Alternative dipole subtraction using Nagy-Soper dipoles

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Aim of the talk

- will not talk for hours on importance of NLO at LHC (you know this anyway); instead:
- give a schematic overview on sources of divergencies and treatment for NLO calculations
- explain idea and setup of subtraction scheme(s)
- show this explicitly for single W at LHC
- explain why we develop a new scheme
- (only very few words about parton showers)

I was told to...

“Please keep in mind that most people in the audience are not experts on your particular topic and there are also many PhD students present” (O. Brein, 12/08)

$\Rightarrow$ some introduction (sorry for the experts...)

Introduction and Motivation

- era of LHC approaching:
  "real" data-taking hopefully early next year
- LHC: hadron collider $\Rightarrow$ many things to be taken into account
- hadrons $\longrightarrow$ partons: use parton distribution functions
- processes governed by QCD:
  large NLO corrections (up to 50%)
- processes governed by QCD: also need parton showers
- many more issues (background, underlying events, scale dependencies, ...)
- same for experimental uncertainties
- here:
  talk about treatment of **NLO corrections on parton level**
- few words about **parton showers**
- many other issues are equally important

Structure of NLO calculations

General structure of NLO cross sections (1)

Contributions to a fixed order cross section $\sigma(\alpha^\kappa)$

- Leading order (LO) cross section, contributions to $\mathcal{O}\left(\alpha^\kappa\right)$:

$$\sigma_{\text{Born}} = \int d\Gamma_m \, |\mathcal{M}^{(m)}_{\text{Born}}|^2(s)$$

with $d\Gamma_m$ phase space for m particles in final state,
$\mathcal{M}^{(m)}_{\text{Born}}$ matrix element

General structure of NLO cross sections (2) - NLO part

- NLO contributions, to $\mathcal{O}(\alpha^{\kappa+1})$: virtual (=loops) and real (additional particle emission)
- virtual corrections:

$$\sigma_{\text{virt}} = \int d\Gamma_m \, 2\, Re(\mathcal{M}^{(m)}_{\text{Born}} \, (\mathcal{M}^{(m)}_{\text{virt}})^*)$$

- emission of additional real particles:

$$\sigma_{\text{Born}+1} = \int d\Gamma_{m+1} \, |\mathcal{M}^{(m+1)}|^2$$

one more particle in the final state

- correct power counting requires to take both virtual and real diagrams into account, such that

$$\sigma_{\text{NLO}}(\alpha^\kappa) = \sigma^{(m)}_{\text{Born}}(\alpha^\kappa) + \sigma^{(m)}_{\text{virt}}(\alpha^{\kappa+1}) + \sigma^{(m+1)}_{\text{real}}(\alpha^{\kappa+1})$$

NLO corrections - sources of divergencies

- NLO calculations involve integrals over undetermined loop momenta

$$\mathcal{M}_{\text{virt}} \propto \int \prod_i d^4 k_i \, F(k_i \, p_1 \, p_2, ...)$$

(F: general function of internal and external momenta; depends on Lorentz structure, ...)

- poles for $k \longrightarrow \infty$: ultraviolett divergency treated by renormalizing the parameters of the theory (masses, couplings, ...), $\Rightarrow$ not my topic here
- poles for $k \longrightarrow 0$: infrared divergencies

$\Rightarrow$ will attack these in the rest of my talk

Infrared divergencies (1)

- not everything lost: for well defined variables, infrared singularities cancel for fixed $\mathcal{O}(\alpha^n)$ calculations (Kinoshita, Lee, Nauenberg, 1964)
- (aside: we here assume that the Born cross section σ_{Born} is infrared finite)
- source of infrared divergency: emission of massless particles
- appears as terms

$$\frac{1}{p_i p_j}$$

in matrix elements

Infrared divergencies (2)

- $p_{i,j}$: four momenta; p_i emitter, p_j emitted particle
- $$p_i p_j \;=\; E_i\, E_j\, (1\,-\cos\,\theta_{ij})$$
  for massless particles
- $E_j \;\longrightarrow\; 0$: soft divergence
- $\cos\theta_{ij} \;\longrightarrow\; 1$: collinear divergence
- both can appear at the same time: douple poles

Infrared divergencies: Treatments (1)

- KNL theorem: infrared divergencies cancel between real and vitrual contributions: eg

$$\sigma_{\text{real}} = \frac{1}{\varepsilon} A + ..., \sigma_{\text{virt}} = -\frac{1}{\varepsilon} A + ..., \sum = \text{finite}$$

  (example for single poles)
  $\Rightarrow$ need to have a good (analytical) parametrization
- one option: give a fictitous mass to the emitted particles
  $\Rightarrow$ divergencies appear as $\ln m_{\text{fict}}$, let $m \rightarrow 0$ in the end
- typically used in QED calculations

Infrared divergencies: Treatments (2)

- second option: go from $D = 4$ to $D = 4 - 2\,\varepsilon$ dimensions
- poles then appear as

$$\sigma_{\text{real}} = \frac{A}{\varepsilon^2} + \frac{B}{\varepsilon} + \ldots.$$

- A, B depend on the splitting process
- eg in QCD $\tilde{p}_i \longrightarrow p_i + p_j$ (omitted color factors etc)

$$\begin{aligned} q \longrightarrow q\,g \quad &: \quad \propto \frac{1}{\varepsilon^2} + \frac{3}{2\,\varepsilon} \\ g \longrightarrow q\,\bar{q} \quad &: \quad \propto -\frac{1}{3\varepsilon} \end{aligned}$$

- poles arise from **integration** of phase space of p_j
- important: **this behaviour is the same for all processes**

Subtraction Schemes

Dipole subtraction: general idea

- know that pole structure always the same
- can also show: in the singular limits,

$$|\mathcal{M}^{(m+1)}|^2 \longrightarrow D_{ij}(p_i, p_j)\,|\mathcal{M}^{(m)}|^2, \quad D_{ij} \sim \frac{1}{p_i p_j} \qquad (1)$$

- D_{ij}: **dipoles**, contain complete singularity structure
- also means that

$$\int d\Gamma_{m+1} \left(|\mathcal{M}^{(m+1)}|^2 - \sum_{ij} D_{ij} |\mathcal{M}^{(m)}|^2 \right) = \text{finite}$$

- **general idea of dipole subtraction:** make use of (1), shift singular parts from $m+1$ to m particle phase space

Dipole subtraction for total cross sections

Master formula

$$\sigma_{NLO} = \int d\Gamma_m \left(|\mathcal{M}|^2_{\text{Born}} + 2\,\text{Re}(\mathcal{M}_{\text{Born}}\mathcal{M}^*_{\text{virt}}) + \widetilde{F}_{\text{sing}}|\mathcal{M}|^2_{\text{Born}}\right) J^{(m)} + \int d\Gamma_{m+1} \left(|\mathcal{M}|^2_{\text{real}} - F_{\text{sing}}|\mathcal{M}|^2_{\text{Born}}\right) J^{(m+1)}$$

$\Rightarrow$ effectively added "0"; both integrals finite
(major work: Catani, Seymour, 1996)

- Js: define the quantities which are measured; here you put in guarantee that Born part is infrared finite
(eg require that $p_k p_l > p^2_{min}$)
- $\tilde{F} = \int dp_j\, F$; $\Longleftarrow$ this is where all the work is

Interlude: energy momentum conservation (1)

...unfortunately, some complications are involved...

- previous slide: add and subtract "0" in terms of

$$\int d\Gamma_m \widetilde{F}_{\text{sing}} |\mathcal{M}^{(m)}_{\text{Born}}|^2 - \int d\Gamma_{m+1} F_{\text{sing}} |\mathcal{M}^{(m)}_{\text{Born}}|^2$$

- addition and subtraction takes place in different phase spaces

$$m \longleftrightarrow m+1$$

- somehow need to define a matching $(m+1) \Rightarrow (m)$
- why ?? want all external particles to be onshell

  here: $p^2 = 0$ for massless particles

Interlude: energy momentum conservation (2)

- example: $q \longrightarrow q+g$ splitting

$$m+1: q(p_i)+g(p_j), \quad m: q(p_{\tilde{i}})$$

- everything onshell: $p_{\tilde{i}}^2 = 0,\ p_i^2 = 0,\ p_j^2 = 0$
- not possible if $p_{\tilde{i}} = p_i + p_j$!!
- $\Rightarrow$ need to redistribute the momenta somehow

$$p_{\tilde{a}}^{(m)} = F\left(p_a^{(m+1)}, p_b^{(m+1)}, \dots\right)$$

- also need to keep total energy/ momentum conserved:

$$\sum_m p_{\tilde{a}} \stackrel{!}{=} \sum_{m+1} p_a$$

(sum over outgoing particles only)

Second ingredient: Parametrization of integration variables

- again: remember you have

$$F_{\text{sing}} \propto D_{ij},\ \widetilde{F}_{sing} = \int d\Gamma_1 \, D_{ij},\ d\Gamma_1 \propto d^4 p_j \, \delta(p_j^2)$$

$$\Longrightarrow \widetilde{F}_{sing} \propto \int d^4 p_j \, \delta(p_j^2) \, D_{ij}$$

- 3 free variables (in D dimensions: $D-1$)

  !! need to be written in terms of m particle variables !!
- now all ingredients:
  total energy momentum conservation, onshellness of external particles, need for integration variables

So far (1):

- shown that NLO corrections are important for hadronic processes
- singularities can occur from ultraviolett and infrared divergencies
- ultraviolett: handled by renormalizing the theory
- infrared: cancel in fixed order calculations
- for the latter: need good pole parametrizations

So far (2):

- one way to handle this: dipole subtraction schemes
- infinities are "shifted" around in m, $m+1$ contributions of σ_{tot}
- complications: energy momentum conservation, onshellness or external particles, parametrization of integration variables

master formula

$$\sigma_{NLO} = \int d\Gamma_m \left(|\mathcal{M}|^2_{\text{Born}} + 2\,\text{Re}(\mathcal{M}_{\text{Born}}\mathcal{M}^*_{\text{virt}}) + \widetilde{F}_{\text{sing}}|\mathcal{M}|^2_{\text{Born}} \right) J^{(m)} + \int d\Gamma_{m+1} \left(|\mathcal{M}|^2_{\text{real}} - F_{\text{sing}}|\mathcal{M}|^2_{\text{Born}} \right) J^{(m+1)}$$

In the following:

- talk about Catani Seymour dipole scheme
- talk about (alternative) Nagy Soper dipole scheme
- show some results and comparisons for the latter
- will NOT talk about parton showers (work done by collaborators)

...work in progress...

Catani Seymour subtraction scheme

Catani Seymour Dipoles (the praise)

- Catani, Seymour 1996: suggested dipole subtraction scheme for NLO calculations, massless particles
- became "standard" for many NLO calculations
- paper in general handled as a "toolbox", ie formulas can be extracted without actually understanding how to get there and where they come from
- some difficulty: understanding the notation
- has been applied in numerous calculations (8. Jan. 09: 537 citations)
- also **very** helpful for (becoming) experts
- follow up work: massive particles (Catani, Dittmaier, Seymour, Trocsanyi, 02), combined with phase space slicing (Nagy, 03)

Subtraction schemes

- also other subtraction schemes around (as eg implemented in RacoonWW, ...)

important message:

poles have to be the same; finite parts can differ

$=$ behaviour in the singular regions is unique

Single W using Catani Seymour dipoles

(and more details about subtraction schemes)

Processes at hadron colliders: general

- hadron colliders (as Tevatron, LHC) collide **hadrons**
- QCD: talks about **partons**
- transition: parton distribution functions (PDFs) $f_l(x, \mu_F)$;
  $l = q, \bar{q}, g$ flavour, x momentum fraction, (μ_F factorization scale)

masterformula

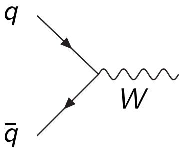
$$\sigma_{\text{hadr}}(p\,\bar{p} \to X) = \sum_{l_1,l_2} \int dx_1 \int dx_2\, f_{l_1}(x_1) f_{l_2}(x_2)\, \sigma_{\text{part}}(x_1, x_2;\, l_1 l_2 \to X)$$

- perturbative, nonperturbative part

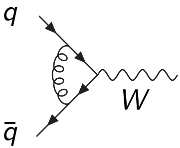
Sample process: single W production

(some transparencies: courtesy of Chenghan Chung, RWTH Aachen)

- now: do everything at parton level, worry about PDFs later
- tree level: $q\,\bar{q} \longrightarrow W$

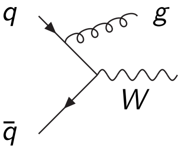


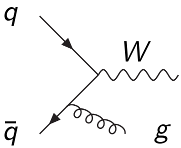
- virtual corrections: $q\,\bar{q} \longrightarrow W$



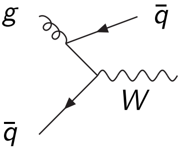
Single W production: real emissions (W + jet)

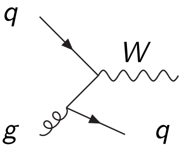
- real corrections: $q\,\bar{q} \;\rightarrow\; W + g$





- real corrections: $g\,\bar{q} \;\rightarrow\; W\,\bar{q},\;\; g\,q \;\rightarrow\; W\,q$ (same order in α_s !!)





(+ 2 more diagrams...)

Dipoles à la Catani Seymour

In the following:

Concentrate on $q\bar{q}$ induced processes

(Rest simultaneously)

- first: define general kinematics

$$q(p_1) + \bar{q}(p_2) \rightarrow W(p_3)\,(+\,g(p_4))$$

  where we need to keep track of $m, m+1$ particle space
- in the following: m:$\tilde{p}_i$

Single W: squared amplitudes

- Born:

$$\frac{1}{4}\,\frac{1}{9}\sum_{\text{spins, colors}}|\mathcal{M}_{\text{Born}}|^2 = \frac{g^2}{12}\,m_W^2$$

- real emission

$$\frac{1}{4}\,\frac{1}{9}\sum_{\text{spins, colors}}|\mathcal{M}_{\text{real}}|^2 = \frac{8}{9}\,g^2\,\alpha_s\,\pi\,\frac{t^2+u^2+2s\,p_3^2}{t\,u}$$

- virtual contribution (in $D = 4 - 2\,\varepsilon$ dimensions)

$$|\widetilde{\mathcal{M}}_v|^2 = |\mathcal{M}_{\text{Born}}|^2\,\frac{2\,\alpha_s}{3\,\pi}\,\frac{1}{\Gamma(1-\varepsilon)}\left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon}\left\{-\frac{2}{\varepsilon^2}-\frac{3}{\varepsilon}-8+\pi^2\right\}$$

Single W: pole structures

- real emission

$$\frac{1}{4}\frac{1}{9}\sum_{\text{spins, colors}}|\mathcal{M}_{\text{real}}|^2 = \frac{8}{9}\,g^2\,\alpha_s\,\pi\,\frac{t^2+u^2+2s\,p_3^2}{t\,u}$$

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single poles,

Single W: pole structures

- real emission

$$\frac{1}{4}\,\frac{1}{9}\sum_{\text{spins, colors}}|\mathcal{M}_{\text{real}}|^2 \;=\; \frac{8}{9}\,g^2\,\alpha_s\,\pi\,\frac{t^2+u^2+2s\,p_3^2}{t\,u}$$

- virtual contribution (in $D=4-2\,\varepsilon$ dimensions)

$$|\widetilde{\mathcal{M}}_v|^2 \;=\; |\mathcal{M}_{\text{Born}}|^2\,\frac{2\,\alpha_s}{3\,\pi}\,\frac{1}{\Gamma(1-\varepsilon)}\,\left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon}\left\{-\frac{2}{\varepsilon^2}-\frac{3}{\varepsilon}-8+\pi^2\right\}$$

single poles, double poles

Subtraction terms

- 2 particle phase space (real emission)

$$\mathcal{D}^{14,2} + \mathcal{D}^{24,1} = \frac{8}{9}\,\pi\,\alpha_s\,g^2\left(\frac{s^2+(s+t+u)^2}{t\,u}\right) = \underbrace{\frac{1}{4}\frac{1}{9}\sum|\mathcal{M}_{\text{real}}|^2}_{\text{singular}} + \underbrace{\frac{16}{9}g^2\,\alpha_s\,\pi}_{\text{finite}}$$

- 1 particle phase space (virtual contribution)

$$\mathbf{I}(\varepsilon)|\mathcal{M}_b|^2 = \underbrace{\frac{2\alpha_s}{3\pi}\frac{1}{\Gamma(1-\varepsilon)}\left(-8+\frac{2}{3}\pi^2\right)|\mathcal{M}_b|^2}_{\text{finite}} - \underbrace{|\widetilde{\mathcal{M}}_v|^2}_{\text{singular (+finite)}}$$

all singular terms will disappear in subtraction

Integrated Dipoles in more details: I, K, P (1)

$m+1$ **phase space: in principle easy**

$$\int d\Gamma_{m+1}\left(|\mathcal{M}_{\text{real}}|^2-\sum D\right),\quad \text{finite}$$

m **particle phase space: more complicated**

need integration variables (emission from p_1):

$$x\;=\;1-\frac{p_4(p_1+p_2)}{p_1p_2}\;\text{softness},\;\;\tilde{v}\;=\;\frac{p_1p_4}{p_1p_2}\;\text{collinearity}$$

Integrated Dipoles in more details: I, K, P (2)

- in principle, obtain $\int d\Gamma_1 \, D \;=\; \int_0^1 dx \, \left(\mathbf{I}(\varepsilon) + \tilde{\mathbf{K}}(x,\varepsilon)\right)$
- $\mathbf{I}(\varepsilon) \;\propto\; \delta(1-x)$: corresponds to loop part
- $\tilde{\mathbf{K}}(x,\varepsilon)$ contains finite parts as well as **collinear singularities**
- latter need to be cancelled by adding **collinear counterterm**

$$\frac{1}{\varepsilon}\left(\frac{4\pi\mu^2}{\mu_F^2}\right)^{\varepsilon} P^{qq}(x)$$

  depends on factorization scale μ_F ($P^{qq}(x)$ splitting function)
- PDFs come in again: term already accounted for by folding w PDF, needs to be subtracted
- for $q\,g \;\rightarrow\; W\,q$ like processes, only singularity which appears

Dipole subtraction: Master formula (1)

Symbolic Masterformula

$$\sigma = \sigma^{LO} + \sigma^{NLO}$$

$$\sigma^{NLO} = \int_{m+1} d\sigma^R + \int_m d\sigma^V + \int d\sigma^C$$

$$= \int_{m+1} (d\sigma^R - d\sigma^A) + \int_m \left(d\sigma^{\tilde{A}} + d\sigma^V + d\sigma^C \right),$$

$$\sigma_m^{\mathrm{NLO}}(s) = \int_m \Big\{ |\widetilde{\mathcal{M}}_{\mathrm{virt}}(s;\varepsilon)|^2 + \mathbf{I}(\varepsilon) |\mathcal{M}_{\mathrm{Born}}(s)|^2$$

$$+ \int_0^1 dx \, (\mathbf{K}(x) + \mathbf{P}(x;\mu_F)) \, |\mathcal{M}_{\mathrm{Born}}(x,s)|^2 \Big\}$$

Dipole subtraction: Master formula (1)

Symbolic Masterformula

$$\sigma = \sigma^{LO} + \sigma^{NLO}$$

$$\sigma^{NLO} = \int_{m+1} d\sigma^R + \int_m d\sigma^V + \int d\sigma^C$$

$$= \int_{m+1} (d\sigma^R - d\sigma^A) + \int_m \left(d\sigma^{\tilde{A}} + d\sigma^V + d\sigma^C \right),$$

$$\sigma_m^{\mathrm{NLO}}(s) = \int_m \left\{ |\widetilde{\mathcal{M}}_{\mathrm{virt}}(s;\varepsilon)|^2 + \mathbf{I}(\varepsilon)|\mathcal{M}_{\mathrm{Born}}(s)|^2 + \int_0^1 dx\, (\mathbf{K}(x) + \mathbf{P}(x;\mu_F))\, |\mathcal{M}_{\mathrm{Born}}(x,s)|^2 \right\}$$

Dipole subtraction: Master formula (1)

Symbolic Masterformula

$$
\begin{aligned}
\sigma &= \sigma^{LO} + \sigma^{NLO} \\
\sigma^{NLO} &= \int_{m+1} d\sigma^{R} + \int_{m} d\sigma^{V} + \int d\sigma^{C} \\
&= \int_{m+1} \left(d\sigma^{R} - d\sigma^{A}\right) + \int_{m} \left(d\sigma^{\tilde{A}} + d\sigma^{V} + d\sigma^{C}\right), \\
\sigma^{\mathrm{NLO}}_{m}(s) &= \int_{m} \Big\{ |\widetilde{\mathcal{M}}_{\mathrm{virt}}(s;\varepsilon)|^2 + \mathbf{I}(\varepsilon)|\mathcal{M}_{\mathrm{Born}}(s)|^2 \\
&\quad + \int_0^1 dx\,\left(\mathbf{K}(x) + \mathbf{P}(x;\mu_F)\right)|\mathcal{M}_{\mathrm{Born}}(x,s)|^2 \Big\}
\end{aligned}
$$

Dipole subtraction: Master formula (2)

Real Masterformula $(s = (p_a + p_b)^2)$

$$
\begin{aligned}
\sigma(s) \;=\; & \int_m d\Phi^{(m)}(s)\frac{1}{n_c(a)n_c(b)}\,|\mathcal{M}^{(m)}|^2(s)F_J^{(m)} \\
& + \int d\Phi^{(m+1)}(s)\left\{\frac{1}{n_c(a)n_c(b)}\,|\mathcal{M}^{(m+1)}|^2(s))F_J^{(m+1)} \;-\; \sum_{\text{dipoles}}(\mathcal{D}\,\cdot\,F_J^{(m)})\right\} \\
& + \int d\Phi^{(m)}(s)\left\{\frac{1}{n_c(a)n_c(b)}|\mathcal{M}^{(m)}|^2_{1\text{ loop}}(p_a,p_b) \;+\; \mathbf{I}(\varepsilon)|\mathcal{M}^{(m)}|^2(s)\right\}_{\varepsilon=0} F_J^{(m)} \\
& + \left\{\int dx_a\, dx_b\delta(x-x_a)\,\delta(x_b-1)\int d\Phi^{(m)}(x_a p_a, x_b p_b)|\mathcal{M}^{(m)}|^2(x_a p_a, x_b p_b)\right. \\
& \times \left.\left(\mathbf{K}^{a,a'}(x) + \mathbf{P}^{a,a'}(x_a p_a, x_b p_b, x;\mu_F^2)\right)\right\} + (a \leftrightarrow b)
\end{aligned}
$$

where all colour/ phase space factors have been accounted for

Single W using Nagy Soper Dipoles

(and some details about parton shower matching)

Nagy Soper dipoles: shower algorithm (commercial slide)

- Nagy Soper dipoles: suggested in 2007 (arXiv:0706.0017) in the context of parton showers
- "Parton showers with quantum interference"
  $\Rightarrow$ aim is to treat parton showers on a quantum-mechanical level (usual treatment: classical, ie averaging over spins, no interference effects, only leading color)

follow up work: (same authors)

- "Parton showers with quantum interference: Leading color, spin averaged" (arXiv:0801.1917)
  shows equivalence to standard showers in singular limit
- "Parton showers with quantum interference: Leading color, with spin" (arXiv:0805.0216)
  inclusion of spin effects

From parton showers to subtraction schemes

- basic idea: can use the splitting functions in the parton shower as dipole subtraction terms

  $\Rightarrow$ have same behaviour in singular limits
- "turn around" of idea suggested by Nagy, Soper (hep-ph/0503053):
  use Catani Seymour Dipoles for shower algorithm
- has lead to development of Catani Seymour-like showers: Dinsdale, Ternick, Weinzierl (arXiv:0709.1026), Schumann, Krauss (arXiv:0709.1027)
- advantage: simplify treatment of double counting for combination with NLO calculations

Interlude: Double counting for showers and NLO (1)

Very very short...

- double counting: hard real emissions are described in both shower and "real emission" matrix element
- want:hard: matrix element, soft: shower (always talk about 1 jet)
- can be achieved by adding and subtracting a counterterm

$$-\int_{m+1} d\sigma^{\mathrm{PS}}|_{m+1} + \int_{m+1} d\sigma^{\mathrm{PS}}|_{m}$$

details in hep-ph/0204244: "Matching NLO QCD computations and parton shower simulations" (Frixione, Webber), MC@NLO

Interlude: Double counting for showers and NLO (2)

- important: have new terms in $m+1$ phase space

$$\int_{m+1}\left(d\sigma^{R}\underbrace{-d\sigma^{A}+d\sigma^{PS}|_{m}}_{=0}-d\sigma^{PS}|_{m+1}\right)$$

- same splitting functions: second and third term cancel !! left with

$$\int_{m+1}\left(d\sigma^{R}-d\sigma^{PS}|_{m+1}\right)$$

$\Rightarrow$ improves numerical efficiency

Single W using NS Dipoles

Results

Nagy Soper subtraction terms

- 2 particle phase space (real emission)

$$\mathcal{D}^{14,2} + \mathcal{D}^{24,1} = \frac{8}{9}\pi\,\alpha_s\,g^2\left(\frac{t^2+u^2+2\,s\,p_3^2}{t\,u}\right) = \underbrace{\frac{1}{4}\frac{1}{9}\sum|\mathcal{M}_{\text{real}}|^2}_{\text{singular}}$$

- 1 particle phase space (virtual contribution)

$$\mathbf{I}(\varepsilon)|\mathcal{M}_b|^2 = \underbrace{\frac{2\alpha_s}{3\pi}\frac{1}{\Gamma(1-\varepsilon)}(-8+\frac{2}{3}\pi^2)|\mathcal{M}_b|^2}_{\text{finite}} - \underbrace{|\widetilde{\mathcal{M}}_v|^2}_{\text{singular (+finite)}}$$

- difference to Catani Seymour:
  subtraction in m particle phase space, $\mathbf{K}(x)$ terms

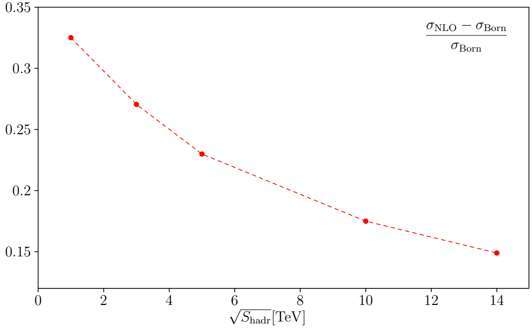
  pole structure the same, finite terms differ ✓

Numerical results for single W (complete), NLO corrections in percent (slide by C. Chung)

$\frac{\sigma_{NLO}-\sigma_{LO}}{\sigma_{LO}}$ as a function of $\sqrt{S_{\text{hadr}}}$

input: $M_W = 80.35$ GeV, PDF $\Rightarrow$ cteq6m, $\alpha_s(M_W) = 0.120299$

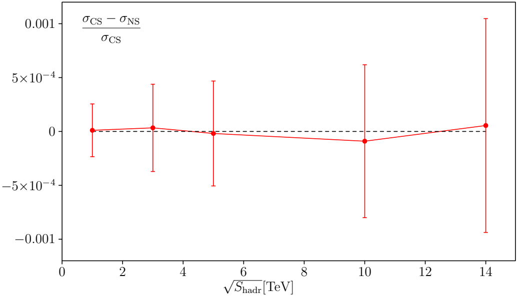
corrections up to 30%



Numerical results for single W (complete), relative difference (slide by C. Chung)

plot relative difference between CS and NS: $\frac{\sigma_{CS}-\sigma_{NS}}{\sigma_{CS}}$

input: $M_W = 80.35$ GeV, PDF $\Rightarrow$ cteq6m, $\alpha_s(M_W) = 0.120299$



agree on the sub-permill level ✓

Why bother ??

- 2 different schemes, giving same results, one well established: why bother ??
- first reason: have a parton shower with some additional features nearly ready $\Rightarrow$ matching
- second reason: remember: onshell-ness for external particles required shifts of momenta when going from $m+1$ to m particle phase space !!

!! biggest difference !!

Shifting momenta

- Catani Seymour: define emitter-spectator pair, momentum goes to 1 additional particle only

$\Rightarrow$ quite easy integrations

$\Rightarrow$ for increasing number of particles, huge number of transformation necessary $\Rightarrow$ code instabilities, long runtimes, ...

- Nagy Soper: shift momenta to **all** non-emitting external particles
- number of transformations = number of emitters
- eq $\gamma^* \rightarrow q\,\bar{q}\,g$ (m=3 at LO): 12 (CS) vs 6(NS)
- leads (unfortunately) to more complicated integrals for the m phase space integrals, some purely numerical
- in general: # of transformations: CS $\sim N^3_{\text{jets}}/2$, NS$\sim N^2_{\text{jets}}/2$

Status Quo and Outlook

Status quo (instead of Summary)

- goal: establish NS dipole formalism
- need to countercheck a) singularities, b) finite terms
- a) almost completely done; missing: processes w more than 2 gluons in the final state (complicated integration measure)
- b) counterchecked for all processes with initial state partons only, rest needs checks (some integrals still missing; see a))

Checked processes

- single W at hadron colliders:
  complete equivalence, agreement with MCFM (see talk)
- Dijet production at lepton colliders:
  singularity structure checked, rest underway
- deep inelastic scattering:
  singularity cancellation for virtual parts checked

Outlook

Outlook

- finish integration of missing parts, check by application to simple processes for unchecked splitting functions ($g \rightarrow gg$, $m > 2$ in final state)
- implement on matrix element level
- match with parton shower (Z. Nagy; underway)
- apply in (new) higher order calculations
- .... (more to come)

! Thanks for listening !

Appendix

$q \longrightarrow q\,g$ for initial state quarks: Catani Seymour (1)

- $q(\tilde{p}_1) \longrightarrow q(p_1) + g(p_4)$, q enters hard interaction
- Dipole:

$$D^{14,2} = -\frac{8\,\pi\,\mu^2\,\alpha_s\,C_F}{s+t+u}\left(\frac{2\,s\,(s+t+u)}{t\,(t+u)} + (1-\varepsilon)\,\frac{t+u}{t}\right)$$

- matching ($\tilde{p}_2 = p_2$)

$$\tilde{p}_1 = x\,p_1, \quad x = 1 - \frac{p_4\,(p_1+p_2)}{(p_1\,p_2)}$$

$$\tilde{p}_k^\mu = \Lambda^\mu{}_\nu\,p_k^\nu, \quad (k\text{: final state particles})$$

$$\Lambda^{\mu\nu} = -g^{\mu\nu} - \frac{2\,(K+\widetilde{K})^\mu(K+\widetilde{K})^\nu}{(K+\widetilde{K})^2} + \frac{2\,K^\mu\,\widetilde{K}^\nu}{K^2}$$

$$K = p_1 + p_2 - p_4, \quad \widetilde{K} = \tilde{p}_1 + p_2$$

$q \longrightarrow q\,g$ for initial state quarks: Catani Seymour (2)

- integration variables:

$$v \;=\; \frac{p_1 p_4}{p_1 p_2},\; x \;=\; 1 - \frac{p_4\,(p_1 + p_2)}{(p_1 p_2)}$$

- in p_1, p_2 cm system: $E_4 \;\longrightarrow\; 0 \;\Rightarrow\; x \;\longrightarrow\; 1$ (softness)
  $\cos\theta_{14} \;\longrightarrow\; 1 \;\Rightarrow\; v \;\longrightarrow\; 0$ (collinearity)
- Dipole in terms of integration variables

$$D^{14,2} \;=\; -\frac{8\,\pi\,\alpha_s\,C_F}{v\,x\,s}\,\left(\frac{1+x^2}{1-x} - \varepsilon(1-x)\right)$$

- integration measure

$$[dp_j] \;=\; \frac{(2\,p_1 p_2)^{1-\varepsilon}}{16\,\pi^2}\,\frac{d\Omega_{d-3}}{(2\,\pi)^{1-\varepsilon}}\,dv\,dx\,(1-x)^{-2\,\varepsilon}\,\left[\frac{v}{1-x}\,\left(1-\frac{v}{1-x}\right)\right]^{-\varepsilon}$$

where $v \leq\; 1 - x$ and all integrals between 0 and 1

$q \longrightarrow q\,g$ for initial state quarks: Catani Seymour (3)

- result

$$\mu^{2\varepsilon}\int [dp_j]\, D^{14,2} = \frac{\alpha_s}{2\,\pi}\,\frac{1}{\Gamma(1-\varepsilon)}\,C_F\,\left(\frac{2\mu^2\pi}{p_1 p_2}\right)^{\varepsilon}$$

$$\times \int_0^1 dx\,\left(\mathbf{I}(\varepsilon)\delta(1-x) + \tilde{\mathbf{K}}(x,\varepsilon)\;\underbrace{-\frac{1}{\varepsilon}P^{qq}(x)}_{\text{killed by coll CT}}\right)$$

with

$$\mathbf{I}(\varepsilon) = \frac{1}{\varepsilon^2}+\frac{3}{2\varepsilon}-\frac{\pi^2}{6}$$

$$\mathbf{K}(x) = (1-x) - 2\,(1+x)\ln(1-x) + \left(\frac{4}{1-x}\ln(1-x)\right)_+$$

$$P^{qq}(x) = \left(\frac{1+x^2}{1-x}\right)_+ \quad \text{regularized splitting function}$$

$q \longrightarrow q\,g$ for initial state quarks: Nagy Soper (1)

- $q(\tilde{p}_1) \longrightarrow q(p_1) + g(p_4)$, q enters hard interaction
- Dipole:

$$D^{14,2} = -\frac{8\,\pi\,\mu^2\,\alpha_s\,C_F}{s+t+u}\left(\frac{2\,s\,u\,(s+t+u)}{t\,(t^2+u^2)} + (1-\varepsilon)\,\frac{u}{t}\right)$$

as CS, same pole structure as CS

- matching, integration variables, integration measure: as Catani Seymour($v \leftrightarrow y$)
- Dipole in terms of integration variables

$$D^{14,2} = -\frac{8\pi\alpha_s\,C_F}{x\,s} \times \left(\frac{1-x-y}{y}(1-\varepsilon) + \frac{2\,x}{y\,(1-x)} - \frac{2\,x\,[2\,y-(1-x)]}{(1-x)[y^2+(1-x-y)^2]}\right)$$

$q \longrightarrow q\,g$ for initial state quarks: Nagy Soper (2)

- result

$$\mu^{2\varepsilon} \int [dp_j]\, D^{14,2} = \frac{\alpha_s}{2\,\pi} \frac{1}{\Gamma(1-\varepsilon)}\, C_F \left(\frac{2\,\mu^2\pi}{p_1 p_2}\right)^{\varepsilon} \times \int_0^1 dx \left(\mathbf{I}(\varepsilon)\delta(1-x) + \tilde{\mathbf{K}}(x,\varepsilon) \underbrace{-\frac{1}{\varepsilon} P^{qq}(x)}_{\text{killed by coll CT}} \right)$$

with

$$\mathbf{K}(x) = (1-x) - 2\,(1+x)\ln(1-x) + \left(\frac{4}{1-x}\ln(1-x)\right)_+ - (1-x)$$

- equivalence of dipoles schemes checked analytically

Final state $g \to q\,\bar{q}$ using Catani Seymour (1)

- $g(\tilde{p}_i) \to q(p_i) + \bar{q}(p_j)$,
  spectator: any other final state parton, p_k
- Dipole:

$$D^{ij,k} \propto \frac{1}{p_i p_j}\left[1 - \frac{2\,(p_i p_k)\,(p_j p_k)}{(1-\varepsilon)\,(p_i p_k + p_j p_k)^2}\right]$$

- matching

$$\tilde{p}_i = p_i + p_j - \frac{y}{1-y} p_k \quad y = \frac{p_i p_j}{p_i p_j + p_i p_k + p_j p_k}$$

$$\tilde{p}_k^\mu = \frac{1}{1-y} p_k$$

all other final state particles untouched

Final state $g \longrightarrow q\,\bar{q}$ using Catani Seymour (2)

- integration variables: y, z

$$z = \frac{p_i p_k}{p_i p_k + p_j p_k}$$

y: softness, z collinearity

- Dipole in terms of integration variables

$$D^{ij,k} \propto \frac{1}{y}\left[1 - \frac{z\,(1-z)}{1-\varepsilon}\right]$$

- integration measure

$$[dp_j] = \frac{(2\,\tilde{p}_i\tilde{p}_k)^{1-\varepsilon}}{16\,\pi^2}\,\frac{d\Omega_{d-3}}{(2\,\pi)^{1-\varepsilon}}\,dz\,dy\,(1-y)^{1-2\,\varepsilon}y^{-\varepsilon}\,\left[z\,(1-z)\right]^{-\varepsilon}$$

where all integrals between 0 and 1

Final state $g \longrightarrow q\,\bar{q}$ using Catani Seymour (3)

- result

$$\mu^{2\varepsilon}\int [dp_j]\,D^{ij,k} \;=\; \frac{\alpha_s}{2\,\pi}\,\frac{1}{\Gamma(1-\varepsilon)}\,T_R\left(\frac{2\mu^2\pi}{\tilde{p}_i\tilde{p}_k}\right)^{\varepsilon}\left[-\frac{2}{3\,\varepsilon}-\frac{16}{9}\right]$$

Final state $g \longrightarrow q\,\bar{q}$ using Nagy Soper (1)

- $g(\tilde{p}_i) \longrightarrow q(p_i) + \bar{q}(p_j)$,
  spectator: all other final state partons
- Dipole:

$$D^{ij} \propto \frac{1}{p_i p_j} \left[1 - \frac{2\,(p_i \tilde{n})\,(p_j \tilde{n})}{(1-\varepsilon)\,(p_i \tilde{n} + p_j \tilde{n})^2} \right]$$

with

$$\begin{aligned} \tilde{n} &= \frac{1+y+\lambda}{2\,\lambda}\,Q - \frac{a}{\lambda}\,(p_i + p_j) \\ y &= \frac{p_i p_j}{(p_i + p_j)Q - p_i p_j} \\ \lambda &= \sqrt{(1+y)^2 - 4\,a\,y}, \\ a &= \frac{Q^2}{(p_i + p_j)Q - p_i p_j} \end{aligned}$$

and Q cm energy of the incoming particles

Final state $g \longrightarrow q\,\bar{q}$ using Nagy Soper (2)

- matching

$$\tilde{p}_i \;=\; \frac{1}{\lambda}(p_i + p_j) - \frac{1-\lambda+y}{2\,\lambda\,a}\,Q$$

$$\tilde{p}_k^{\mu} \;=\; \Lambda^{\mu}{}_{\nu}\,p_k^{\nu} \quad \textbf{all } \text{final state particles}$$

$$\Lambda^{\mu\nu} \;=\; -g^{\mu\nu} \;-\; \frac{2\,(K+\widetilde{K})^{\mu}(K+\widetilde{K})^{\nu}}{(K+\widetilde{K})^2} + \frac{2\,K^{\mu}\,\widetilde{K}^{\nu}}{K^2}$$

$$K = Q - p_i - p_j,\quad \widetilde{K} = Q - \tilde{p}_i$$

- integration variables: y, z

$$z \;=\; \frac{p_j\,\tilde{n}}{p_i\,\tilde{n} + p_j\,\tilde{n}}$$

y: softness, z collinearity

Final state $g \longrightarrow q\,\bar{q}$ using Nagy Soper (3)

- Dipole in terms of integration variables

$$D^{ij} \propto \frac{1}{y}\left[1-\frac{z\,(1-z)}{1-\varepsilon}\right]$$

- integration measure

$$[dp_j] = \frac{(2\,\tilde{p}_i\,Q)^{1-\varepsilon}}{16\,\pi^2}\,\frac{d\Omega_{d-3}}{(2\,\pi)^{1-\varepsilon}}\,dz\,dy\,\lambda^{1-2\,\varepsilon}y^{-\varepsilon}\,[z\,(1-z)]^{-\varepsilon}$$

z: between 0 and 1
y: between 0 and $y_{\max} = \left(\sqrt{a}-\sqrt{a-1}\right)^2$

Final state $g \longrightarrow q\,\bar{q}$ using Nagy Soper (4)

- result

$$\mu^{2\varepsilon}\int[dp_j]\,D^{ij} \;=\; T_R\,\frac{\alpha_s}{2\,\pi}\,\frac{\alpha_s}{\Gamma(1-\varepsilon)}\left(\frac{2\,\pi\,\mu^2}{p_{\tilde{i}}Q}\right)^{\varepsilon} \times\left[-\frac{2}{3\,\varepsilon}-\frac{16}{9}+\frac{2}{3}\left[(a-1)\,\ln(a-1)-a\,\ln\,a\right]\right],$$

- for $a = 1$, reduces completely to Catani Seymour result
- (reason: $a = 1$ implies only 2 particles in the final state, $\tilde{n} \longrightarrow p_k$, $\Rightarrow$ complete equivalence)
- tradeoff: all final state particles get additional momenta: integral more complicated, but fewer transformations necessary
- however: integrals with gluons in $m+1$ final state even more complicated !!! $\Rightarrow$ next time....