Supergravity origin of the MSSM inflation

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Cosmo 07, Brighton
August 21 2007

Introduction

- MSSM flat directions can succesfully act as the inflaton [1]
- However, the low scale ($H \sim 1 - 10 \text{ Gev}$) requires an extremely flat potential $\implies$ fine-tuning ($\sim 10^{-18}$)
- Can we understand the flatness within some supergravity model?

[1] R. Allahverdi, K. Enqvist, J. Garcia-Bellido, A. Mazumdar
R. Allahverdi, K. Enqvist, J. Garcia-Bellido, A. Jokinen, A. Mazumdar

Flat directions

- Supersymmetric vacuum degenerate, potential vanishes along flat directions
- Flatness lifted by SUSY breaking and non-renormalizable terms in the potential

MSSM inflation

- $d = 6$ flat directions **LLe** and **udd** candidates for the inflaton
- After SUSY breaking the potential to lowest order becomes

$$V(\phi) = \frac{1}{2} m^2 |\phi|^2 - \frac{A\lambda}{6} |\phi|^6 + \lambda^2 |\phi|^{10}$$

- If $m^2 = 40A^2$, saddle point at $|\phi_0| = \left(\frac{A}{20\lambda}\right)^{1/4}$

$\Longrightarrow$ Inflation with:

$$\begin{aligned} \delta &\approx \frac{4}{5\pi}\sqrt{20}\left(\frac{m}{|\phi_0|}\right) N_{\rm COBE}^2 \\ n_s &\approx 1 - 4/N_{\rm COBE} \end{aligned}$$

Fine-tuning

- The condition $m^2 = 40A^2$ must be satisfied to the precision $\sim 10^{-18}$
- Can we obtain this naturally in some SUGRA model?

Scalar potential

- The SUGRA scalar potential reads

$$V = e^{G}(K^{M\bar{N}} G_M G_{\bar{N}} - 3) \ ,$$

where $G = K + \ln|W|^2$, K is the Kähler potential and W the superpotential

- F-term SUSY breaking if $\langle G^M \rangle \neq 0$ for some "hidden sector" fields

$\Longrightarrow$ SUSY breaking parameters

The saddle point condition

- Consider a model with [2]

$$
\begin{aligned}
W &= \hat{W}(h_m, h_m^*) + \frac{\hat{\lambda}(h_m, h_m^*)}{6}|\phi|^6 \\
K &= \hat{K}(h_m, h_m^*) + \hat{Z}_2(h_m, h_m^*)|\phi|^2 + \hat{Z}_4(h_m, h_m^*)|\phi|^4 + \ldots
\end{aligned}
$$

- In the vicinity of the eventual saddle point the potential becomes

$$
V = e^G(K^{M\bar{N}}G_M G_{\bar{N}} - 3) = \frac{1}{2}m^2|\phi|^2 - \frac{A\lambda}{6}|\phi|^6 + \lambda^2|\phi|^{10} + \mathcal{O}(|\phi_0|^{12})
$$

- The lowest order saddle point condition $A^2 = 40m^2$ reads

$$
\begin{aligned}
&|\hat{K}^m\hat{K}_m - 6\hat{Z}_2^{-1}\hat{K}^{\bar{m}}\hat{Z}_{2\bar{m}} + 3|^2 \\
&= 20(\hat{K}^m\hat{K}_m + \hat{K}^m\hat{K}^{\bar{n}}(\hat{Z}_2^{-2}\hat{Z}_{2m}\hat{Z}_{2\bar{n}} - \hat{Z}_2^{-1}\hat{Z}_{2m\bar{n}}) - 2)
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[2] K. Enqvist, L. Mether, S.N.

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1-D hidden sector

- Consider 1-D hidden sector as a simple example
- Assume $\hat{K} = \hat{K}(h+h^*)$, $\hat{Z}_2 = \hat{Z}_2(h+h^*)$ and treat $\hat{K}$ and $\hat{Z}_2$ as independent variables
- The general solution of $A^2 = 40m^2$ becomes

$$\begin{aligned} \hat{K} &= \beta \ln(h+h^*) \\ \hat{Z}_2 &= (h+h^*)^{-2/9+\beta/6}\left[c_1(h+h^*)^{\omega(\beta)} + c_2(h+h^*)^{-\omega(\beta)}\right]^{5/9}, \end{aligned}$$

where c_1, c_2 are constants and $\omega(\beta) = 1/2\sqrt{-17-6\beta}$

Generic hidden sector

- Make an Ansatz motivated by the 1-D case:

$$K = \sum_m \beta_m \ln(h_m + h_m^*) + \kappa \prod_m (h_m + h_m^*)^{\alpha_m} |\phi|^2 + \dots \ ,$$

with α_m, β_m and κ constants

- Solves $A^2 = 40m^2$ if

$\beta = \sum \beta_m$	$\alpha = \sum \alpha_m$
-3	$-\frac{4}{9}$
-7	0
-7	$-\frac{25}{9}$
-11	$-\frac{1}{9}$
-11	-4

Higher order corrections

- The higher order corrections in

$$V = \frac{1}{2}m^2|\phi|^2 - \frac{A\lambda}{6}|\phi|^6 + \lambda^2|\phi|^{10} + \mathcal{O}(|\phi_0|)^{12}$$

affect the existence of the saddle point

- The condition $A^2 = 40m^2$ is $\mathcal{O}(|\phi_0|^{10})$
- In addition, the corrections $\mathcal{O}(|\phi_0|^{12})$ and $\mathcal{O}(|\phi_0|^{14})$ relevant but higher orders negligible

Higher order corrections

- The saddle point condition satisfied to required precision for

$$K = \sum_m \beta_m \ln(h_m + h_m^*) + \kappa \prod_m (h_m + h_m^*)^{\alpha_m} \phi^2 + \mu\Big(\kappa \prod_m (h_m + h_m^*)^{\alpha_m}\Big)^2 \phi^4 + \nu\Big(\kappa \prod_m (h_m + h_m^*)^{\alpha_m}\Big)^3 \phi^6 + \ldots$$

with $(\delta \equiv \sum_m \alpha_m^3/\beta_m^2)$

$\beta = \sum \beta_m$	$\alpha = \sum \alpha_m$	$\gamma = \sum \frac{\alpha_m^2}{\beta_m}$	μ	ν
-3	$-\frac{4}{9}$	$\frac{29}{81}$	$\frac{19}{108}$	$-\frac{391}{10935} + \frac{8}{45}\delta$
-7	0	$\frac{20}{9}$	$-\frac{7}{36}$	$-\frac{71}{972} + \frac{1}{6}\delta$
-7	$-\frac{25}{9}$	$-\frac{770}{81}$	$\frac{26}{27}$	$-\frac{31}{2187} - \frac{1}{18}\delta$
-11	$-\frac{1}{9}$	$\frac{7613}{2673}$	$-\frac{559}{1782}$	$\frac{182275}{5233734} + \frac{259}{1602}\delta$
-11	-4	$-\frac{47}{3}$	$\frac{31}{12}$	$\frac{34}{27}$

Conclusions

- There exist Kähler potentials naturally yielding to the saddle point condition of the MSSM inflation
- The form of the Kähler potentials common in string theory compactifications
- Open questions: obtaining exactly the correct numbers, dynamics of the moduli fields, loop corrections...

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