Automation of Dipole Subtraction Method in MadGraph

Nicolas Greiner
in collaboration with R.Frederix,T.Gehrmann

Durham, 05.03.2009

Outline

1. Introduction
   - Status of Automation
   - Dipole Subtraction Method
   - MadGraph/MadEvent
2. MadDipole
3. Integrated dipoles

LO event generator tools:

- PYTHIA [Sjoestrand,Mrenna,Skands]
- HERWIG/HERWIG++ [Marchesini,Webber],[Baehr et al.]
- MadGraph/MadEvent [Stelzer,Long],[Maltoni,Stelzer],[Alwall et al.]
- CompHep/CalcHep [Boos et al.],[Pukhov]
- SHERPA [Gleisberg et al.]
- WHIZARD [Kilian,Ohl,Reuter]
- ALPGEN [Mangano,Moretti,Piccinini,Pittau,Polosa]
- HELAC [Kanaki,Papadopoulos]
- ...

NLO calculation programs:

- MCFM [Campbell,Ellis]
- NLOJET++ [Nagy]
- MC@NLO [Frixione,Webber]
- POWHEG [Nason et al.]
- ...

Automation of loop calculations:

Enormous progress in recent years : Unitarity methods, recursion relations, generalized unitarity, OPP-method,twistor-inspired methods...

$\longrightarrow$ Packages like

- CutTools [Ossola,Papadopoulos,Pittau]
- BlackHat [Berger et al.]
- Rocket [Giele,Zanderighi]
- Golem [Binoth et al.]
- ...

Automation of subtraction methods:

Several algorithms for subtraction terms:

- Dipole subtraction [Catani,Seymour],[Catani,Dittmaier,Seymour,Trocsanyi]
- Residue subtraction [Frixione,Kunszt,Signer]
- Antenna subtraction

  [Kosower],[Campbell,Cullen,Glover],[Gehrmann-DeRidder,Gehrmann,Glover] ,[Daleo,Gehrmann,Maitre]

First automation of dipole subtraction in SHERPA [Gleisberg,Krauss] and TeVJet [Seymour,Tevlin] and attempts for external library interfaced with MadGraph. [Hasegawa,Moch,Uwer]

$\rightarrow$ No general tool available for arbitrary process and massive dipoles.

Why Dipole Subtraction Method?

- Antenna formalism would be less complicated (1 antenna $\sim$ 2 dipoles).
- Antenna formalism can be extended to NNLO.

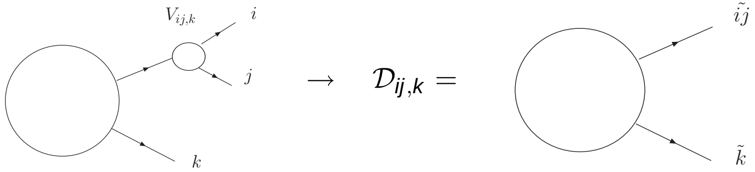
+ Dipole Method: straightforward Feynman diagrammatic approach
+ 1 dipole $\hat{=}$ 1 Feynman diagram
  $\longrightarrow$ color treatment 'easy', would be more difficult in antenna formalism.

Dipole Subtraction Method: [Catani,Seymour] [Catani,Dittmaier,Seymour,Trocsanyi]

Find expressions $d\sigma^A$ for infrared singularities and subtract/add them

$$\Rightarrow \sigma^{NLO} = \int_{m+1} \left[\left(d\sigma^R \right) - \left(d\sigma^A \right) \right] + \int_m \left[d\sigma^V + \int_1 d\sigma^A \right]$$

- Dipoles contain all infrared singularities occuring in specific process.
- Cross sections for real emission and virtual corrections are finite and can be calculated independently.



$$\mathcal{D}_{ij,k}\ (p_1,...,p_{m+1}) = -\frac{1}{2p_i \cdot p_j}$$

$$\cdot\, {}_m\!< 1,..,\widetilde{ij},..,\widetilde{k},..,m+1|\ \frac{\boldsymbol{T}_k \cdot \boldsymbol{T}_{ij}}{\boldsymbol{T}_{ij}^2}\ \boldsymbol{V}_{ij,k}\ |1,..,\widetilde{ij},..,\widetilde{k},..,m+1>_m \quad .$$

with emitter i and spectator k and dipole splitting function $V_{ij,k}$.

$$\tilde{p}_k^\mu = \frac{1}{1-y_{ij,k}}\, p_k^\mu \quad , \quad \tilde{p}_{ij}^\mu = p_i^\mu + p_j^\mu - \frac{y_{ij,k}}{1-y_{ij,k}}\, p_k^\mu, \quad y_{ij,k} = \frac{p_i p_j}{p_i p_j + p_j p_k + p_k p_i}.$$

Note: $p_i^\mu + p_j^\mu + p_k^\mu = \tilde{p}_{ij}^\mu + \tilde{p}_k^\mu$ and $\tilde{p}_{ij}^2 = \tilde{p}_k^2 = 0$.

MadGraph: [Stelzer,Long]

Type in process: e.g. **e+ e- $\rightarrow$ u u~**

$\Rightarrow$ MadGraph provides a Fortran code that calculates $|\mathcal{M}|^2$ for a given phase space point, summed over colors and helicities.

MadEvent: [Maltoni,Stelzer]

- Takes MadGraph output and integrates over phase space.
- Event generator.

MadGraph/MadEvent public available:
http://madgraph.hep.uiuc.edu/

MadDipole: [Frederix,Gehrmann,NG]
Type in real emission process: e.g. **e+ e- → u u~ g**
⇒ Analogous to MadGraph, MadDipole returns Fortran code for:

- Matrixelement for real emission.
- All possible dipoles for all possible born processes.

Further information and download:
http://madgraph.hep.uiuc.edu/

$$\mathcal{D}_{ij,k} \sim {}_m\!< 1,..,\widetilde{ij},..,\widetilde{k},..,m+1| \frac{\boldsymbol{T}_k \cdot \boldsymbol{T}_{ij}}{\boldsymbol{T}_{ij}^2} \boldsymbol{V}_{ij,k} |1,..,\widetilde{ij},..,\widetilde{k},..,m+1>_m$$

Splitting function $V_{ij,k}$ is tensor in helicity space. $V_{ij,k} = V_{ij,k}^{\mu\nu}$

$\Rightarrow$ Need modification of color and helicity management.

1. Color management:

- Use already existing routines $\rightarrow$ fast and correct.
- Insert additional operators in existing color calculation. Note: $\boldsymbol{T}_k \cdot \boldsymbol{T}_{ij}$ connect bra and ket $\rightarrow$ need different labelling. $\rightarrow$ new routines for squaring. $\rightarrow$ large objects.

2. Helicity management:

$V_{ij,k} = V_{ij,k}^{\mu\nu}$ combines different helicity combinations.

$$\begin{aligned}
\mathcal{D}_{ij,k} &\sim {}_m\langle 1, ...\tilde{ij}, ..., \tilde{k}, ..., m+1|_\mu \mathbf{V}_{ij,k}^{\mu\nu}\, {}_\nu|1, ...\tilde{ij}, ..., \tilde{k}, ..., m+1\rangle_m \\
&= {}_m\langle 1, ...\tilde{ij}, ..., \tilde{k}, ..., m+1|_{\mu'} \left(-g^{\mu'}_{\mu}\right) \mathbf{V}_{ij,k}^{\mu\nu} \left(-g_\nu^{\ \nu'}\right) {}_{\nu'}|1, ...\tilde{ij}, ..., \tilde{k}, ..., m+1\rangle_m \\
&= \sum_{\lambda_a, \lambda_b} {}_m\langle \ldots |_{\mu'}\, \epsilon^{*\mu'}(\lambda_b)\epsilon_\mu(\lambda_b)\mathbf{V}_{ij,k}^{\mu\nu}\, \epsilon^*_\nu(\lambda_a)\epsilon^{\nu'}(\lambda_a)\, {}_{\nu'}| \ldots \rangle_m \\
&= \sum_{\lambda_a, \lambda_b} {}_m\langle \ldots |_{\lambda_b}\, V(\lambda_b, \lambda_a)\, {}_{\lambda_a}| \ldots \rangle_m
\end{aligned}$$

with $V(\lambda_b, \lambda_a) = \epsilon_\mu(\lambda_b)\, \mathbf{V}_{ij,k}^{\mu\nu}\, \epsilon^*_\nu(\lambda_a)$ and $\epsilon^\mu(\lambda)\, {}_\mu| \ldots \rangle_m = {}_\lambda| \ldots \rangle_m$.

Phase space restrictions

Subtraction only needed when approaching divergency. $\Rightarrow$ Cut away non-singular parts of phase space by additional parameter $\alpha \in [0,1]$.
[Nagy,Trocsanyi]

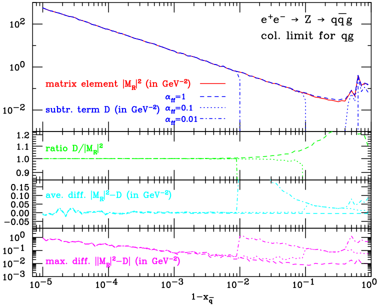
$$
\begin{aligned}
d\sigma_{ab}^{A} = & \sum_{\{n+1\}} d\Gamma^{(n+1)}(p_a, p_b, p_1, \ldots, p_n + 1) \frac{1}{S_{\{n+1\}}} \\
& \times \Bigg\{ \sum_{\substack{\text{pairs} \\ i,j}} \sum_{k \neq i,j} \mathcal{D}_{ij,k}(p_a, p_b, p_1, \ldots, p_{n+1}) F_J^{(n)}(p_a, p_b, p_1, .., \tilde{p}_{ij}, \tilde{p}_k, .., p_{n+1}) \Theta(y_{ij,k} < \alpha) \\
& + \sum_{\substack{\text{pairs} \\ i,j}} \Big[\mathcal{D}_{ij}^{a}(p_a, p_b, p_1, \ldots, p_{n+1}) F_J^{(n)}(\tilde{p}_a, p_b, p_1, .., \tilde{p}_{ij}, .., p_{n+1}) \Theta(1 - x_{ij,a} < \alpha) \quad + (a \leftrightarrow b) \Big] \\
& + \sum_{i \neq k} \Big[\mathcal{D}_{k}^{ai}(p_a, p_b, p_1, \ldots, p_{n+1}) F_J^{(n)}(\tilde{p}_a, p_b, p_1, .., \tilde{p}_k, .., p_{n+1}) \Theta(u_i < \alpha) + (a \leftrightarrow b) \Big] \\
& + \sum_{i} \Big[\mathcal{D}^{ai,b}(p_a, p_b, p_1, \ldots, p_{n+1}) F_J^{(n)}(\tilde{p}_a, p_b, \tilde{p}_1, \ldots, \tilde{p}_{n+1}) \Theta(\tilde{v}_i < \alpha) + (a \leftrightarrow b) \Big] \Bigg\} .
\end{aligned}
$$

$\rightarrow$ 4 parameters: `alpha_ff`, `alpha_fi`, `alpha_if`, `alpha_ii`, adjustable by user.

Massive particles

- Motivation: collinear radiation off massive particle finite, but source of possibly large logs.
- Finite dipoles put in separate routine `dipolsumfinite(...)`.
  Not evaluated by default but can be switched on if needed.
- Recover massless results in the limit of vanishing masses.

Check: In the limit $s_{ij} = p_i \cdot p_j \to 0$ dipoles approach matrixelement.



Ratio $|\mathcal{M}|^2 / \sum_{dipoles} \to 1$.
Difference integrable.

Package contains routine that checks all limits.

```
-----------------------------------------------------------------------------------------
Limit:  p( 4 ).p( 5 ) goes to zero
-----------------------------------------------------------------------------------------
p( 4).p( 5)/s(1,2) , sqrt(s( 4, 5)),  |M|^2 ,     |Sub.term|^2 ,|M|^2/|Sub.term|^2
-----------------------------------------------------------------------------------------
0.315364E+00 ,      0.561573E+03 , 0.226532E-06 , 0.000000E+00 , +Infinity
0.737460E-01 ,      0.271562E+03 , 0.712382E-06 , 0.000000E+00 , +Infinity
0.537588E-01 ,      0.231859E+03 , 0.300252E-05 , 0.000000E+00 , +Infinity
0.332833E-01 ,      0.182437E+03 , 0.114455E-05 , 0.000000E+00 , +Infinity
0.143302E-01 ,      0.119709E+03 , 0.837228E-05 , 0.000000E+00 , +Infinity
0.113982E-01 ,      0.106762E+03 , 0.174303E-04 , 0.000000E+00 , +Infinity
0.101978E-01 ,      0.100984E+03 , 0.236247E-03 , 0.000000E+00 , +Infinity
0.573922E-02 ,      0.757576E+02 , 0.253141E-03 , 0.244672E-03 , 0.103461E+01
0.253423E-02 ,      0.503411E+02 , 0.236898E-04 , 0.233953E-04 , 0.101259E+01
0.251508E-02 ,      0.501505E+02 , 0.503096E-03 , 0.488450E-03 , 0.102999E+01
0.192366E-02 ,      0.438595E+02 , 0.524883E-03 , 0.502630E-03 , 0.104427E+01
0.479893E-03 ,      0.219065E+02 , 0.526637E-03 , 0.522605E-03 , 0.100771E+01
0.317967E-03 ,      0.178316E+02 , 0.222351E-02 , 0.221944E-02 , 0.100184E+01
0.168300E-03 ,      0.129730E+02 , 0.578656E-03 , 0.577545E-03 , 0.100192E+01
0.145372E-03 ,      0.120571E+02 , 0.152367E-02 , 0.152807E-02 , 0.997118E+00
0.609018E-04 ,      0.780396E+01 , 0.303571E-01 , 0.302411E-01 , 0.100384E+01
0.259173E-04 ,      0.509091E+01 , 0.580746E-02 , 0.581356E-02 , 0.998952E+00
0.186237E-04 ,      0.431552E+01 , 0.568871E-01 , 0.568692E-01 , 0.100031E+01
0.178298E-04 ,      0.422253E+01 , 0.371004E-02 , 0.371583E-02 , 0.998442E+00
0.910116E-05 ,      0.301681E+01 , 0.480914E-02 , 0.480739E-02 , 0.100036E+01
0.156505E-05 ,      0.125102E+01 , 0.134983E+00 , 0.135049E+00 , 0.999514E+00
0.152407E-05 ,      0.123453E+01 , 0.357109E-01 , 0.357250E-01 , 0.999606E+00
0.140303E-05 ,      0.118449E+01 , 0.140632E+00 , 0.140663E+00 , 0.999781E+00
0.353217E-06 ,      0.594321E+00 , 0.804008E+00 , 0.804034E+00 , 0.999968E+00
```

```
-----------------------------------------------------------------------------------------
Limit:  p( 5 ) goes soft
-----------------------------------------------------------------------------------------
p(0, 5)^2/s(1,2),   |M|^2,     |Sub.term|^2,  |M|^2/|Sub.term|^2
-----------------------------------------------------------------------------------------
0.869211E-03 ,    0.667915E-06 , 0.000000E+00 ,    +Infinity
0.688787E-03 ,    0.173590E-06 , 0.000000E+00 ,    +Infinity
0.421426E-03 ,    0.641739E-06 , 0.000000E+00 ,    +Infinity
0.387974E-03 ,    0.135239E-05 , 0.000000E+00 ,    +Infinity
0.336094E-03 ,    0.942989E-05 , 0.000000E+00 ,    +Infinity
0.185406E-03 ,    0.122405E-04 , 0.000000E+00 ,    +Infinity
0.137483E-03 ,    0.423950E-04 , 0.000000E+00 ,    +Infinity
0.103854E-03 ,    0.168496E-03 , 0.000000E+00 ,    +Infinity
0.637535E-04 ,    0.679828E-04 , 0.000000E+00 ,    +Infinity
0.376601E-04 ,    0.707965E-03 , 0.000000E+00 ,    +Infinity
0.191145E-04 ,    0.486171E-02 , 0.317993E-02 ,    0.152887E+01
0.179479E-04 ,    0.696190E-03 , 0.479683E-03 ,    0.145135E+01
0.129797E-04 ,    0.415716E-02 , 0.321313E-02 ,    0.129380E+01
0.105788E-04 ,    0.125458E-01 , 0.125451E-01 ,    0.100005E+01
0.774787E-05 ,    0.299737E-02 , 0.299749E-02 ,    0.999957E+00
0.563720E-05 ,    0.574145E-02 , 0.574158E-02 ,    0.999977E+00
0.343292E-05 ,    0.147020E+00 , 0.147017E+00 ,    0.100002E+01
0.909122E-06 ,    0.216597E+01 , 0.216591E+01 ,    0.100003E+01
```

Further checks against MCFM: [Campbell,Ellis]

process	subprocesses
Drell-Yan (W)	$q\bar{q}' \to W^+(\to e^+\nu_e)g$ $qg \to W^+(\to e^+\nu_e)q'$
Drell-Yan (Z)	$q\bar{q} \to Z(\to e^+e^-)g$ $qg \to Z(\to e^+e^-)q$
Drell-Yan (Z+jet)	$q\bar{q} \to Z(\to e^+e^-)q'\bar{q}'$ $q\bar{q} \to Z(\to e^+e^-)q\bar{q}$ $q\bar{q} \to Z(\to e^+e^-)gg$ $q\bar{g} \to Z(\to e^+e^-)qg$ $g\bar{g} \to Z(\to e^+e^-)q\bar{q}$
top quark pair ($t\bar{t}$)	$q\bar{q} \to t(\to bl^+\nu_l)\bar{t}(\to \bar{b}l^-\bar{\nu}_l)g$ $qg \to t(\to bl^+\nu_l)\bar{t}(\to \bar{b}l^-\bar{\nu}_l)q$ $gg \to t(\to bl^+\nu_l)\bar{t}(\to \bar{b}l^-\bar{\nu}_l)g$
t-channel single top with massive b-quark	$gg \to t\bar{b}q\bar{q}'$ $qq' \to t\bar{b}q'q''$ $qq' \to t\bar{b}q'q''$ $qg \to t\bar{b}q'g$

Compare dipoles in single phase space points. No inconsistencies found.

$$\Rightarrow \sigma^{NLO} = \int_{m+1} \left[\left(d\sigma^R \right) - \left(d\sigma^A \right) \right] + \int_m \left[d\sigma^V + \int_1 d\sigma^A \right]$$

Fully automated integration over one-particle phase space would be more convient for user.
Phase space factorization:

$$d\phi(p_i, p_j, p_k; Q) = d\phi(\widetilde{p}_{ij}, \widetilde{p}_k; Q) \; \left[dp_i(\widetilde{p}_{ij}, \widetilde{p}_k) \right]$$

Integration over dipole:

$$\int \left[dp_i(\widetilde{p}_{ij}, \widetilde{p}_k) \right] \; \mathcal{D}_{ij,k}(p_1, ..., p_{m+1})$$
$$= - \mathcal{V}_{ij,k} \;\; {}_m< 1, .., \widetilde{ij}, .., \widetilde{k}, .., m+1 | \frac{\boldsymbol{T}_k \cdot \boldsymbol{T}_{ij}}{\boldsymbol{T}_{ij}^2} | 1, .., \widetilde{ij}, .., \widetilde{k}, .., m+1 >_m \;,$$

with $\mathcal{V}_{ij,k} = \int \left[dp_i(\widetilde{p}_{ij}, \widetilde{p}_k) \right] \; \frac{1}{2p_i \cdot p_j} \; < \boldsymbol{V}_{ij,k} > \equiv \; \frac{\alpha_S}{2\pi} \frac{1}{\Gamma(1-\epsilon)} \left(\frac{4\pi\mu^2}{2\widetilde{p}_{ij}\widetilde{p}_k} \right)^{\epsilon} \mathcal{V}_{ij}(\epsilon)$

Several non-trivial details:

- Integrated splitting function with initial state particles contains distributions ,e.g.

$$\mathcal{V}_{qg}(x;\epsilon) = C_F\left[\left(\frac{2}{1-x}\ln\frac{1}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ + \frac{2}{1-x}\ln(2-x)\right] + \delta(1-x)\left[\mathcal{V}_{qg}(\epsilon) - \frac{3}{2}C_F\right] + \mathcal{O}(\epsilon) \ ,$$

  with $\int_0^1 dx\, g(x)\,[\,\mathcal{V}(x)\,]_+ \equiv \int_0^1 dx\,[g(x) - g(1)]\,\mathcal{V}(x)$.

- $\Rightarrow$ Need to calculate $|\mathcal{M}|^2$ at x and at $x = 1$.
- For massive particles also $\delta(x_+ - x)$ and $(..)_{x_+}$ contributions with $x_+ = 1 - 4\frac{m_f^2}{Q^2}$ and $\int_0^1 \mathrm{d}x\left(f(x)\right)_{x_+} g(x) \equiv \int_0^1 \mathrm{d}x\, f(x)\Theta(x_+ - x)\,[g(x) - g(x_+)]$.

- Inclusion of α-parameter leads to nontrivial dependence on α [Nagy,Trocsanyi],[Campbell,Ellis].
  New integrals required.
- Assume only one mass scale.
- Inclusion of pdfs.
- Checks involve sampling over pdf $\rightarrow$ more involved
- Results dependend on regularisation scheme

$\rightarrow$ **How should the implementation look like?**

- Inclusion of α-parameter leads to nontrivial dependence on α [Nagy,Trocsanyi],[Campbell,Ellis].

  New integrals required.
- Assume only one mass scale.
- Inclusion of pdfs.
- Checks involve sampling over pdf $\rightarrow$ more involved
- Results dependend on regularisation scheme

$\rightarrow$ **How should the implementation look like?**

Introduce only one new subroutine:

```
intdipoles(p,x,epssq,eps,finite)
```

Input: phase space point $p = p(0:3, nexternal)$
momentum fraction $x = x(2)$

Output: Coefficients of $\frac{1}{\epsilon^2}$- ,$\frac{1}{\epsilon}$-, and finite-terms.

Coefficients as 5-dimensional vectors:

1. $\delta(1-x)$ terms
2. terms regular in x
3. $+$-distribution terms (singular at $x = 1$)

and for the massive case:

4. $\delta(x_+ - x)$ terms
5. x_+-distribution terms

Which parts should be expanded?

$$\mathcal{V}_{ij,k} = \int \left[dp_i(\widetilde{p}_{ij},\widetilde{p}_k)\right] \frac{1}{2p_i \cdot p_j} < \boldsymbol{V}_{ij,k} > \equiv \frac{\alpha_S}{2\pi} \underbrace{\frac{1}{\Gamma(1-\epsilon)}\left(\frac{4\pi\mu^2}{s_{ij,k}}\right)^{\epsilon}}_{=1+\left(\log\left(\frac{4\pi\mu}{s_{ij,k}}\right)-\gamma\right)\epsilon+O(\epsilon^2)} \mathcal{V}_{ij}(\epsilon)$$

$\Rightarrow$ Expansion of the whole expression leads to artificial dependence of the finite terms on the renormalisation scale μ and has to be cancelled by virtual corrections

What is the 'best' expansion ?

- At the moment the whole expression on the previous slide is expanded in the implementation, using $\overline{MS}$ scheme.
- Second implementation planned: Factorize out $I^{(1)}$-operator: [Catani]

Write 1-loop matrixelement as:

$$|\mathcal{M}_m^{(1)}(\mu^2;\{p\})\rangle_{\rm RS} = \boldsymbol{I}^{(1)}(\epsilon,\mu^2;\{p\})\,|\mathcal{M}_m^{(0)}(\mu^2;\{p\})\rangle_{\rm RS} + |\mathcal{M}_m^{(1)\,{\rm fin}}(\mu^2;\{p\})\rangle_{\rm RS} \quad .$$

Singularities are in $I^{(1)}$:

$$\boldsymbol{I}^{(1)}(\epsilon,\mu^2;\{p\}) = \frac{1}{2}\frac{e^{-\epsilon\psi(1)}}{\Gamma(1-\epsilon)}\sum_i \frac{1}{\boldsymbol{T}_i^2}\,\mathcal{V}_i^{\rm sing}(\epsilon)\,\sum_{j\neq i}\boldsymbol{T}_i\cdot\boldsymbol{T}_j\,\left(\frac{\mu^2 e^{-i\lambda_{ij}\pi}}{2p_i\cdot p_j}\right)^{\epsilon} \quad ,$$

with $\psi(1) = -\gamma_E$, $e^{-i\lambda_{ij}\pi}$ unitarity phase.

$$\mathcal{V}_i^{\rm sing}(\epsilon) = \boldsymbol{T}_i^2\frac{1}{\epsilon^2} + \gamma_i\,\frac{1}{\epsilon} \quad ,$$

$\boldsymbol{T}_i^2 = C_F, C_A$. $\gamma_i = \frac{3}{2}C_F, \frac{11}{6}C_A - \frac{2}{3}T_R N_f$.

Advantages when using $I^{(1)}$-operator:

- Clear and well defined way used by many people
- No need to calculate singular terms numerically, because a priori clear how they look like

Disadvantage:

- Generalization for massive case gets slightly more complicated [Catani, Dittmaier, Trocsanyi]

$$\boldsymbol{I}_m^{\rm R.S.}(\epsilon,\mu^2;\{p_i,m_i\}) = \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\Bigg\{ q\,\frac{1}{2}\left(\frac{\beta_0}{\epsilon}-\tilde{\beta}_0^{\rm R.S.}\right) + \sum_{\substack{j,k=1\\k\neq j}}^{m} \boldsymbol{T}_j\cdot\boldsymbol{T}_k\left(\frac{\mu^2}{|s_{jk}|}\right)^{\epsilon}\left[\mathcal{V}_{jk}^{(\rm cc)}(s_{jk};m_j,m_k;\epsilon)+\frac{1}{v_{jk}}\left(\frac{1}{\epsilon}\,\mathrm{i}\pi-\frac{\pi^2}{2}\right)\Theta(s_{jk})\right] - \sum_{j=1}^{m}\Gamma_j^{\rm R.S.}(\mu,m_j;\epsilon)\Bigg\}\,.$$

β_0 is the first coefficient of the QCD beta function:

$$\beta_0 = \frac{11}{3}\,C_A - \frac{4}{3}\,T_R(N_f + N_F)\,,$$

$\mathcal{V}^{(\mathrm{cc})}_{jk}$ controls colour correlation. For non-vanishing masses:

$$\mathcal{V}^{(\mathrm{cc})}_{jk}(s_{jk};m_j,m_k;\epsilon) = \frac{1}{2\epsilon}\,\frac{1}{v_{jk}}\,\ln\frac{1-v_{jk}}{1+v_{jk}} - \frac{1}{4}\left(\ln^2\frac{m_j^2}{|s_{jk}|} + \ln^2\frac{m_k^2}{|s_{jk}|}\right) - \frac{\pi^2}{6},$$

for one or two vanishing masses:

$$\mathcal{V}^{(\mathrm{cc})}_{jk}(s_{jk};m_j,0;\epsilon) = \frac{1}{2\epsilon^2} + \frac{1}{2\epsilon}\ln\frac{m_j^2}{|s_{jk}|} - \frac{1}{4}\ln^2\frac{m_j^2}{|s_{jk}|} - \frac{\pi^2}{12}\,,$$

$$\mathcal{V}^{(\mathrm{cc})}_{jk}(s_{jk};0,0;\epsilon) = \frac{1}{\epsilon^2}\,.$$

$\Gamma_j^{\rm R.S.}$ depend on parton flavour and masses.
For massive quarks:

$$\Gamma_q(\mu, m_q; \epsilon) = \boldsymbol{T}_q^2 \left(\frac{1}{\epsilon} - \ln \frac{m_q^2}{\mu^2} - 2 \right) + \gamma_q \ln \frac{m_q^2}{\mu^2} = C_F \left[\frac{1}{\epsilon} + \frac{1}{2} \ln \frac{m_q^2}{\mu^2} - 2 \right] .$$

For gluons and massless quarks:

$$\Gamma_g^{\rm R.S.}(\mu, m_{\{F\}}; \epsilon) = \frac{1}{\epsilon} \gamma_g - \tilde{\gamma}_g^{\rm R.S.} - \frac{2}{3} T_R \sum_{F=1}^{N_F} \ln \frac{m_F^2}{\mu^2} ,$$

$$\Gamma_q^{\rm R.S.}(\mu, 0; \epsilon) = \frac{1}{\epsilon} \gamma_q - \tilde{\gamma}_q^{\rm R.S.} ,$$

$\tilde{\gamma}_j^{\rm R.S.}$ are contributions depending on regularization scheme:

$$\tilde{\gamma}_j^{\rm CDR} = 0 , \qquad \tilde{\gamma}_{j=q,\bar{q}}^{\rm DR} = \frac{1}{2} C_F , \qquad \tilde{\gamma}_{j=g}^{\rm DR} = \frac{1}{6} C_A .$$

Regularization scheme dependence:

- Sum (real emission + virtual corrections) independent of regularization scheme
- But several parts depend on regularization scheme.

MadGraph: external particles 4-dimensional, integration over 1-particle phase space d-dimensional
$\rightarrow$ 't Hooft-Veltman scheme (tHV) (similar to conventional dimensional regularization (CDR)).
On the other hand dimensional reduction (DR) often used.
$\Rightarrow$ Differences in finite terms. $\rightarrow$ Both methods are implemented.

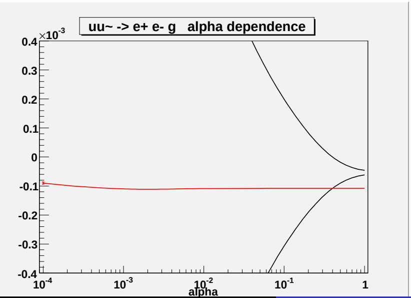
Phase space restriction:

Integrated dipoles depend on α-parameter, but final result must be independent.

$$\int_{n+1} \left(d\sigma^R - d\sigma^A\right) + \int_n \text{(finite parts of int. dip.)} = \text{const}$$

$\Rightarrow$ Must be checked for all possible dipoles.

Example:



What about the pdfs?

- Initial state emitter/spectator involve integration over momentum fraction. $\rightarrow$ Good point to introduce pdfs.

But maybe user wants to use his/her own pdfs ?!

$\Rightarrow$ `GETPDF(x1, x2, u, ubar, PDF)`

'Dummy' routine GETPDF can contain link to MadEvent pdfs or user can link his/her own pdfs.

What about the pdfs?

- Initial state emitter/spectator involve integration over momentum fraction. $\rightarrow$ Good point to introduce pdfs.

But maybe user wants to use his/her own pdfs ?!

$\Rightarrow$ `GETPDF(x1, x2, u, ubar, PDF)`

'Dummy' routine GETPDF can contain link to MadEvent pdfs or user can link his/her own pdfs.

What about the pdfs?

- Initial state emitter/spectator involve integration over momentum fraction. $\rightarrow$ Good point to introduce pdfs.

But maybe user wants to use his/her own pdfs ?!

$\Rightarrow$ `GETPDF(x1, x2, u, ubar, PDF)`

'Dummy' routine GETPDF can contain link to MadEvent pdfs or user can link his/her own pdfs.

To Do:

- Finish checking of alpha dependence
- Implementation of the $I^{(1)}$-operator
- Inclusion of pdf's
- Check the implementation of singular/finite terms

Conclusions

- Dipole subtraction formalism ensures finiteness of real emission terms and virtual corrections.
- MadDipole: Fully automated implementation of dipole formalism.
- Numerous checks to ensure correctness.
- Automated integration over one particle phase space desirable.
  $\longrightarrow$ In progress.
- Principle implementation done.
  Needs futher testing/improvements.