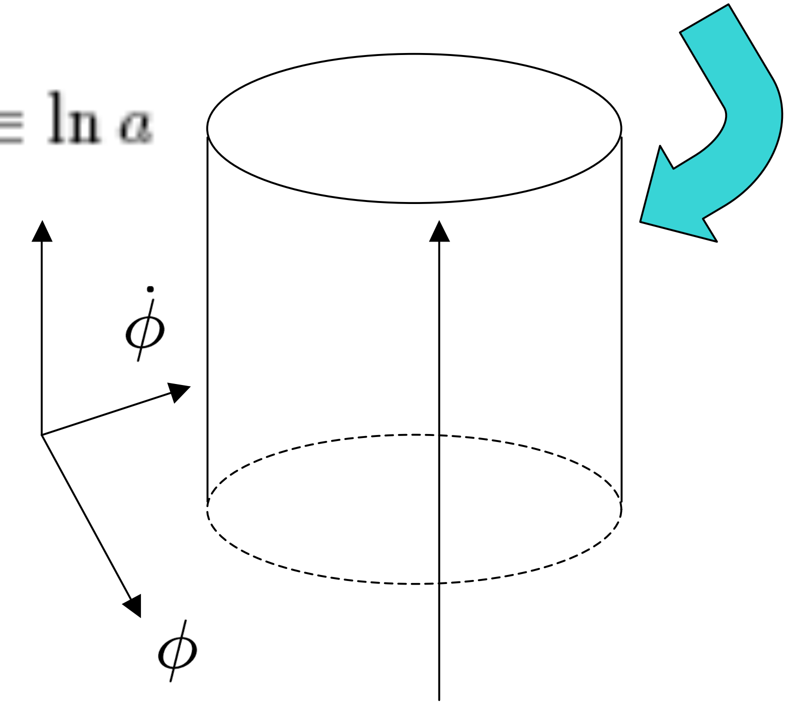
Gibbons & Turok
hep-th/0609095

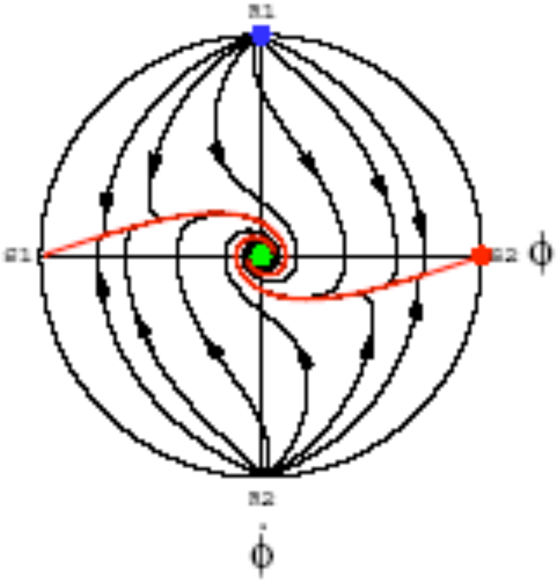
$$\int \mathbf{B}\cdot d\mathbf{S} = \oint \mathbf{A}\cdot d\mathbf{l},$$

$$\int d\mu = \int d\dot{\phi}\wedge d\phi$$



$$\lambda \equiv \ln a$$

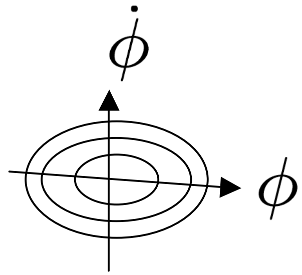




$$2\int a^3|\dot{\phi}|d\phi \equiv \oint p_\phi d\phi$$

$$\int d\phi\, p_\phi \equiv J$$

$$(J,\theta)$$



GT: Measure the priors of trajectories at the stage of long lasting scalar field oscillations, in terms of a uniform distribution of the phase θ of the oscillations. Results: probability $\propto e^{-3N}$

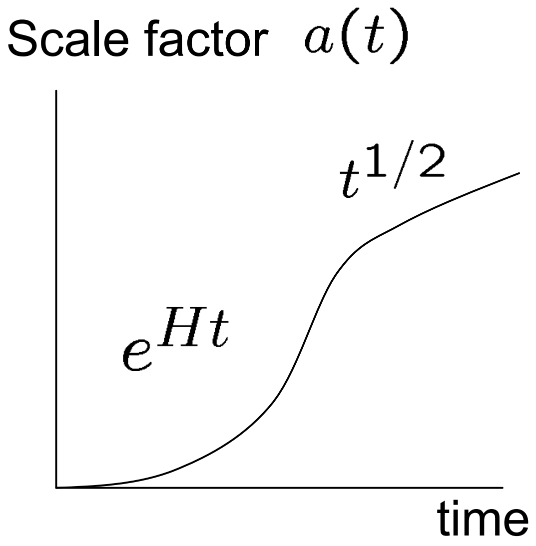
KL: J is not canonical invariant measure but an artifact of the cut of the hypersurface $\int d\mu$. For preheating the stage of oscillations is very short and cannot be used for the choise of priors.

Physical choise is not $\int d\mu$ but $\int \rho d\mu$, where ρ is the "weight", e.g. $\rho = |\Psi|^2$.

Typically ρ decreases as $\dot{\phi}$ increases.

Equally good is the cut of $\int d\mu$ before inflation, where $\int d\phi p_\phi$ is also an invariant

Early Universe Inflation



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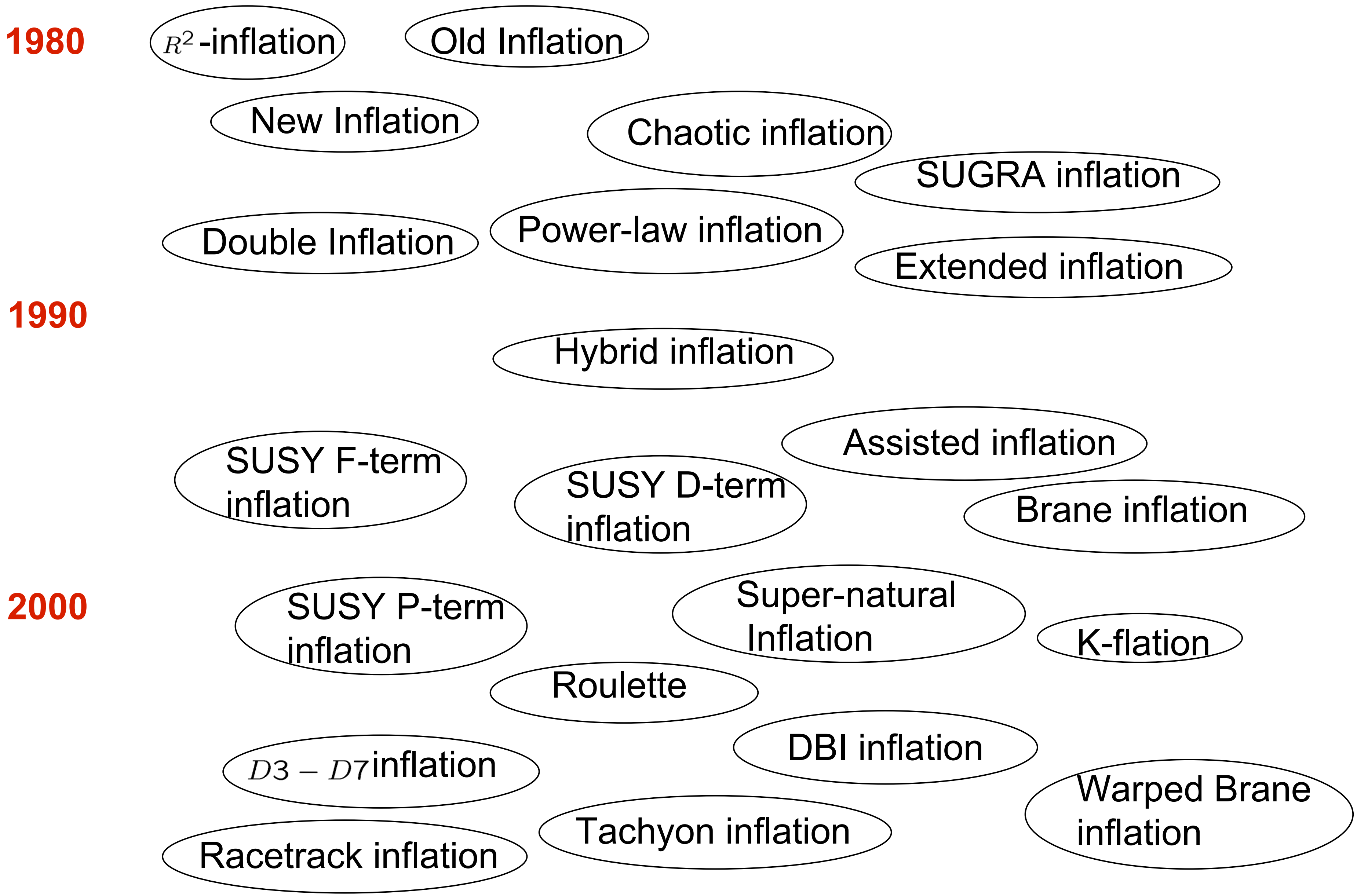
slow roll $\dot{\phi}^2 \ll V$

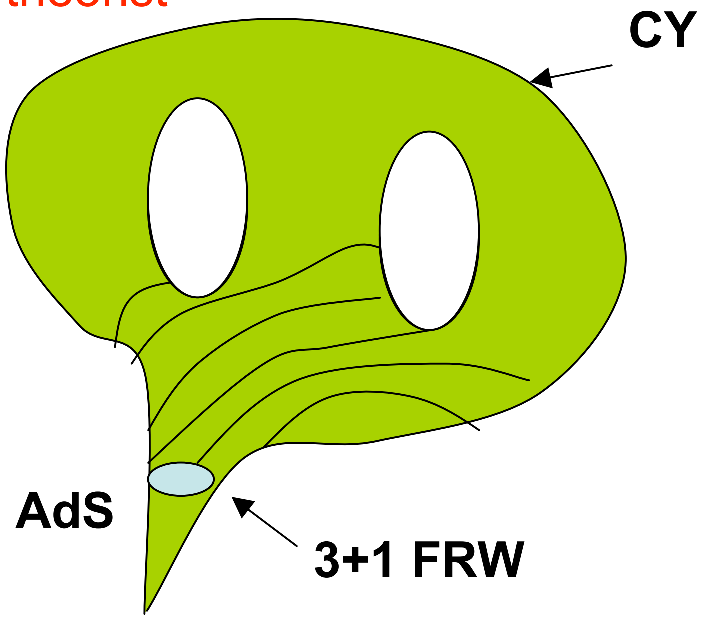
choise of $V(\phi)$

Inflation in the context of ever changing fundamental theory

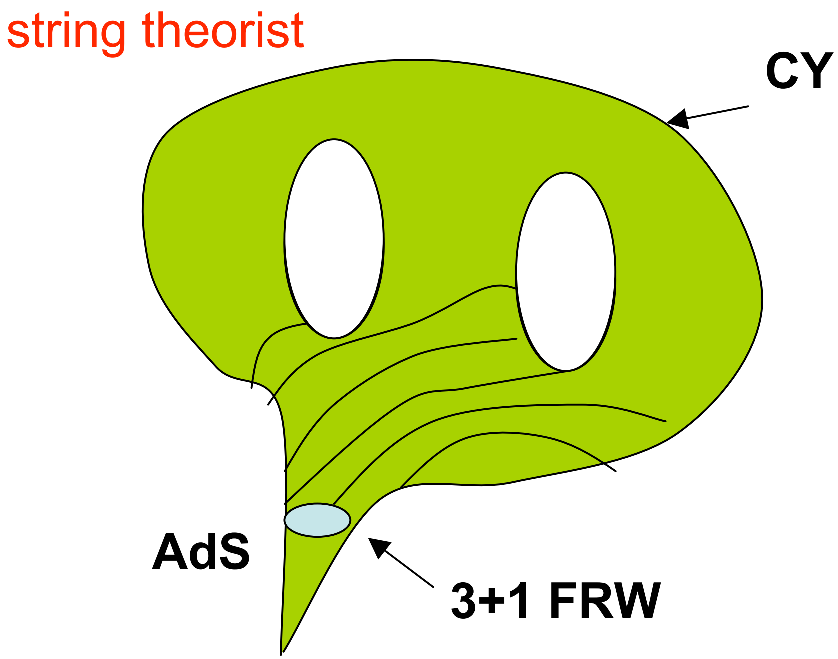


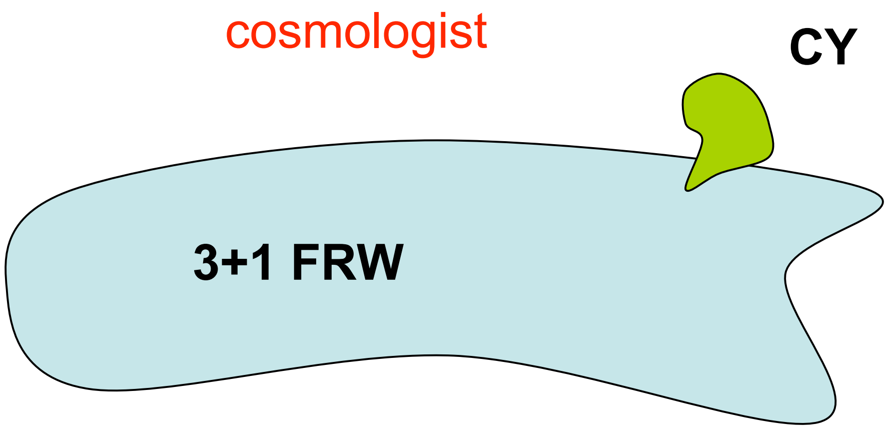
Inflation in String Theory:
Cosmology with Compactification

string theorist



Inflation in String Theory:
Cosmology with Compactification



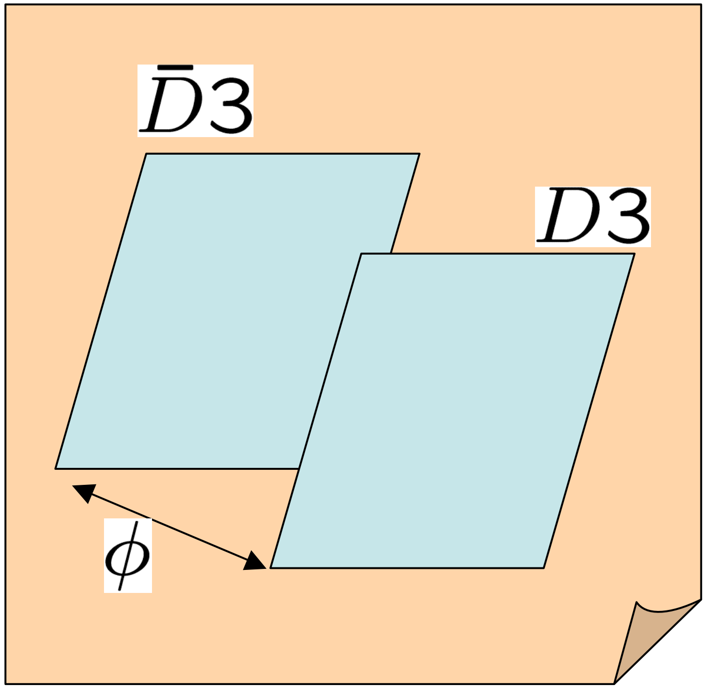


String Theory inflation models

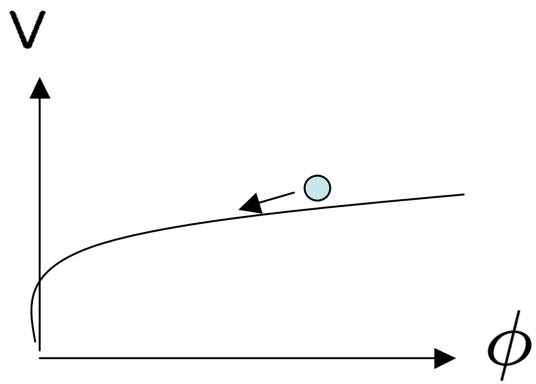
Modular Inflation. They use Kahler moduli/axion like the fields that are a present in the KKLT stabilization.

Brane inflation. The inflaton field corresponds to the distance between branes in Calabi-Yau space. Historically, this was the first class of string inflation models.

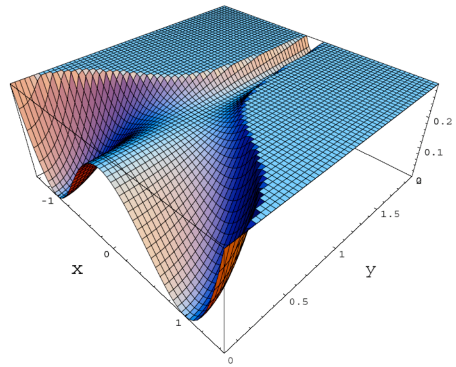
Inflaton with branes in String Theory

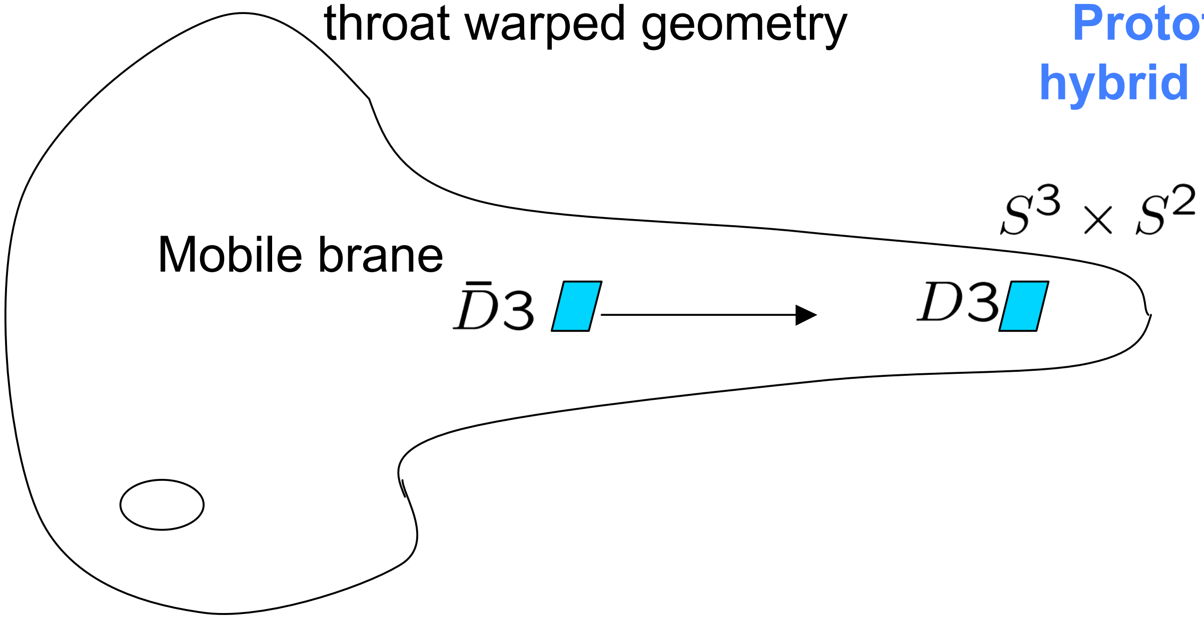


4-dim picture

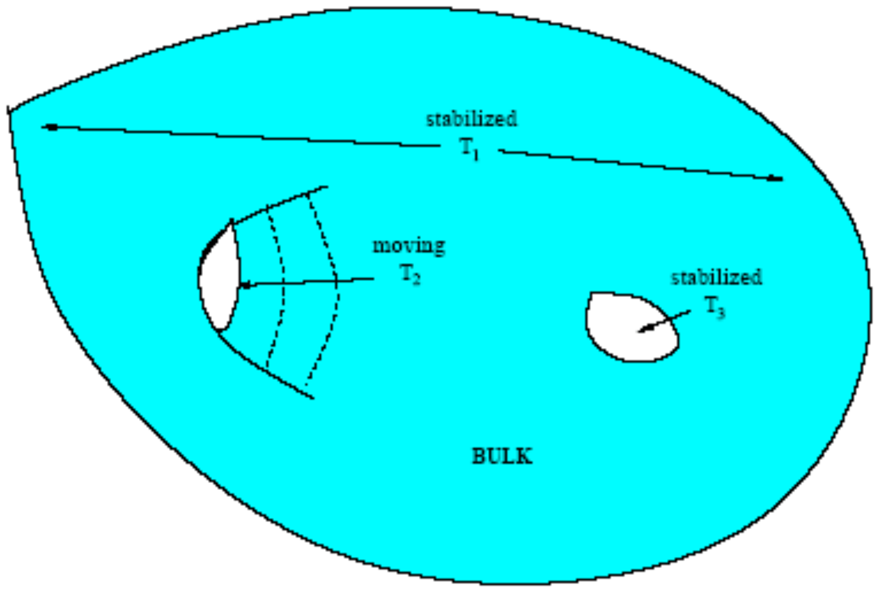


Prototype of
hybrid inflation





Kahler moduli Inflation (Conlon&Quevedo hep-th/050912)
Roulette Inflation-Kahler moduli/axion
(Bond, LK, Prokushkin&Vandrevange hep-th/0612197)



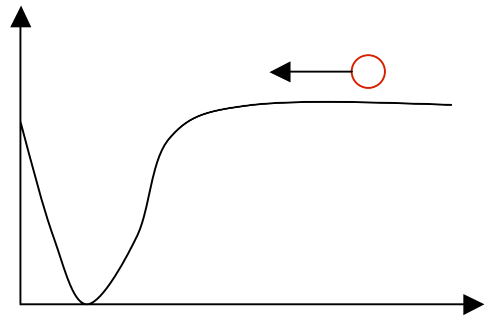


Figure 1: Schematic illustration of the ingredients in Kähler moduli inflation. The four-cycles of the CY are the Kähler moduli T_i which govern the sizes of different holes in the manifold. We assume T_3 and the overall scale T_1 are already stabilized, while the last modulus to stabilize, T_2, drives inflation while settling down to its minimum. The imaginary parts of T_i have to be left to the imagination. The outer $3 + 1$ observable dimensions are also not shown.

$$V(\phi,\bar\phi) = e^{\mathcal{K}/M_P^2}\left(\mathcal{K}^{i\bar j} D_i \hat W D_{\bar j}\bar{\hat W} - \frac{3}{M_P^2}\hat W \bar{\hat W}\right) + \text{ D-terms}$$

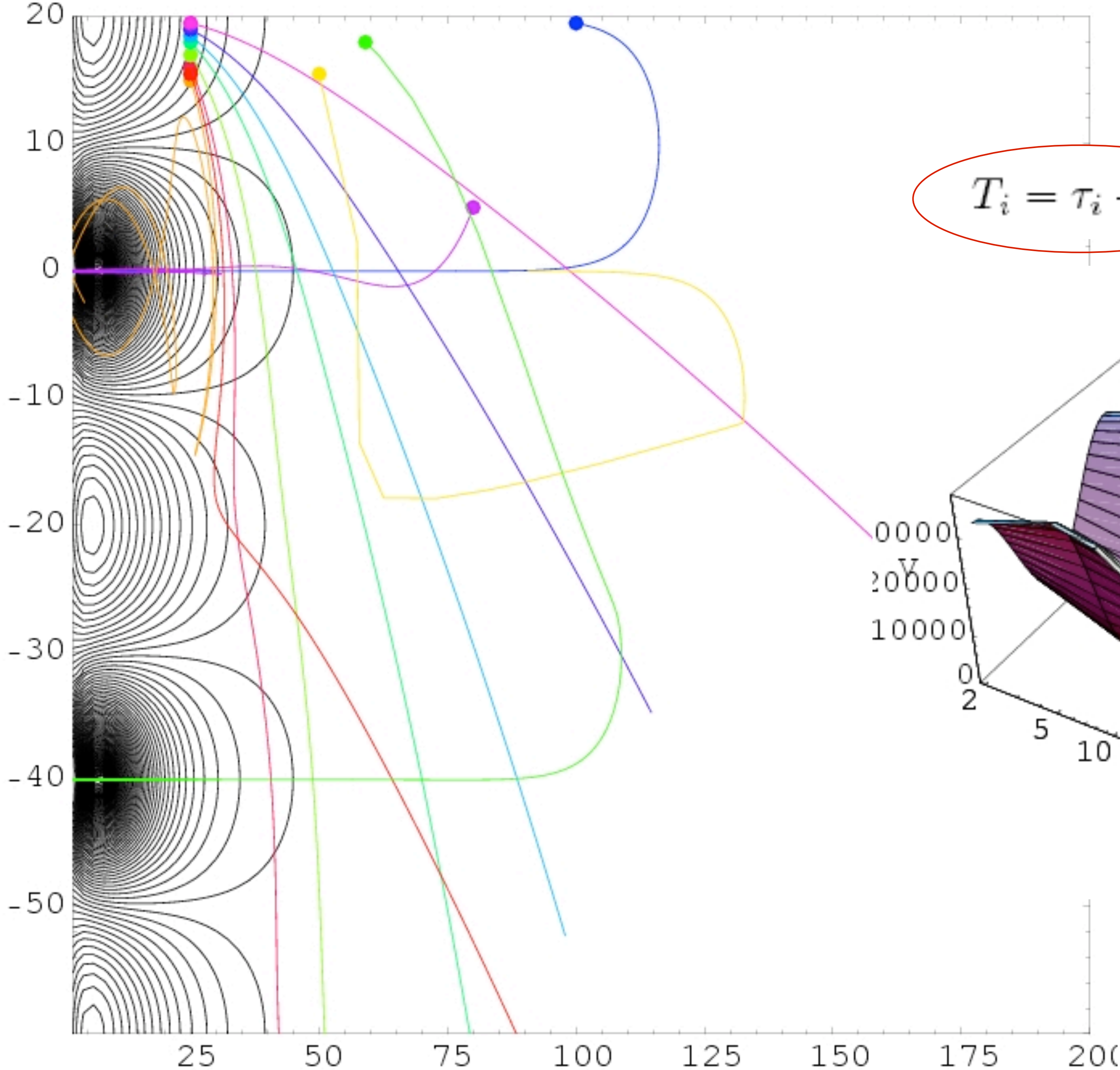
$$\frac{\mathcal{K}}{M_P^2} = -2\ln\left(\mathcal{V} + \frac{\xi}{2}\right) + \ln g_s + \mathcal{K}_{cs}$$

$$\hat W = \frac{g_s^{\frac{3}{2}} M_P^3}{\sqrt{4\pi}}\left(W_0 + \sum_{i=1}^{h^{1,1}} A_i e^{-a_i T_i}\right), \quad W_0 = \frac{1}{l_s^2}\int_M G_3 \wedge \Omega$$

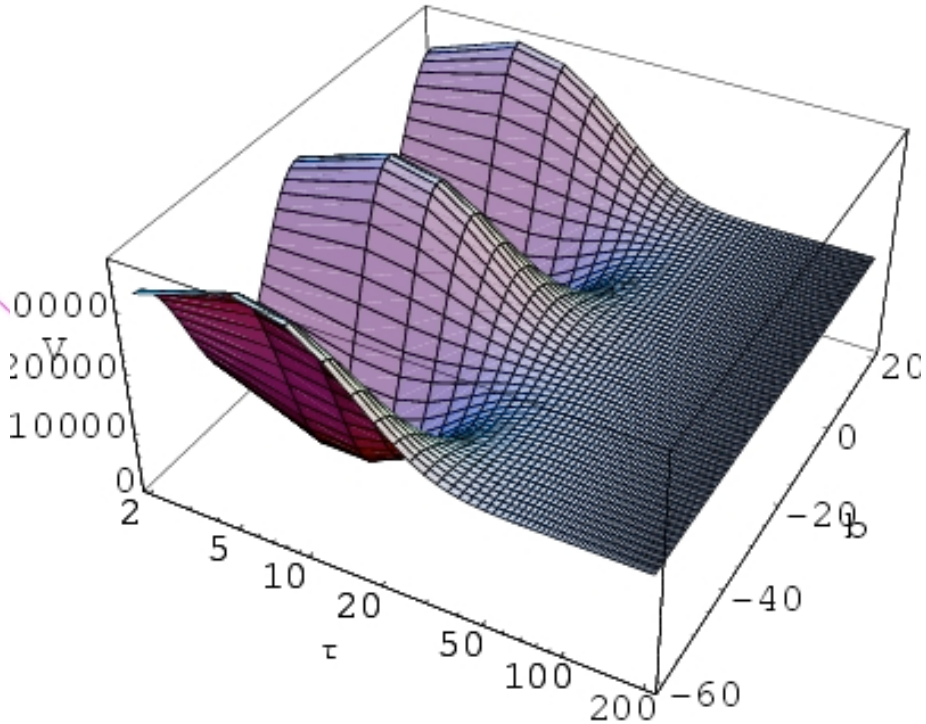
$$T_i = \tau_i + i\theta_i$$

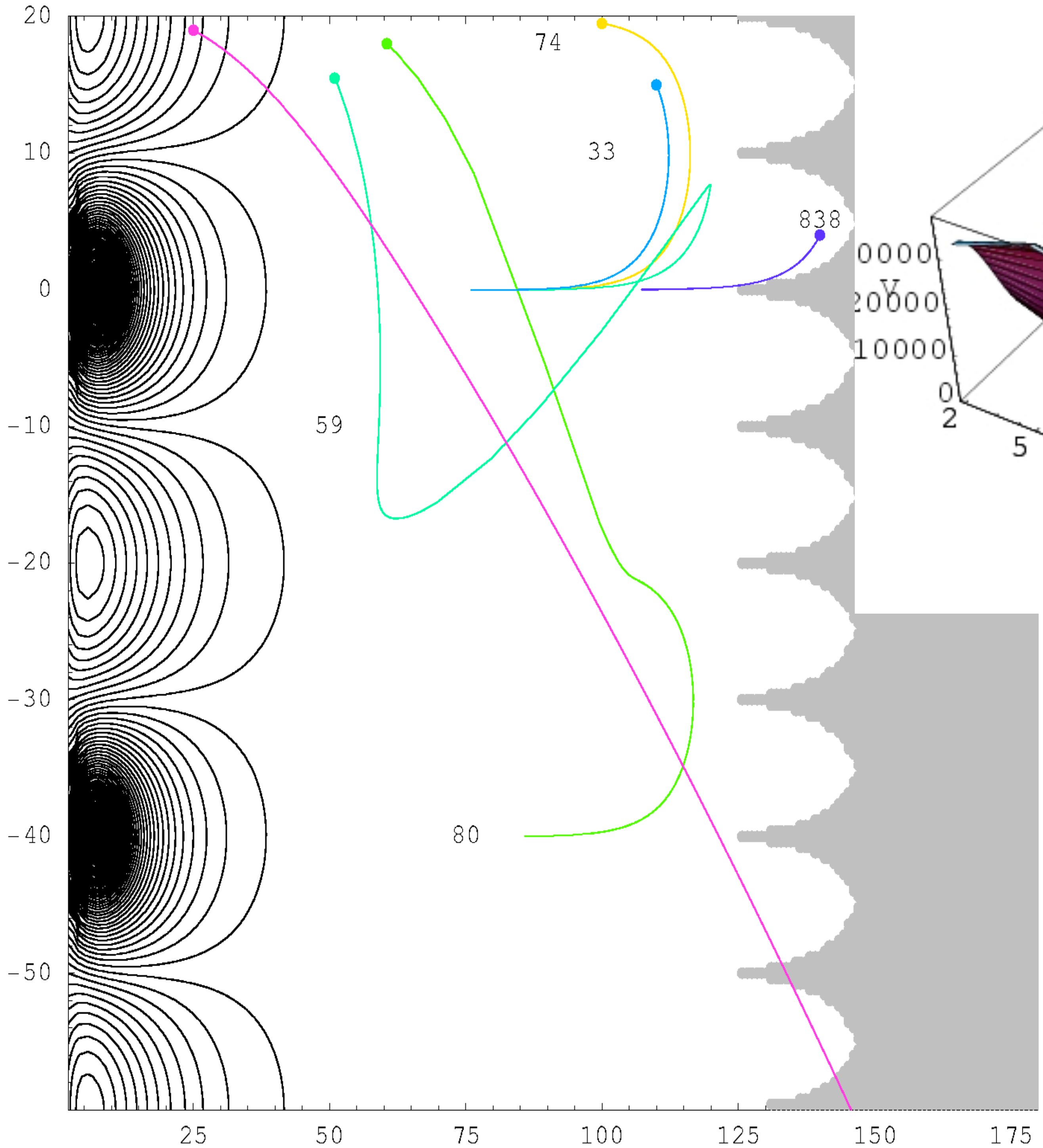
$$\begin{aligned} V(T_1,...,T_n) \quad = \quad & \frac{12W_0^2\xi}{(4\mathcal{V}-\xi)(2\mathcal{V}+\xi)^2} + \sum_{i=2}^{n}\frac{12e^{-2a_i\tau_i}\xi A_i^2}{(4\mathcal{V}-\xi)(2\mathcal{V}+\xi)^2} + \frac{16(a_iA_i)^2\sqrt{\tau_i}e^{-2a_i\tau_i}}{3\alpha\lambda_2(2\mathcal{V}+\xi)} \\ & + \frac{32e^{-2a_i\tau_i}a_iA_i^2\tau_i(1+a_i\tau_i)}{(4\mathcal{V}-\xi)(2\mathcal{V}+\xi)} + \frac{8W_0A_ie^{-a_i\tau_i}\cos(a_i\theta_i)}{(4\mathcal{V}-\xi)(2\mathcal{V}+\xi)}\left(\frac{3\xi}{(2\mathcal{V}+\xi)} + 4a_i\tau_i\right) \\ & + \sum_{\substack{i,j=2\\ i<j}}^{n}\frac{A_iA_j\cos(a_i\theta_i - a_j\theta_j)}{(4\mathcal{V}-\xi)(2\mathcal{V}+\xi)^2}e^{-(a_i\tau_i+a_j\tau_j)}\left[32(2\mathcal{V}+\xi)(a_i\tau_i + a_j\tau_j\right. \\ & \left. +2a_ia_j\tau_i\tau_j) + 24\xi\right] + V_{\text{uplift}} \ . \end{aligned} \tag{18}$$

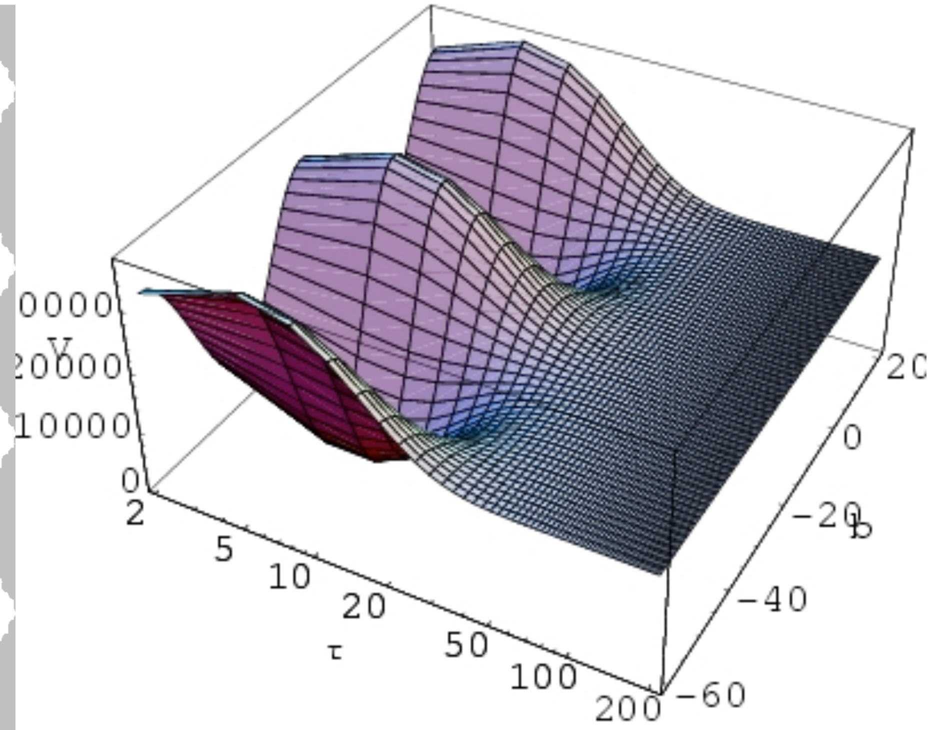
String theory landscape of the Kahler moduli/axion Inflation



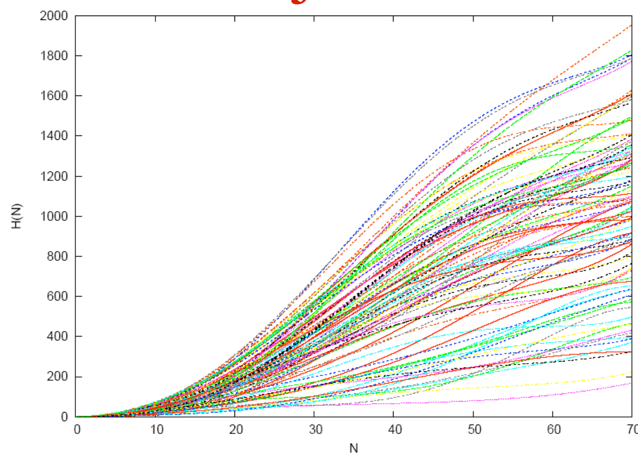
$$T_i = \tau_i + i\theta_i$$







Ensemble of Inflationary trajectories



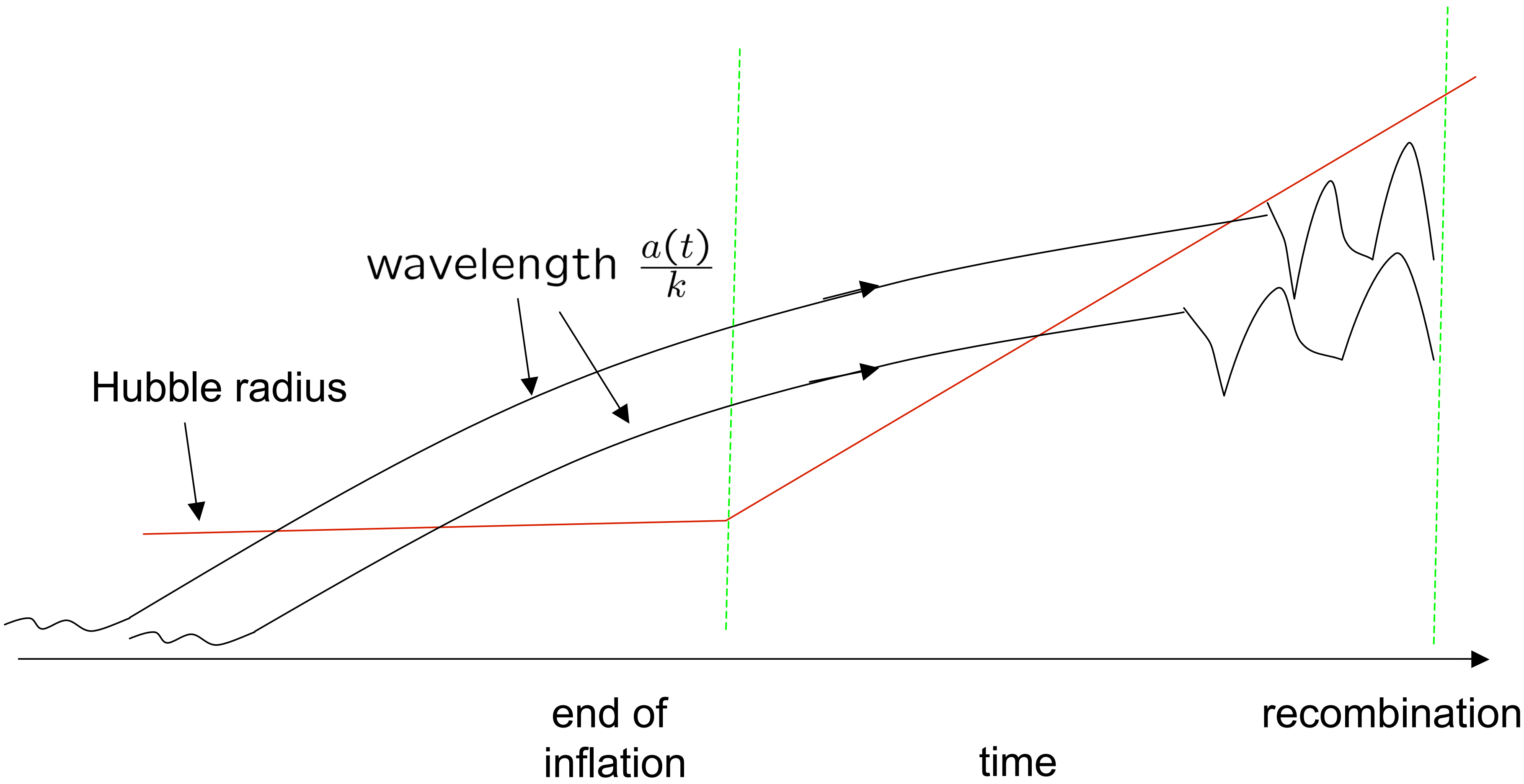
Lessons:

Miltiple fields Inflation

Ensemble of acceleration histories (trajectories) for the same underlying theory

Prior probabilities of trajectories P(H(t))

Small amplitude of gravity waves r from inflation $r \sim 10^{-5}$


wavelength $\frac{a(t)}{k}$
Hubble radius
end of
inflation
time
recombination

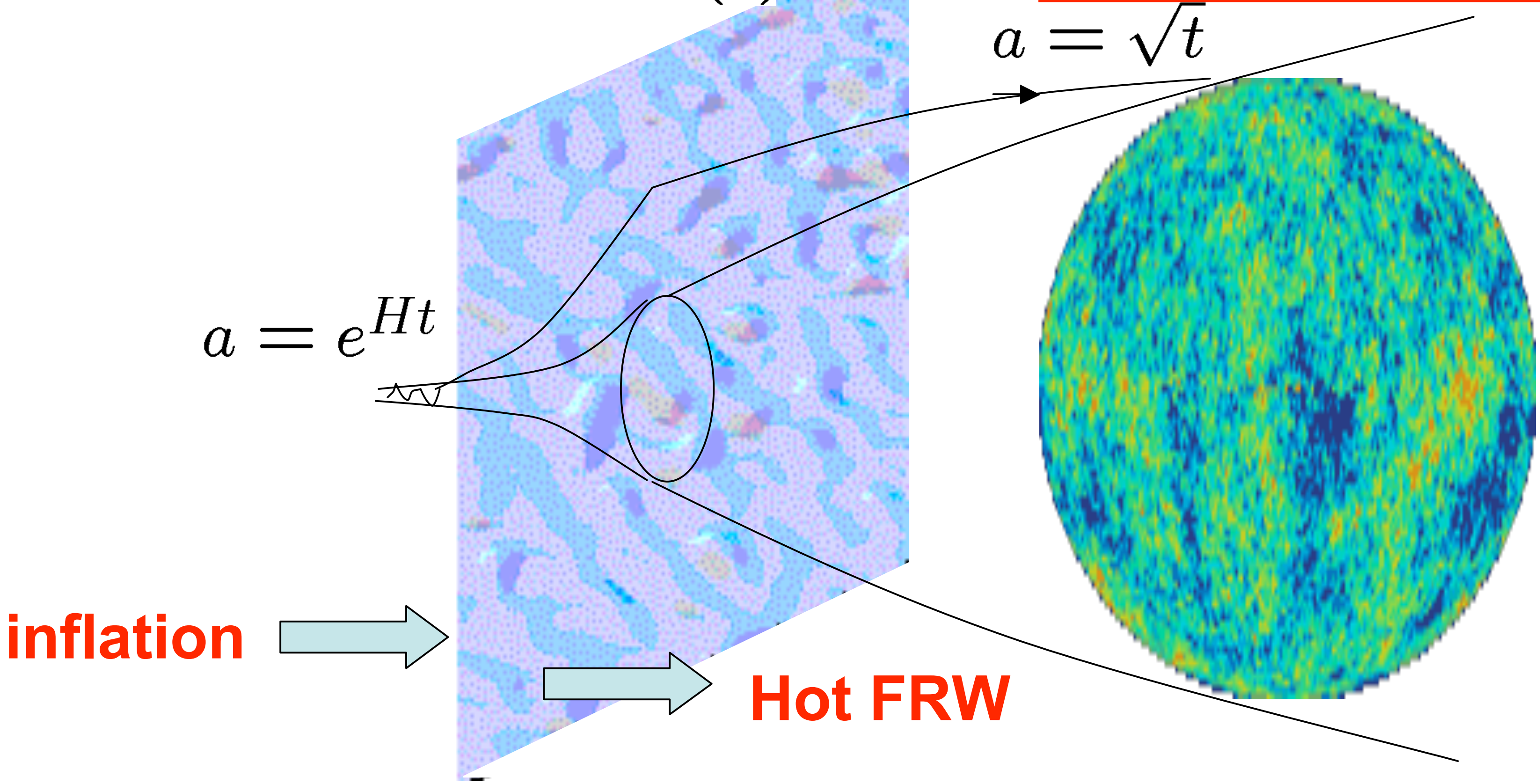
Scalar metric Fluctuations from Inflation

$$ds^2 = -(1-2\Phi)dt^2 + (1+2\Phi)a^2 d\vec{x}^2$$

Initial conditions from Inflation

$$\Omega_{tot} = 1$$
$$k^3 \Phi_k^2 \to P_s = A_s k^{n_s - 1}$$
$$P_T = \frac{H^2}{M_p^2} k^{n_T}$$
$$N = 62 - \ln \frac{10^{16} Gev}{V_{inf}^{1/4}}$$

Random Gaussian Field $\Phi(\vec{x})$



Generation of gravitational waves from inflation

$$ds^2 = -dt^2 + a(t)^2 \left(\delta_{ij} + h_{ij}\right) dx^i dx^j$$

$$h^i_i = 0,\ h^i_{j;i} = 0,\ i, j = 1, 2, 3.$$

$$\Box h_{ij} = \frac{8\pi}{M_p^2} T_{ij}^{TT}$$

free grav waves during inflation

$$\delta R^\mu_\nu = 0$$

$$T_{ij}^{TT} = 0$$

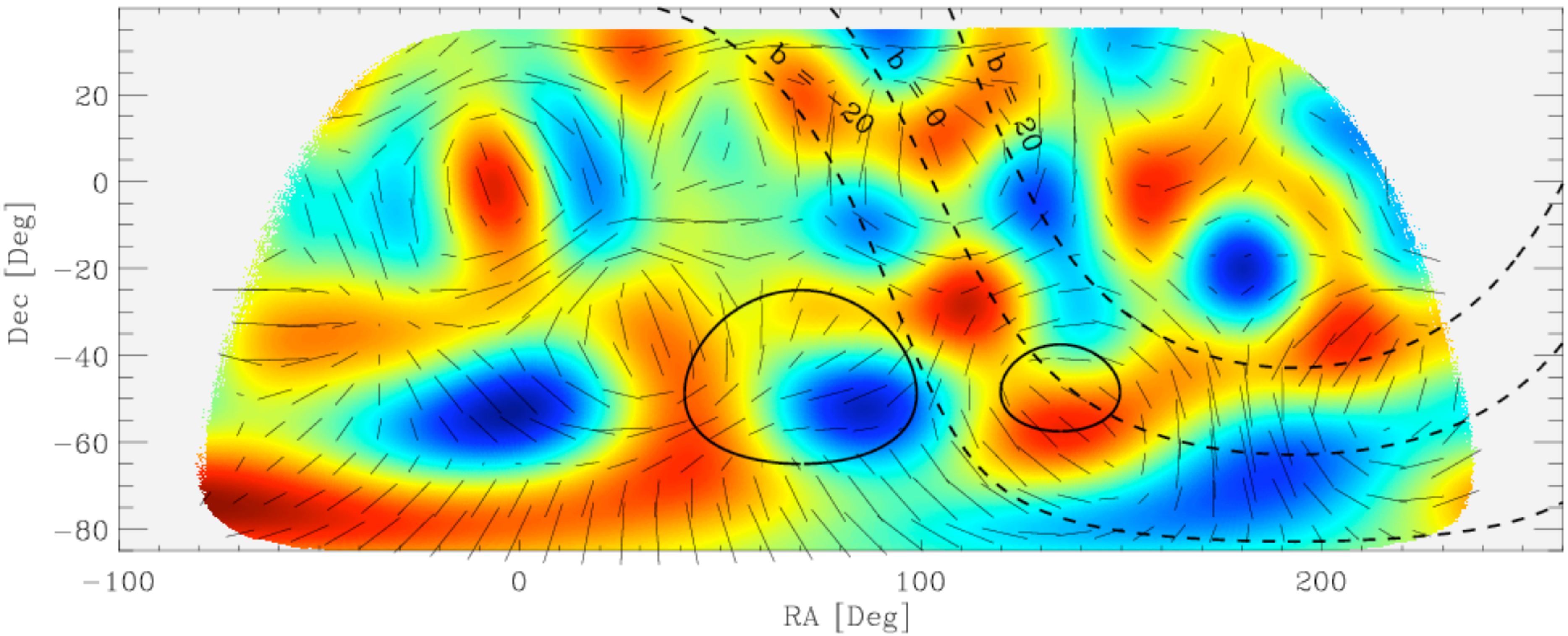
$$h''_k + \left(k^2 - \frac{a''}{a}\right) h_k = 0$$

SPIDER Tensor Signal

- **Simulation of large scale polarization signal**

$$\boxed{\frac{A_T}{A_S} = 0.1}$$

No Tensor
Tensor

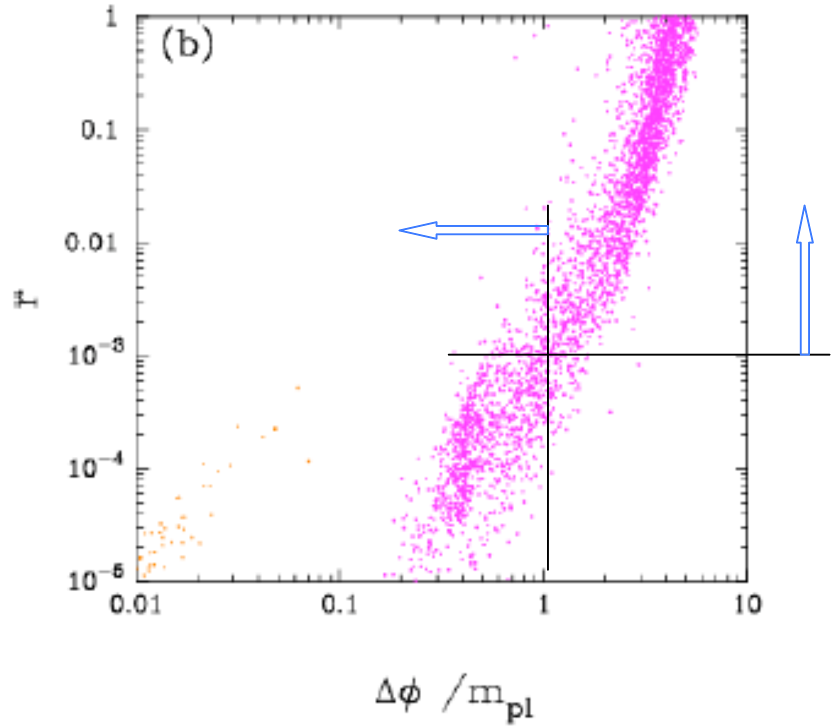


http://www.astro.caltech.edu/~lgg/spider_front.htm

Lyth hep-ph/9606387

$$\Delta\phi \sim \frac{m_{\rm pl}}{8}\left(\frac{r}{\pi}\right)^{1/2}\Delta N$$

Efstathiou &Mack
astro-ph/0503360



Baumann&Mcallister
hep-th/0610285

for the brane inflation $\dfrac{\Delta\phi}{M_p} < \dfrac{2}{\sqrt{n}}$

$$r \leq 0.01(4/n)\left(\frac{60}{N}\right)^2$$

Amplitudeof GW and gravitino mass

$$r \sim 10^8 H^2$$

for inflationary perturbations

KKLT

$$H \leq M_{3/2}$$

Kallosh, Linde
hepth/0704.0647

$$r \leq 10^8 M_{3/2}^2$$

$$r \sim 10^{-2} \longrightarrow M_{3/2} \sim 10^{13} GeV$$

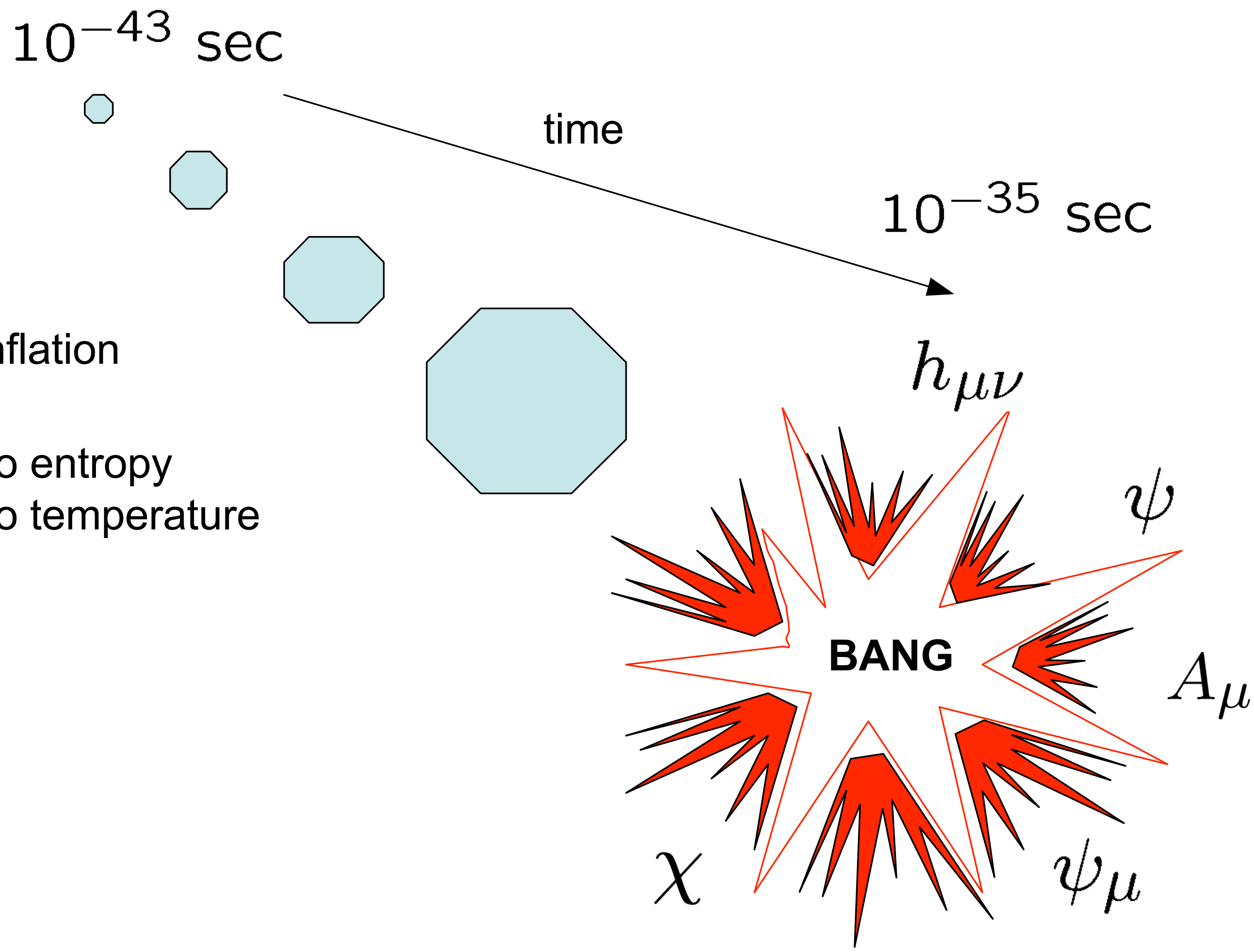
superheavy gravitino

$$M_{3/2} \sim 1 TeV \longrightarrow r \sim 10^{-24}$$

unobservable

A discovery or non-discovery of tensor modes would be a crucial test for string theory and particle phenomenology

Particlegenesis

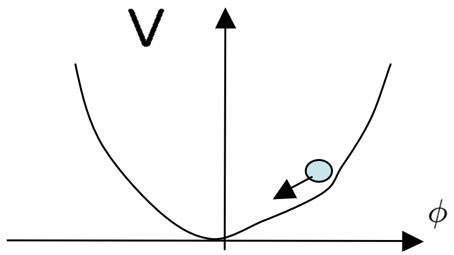


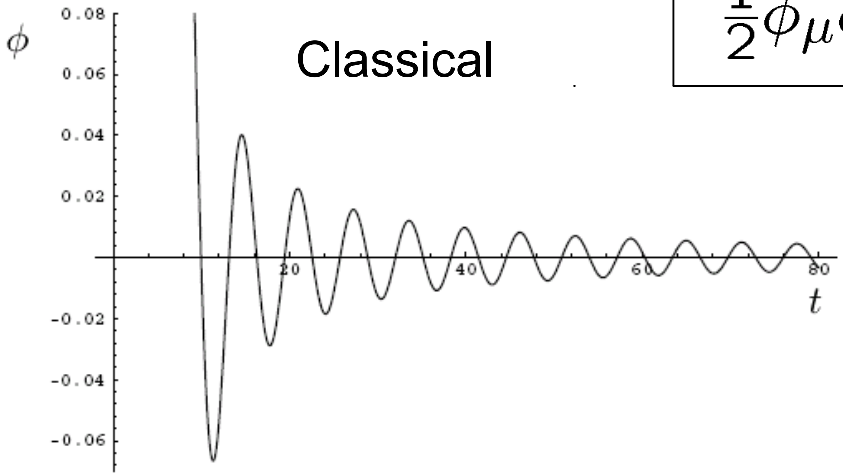
Inflation

no entropy
no temperature

$$\mathcal{L}(\phi, \chi, \psi, A_\mu, \psi_\mu, h_{\mu\nu})$$

Resonant Preheating in Chaotic Inflation





Classical

$$\frac{1}{2}\phi_\mu\phi^\mu + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\chi_\mu\chi^\mu + g^2\phi^2\chi 2$$

Quantum

$$\hat{\chi}(t,\mathbf{x}) = \frac{1}{(2\pi)^{3/2}}\int d^3k\,\left(\hat{a}_k\chi_k(t)\,e^{-i\mathbf{kx}} + \hat{a}_k^+\chi\right.$$

$$\ddot{\chi}_k + 3H\dot{\chi}_k + \left(\frac{k^2}{a^2} + g^2\phi^2\right)\chi_k = 0$$

$$\tau = mt$$

$$\boxed{\ddot{X}_k + \left(\frac{k^2}{m^2} + q\sin^2\tau\right)X_k = 0}$$

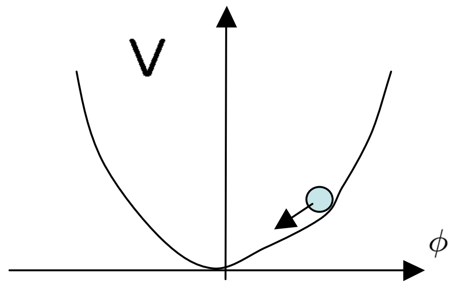
$$\text{parameter } q = \frac{g^2\phi_0^2}{m^2} \sim g^2 10^{10}$$

Occupation number

$$n_k = \frac{\omega_k}{2}\left(\frac{|\dot{\chi}_k|^2}{\omega_k^2} + |\chi_k|^2\right) - \frac{1}{2}$$

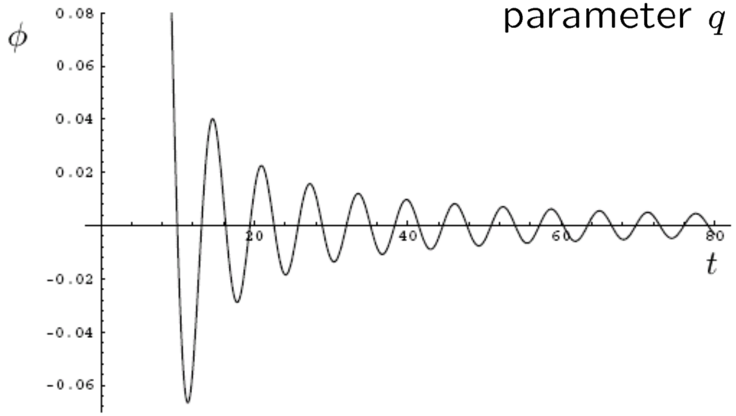
$$n_k \sim e^{\mu t}$$

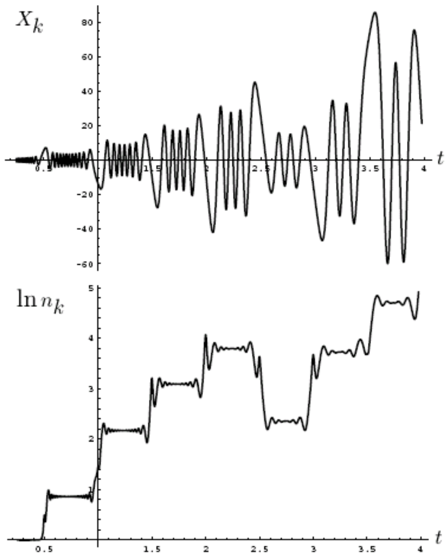
Resonant Preheating in Chaotic Inflation



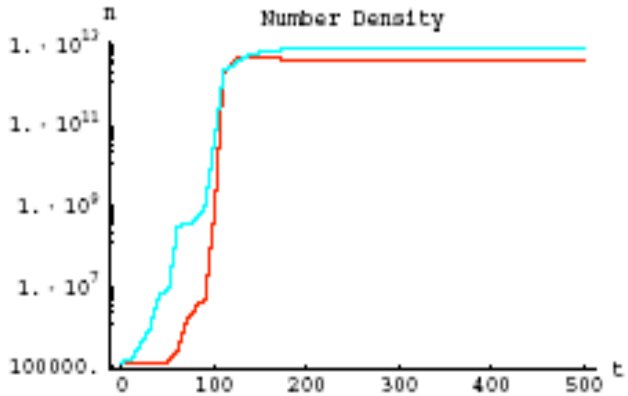
$$g^2\phi^2\chi^2$$

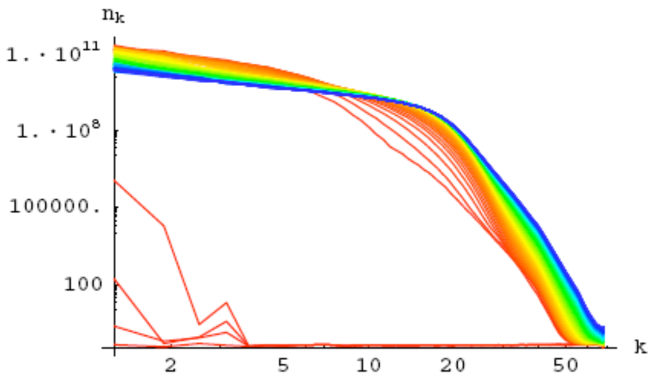
parameter $q = \frac{g^2\phi_0^2}{m^2} \sim g^2 10^{10} \gg 1$

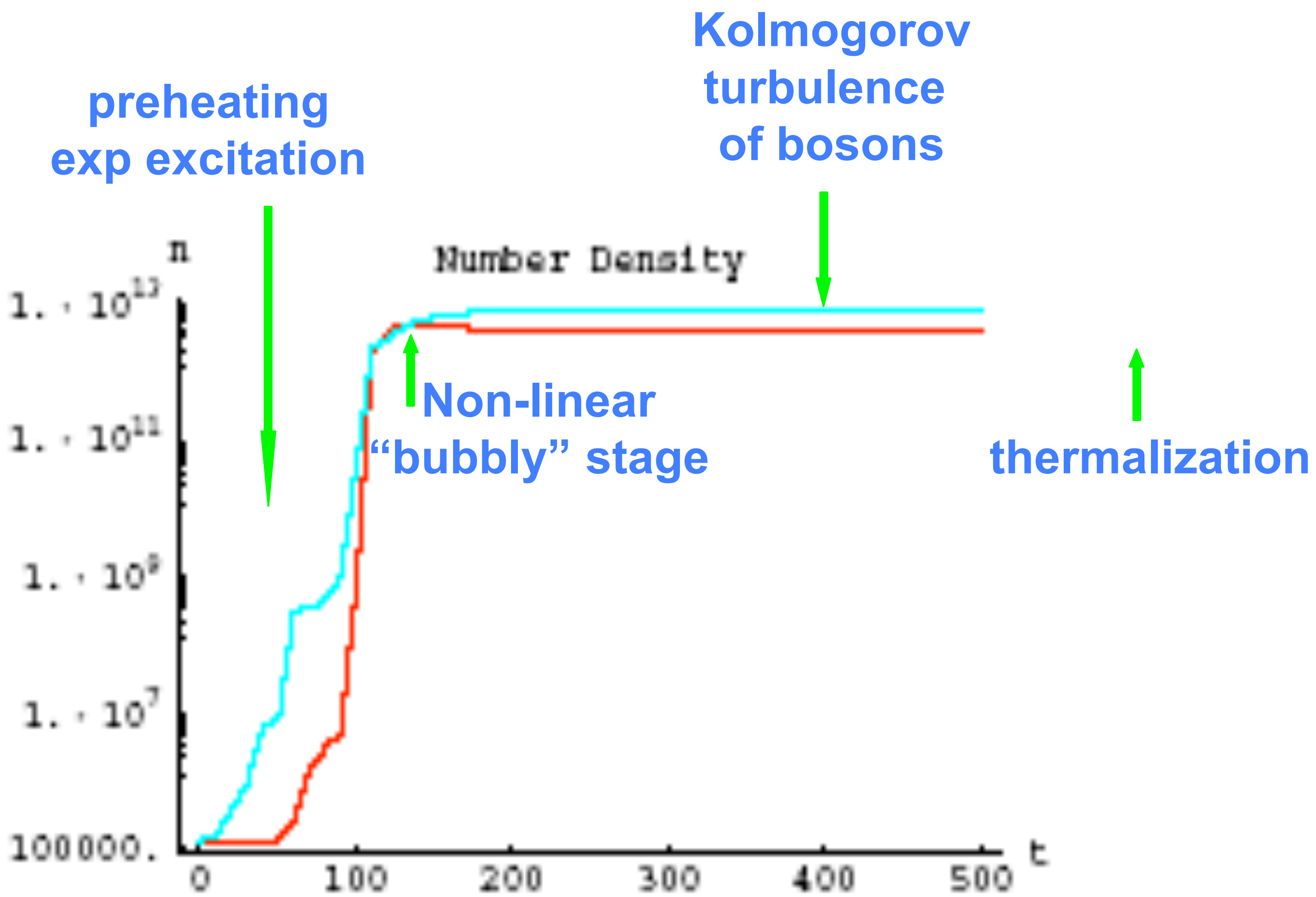




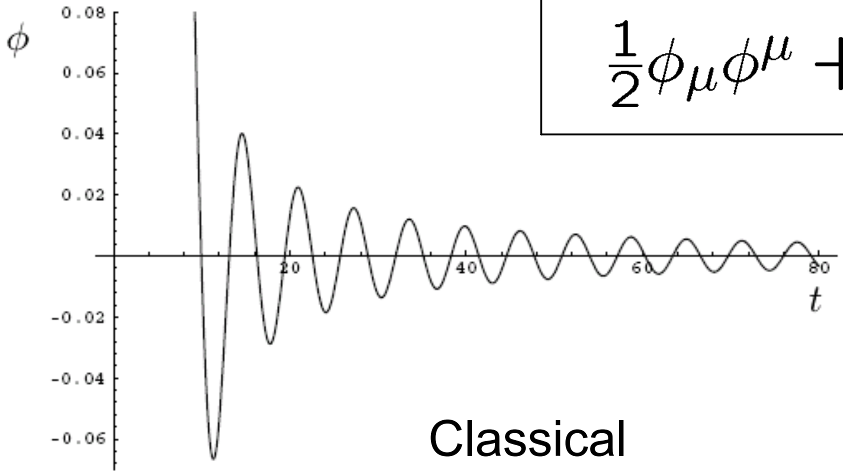
$$\delta\chi$$







preheating
exp excitation
Kolmogorov
turbulence
of bosons
n
Number Density
1. · 10^13
1. · 10^11
1. · 10^9
1. · 10^7
100000.
0
100
200
300
400
500
t
Non-linear
"bubbly" stage
thermalization



$$\frac{1}{2}\phi_\mu\phi^\mu + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\chi_\mu\chi^\mu + g^2\phi^2\chi 2$$

Decay of inflaton and preheating after inflation

Classical

Quantum

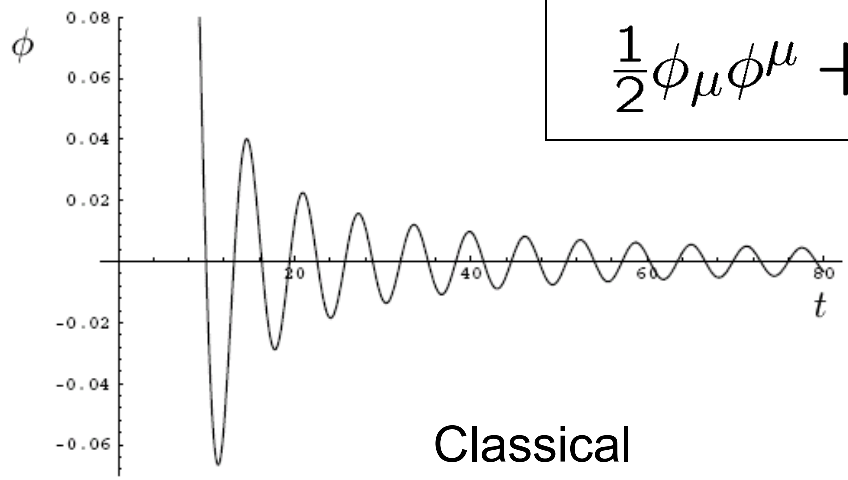
$$\phi_0 + \phi$$

$$\chi$$

Felder, LK
hep-ph/0606256

$$\ddot{\phi} - \nabla^2\phi + g^2\chi^2\phi = 0$$
$$\ddot{\chi} - \nabla^2\chi + g^2\phi^2\chi = 0$$

movie



$$\frac{1}{2}\phi_\mu\phi^\mu + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\chi_\mu\chi^\mu + g^2\phi^2\chi 2$$

Decay of inflaton
and preheating after inflation

Classical

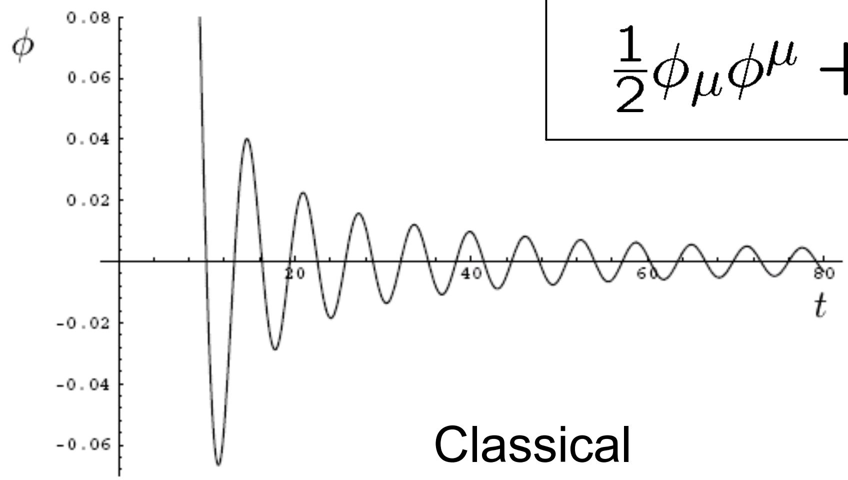
$$\phi_0 + \phi$$

Quantum

$$\chi$$

movie

$$\frac{1}{2}\phi_\mu\phi^\mu + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\chi_\mu\chi^\mu + g^2\phi^2\chi 2$$



Decay of inflaton and preheating after inflation

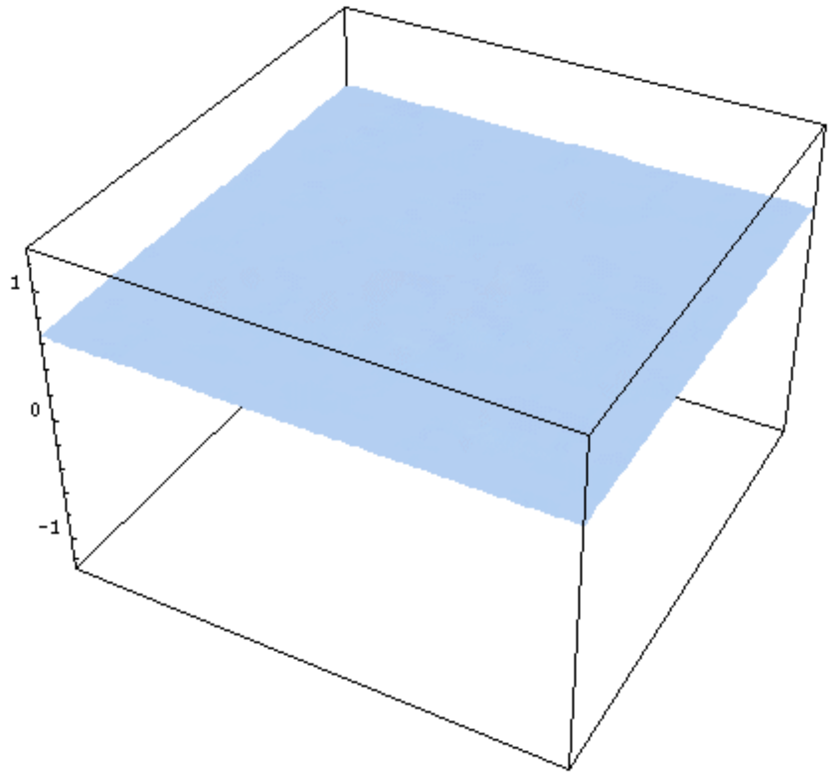
Classical

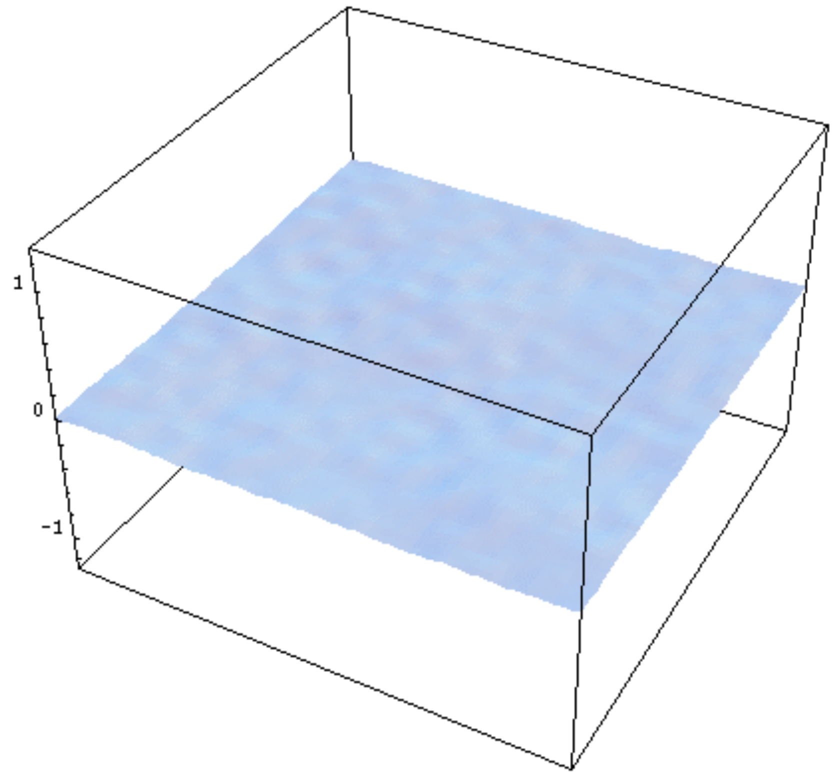
Quantum

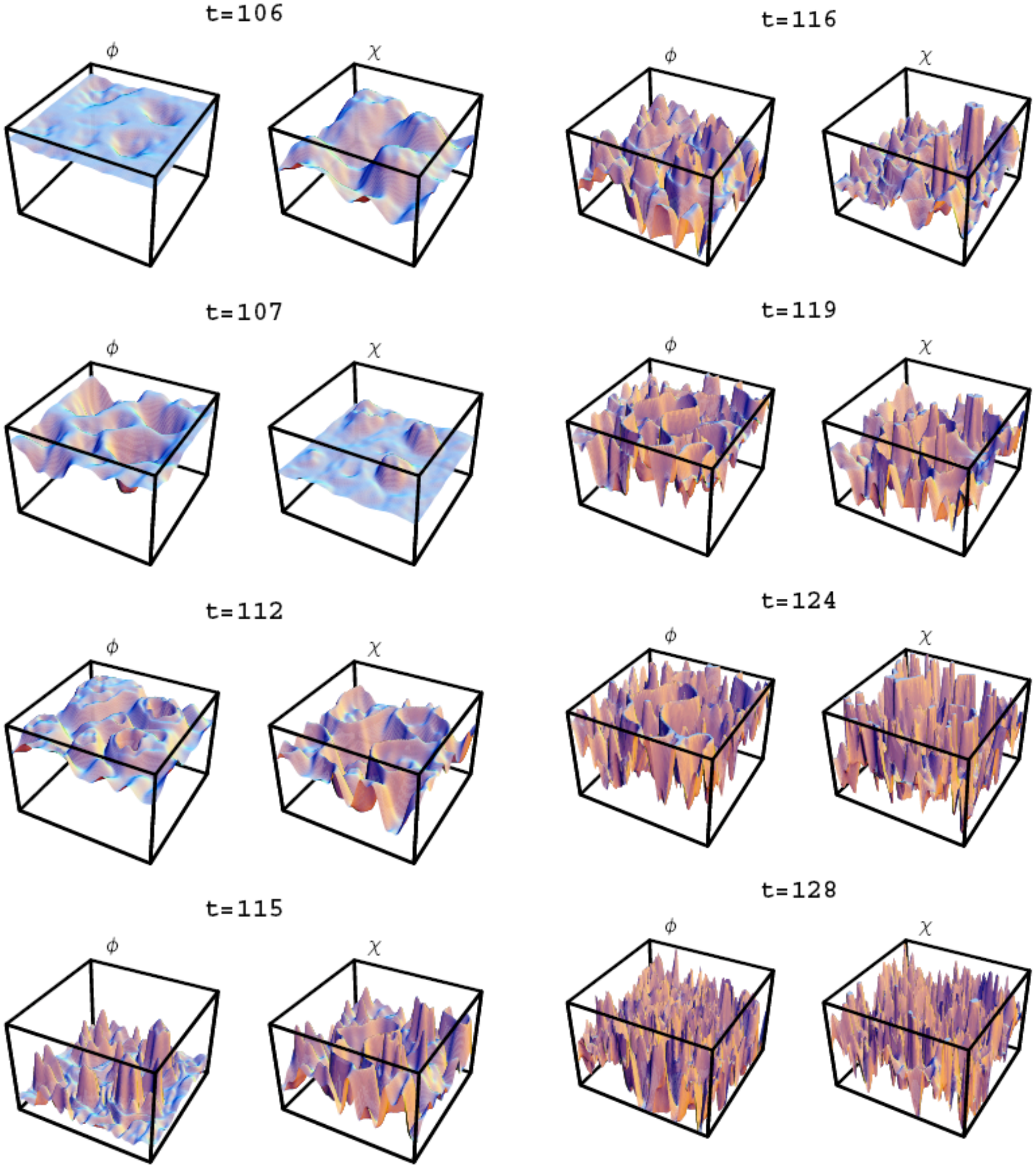
$\phi_0 + \phi$

χ

Slices for t=95.2019






t=106
ϕ
χ
t=116
ϕ
χ
t=107
ϕ
χ
t=119
ϕ
χ
t=112
ϕ
χ
t=124
ϕ
χ
t=115
ϕ
χ
t=128
ϕ
χ

A. Frolov 07

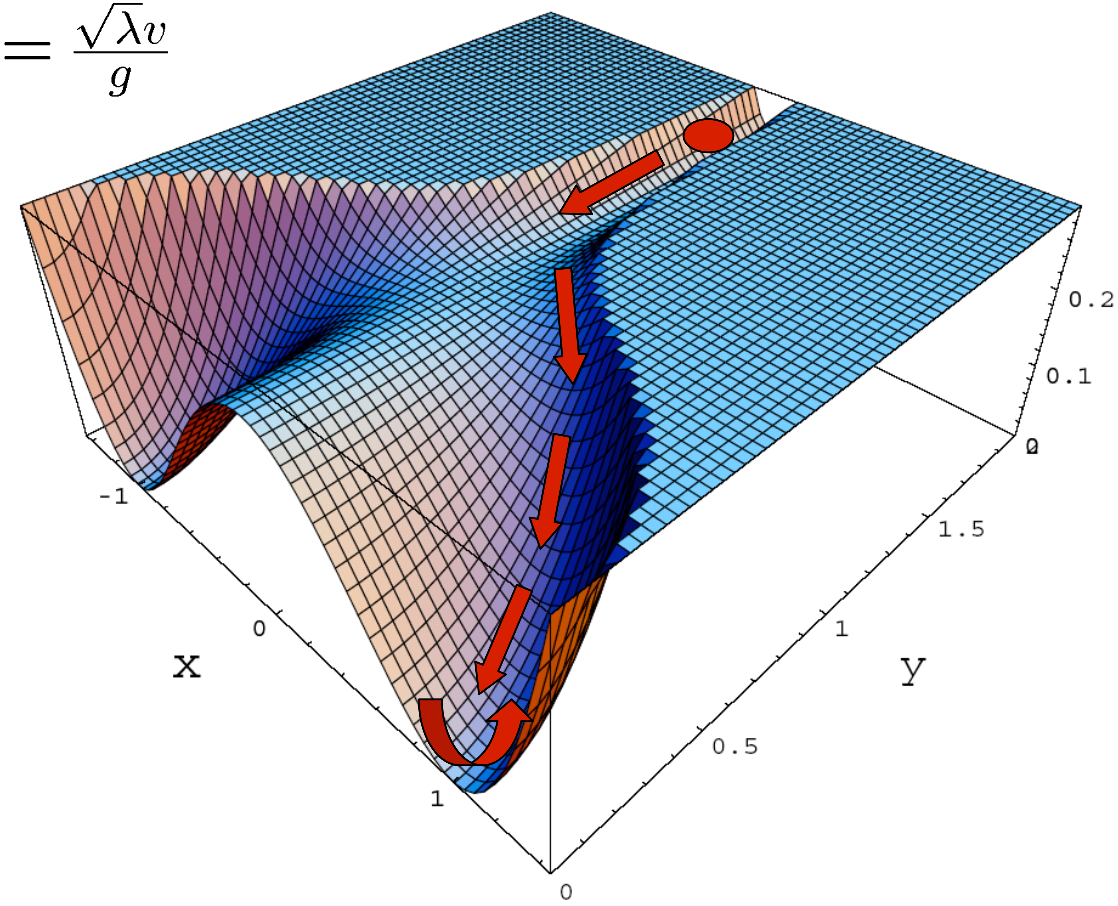
Evolutionof energy density

Evolution of gravitational potential

Tachyonic Preheating in Hybrid Inflation

$$V(\phi,\sigma) = \frac{\lambda}{4}(\sigma^2 - v^2)^2 + \frac{g^2}{2}\phi^2\sigma^2$$

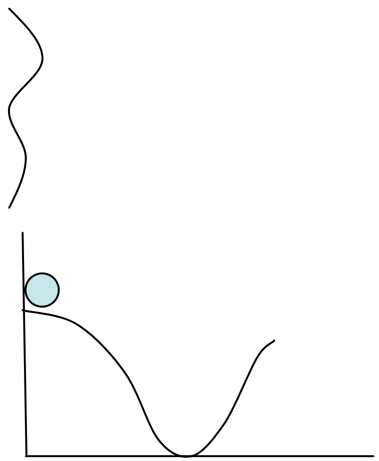
bifurcation point $\phi_c = \frac{\sqrt{\lambda}v}{g}$



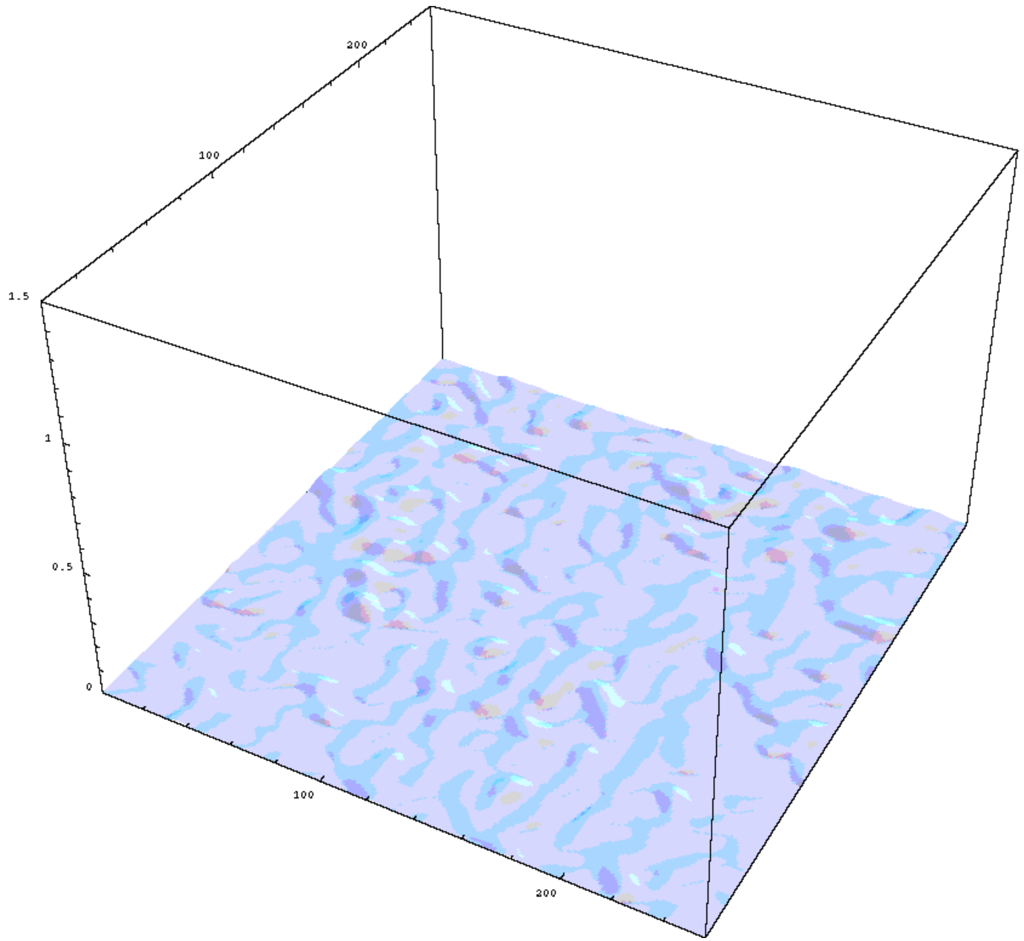
movie

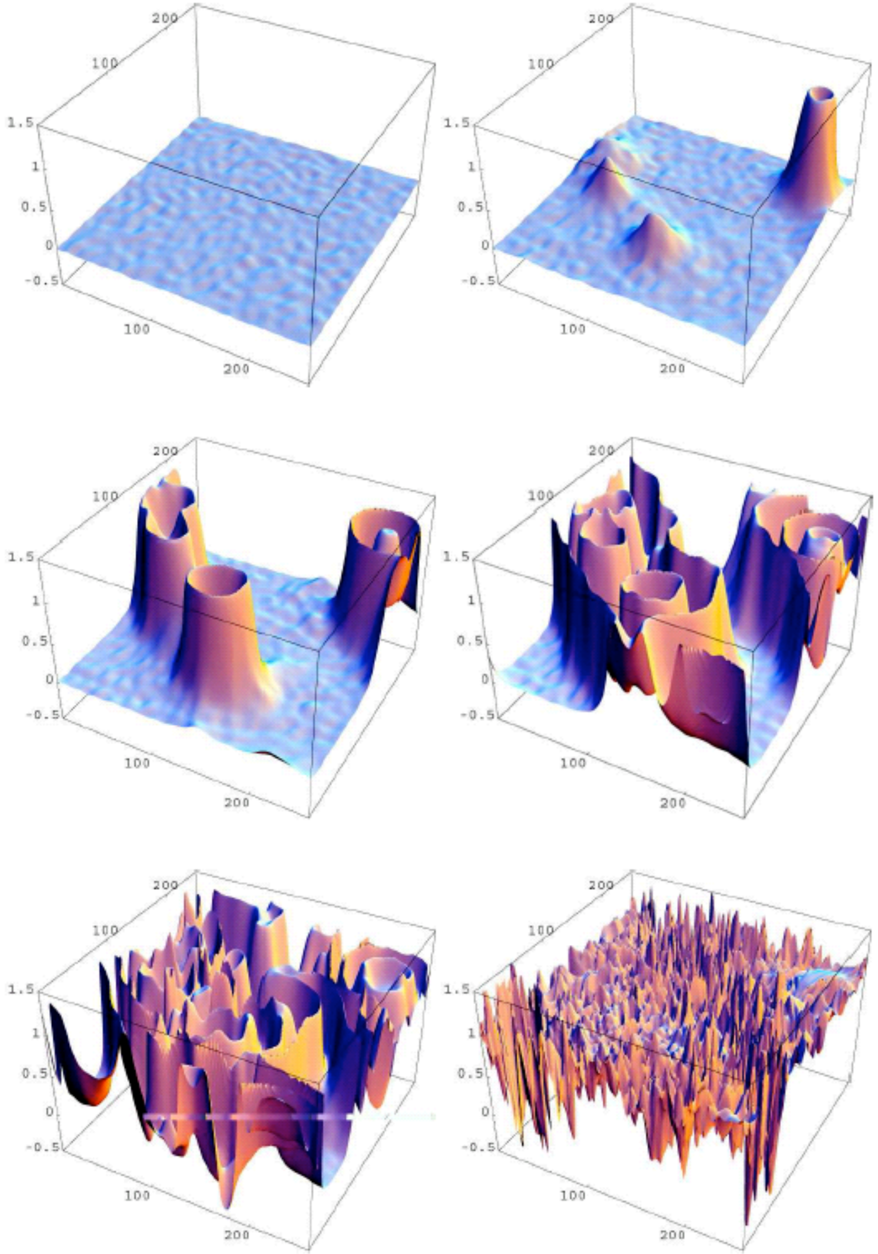
Tachyonic Preheating

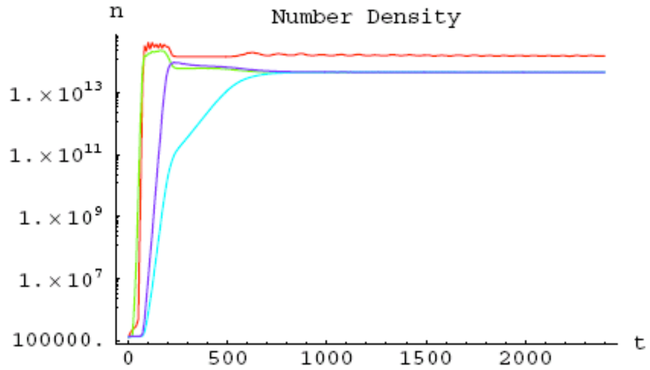
$$V_F = V_0 + \frac{\lambda}{4}\sigma^4 - \frac{\lambda^3}{4}\sigma^3 + \lambda\sigma^2$$



$\sigma(t, \vec{x})$







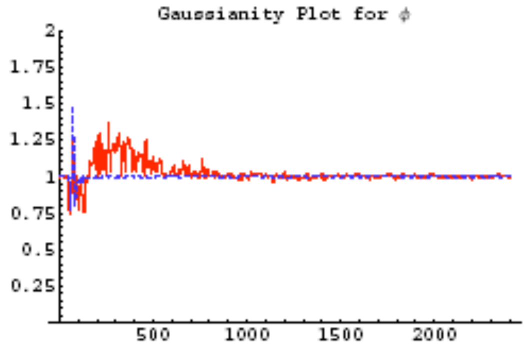


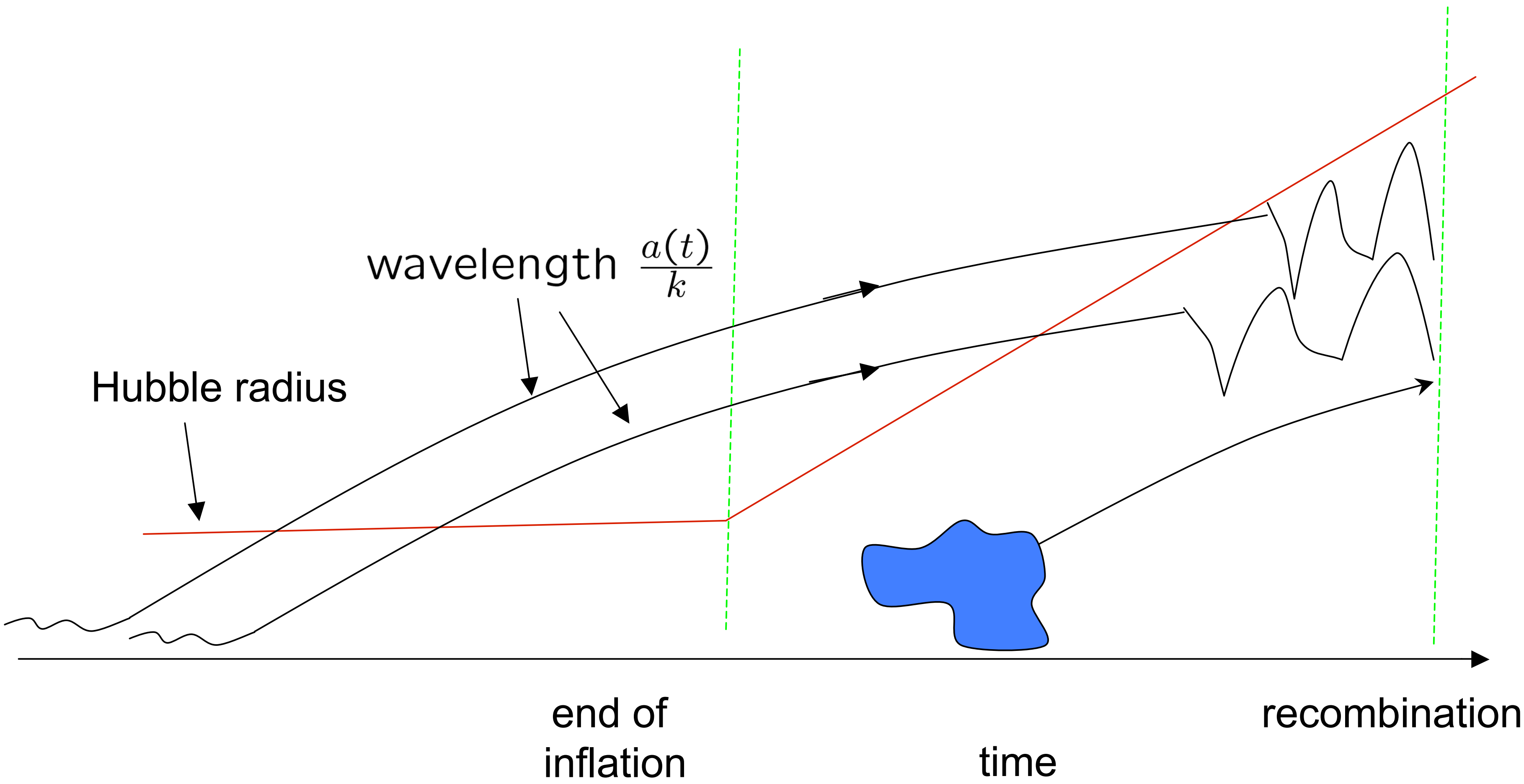
FIG. 10. Deviations from Gaussianity for the field ϕ as a function of time. The solid, red line shows $3\langle\delta\phi^2\rangle^2/\langle\delta\phi^4\rangle$ and the dashed, blue line shows $3\langle\delta\dot{\phi}^2\rangle^2/\langle\delta\dot{\phi}^4\rangle$.

Generation of gravitational waves from random media

$$ds^2 = -dt^2 + a(t)^2 \left(\delta_{ij} + h_{ij}\right) dx^i dx^j$$

$$h^i_i = 0,\ h^i_{j;i} = 0,\ i, j = 1, 2, 3.$$

$$\Box h_{ij} = \frac{8\pi}{M_p^2} T_{ij}^{TT}$$



$$\Box h_{ij} = \frac{8\pi}{M_p^2} T_{ij}^{TT}$$

Stochastic background of gravitational waves emitted from preheating after inflation

Khlebnikov,Tkachev, PRD56(1997)653

Easther and Lim, astro-ph/0601617

Felder and LK, hep-ph/0606256

Easther, Giblin and Lim, astro-ph/0612294

Garcia-Bellido and Figueroa, astro-ph/0701014

Dufaux, LK et al astro-ph/0707:0875

Garcia-Bellido, Figueroa, astro-ph/0707:0839

Emission of stochastic GW by random media

Theory and Numerics of Gravitational Waves from Preheating after Inflation.

Jean-François Dufaux[1], Amanda Bergman[2], Gary Felder[2], Lev Kofman[1] and Jean-Philippe Uzan[3]

astro-ph:0707.0875

$$h''_{ij} + 2\,\frac{a'}{a}\,h'_{ij} - \nabla^2 h_{ij} = 16\pi G\, a^2\, \Pi^{\rm TT}_{ij}$$

$$\bar{h}_{ij} = a\, h_{ij}$$

$$\bar{h}''_{ij}(\mathbf{k}) + \left(k^2 - \frac{a''}{a}\right)\bar{h}_{ij}(\mathbf{k}) = 16\pi G\, a^3\, \Pi^{\rm TT}_{ij}(\mathbf{k})$$

$$\Pi^{\rm TT}_{ij}(\mathbf{k}) = \mathcal{O}_{ij,lm}(\hat{\mathbf{k}})\,\Pi_{lm}(\mathbf{k}) = \left[P_{il}(\hat{\mathbf{k}})\,P_{jm}(\hat{\mathbf{k}}) - \frac{1}{2}\,P_{ij}(\hat{\mathbf{k}})\,P_{lm}(\hat{\mathbf{k}})\right]\Pi_{lm}(\mathbf{k})$$

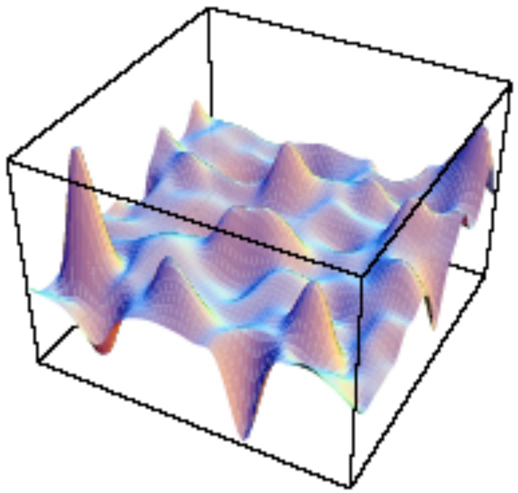
$$P_{ij}(\hat{\mathbf{k}}) = \delta_{ij} - \hat{k}_i\,\hat{k}_j$$

Emission of stochastic GW by random scalar fields

$$\bar{h}''_{ij}(\tau,\mathbf{k}) + k^2\,\bar{h}_{ij}(\tau,\mathbf{k}) = 16\pi G\,a(\tau)\,T^{\rm TT}_{ij}(\tau,\mathbf{k})$$

$$\bar{h}_{ij}(\tau,\mathbf{k}) = \frac{16\pi G}{k}\int_{\tau_i}^{\tau} d\tau'\;\sin\left[k\,(\tau-\tau')\right]\;a(\tau')\,T^{\rm TT}_{ij}(\tau',\mathbf{k})$$

$$T^{\rm TT}_{ij}(\mathbf{k}) = \mathcal{O}_{ij,lm}(\hat{\mathbf{k}})\;\{\partial_l\phi_a\,\partial_m\phi_a\}\,(\mathbf{k}) = \mathcal{O}_{ij,lm}(\hat{\mathbf{k}})\int\frac{d^3\mathbf{p}}{(2\pi)^{3/2}}\,p_l\,p_m\,\phi_a(\mathbf{p})\,\phi_a(\mathbf{k}-\mathbf{p})$$



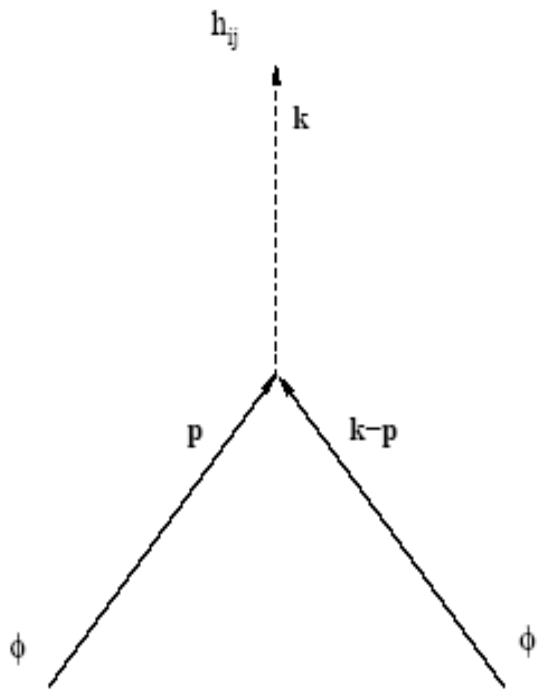
First order phase transitions
Second order phase transitions
Topological defects formation
Thermal bath of scalars
Tachyonic preheating
Resonant preheating

No-go Theorem: No Gravity Waves from Scalar Field Waves

$$\bar{h}_{ij}(\tau,\mathbf{k}) = \frac{16\pi G}{k}\,\mathcal{O}_{ij,lm}(\hat{\mathbf{k}}) \int_{\tau_i}^{\tau} d\tau' \,\sin\left[k\,(\tau-\tau')\right]\, a(\tau') \int \frac{d^3\mathbf{p}}{(2\pi)^{3/2}}\, p_l\, p_m\, \phi_a(\tau',\mathbf{p})\, \phi_a(\tau',\mathbf{k}-\mathbf{p})$$

$$\phi(\mathbf{p},\tau) e^{i\mathbf{p}\mathbf{x}} = b(\mathbf{p})\, e^{\pm i\omega_p\tau + i\mathbf{p}\mathbf{x}} \qquad \omega_p^2 = p^2 + m^2 + g^2\psi^2$$

$$h_{ij}(\tau,\mathbf{k}) \propto e^{\pm ik\tau}\mathcal{O}_{ij,lm}(\hat{\mathbf{k}}) \int d^3\mathbf{p}\, p_l\, p_m\, b(\mathbf{p}) b(\mathbf{k}-\mathbf{p}) \int_{\tau_i}^{\tau} d\tau'\, e^{i\,(\pm\,\omega_p \,\pm\, \omega_{|\mathbf{k}-\mathbf{p}|}\,\pm\, k)\,\tau'}$$



k parallel to **p**

$$\mathcal{O}_{ij,lm}(\hat{\mathbf{k}})\, p_l\, p_m = 0$$

FIG. 1: Would be emission of a graviton h_{ij} with momentum $\mathbf{k}$ from the annihilation of two scalar waves $\phi(\mathbf{p})$ and $\phi(\mathbf{k}-\mathbf{p})$ with momenta $\mathbf{p}$ and $\mathbf{p}-\mathbf{k}$. Helicity 2 of the emitted graviton cannot match the helicity zero of the incoming scalar waves.

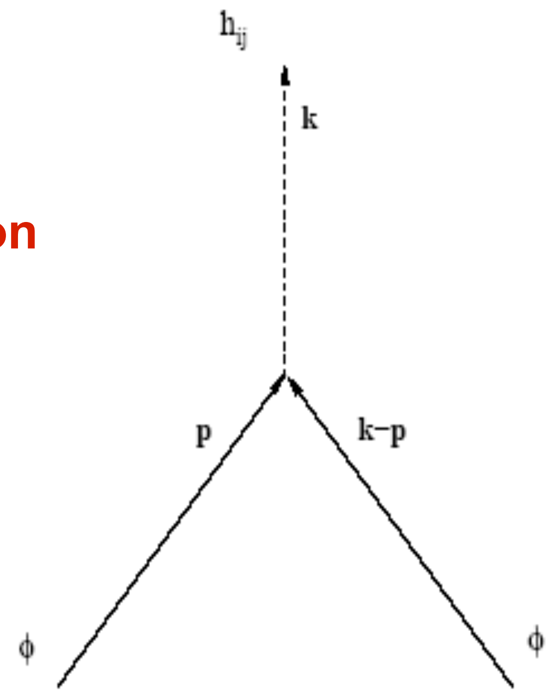
No-go Theorem: No Gravity Waves from Scalar Field Waves

$$\bar{h}_{ij}(\tau, \mathbf{k}) = \frac{16\pi G}{k} \mathcal{O}_{ij,lm}(\hat{\mathbf{k}}) \int_{\tau_i}^{\tau} d\tau' \sin[k(\tau - \tau')] \, a(\tau') \int \frac{d^3\mathbf{p}}{(2\pi)^{3/2}} p_l \, p_m \, \phi_a(\tau', \mathbf{p}) \, \phi_a(\tau', \mathbf{k} - \mathbf{p})$$

$$\phi(\mathbf{p}, \tau) e^{i\mathbf{p}\mathbf{x}} = b(\mathbf{p}) \, e^{\pm i\omega_p \tau + i\mathbf{p}\mathbf{x}} \qquad \omega_p^2 = p^2 + m^2 + g^2\psi^2$$

$$h_{ij}(\tau, \mathbf{k}) \propto e^{\pm ik\tau} \mathcal{O}_{ij,lm}(\hat{\mathbf{k}}) \int d^3\mathbf{p} \, p_l \, p_m \, b(\mathbf{p}) b(\mathbf{k} - \mathbf{p}) \int_{\tau_i}^{\tau} d\tau' \, e^{i(\pm\omega_p \pm \omega_{|\mathbf{k}-\mathbf{p}|} \pm k)\tau'}$$

No GW emission



k parallel to p

$$\mathcal{O}_{ij,lm}(\hat{\mathbf{k}}) \, p_l \, p_m = 0$$

FIG. 1: Would be emission of a graviton h_{ij} with momentum $\mathbf{k}$ from the annihilation of two scalar waves $\phi(\mathbf{p})$ and $\phi(\mathbf{k} - \mathbf{p})$ with momenta $\mathbf{p}$ and $\mathbf{p} - \mathbf{k}$. Helicity 2 of the emitted graviton cannot match the helicity zero of the incoming scalar waves.

No-go Theorem: No Gravity Waves from Scalar Field Waves

$$\bar{h}_{ij}(\tau, \mathbf{k}) = \frac{16\pi G}{k} \mathcal{O}_{ij,lm}(\hat{\mathbf{k}}) \int_{\tau_i}^{\tau} d\tau' \sin\left[k\left(\tau - \tau'\right)\right] a(\tau') \int \frac{d^3\mathbf{p}}{(2\pi)^{3/2}} p_l\, p_m\, \phi_a(\tau', \mathbf{p})\, \phi_a(\tau', \mathbf{k} - \mathbf{p})$$

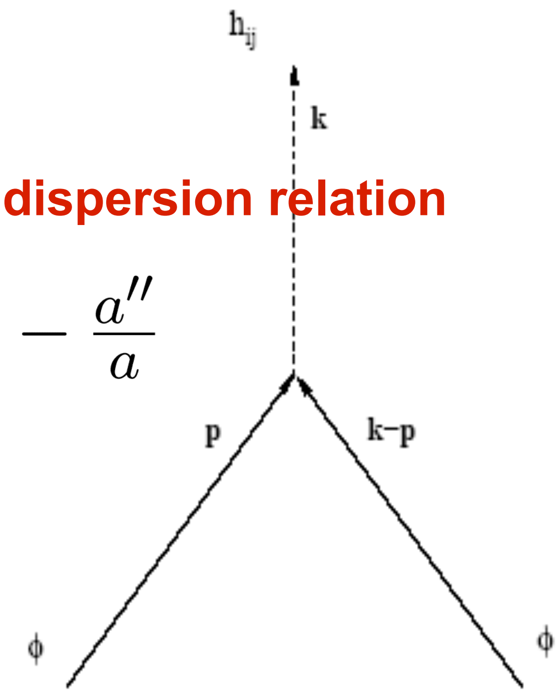
$$\phi(\mathbf{p}, \tau) e^{i\mathbf{p}\mathbf{x}} = b(\mathbf{p})\, e^{\pm i\omega_p \tau + i\mathbf{p}\mathbf{x}}$$

$$\omega_p^2 = p^2 + m^2 + g^2\psi^2$$

$$h_{ij}(\tau, \mathbf{k}) \propto e^{\pm ik\tau} \mathcal{O}_{ij,lm}(\hat{\mathbf{k}}) \int d^3\mathbf{p}\, p_l\, p_m\, b(\mathbf{p}) b(\mathbf{k} - \mathbf{p}) \int_{\tau_i}^{\tau} d\tau'\, e^{i\,(\pm\,\omega_p \pm\, \omega_{|\mathbf{k}-\mathbf{p}|} \pm\, k)\,\tau'}$$

GW generated for non-trivial dispersion relation

$$\omega^2 = \vec{p}^2 + g^2\phi(t)^2 - \frac{a''}{a}$$



k parallel to **p**

$$\mathcal{O}_{ij,lm}(\hat{\mathbf{k}})\, p_l\, p_m = 0$$

FIG. 1: Would be emission of a graviton h_{ij} with momentum $\mathbf{k}$ from the annihilation of two scalar waves $\phi(\mathbf{p})$ and $\phi(\mathbf{k} - \mathbf{p})$ with momenta $\mathbf{p}$ and $\mathbf{p} - \mathbf{k}$. Helicity 2 of the emitted graviton cannot match the helicity zero of the incoming scalar waves.

$$\rho_{\rm gw} = \frac{1}{32\pi G} \langle \dot{h}_{ij}(t,\mathbf{x})\, \dot{h}_{ij}(t,\mathbf{x}) \rangle$$

$$\frac{(16\pi G)^2}{2} \sum_{i,j} \left\{ \left| \int_{\tau_i}^{\tau_f} d\tau' \, \cos\left[k\,(\tau_f - \tau')\right] \, a(\tau')\, T_{ij}^{\rm TT}(\tau',\mathbf{k}) \right|^2 + \left| \int_{\tau_i}^{\tau_f} d\tau' \, \sin\left[k\,(\tau_f - \tau')\right] \, a(\tau')\, T_{ij}^{\rm TT}(\tau',\mathbf{k}) \right|^2 \right\}$$

Random gaussian fields

$$\langle T_{ij}^{\rm TT}(\tau',\mathbf{k})\, T_{ij}^{\rm TT\,*}(\tau'',\mathbf{k}')\rangle =$$

$$\mathcal{O}_{ij,lm}(\hat{\mathbf{k}})\, \mathcal{O}_{ij,rs}(\hat{\mathbf{k}}') \int \frac{d^3\mathbf{p}}{(2\pi)^{3/2}} \int \frac{d^3\mathbf{p}'}{(2\pi)^{3/2}}\, p_l\, p_m\, p'_r\, p'_s\, \langle \phi_a(\mathbf{p},\tau')\, \phi_a(\mathbf{k}-\mathbf{p},\tau')\, \phi_b^*(\mathbf{p}',\tau'')\, \phi_b^*(\mathbf{k}'-\mathbf{p}',\tau'')\rangle$$

$$\left(\frac{d\rho_{\rm gw}}{d\ln k}\right)_{\tau>\tau_f} = \frac{S_k(\tau_f)}{a^4(\tau)}$$

$$S_k(\tau_f) = \frac{2}{\pi}\, G\, k^3 \int \frac{d^3\mathbf{p}}{(2\pi)^3}\, p^4\, \sin^4(\hat{\mathbf{k}},\hat{\mathbf{p}}) \int_{\tau_i}^{\tau_f} d\tau' \int_{\tau_i}^{\tau_f} d\tau''\, \cos\left[k(\tau'-\tau'')\right]\, a(\tau')\, a(\tau'')\, F_{ab}(p,\tau',\tau'')\, F_{ab}(|\mathbf{k}-\mathbf{p}|,\tau',\tau'')$$

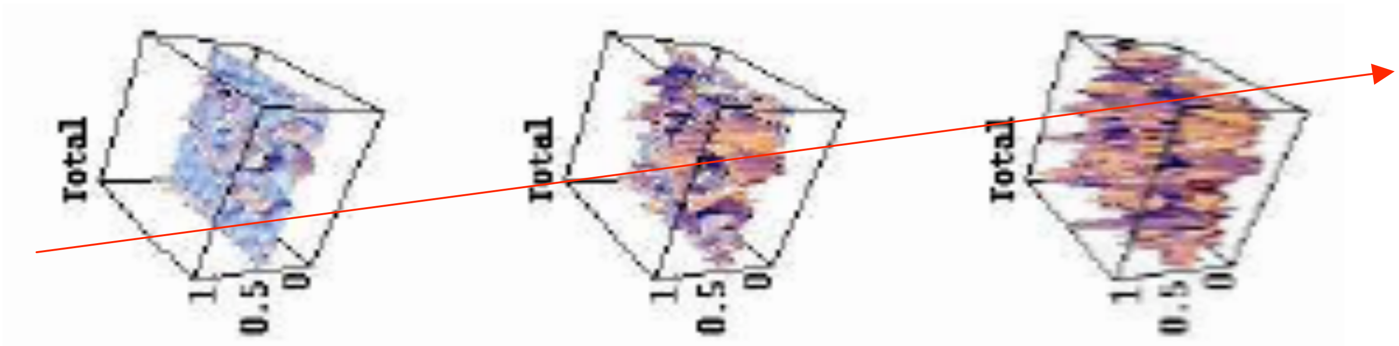
$$\langle \phi_a(\mathbf{p},\tau')\, \phi_b^*(\mathbf{p}',\tau'')\rangle = F_{ab}(p,\tau',\tau'')\, \delta(\mathbf{p}-\mathbf{p}')$$

Emission of GW from preheating

$$\Omega_{\rm gw}\, h^2 = 7.8 \times 10^{-5}\; S_k(\tau_f)\; \frac{a_j^{-4}}{M_{\rm Pl}^2 H_j^2}$$

$$\frac{2}{\pi} G\, k^3 \int \frac{d\mathbf{p}}{(2\pi)^3}\, p^4 \sin^4(\hat{\mathbf{k}}, \hat{\mathbf{p}})$$

$$\left\{ \left| \int_{\tau_i}^{\tau_f} d\tau\; \cos(k\tau)\; a(\tau)\, \chi_p(\tau)\, \chi_{|\mathbf{k}-\mathbf{p}|}(\tau) \right|^2 + \left| \int_{\tau_i}^{\tau_f} d\tau\; \sin(k\tau)\; a(\tau)\, \chi_p(\tau)\, \chi_{|\mathbf{k}-\mathbf{p}|}(\tau) \right|^2 \right\}$$



Numerical calculations of GW emission from Preheating

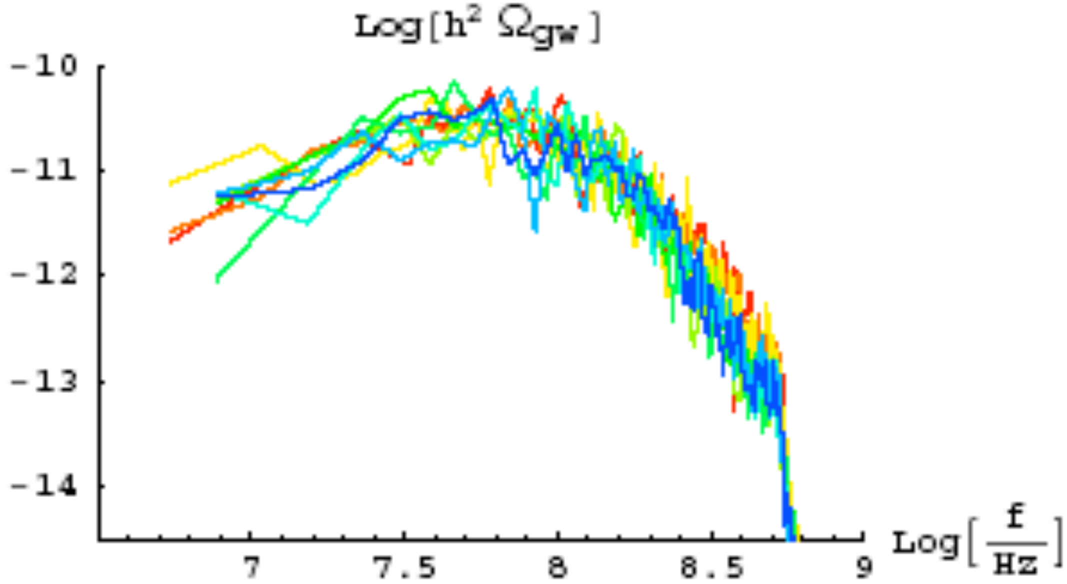


FIG. 3: Spectrum of energy density in gravity waves calculated along nine different directions in k-space. The

$$V = \frac{\lambda}{4}\,\phi^4 + \frac{g^2}{2}\,\phi^2\chi^2 \qquad\qquad q = \frac{g^2}{\lambda}$$

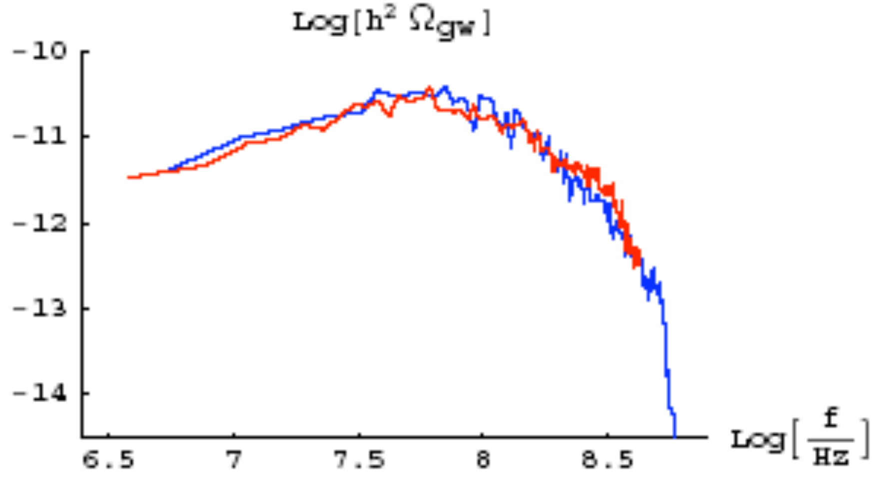


FIG. 1: Spectrum of gravity waves energy density in physical variables today, accumulated up to the time $x_f = 240$, for the model (48) with $q = 120$. The 2 spectra were obtained from simulations with different box sizes, and averaged over different directions in k-space.

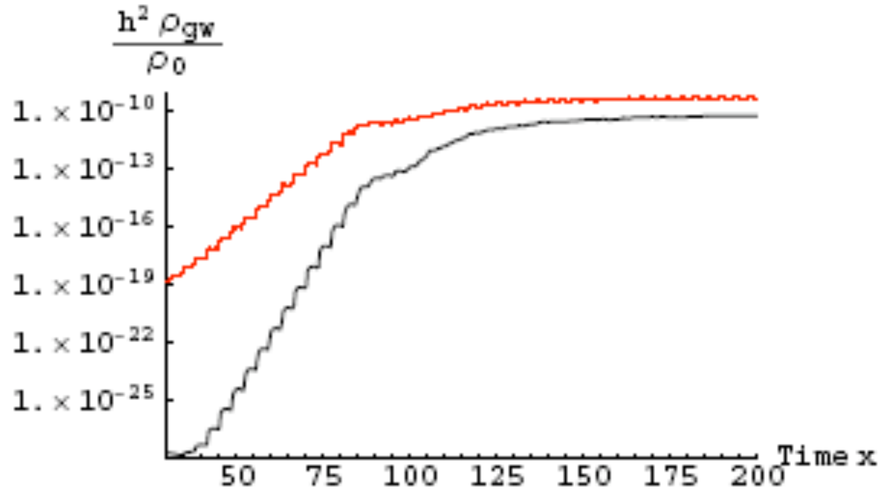


FIG. 2: The thick curve shows the total energy density in gravity waves (53) accumulated up to the time x_f, as a function of x_f. The thin curve shows the evolution with time of the total number density, $n_{\rm tot} = n_\chi + n_\phi$, rescaled to fit on the same figure.

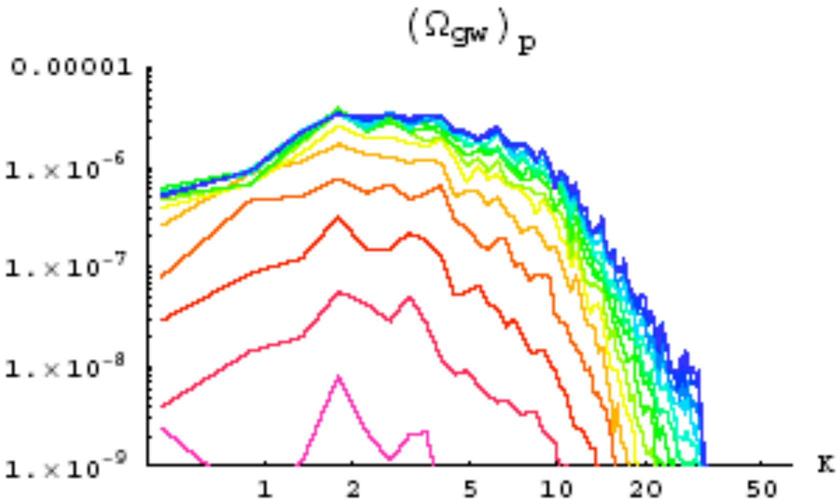


FIG. 3: Spectrum (55) of the gravity waves energy density, accumulated up to different times x_f, as a function of the comoving momentum k (in units of $\lambda\phi_0$). The spectra grow from $x_f = 90$ to $x_f = 240$ with spacing $\Delta x_f = 10$.

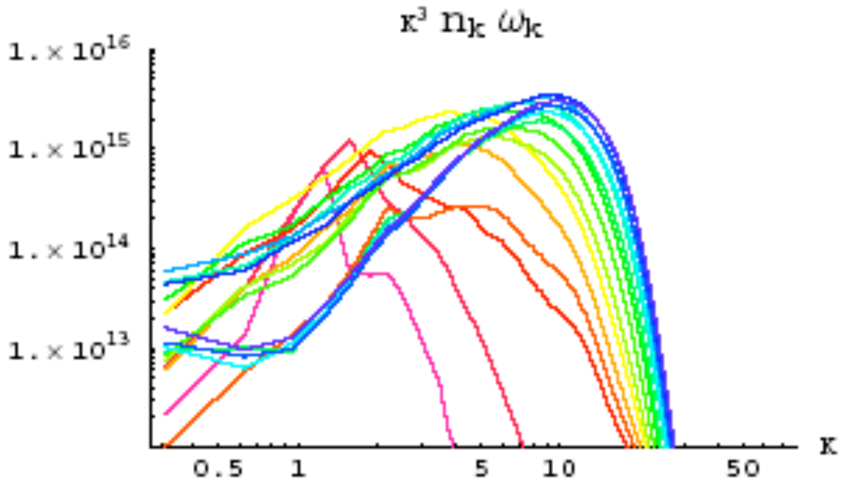
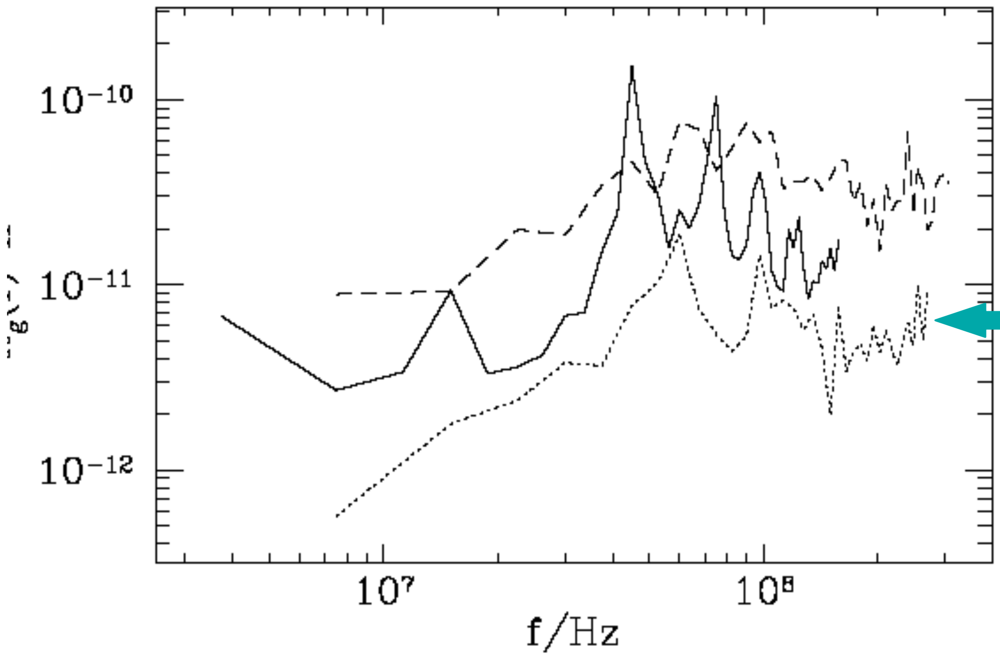


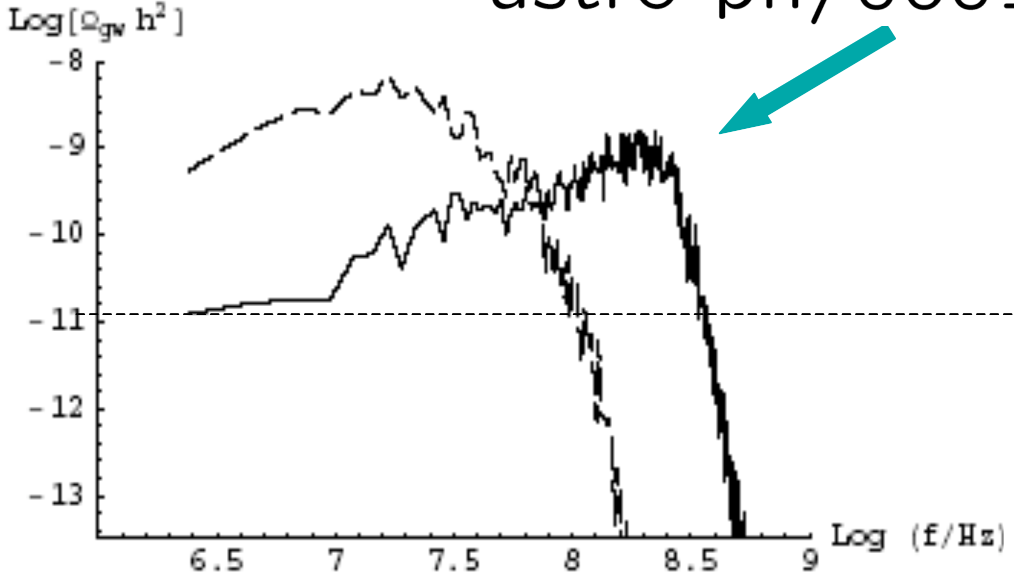
FIG. 4: Measure of the (unnormalised) total energy density in the two scalar fields per logarithmic momentum interval at different moments of time. The same times as in Fig. (3) are shown, the spectra moving towards UV from $x = 90$ to $x = 240$ with spacing $\Delta x = 10$.

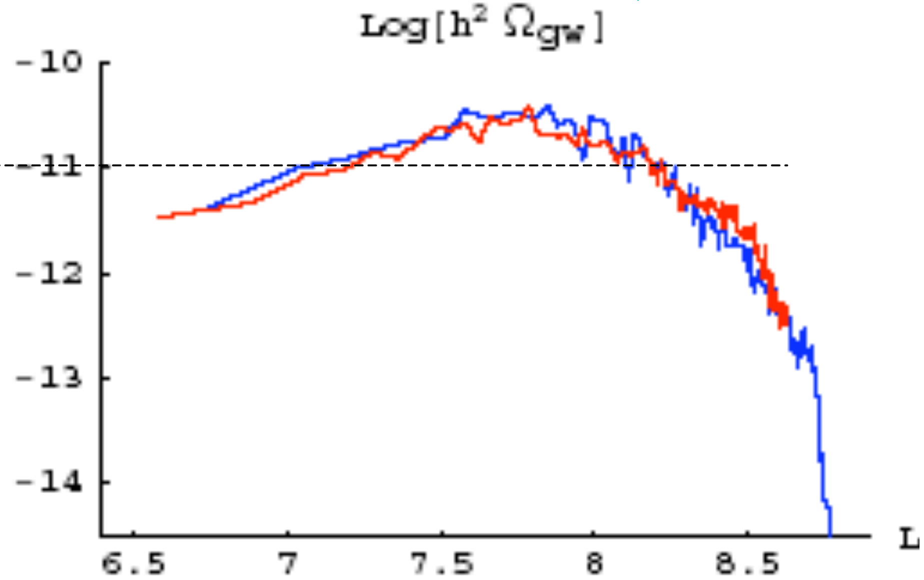


hep-ph/9701423

astro-ph/0707:0875

astro-ph/0601617

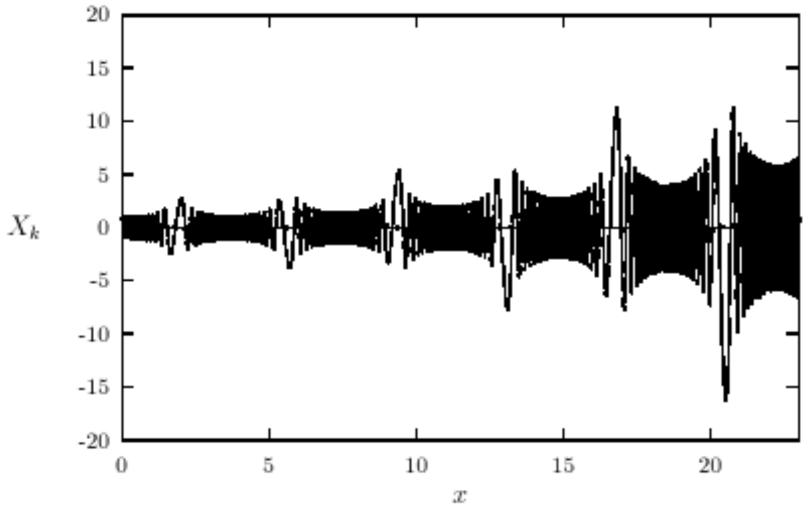


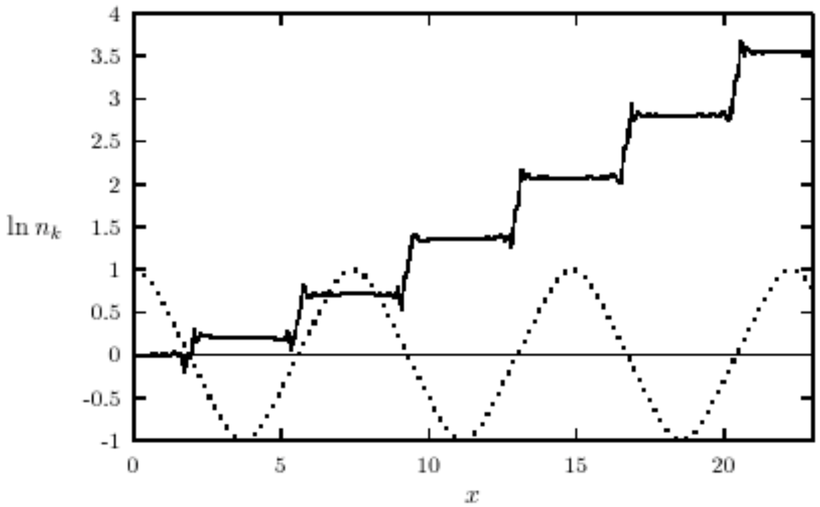


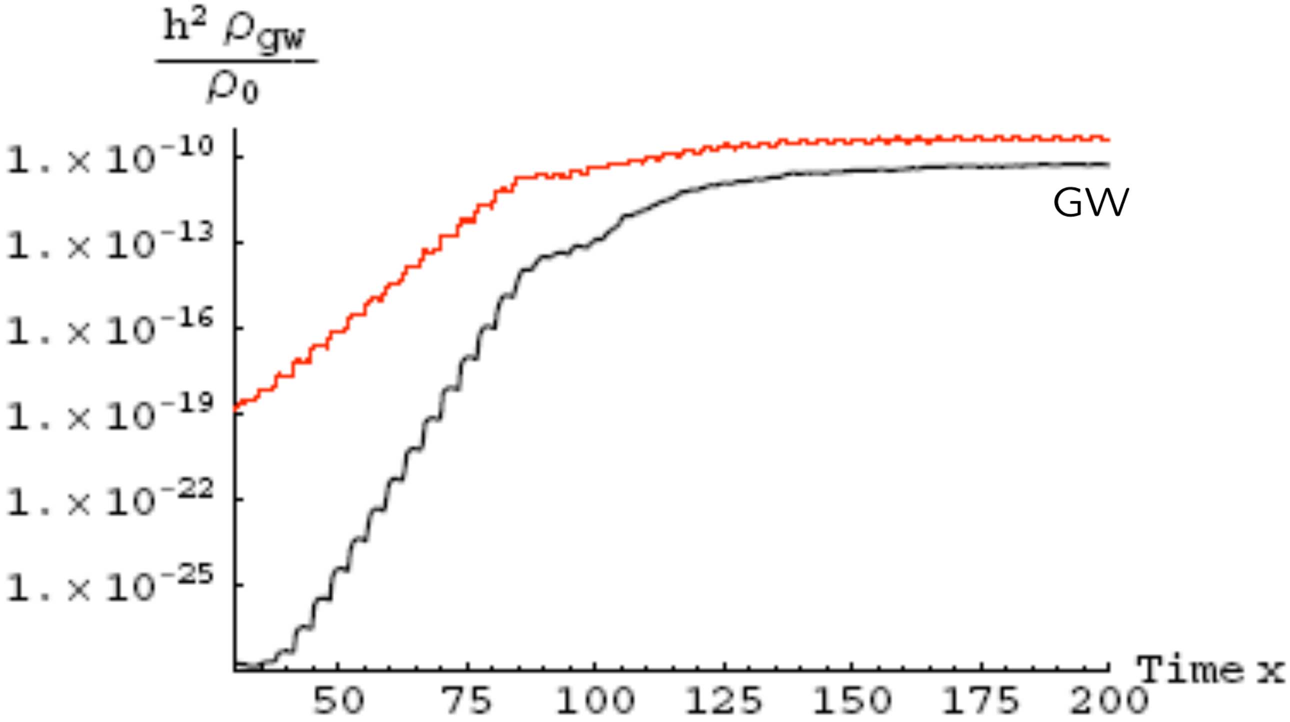
Numerical calculations of GW emission from Preheating

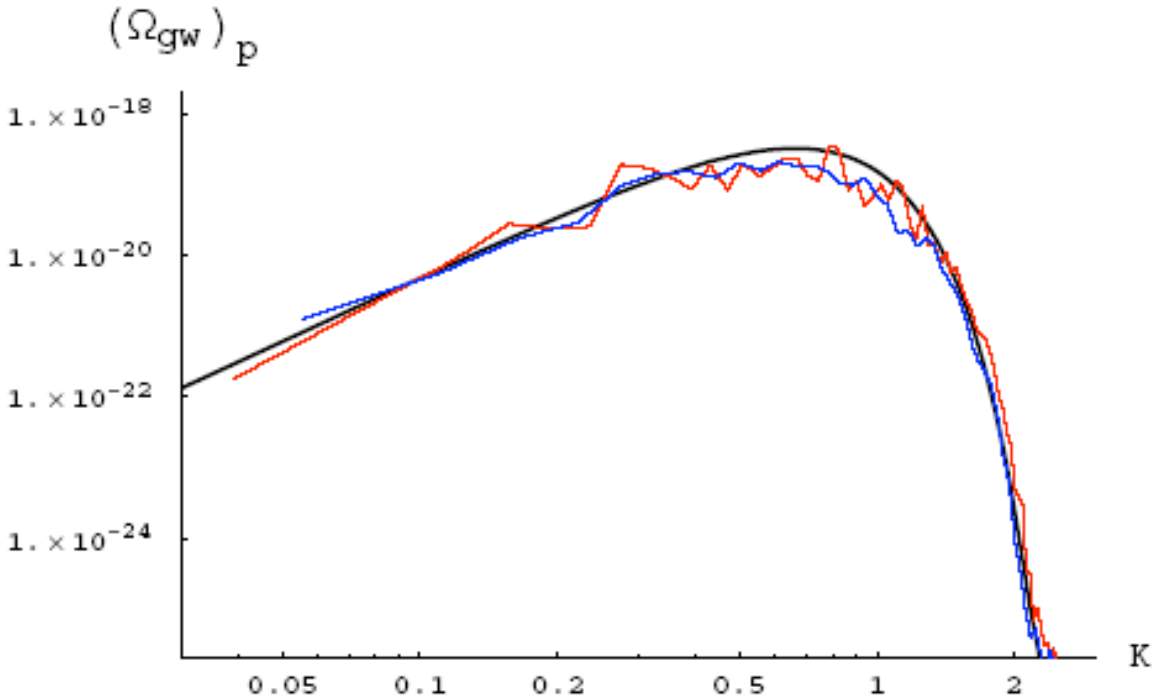
$$\hat{\chi}(\tau, \mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left(\hat{a}_{\mathbf{k}}\, \chi_k(\tau)\, e^{i\mathbf{k}\mathbf{x}} + \hat{a}^{+}_{\mathbf{k}}\, \chi^{*}_k(\tau)\, e^{-i\mathbf{k}\mathbf{x}}\right)$$

$$\begin{aligned} S_k(\tau_f) \;&=\; \frac{2}{\pi}\, G\, k^3 \int \frac{d^3\mathbf{p}}{(2\pi)^3}\, p^4 \sin^4(\hat{\mathbf{k}}, \hat{\mathbf{p}}) \\ &\left\{ \left| \int_{\tau_i}^{\tau_f} d\tau\, \cos(k\tau)\; a(\tau)\, \chi_p(\tau)\, \chi_{|\mathbf{k}-\mathbf{p}|}(\tau) \right|^2 + \left| \int_{\tau_i}^{\tau_f} d\tau\, \sin(k\tau)\; a(\tau)\, \chi_p(\tau)\, \chi_{|\mathbf{k}-\mathbf{p}|}(\tau) \right|^2 \right\} \end{aligned}$$









Analytic check

Shortcut to the answer

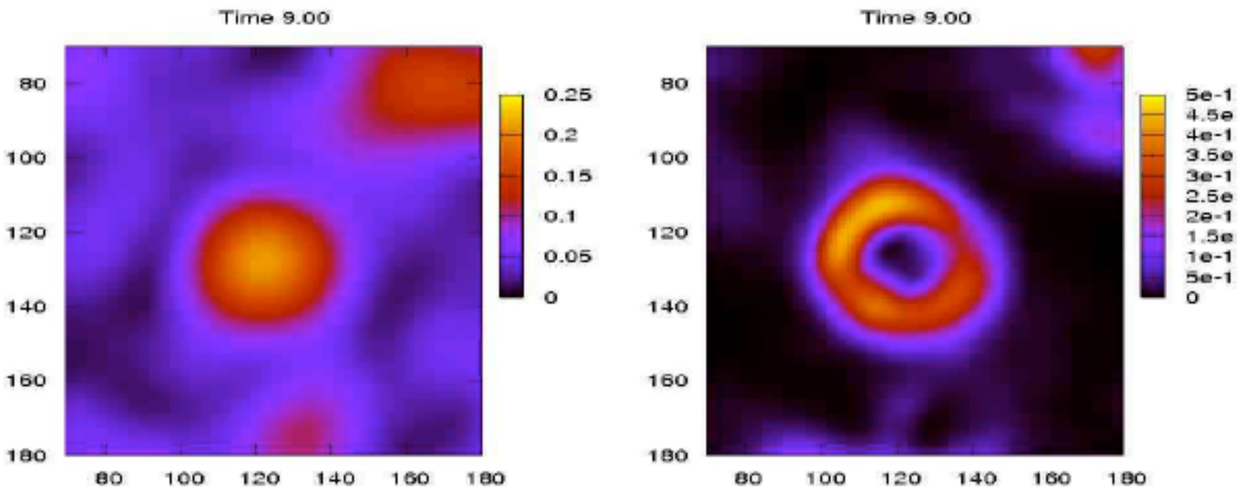
Felder, LK
hep-ph/0606256

estimation

$$\frac{\rho_{gw}}{\rho_r} \sim (RH)^2$$

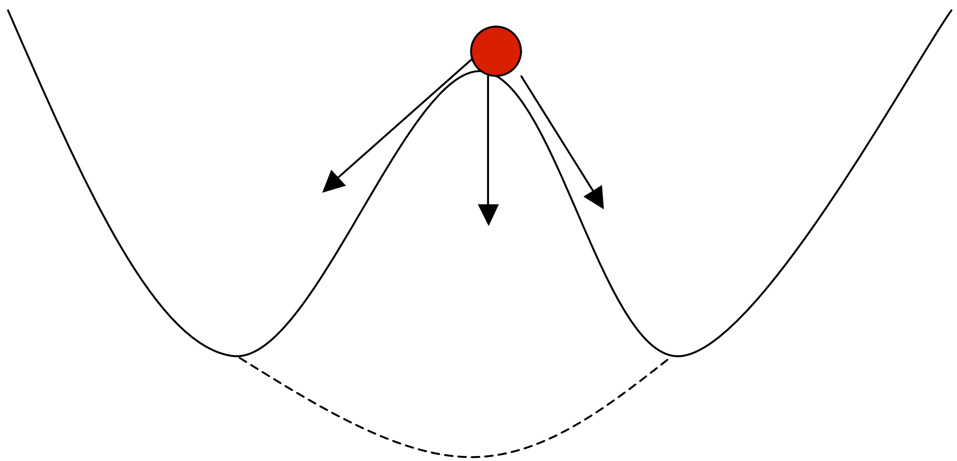
size of structures R vs Hubble radius $1/H$

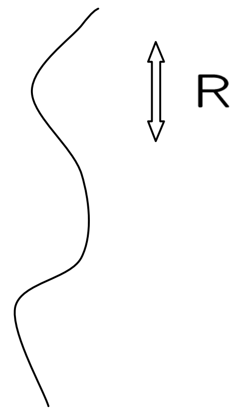
$$f \sim \frac{M}{10^{15} Gev} 10^8 \text{ Hz}$$



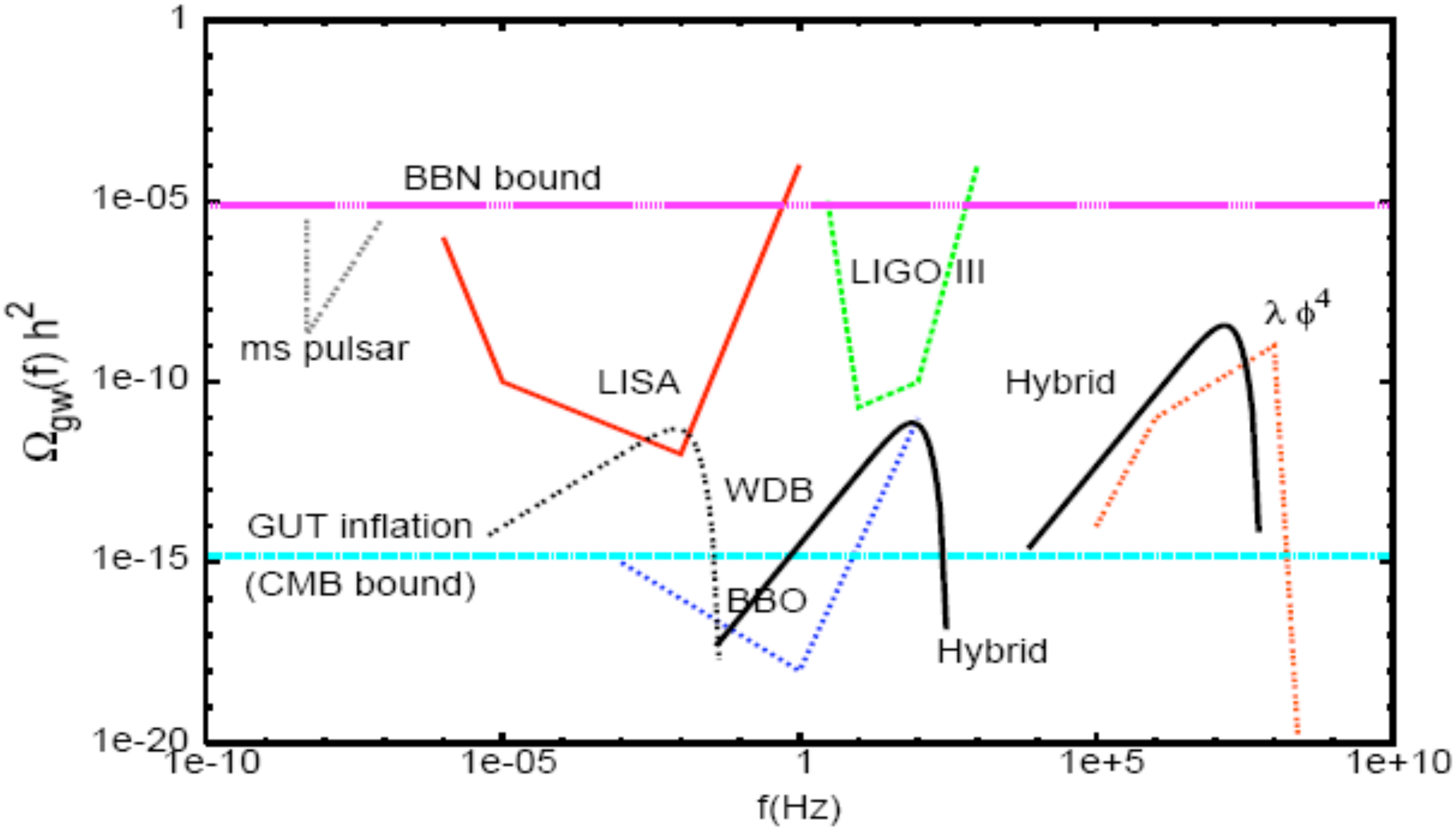
Garcia-Bellido et al
astro-ph/0707:0839

topological effects after hybrid inflation (unstable)
formation of defects results in GW emission

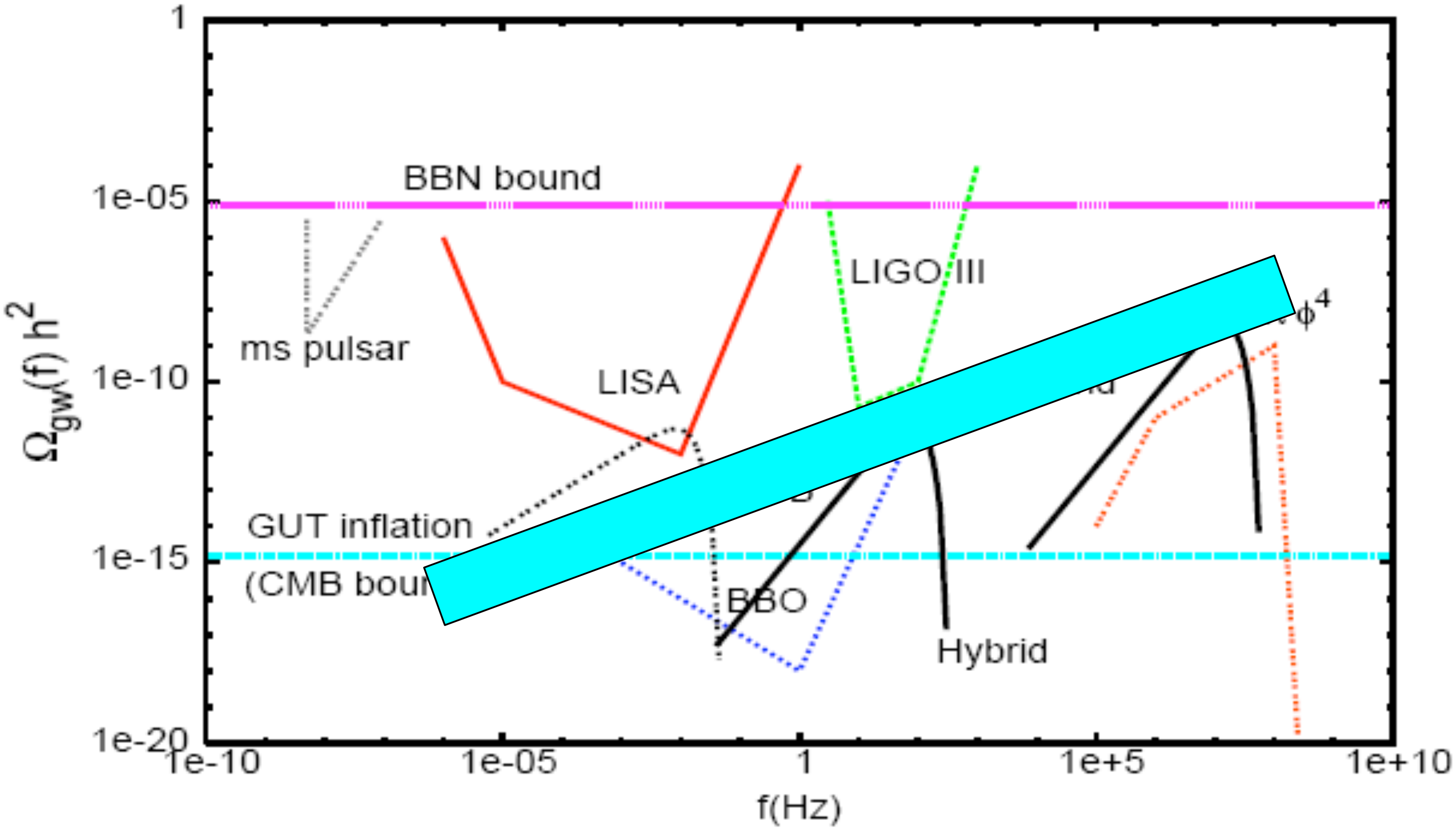




$$\frac{\rho_{gw}}{\rho_r} \sim (RH)^2$$



1
1e-05
1e-10
1e-15
1e-20
1e-10
1e-05
1
1e+5
1e+10
f(Hz)
$\Omega_{gw}(f)\,h^2$
BBN bound
ms pulsar
LISA
LIGO III
$\lambda\,\phi^4$
Hybrid
WDB
GUT inflation
(CMB bound)
BBO
Hybrid



BBN bound
LIGO III
ms pulsar
LISA
GUT inflation
BBO
Hybrid
ϕ^4
1
1e-05
1e-10
1e-15
1e-20
1e-10
1e-05
1
1e+5
1e+10
f(Hz)
$\Omega_{gw}(f)\, h^2$

$$f_* \sim \frac{4 \times 10^{10}\,\mathrm{Hz}}{R_*\,\rho_p^{1/4}}$$

$$h^2\,\Omega^*_{\rm gw} \sim 10^{-6}\,(R_*\,H_p)^2$$

Application to particular hybrid model

$$V = \frac{\lambda}{4}\,(\sigma^2 - v^2) + \frac{g^2}{2}\,\phi^2\,\sigma^2 + \frac{m^2}{2}\,\phi^2$$

$$g^2 >> \lambda$$

$$R_* \sim \frac{5}{\sqrt{\lambda}\,v}\sqrt{\ln\left(\frac{100}{\lambda}\right)}$$

$$f_* \sim \frac{\lambda^{1/4}\,10^{10}\,\mathrm{Hz}}{\sqrt{\ln\left(\frac{100}{\lambda}\right)}}$$

$$h^2\,\Omega^*_{\rm gw} \sim 2 \times 10^{-6}\,\ln\left(\frac{100}{\lambda}\right)\left(\frac{v}{M_{\rm Pl}}\right)^2$$

$$g^2 = 2\,\lambda$$

$$R_* \sim \frac{5}{\lambda\,v}$$

$$f_* \sim \lambda^{3/4}\,10^{10}\,\mathrm{Hz}$$

$$h^2\,\Omega^*_{\rm gw} \sim \frac{2 \times 10^{-6}}{\lambda}\left(\frac{v}{M_{\rm Pl}}\right)^2$$

model with $g^2 = 2\,\lambda = 2 \times 10^{-11}$ and $v = 10^{12}\,\mathrm{GeV}$

would lead to $f_* \sim 50\,\mathrm{Hz}$ and $h^2\,\Omega^*_{\rm gw} \sim 2 \times 10^{-9}$

The story of stochastic gravitational waves is CMB anisotropies of 21 century

GW from high energy inflation are targeted by CMB B-mode polarization experiments

GW from low-energy inflation are targeted by GW astronomy