Fittino: Reverse engineering of supersymmetry

How to get the SUSY blueprint

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Inverse modelling

Often: What you want ≠ what you get

Also often: Problems “easy” to solve in one direction, very difficult in opposite direction (one-way functions)

For many problems in science and engineering, getting what you want requires to follow difficult direction

Inverse modelling: observations → model parameters

Examples:

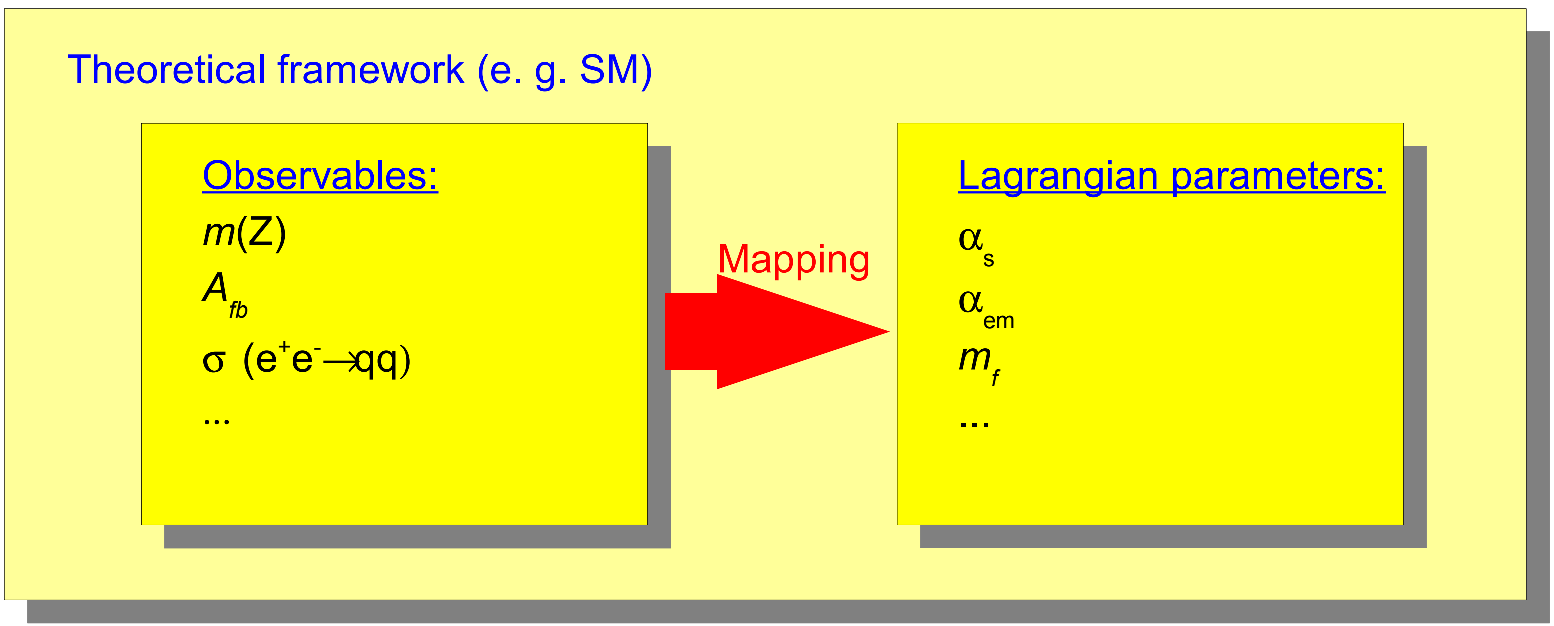
- Reverse engineering Technical device/software → blueprint
- Scattering experiments Angles, energies, particle types → particle/interaction properties
- Remote sensing E. g. multiple 1D/2D meas. → 3D parameter map

Inverse problems are often “ill-posed” (Hadamard)

Inverse modelling in particle physics

Experimentalists provide: σ, BR, asymmetries, ...

Theorists provide: mapping: model parameters → observables for various theories



Need procedure to connect measurements to theory parameters for given theoretical framework

Supersymmetry (SUSY)

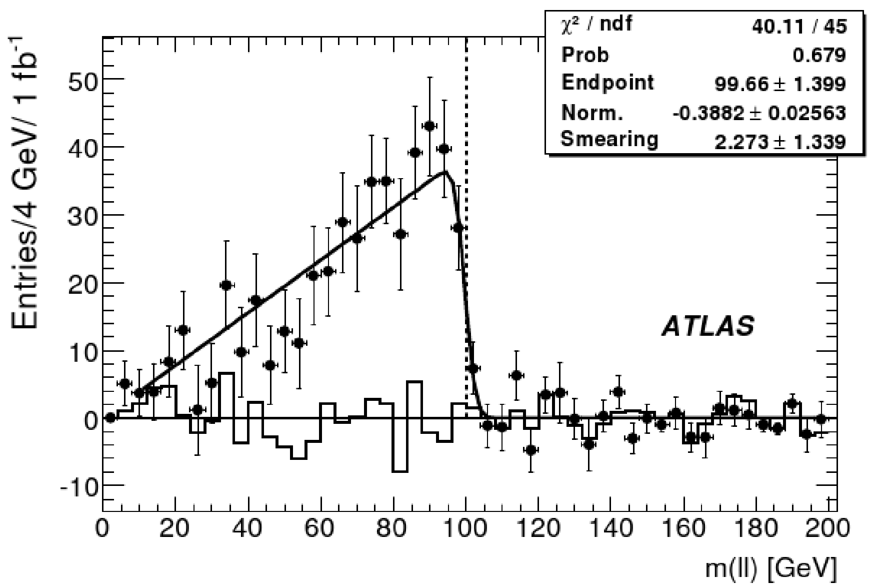
- Attractive candidate for extended SM of particle physics
- Remedies various shortcomings of SM
  - Hierarchy problem
  - No high-scale unification of gauge couplings
  - Lack of dark matter candidate
- SUSY as solution to SM problems only satisfactory if SUSY shows up at the TeV scale

→ Exciting prospects for LHC, ILC

→ Should get prepared for inverse modelling of supersymmetry

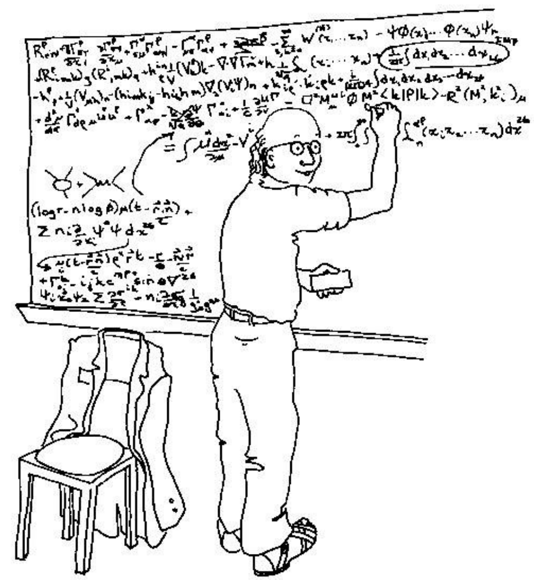
Inverse modelling of supersymmetry

Experiment:



Observables: m_{ll}^{max}, m_h, ...

Theory:



Parameters: tan β, μ, M_1, ...

Questions to answer from measurements:

- What is the underlying (SUSY) model?
  → basically trial and error
- What are the values of its parameters?
  → needs sophisticated techniques

Inverse modelling of supersymmetry

At tree level, some sectors (e. g. chargino, chargino+neutralino) can be treated separately.

At loop level, in principle every observable depends on every parameter.

Complicated mutual dependence of the various parameters.

Approximate picture (not quite correct since non-linear mapping):

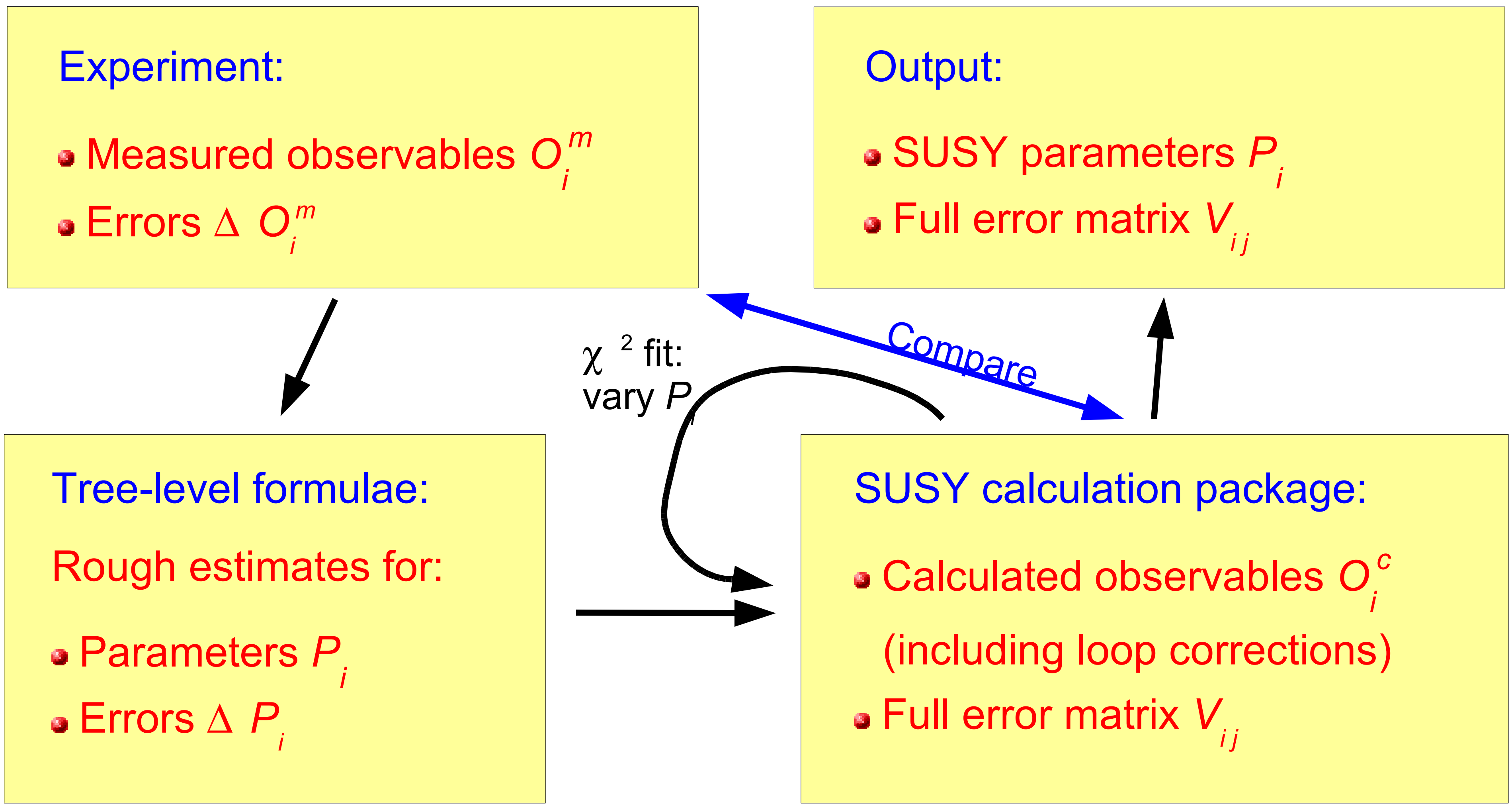
$$\begin{bmatrix} P_1 \\ P_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} \Box & & & 0 \\ & \Box & & \\ & & \Box & \\ 0 & & & \ddots \end{bmatrix} \begin{bmatrix} O_1 \\ O_2 \\ \vdots \end{bmatrix}$$

Tree level

$$\begin{bmatrix} P_1 \\ P_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} \Box & & & \neq 0 \\ & \Box & & \\ & & \Box & \\ \neq 0 & & & \ddots \end{bmatrix} \begin{bmatrix} O_1 \\ O_2 \\ \vdots \end{bmatrix}$$

Loop level

Iterative approach



Programs

Several programs available which allow reconstruction of SUSY parameters from measurements:

- Sfitter (R. Lafaye, T. Plehn, M. Rauch, D. Zerwas)
- Fittino (P. Bechtle, K. Desch, M. Uhlenbrock, P. W.)

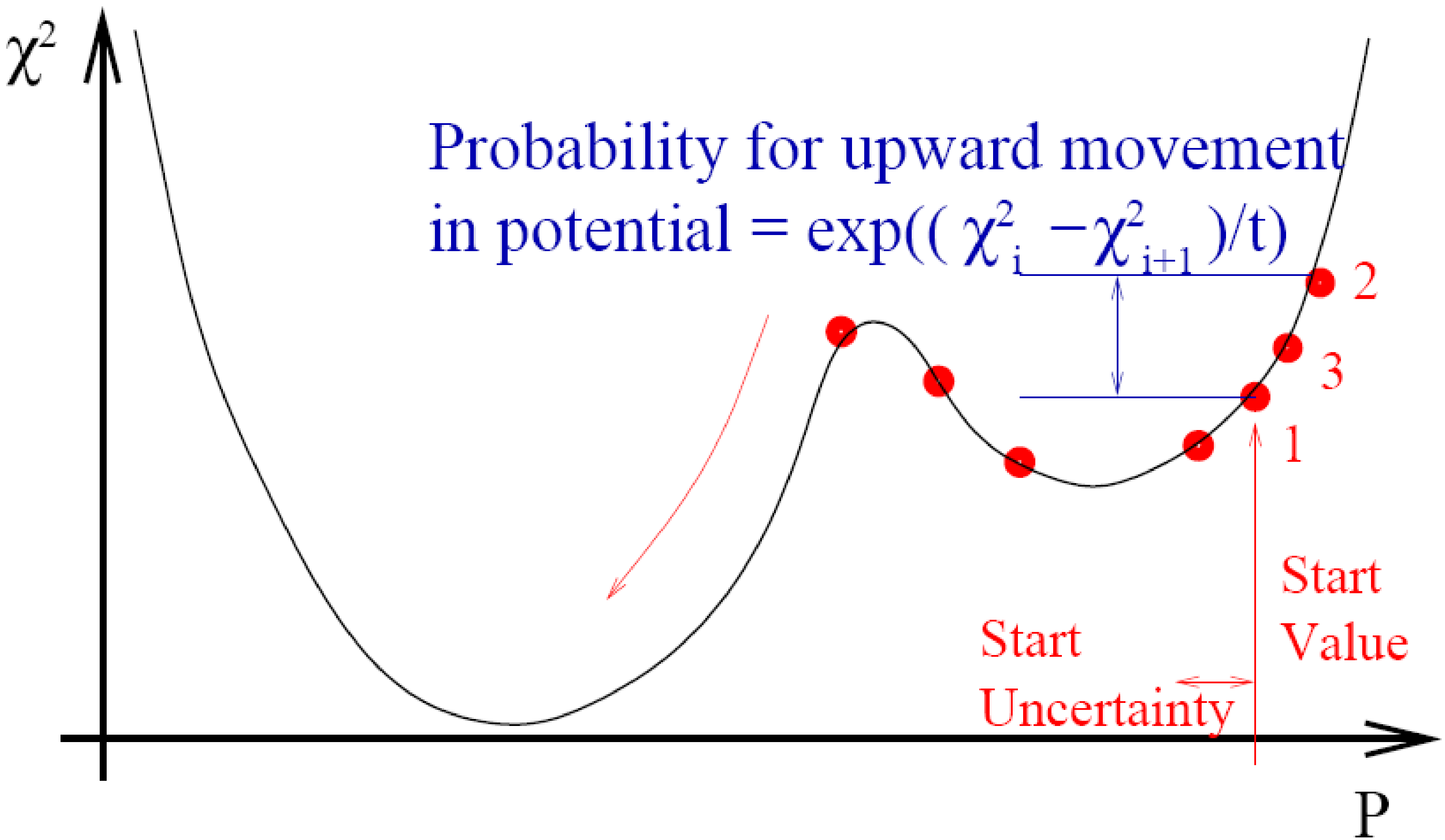
  `http://www-flc.desy.de/fittino`
- Gfitter (H. Flächer, M. Goebel, J. Haller, A. Höcker, K. Mönig, J. Stelzer)

  `http://gfitter.desy.de` (currently only SM and 2HDM)

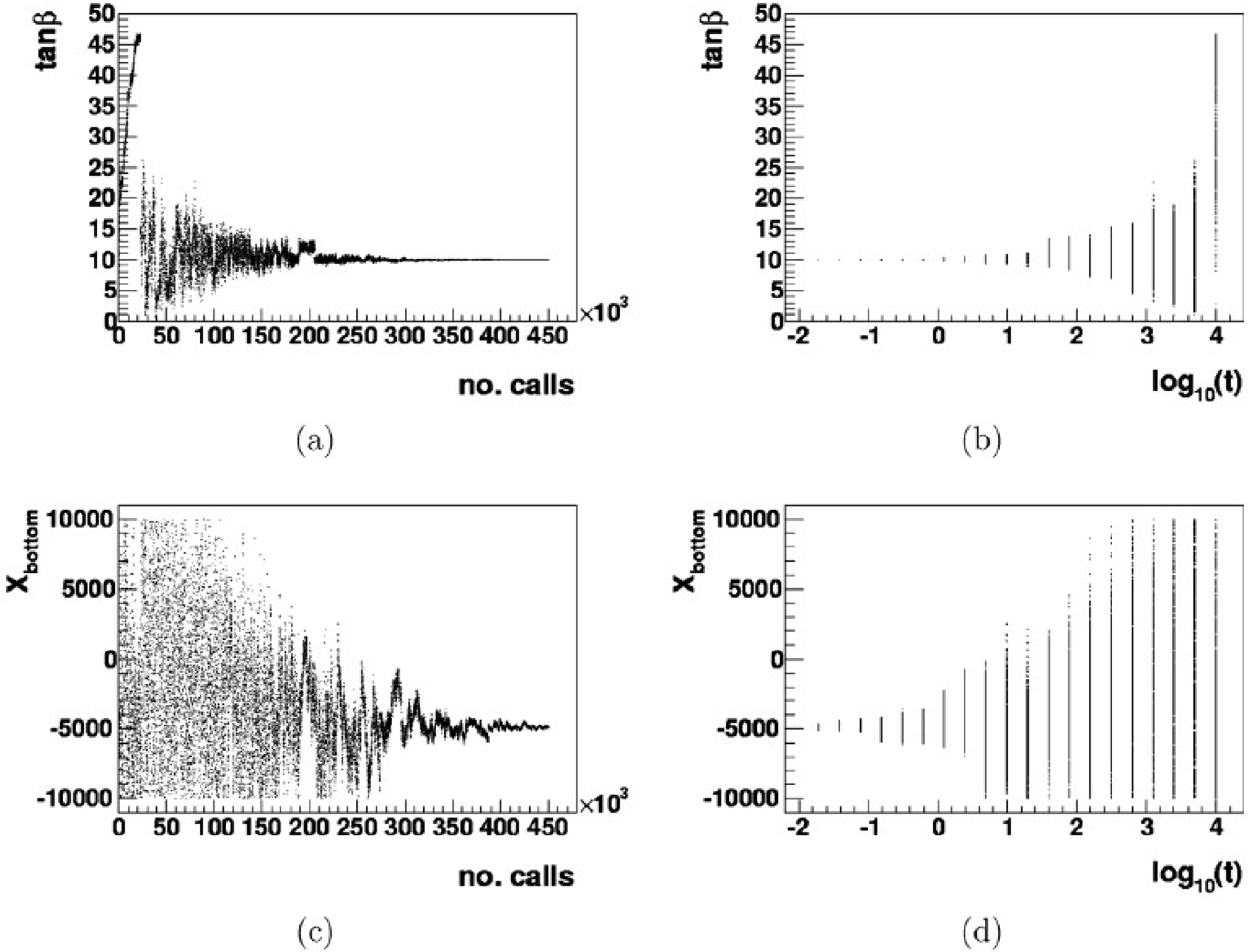
Fittino

- C++ package reconstructing SUSY parameters through χ^2 minimisation (using full correlation information)
- Currently supported SUSY models:
  mSUGRA, GMSB, AMSB, MSSM24, NMSSM
- χ^2 minimisation using MINUIT or simulated annealing
- Calculation of likelihood maps using Markov chain Monte Carlo technique
- Theory predictions from `SPheno` (W. Porod) and `Mastercode` (Buchmüller *et al.*)

Simulated annealing



Simulated annealing in action



Markov chain Monte Carlo

Markov chain = sequence of points x_i (i=1,...n) in parameter space
with associated likelihood

New point x_{n+1} randomly chosen according to proposal PDF is added to chain if $\mathcal{L}(x_{n+1}) > \mathcal{L}(x_n)$

Otherwise it is accepted with probability $\mathcal{L}(x_{n+1})/\mathcal{L}(x_n)$

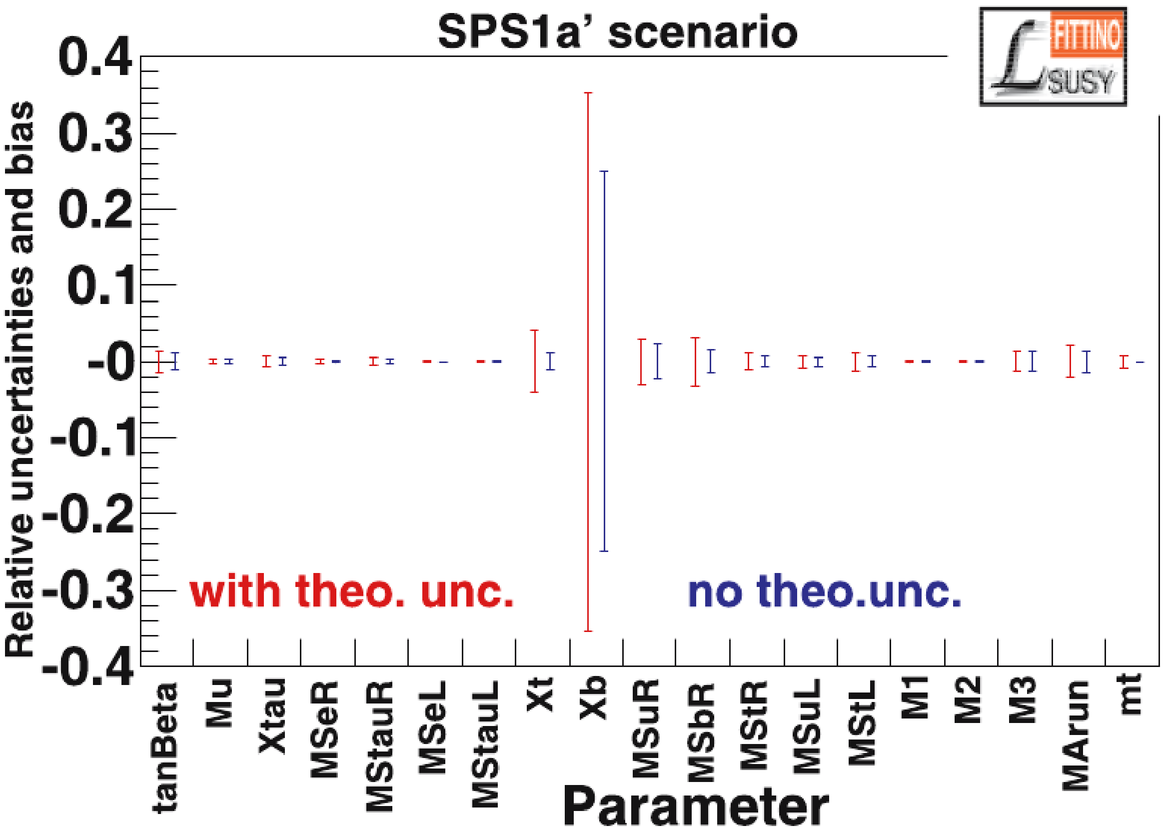
If proposal PDF is chosen properly, sampling density of points x_i in Markov chain proportional to likelihood

History

Fittino originally developed to estimate the potential of combined LHC and ILC measurements (Eur. Phys. J. **C46**, 533-544, 2006)

Relatively far future option

Recently we put focus on presence and near future



Available LE measurements

Wealth of "low" energy (LE) measurements from
past and present experiments:

- LEP, SLC
- B factories
- WMAP
- ...

They put constraints on SUSY

observable	meas. value	constraint	theo. uncert.
α_{em}	127.925	$\pm$ 0.016	
α_S	0.1176	$\pm$ 0.0020	
G_F (GeV^{-2})	1.16637×10^{-5}	$\pm 0.00001 \times 10^{-5}$	
m_Z (GeV)	91.1875	$\pm$ 0.0021	
m_W (GeV)	80.399	$\pm$ 0.025	$\pm$ 0.010
m_c (GeV)	1.27	$\pm$ 0.11	
m_b (GeV)	4.20	$\pm$ 0.17	
m_t (GeV)	172.4	$\pm$ 1.2	
m_τ (GeV)	1.77684	$\pm$ 0.00017	
m_h (GeV)	> 114.4		$\pm$ 3.0
Γ_Z (MeV)	2495.2	$\pm$ 2.3	$\pm$ 1.0
Δa_μ	30.2×10^{-10}	$\pm 8.8 \times 10^{-10}$	$\pm 2.0 \times 10^{-10}$
σ^0_{had} (nb)	41.540	$\pm$ 0.037	
R_ℓ	20.767	$\pm$ 0.025	
R_b	0.21629	$\pm$ 0.00066	
R_c	0.1721	$\pm$ 0.003	
A^ℓ_{FB}	0.01714	$\pm$ 0.00095	
A^b_{FB}	0.0992	$\pm$ 0.0016	
A^c_{FB}	0.0707	$\pm$ 0.0035	
A_ℓ(SLD)	0.1513	$\pm$ 0.0021	
$A_\ell(P_\tau)$	0.1465	$\pm$ 0.0032	
A_b	0.923	$\pm$ 0.020	
A_c	0.670	$\pm$ 0.027	
$\sin^2\theta^\ell_W(Q_{fb})$	0.2324	$\pm$ 0.0012	
Ωh^2	0.1099	$\pm$ 0.0062	$\pm$ 0.012
$BR(B_d \to \mu^+\mu^-)$		$< 2.3 \times 10^{-8}$	$\pm 0.01 \times 10^{-9}$
$BR(B_s \to \mu^+\mu^-)$		$< 4.7 \times 10^{-8}$	$\pm 0.02 \times 10^{-8}$
$R(b \to s\gamma)$	1.117	$\pm 0.076 \pm 0.082$	$\pm$ 0.050
$R(B \to \tau\nu)$	1.15	$\pm$ 0.40	
$R(B \to X_s\ell\ell)$	0.99	$\pm$ 0.32	
$R(K \to \mu\nu)$	1.008	$\pm$ 0.014	
$R(K \to \pi\nu\bar{\nu})$		< 4.5	
$R(\Delta m_{B_s})$	1.11	$\pm$ 0.01	$\pm$ 0.32
$R(\Delta m_{B_s})/R(\Delta m_{B_d})$	1.09	$\pm$ 0.01	$\pm$ 0.16
$R(\Delta\epsilon_K)$	0.92	$\pm$ 0.14	

SM+mSUGRA fit to LE measurements

Fit of α_{em}, α_s, G_F, m_Z, m_b, m_t, m_τ and mSUGRA parameters to LE measurements:

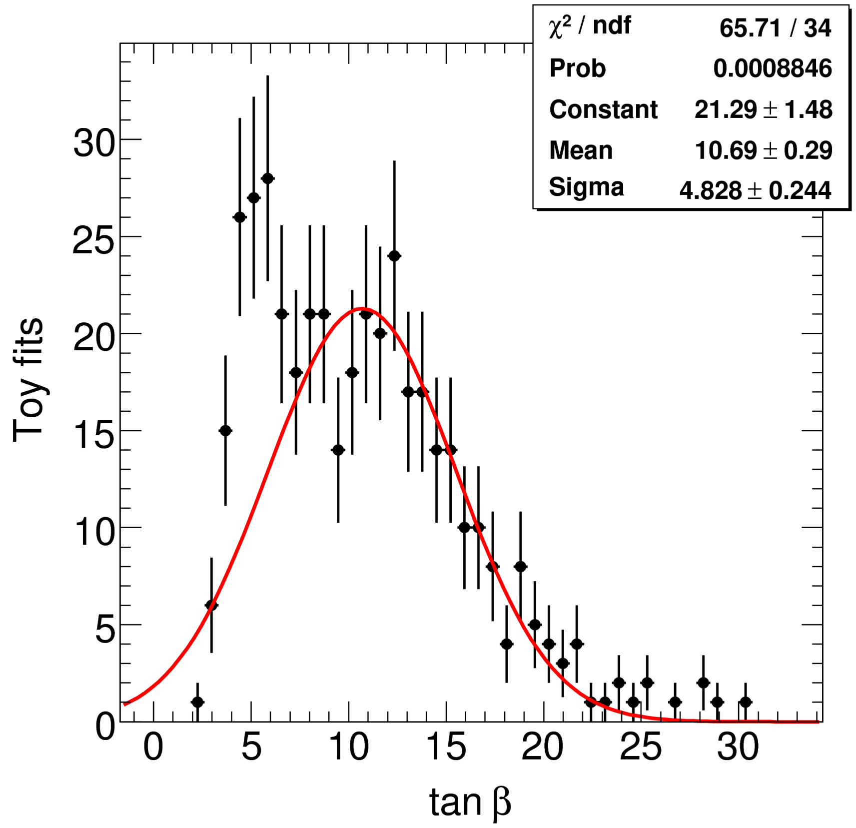
Best agreement for:

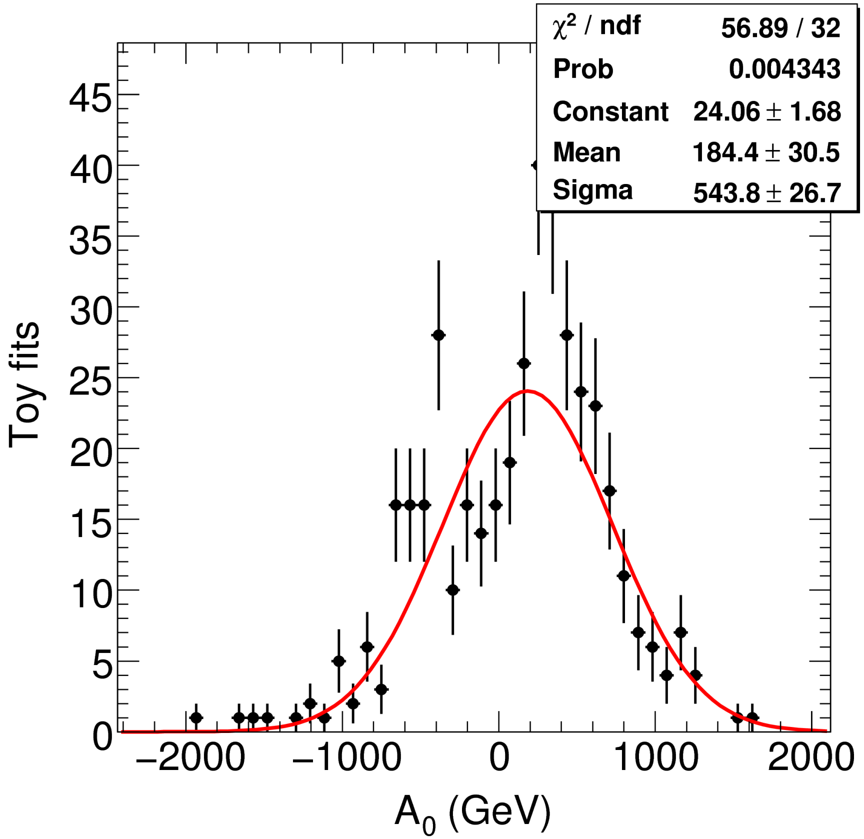
Parameter	Value	Uncertainty
$\tan\beta$	12.4	4.8
A_0 (GeV)	337	544
M_0 (GeV)	71	18
$M_{1/2}$ (GeV)	323	62

Uncertainties obtained from toy fits to observables smeared around nominal values for best fit parameters

SM+mSUGRA fit to LE measurements

Parameter distributions for toy fits:





SM+mSUGRA fit to LE measurements

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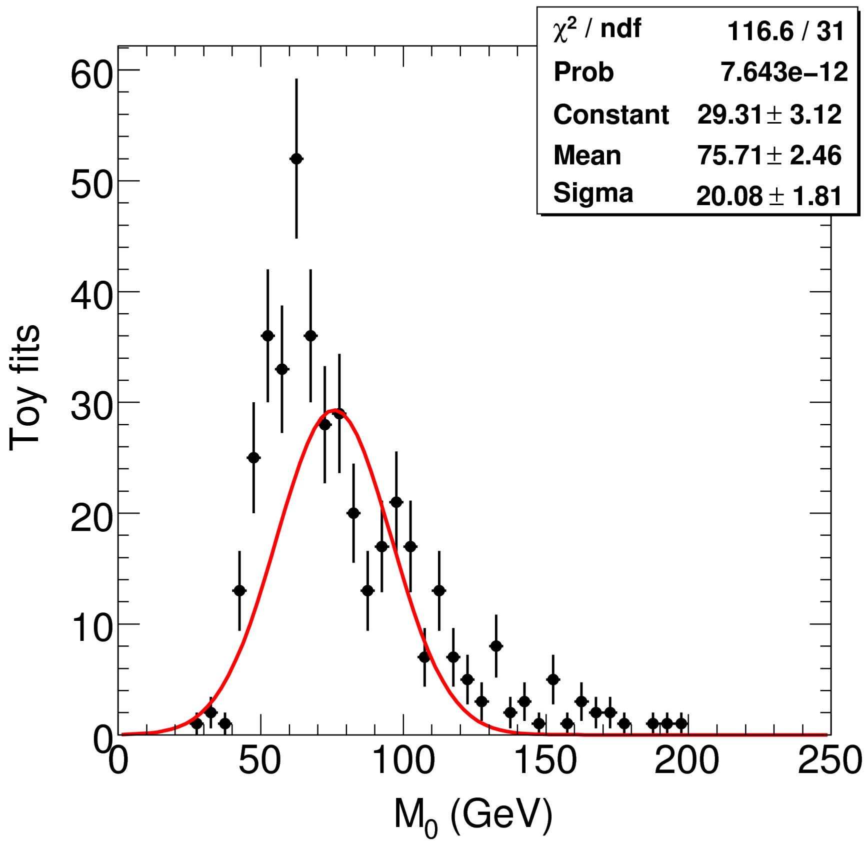
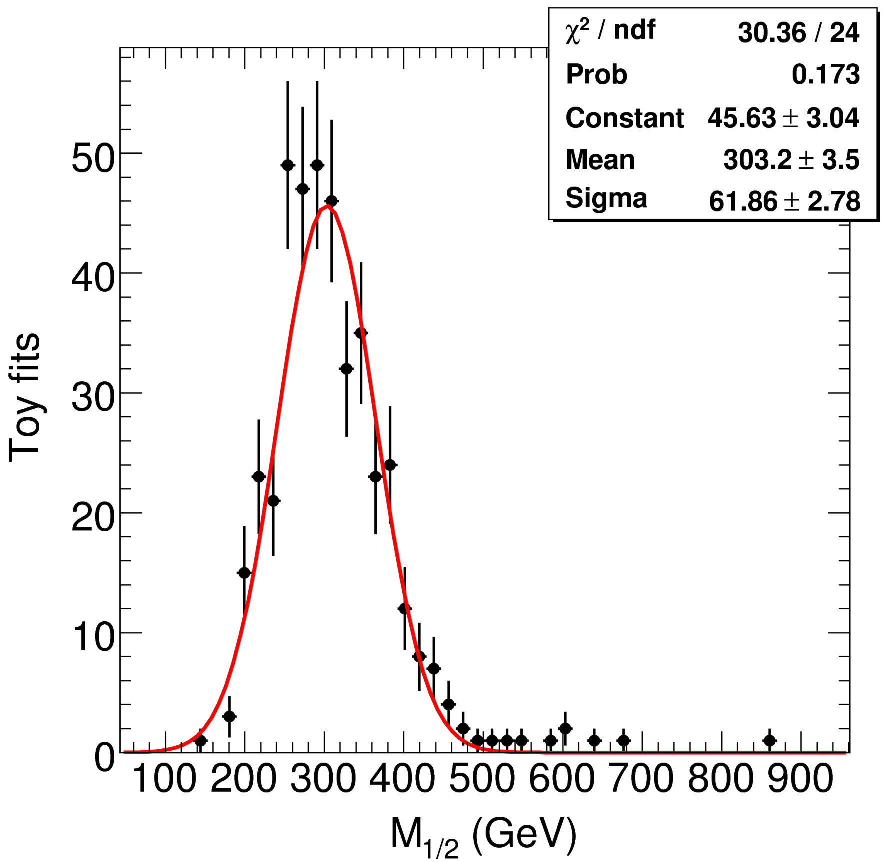
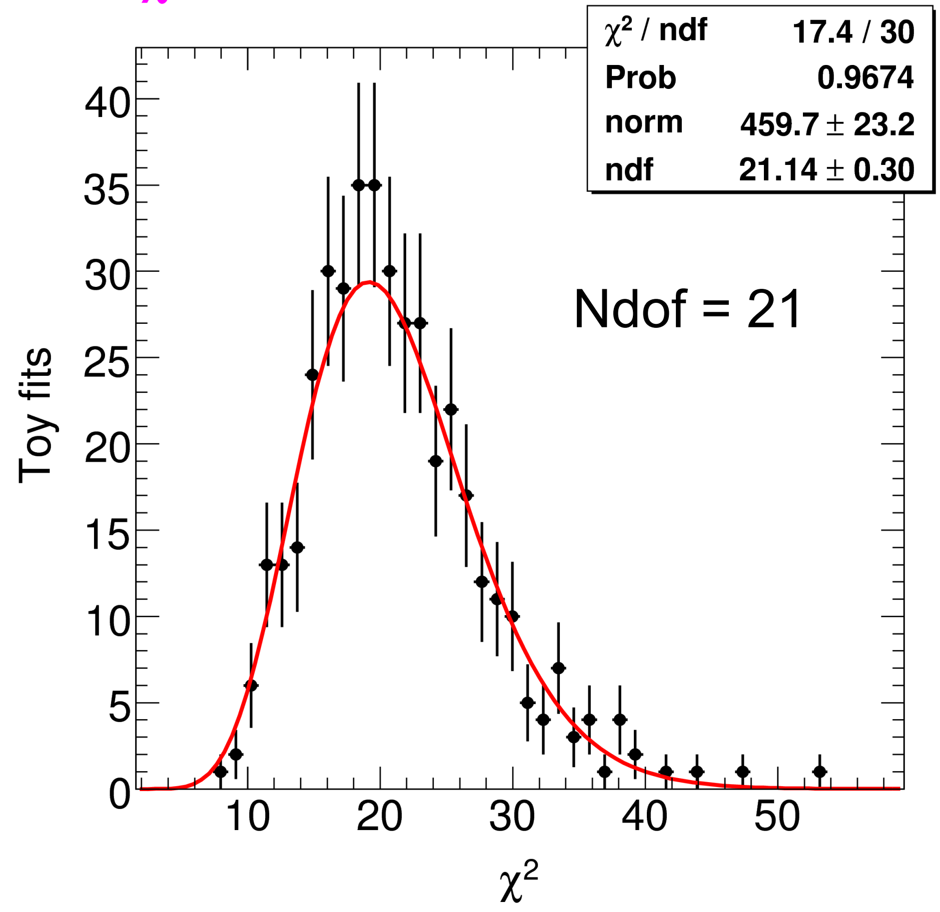




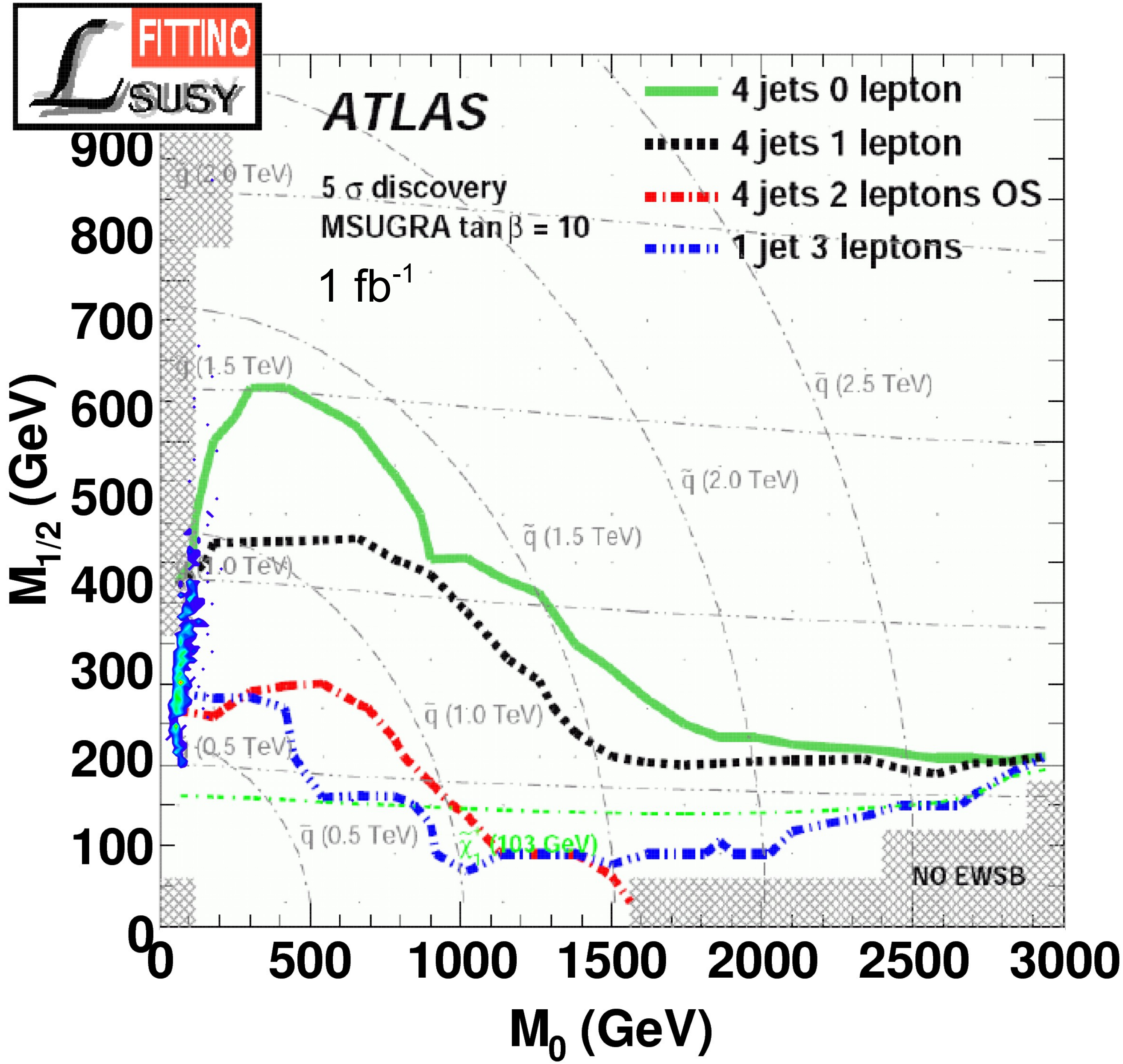
SM+mSUGRA fit to LE measurements

Distributions for toy fits:

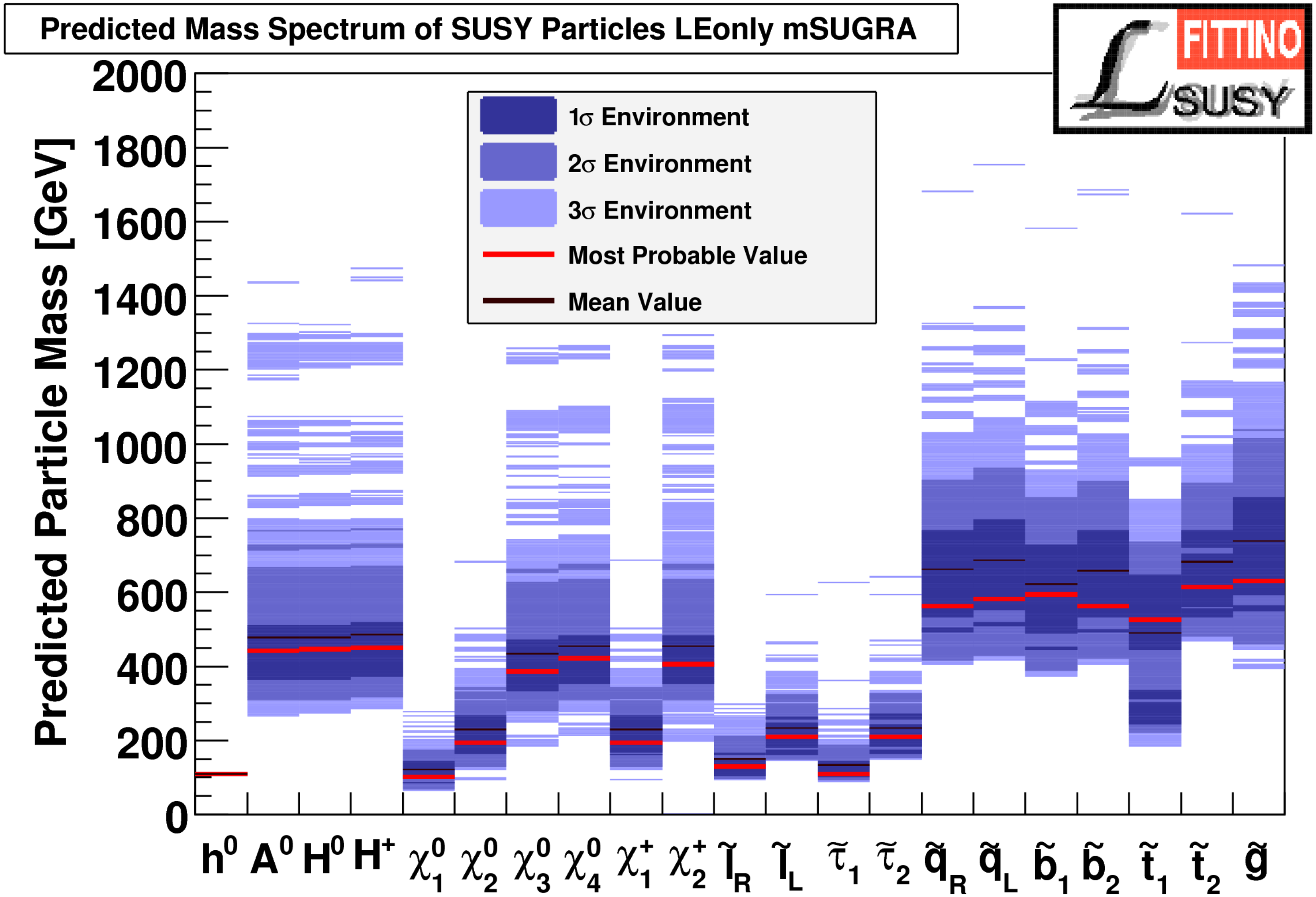
χ^2 distribution:



Comparison: ATLAS discovery potential ↔ mSUGRA fit results

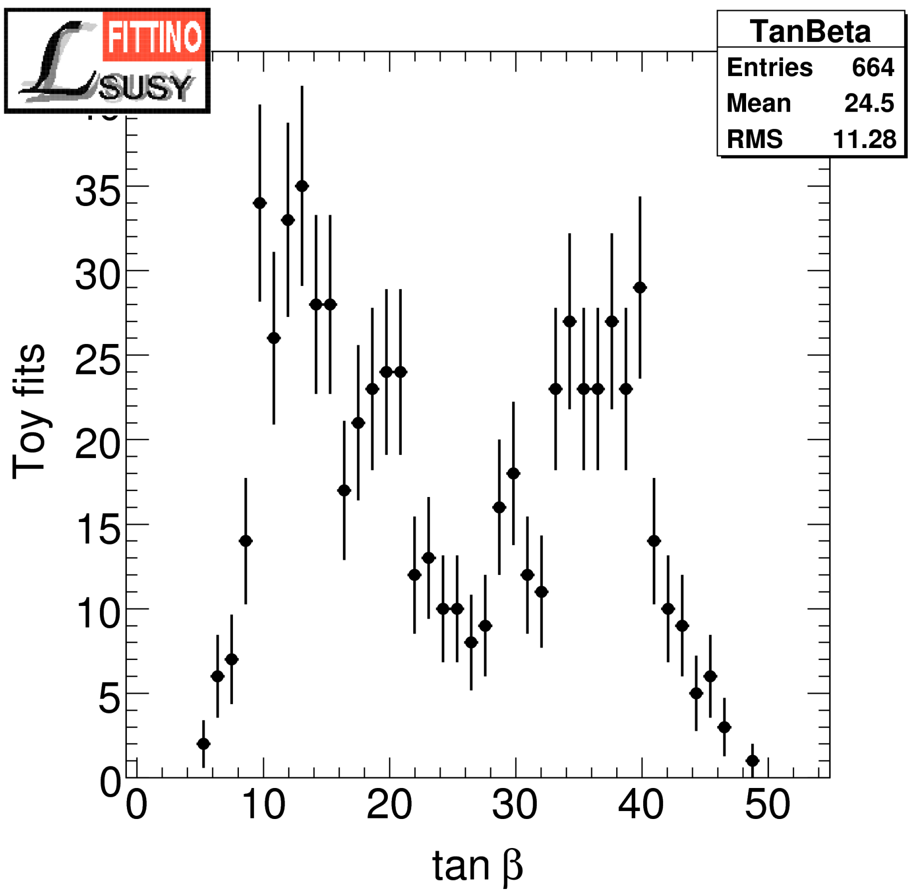


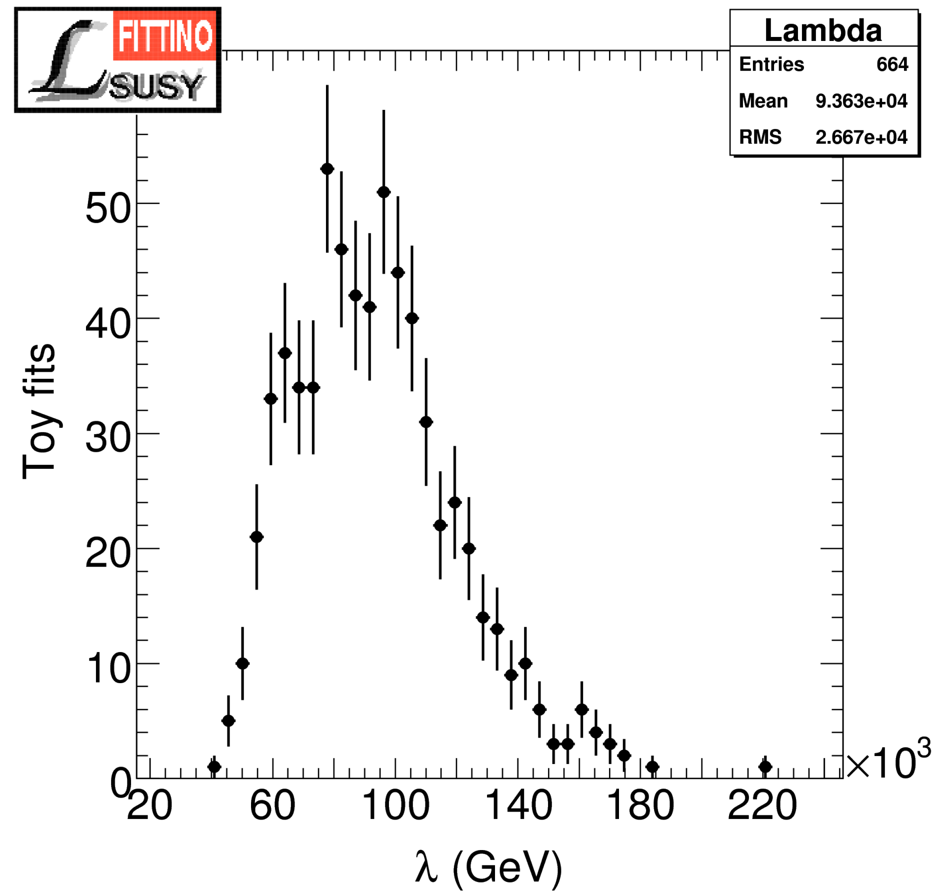
Predicted mass spectrum from mSUGRA fit



SM+GMSB fit to LE measurements

Parameter distributions for toy fits with observables smeared around nominal values for parameters of best fit:

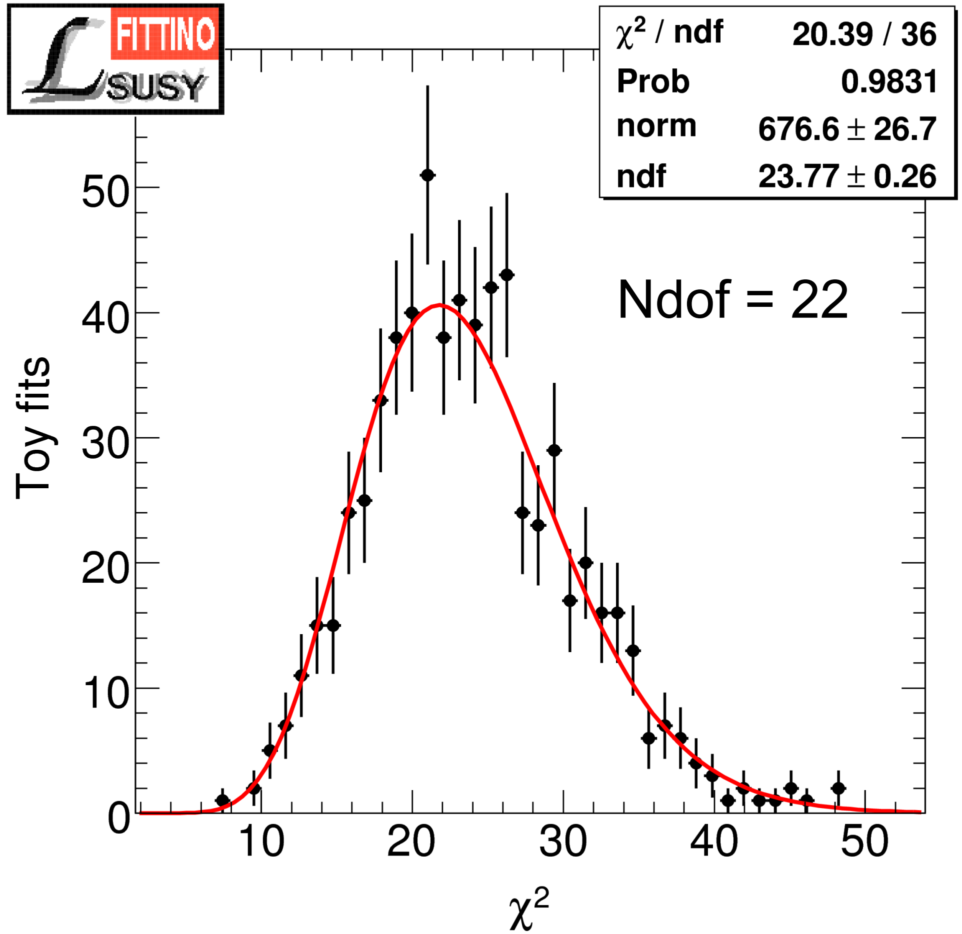




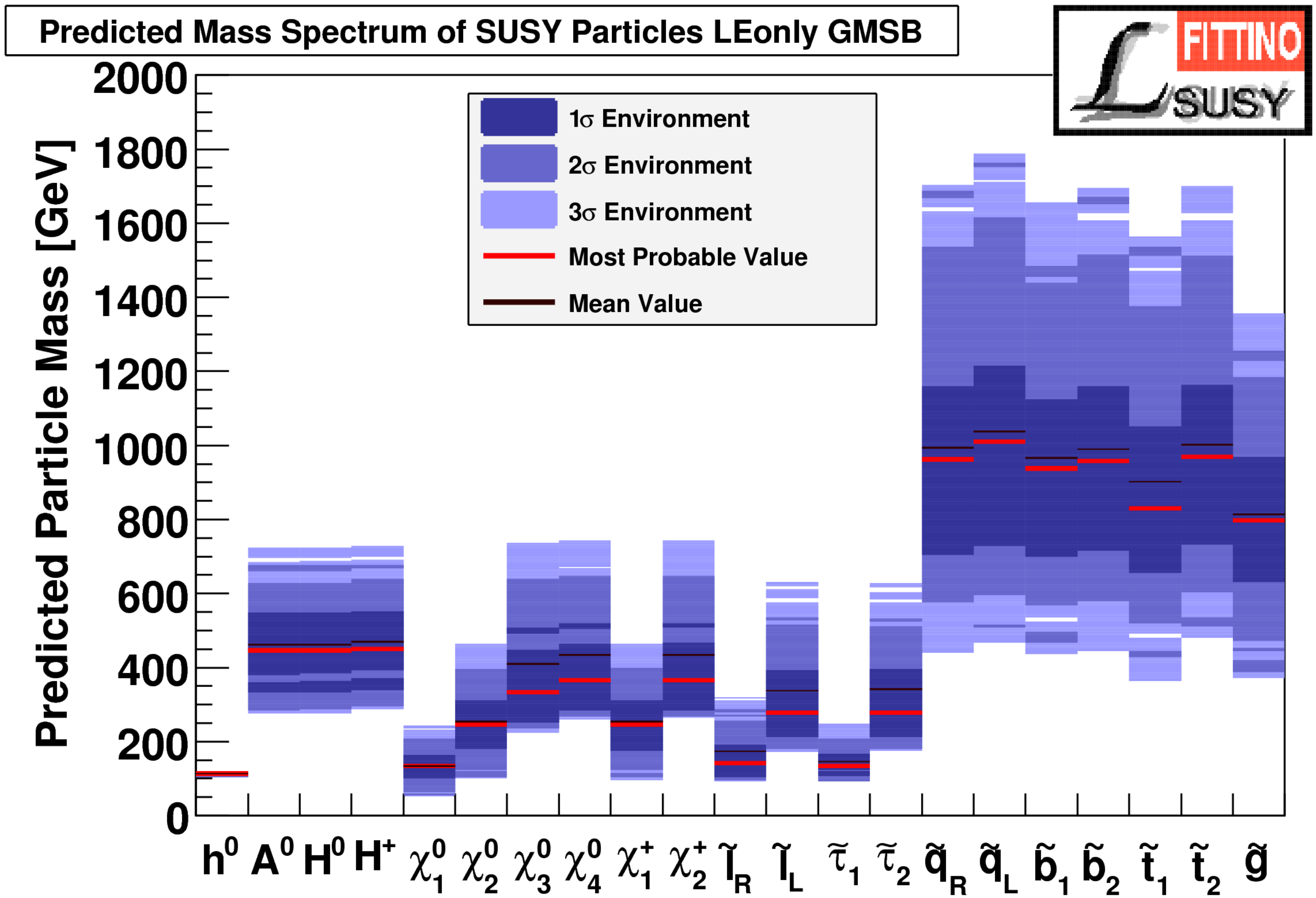
(SUSY breaking scale)

SM+GMSB fit to LE measurements

χ^2 distribution for toy fits:

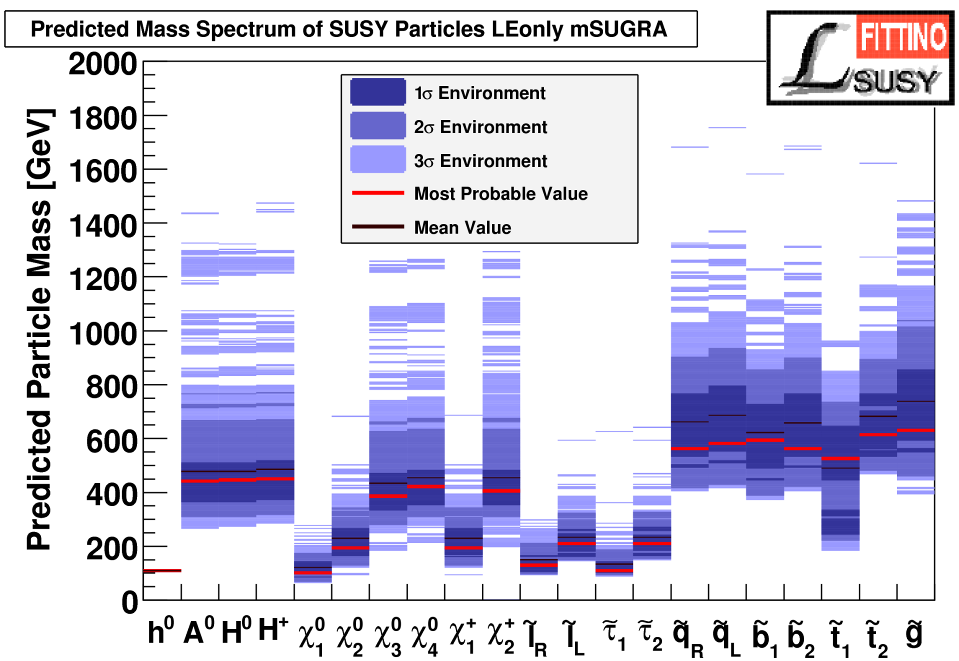


Predicted mass spectrum from SM+GMSB fit

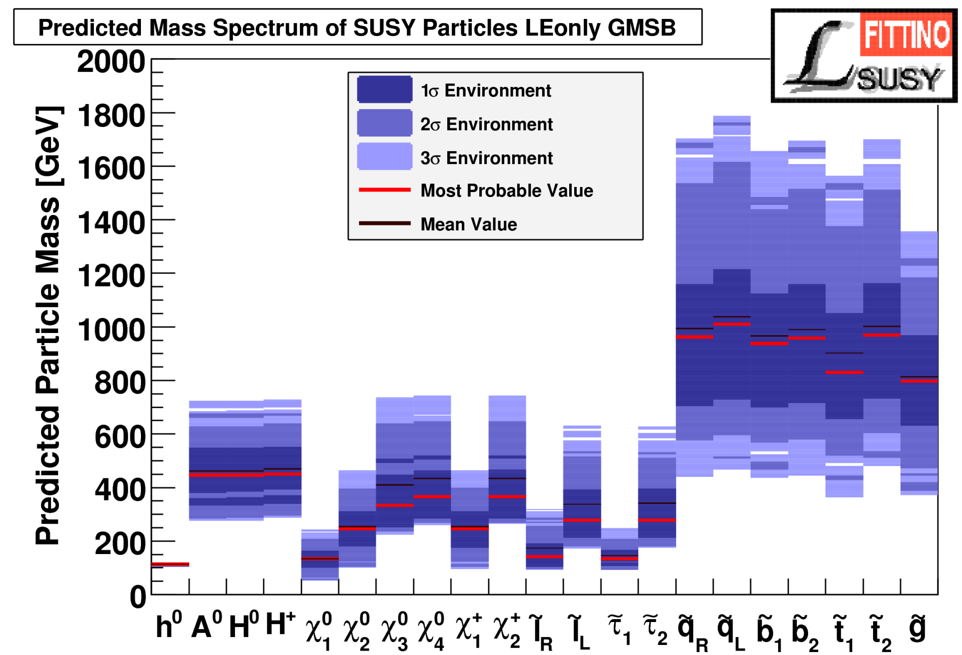


Comparison: mSUGRA ↔ GMSB spectrum

mSUGRA



GMSB



LHC “measurements”

LHC is just around the corner and will hopefully shower us with exciting new measurements.

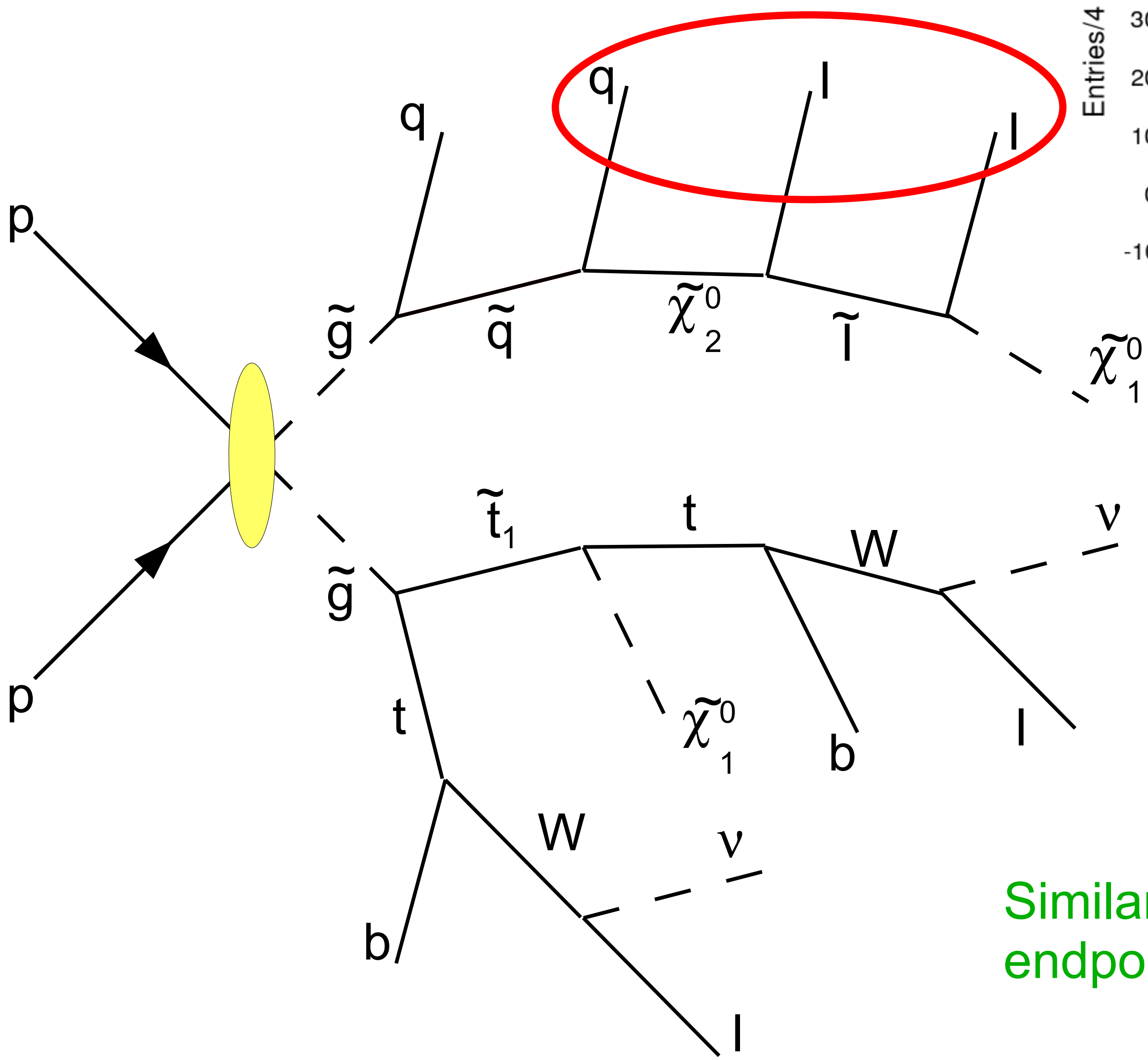
Case study for SPS1a assuming 1 fb^{-1}, 10 fb^{-1} and 300 fb^{-1}:

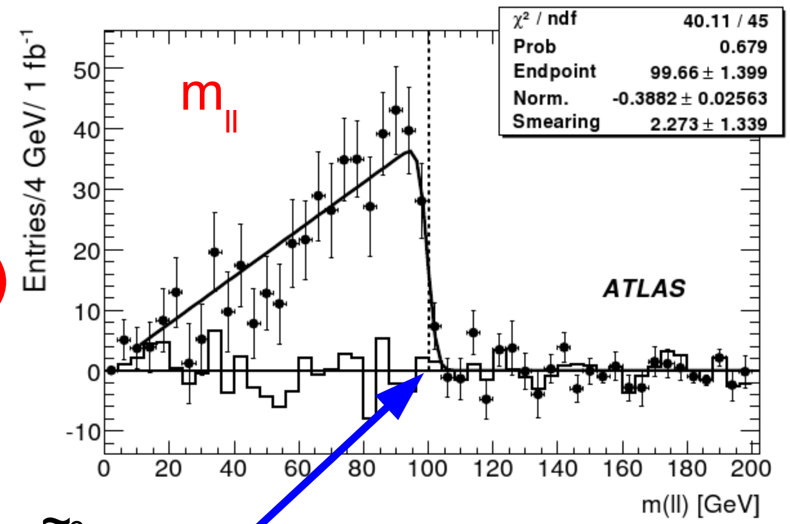
Observable	Nominal Value	Uncertainty							
		1 fb^{-1}	10 fb^{-1}	300 fb^{-1}	LES_1	$LES_{10,300}$	JES_1	$JES_{10,300}$	syst.
m_h	109.1		1.4	0.1		0.1			
m_t	170.9	1.1*	0.05	0.01			1.5*	1.0	
$m_{\tilde{\chi}_1^\pm}$	179.9			11.4				1.8	
$m_{\tilde{\ell}_L} - m_{\tilde{\chi}_1^0}$	105.4			1.7		0.1			6.0
$m_{\tilde{g}} - m_{\tilde{\chi}_1^0}$	510.2		13.7	2.5				5.1	10.0
$m_{\tilde{q}_R} - m_{\tilde{\chi}_1^0}$	454.0	19.6	6.2	1.1			22.7	4.5	10.0
$\langle m_{\tilde{g}} - m_{\tilde{b}_{1,2}} \rangle$	522.6		5.4					5.2	
$m_{\tilde{g}} - m_{\tilde{b}_1}$	89.0			1.5				0.9	
$m_{\tilde{g}} - m_{\tilde{b}_2}$	56.7			2.5				0.6	
$m_{\ell\ell}^{\max} = \epsilon_1(m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\ell}_R})$	80.2	1.7	0.5	0.03	0.16	0.08			
$m_{\ell\ell}^{\max} = \epsilon_1(m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_4^0}, m_{\tilde{\ell}_L})$	279.1		12.6	2.3		0.28			
$m_{\tau\tau}^{\max} = \epsilon_1(m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\tau}_1})$	83.2	12.6	4.0	0.73			4.2	0.8	5.7
$m_{\ell\ell q}^{\max} = \epsilon_1(m_{\tilde{\chi}_1^0}, m_{\tilde{q}_L}, m_{\tilde{\chi}_2^0})$	454.3	13.9	4.2	1.4			22.7	4.5	
$m_{\ell q}^{\mathrm{low}} = \epsilon_1(m_{\tilde{\ell}_R}, m_{\tilde{q}_L}, m_{\tilde{\chi}_2^0})$	324.2	7.6	3.5	0.9			16.2	3.2	
$m_{\ell q}^{\mathrm{high}} = \epsilon_2(m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\ell}_R}, m_{\tilde{q}_L})$	398.3	5.2	4.5	1.0			19.9	4.0	
$m_{\ell\ell q}^{\mathrm{thres}} = \epsilon_3(m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\ell}_R}, m_{\tilde{q}_L})$	216.2	26.5	4.8	1.6			10.8	2.2	
$m_{\ell\ell b}^{\mathrm{thres}} = \epsilon_3(m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\ell}_R}, m_{\tilde{b}_1})$	196.4		19.7	3.6				2.0	
$m_{tb}^{\mathrm{w}} = \epsilon_4(m_t, m_{\tilde{t}_1}, m_{\tilde{\chi}_j^\pm}, m_{\tilde{g}}, m_{\tilde{b}_i})$	360.9	43.0	13.6	2.5			18.0	3.6	
$\frac{\mathrm{BR}(\tilde{\chi}_2^0 \to \tilde{\ell}\ell) \times \mathrm{BR}(\tilde{\ell} \to \tilde{\chi}_1^0 \ell)}{\mathrm{BR}(\tilde{\chi}_2^0 \to \tilde{\tau}_1 \tau) \times \mathrm{BR}(\tilde{\tau}_1 \to \tilde{\chi}_1^0 \tau)}$	0.08	0.009	0.003	0.001					0.008
$\frac{\mathrm{BR}(\tilde{g} \to \tilde{b}_2 b) \times \mathrm{BR}(\tilde{b}_2 \to \tilde{\chi}_2^0 b)}{\mathrm{BR}(\tilde{g} \to \tilde{b}_1 b) \times \mathrm{BR}(\tilde{b}_1 \to \tilde{\chi}_2^0 b)}$	0.16			0.078					

Mass reconstruction at LHC (RPC SUSY)

No mass peaks due to 2 escaping LSPs

Most important mass information source:





Endpoint sensitive to SUSY masses:

$$\left(m_{ll}^2\right)^{\text{edge}} = \frac{\left(m_{\tilde{\chi}_2^0}^2 - m_{\tilde{l}_R}^2\right)\left(m_{\tilde{l}_R}^2 - m_{\tilde{\chi}_1^0}^2\right)}{m_{\tilde{l}_R}^2}$$

Similar formulae for other endpoints, e. g. m_{llq}, m_{lq}^{min}, m_{lq}^{max}, ...

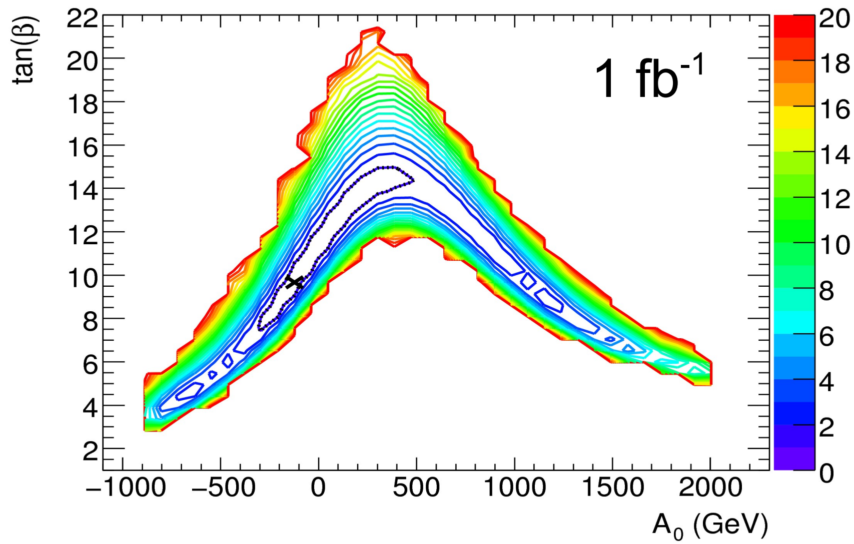
Likelihood maps from LHC “measurements”

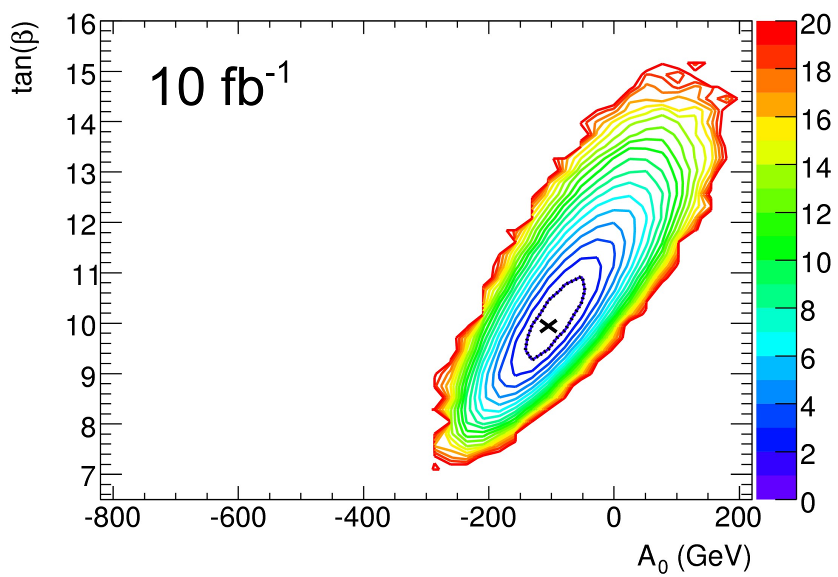
Likelihood maps for mSUGRA parameters, sign(μ) fixed to +

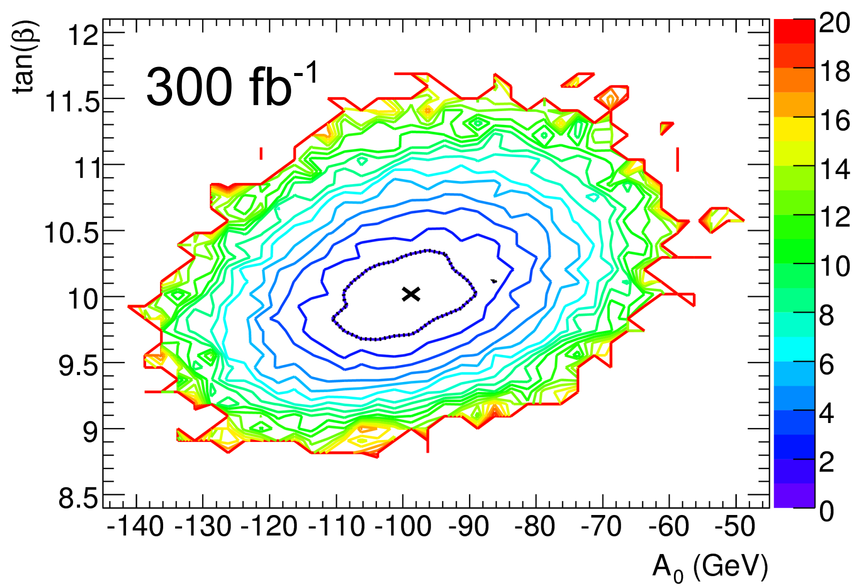
Plots show contours of $2\ln(\mathcal{L}_{\max}/\mathcal{L})$

Note different scales!

SPS1a: $\tan\beta = 10$, $A_0 = -100$ GeV







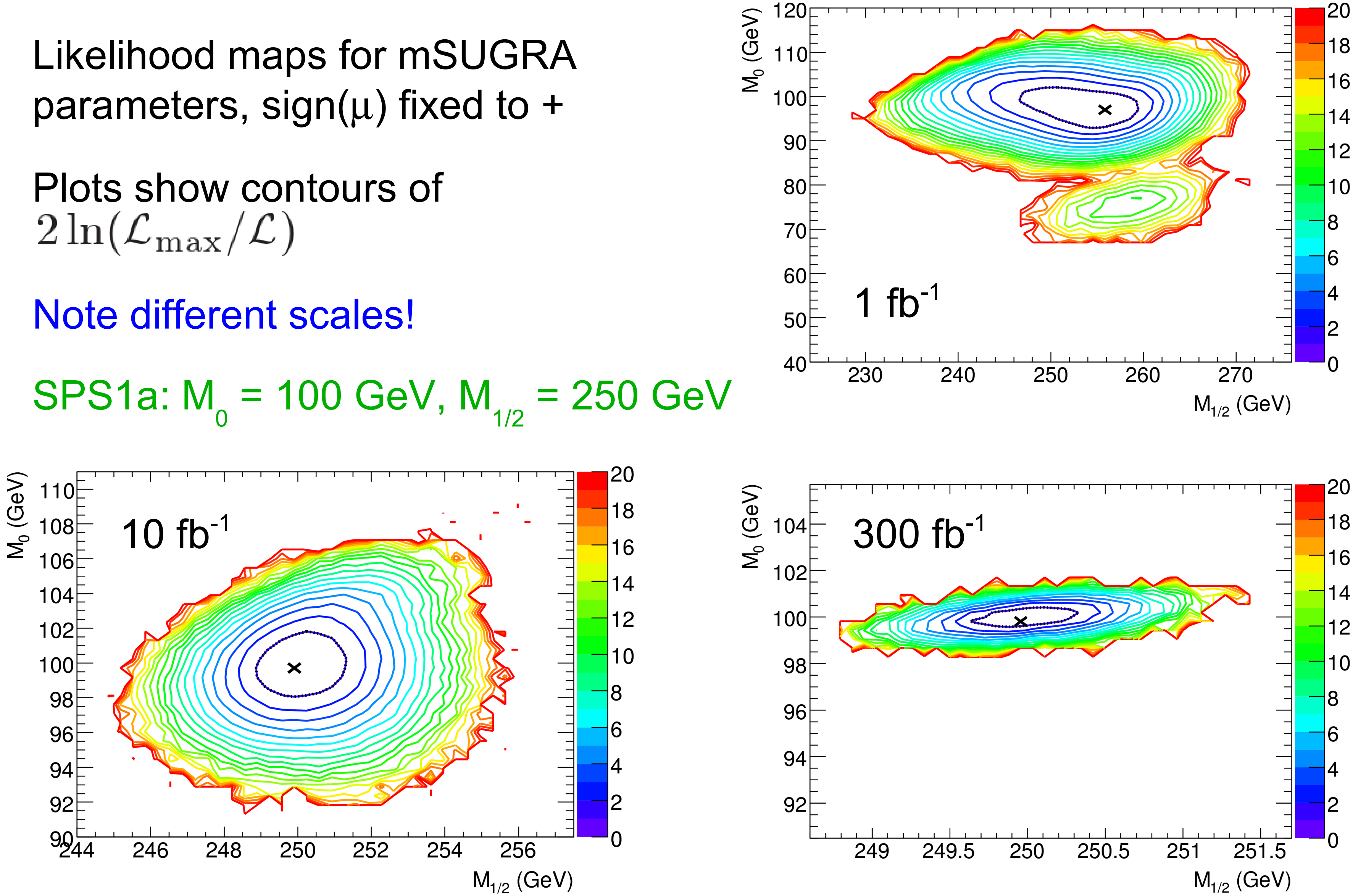
Likelihood maps from LHC “measurements”

Likelihood maps for mSUGRA parameters, sign(μ) fixed to +

Plots show contours of
$2\ln(\mathcal{L}_{\max}/\mathcal{L})$

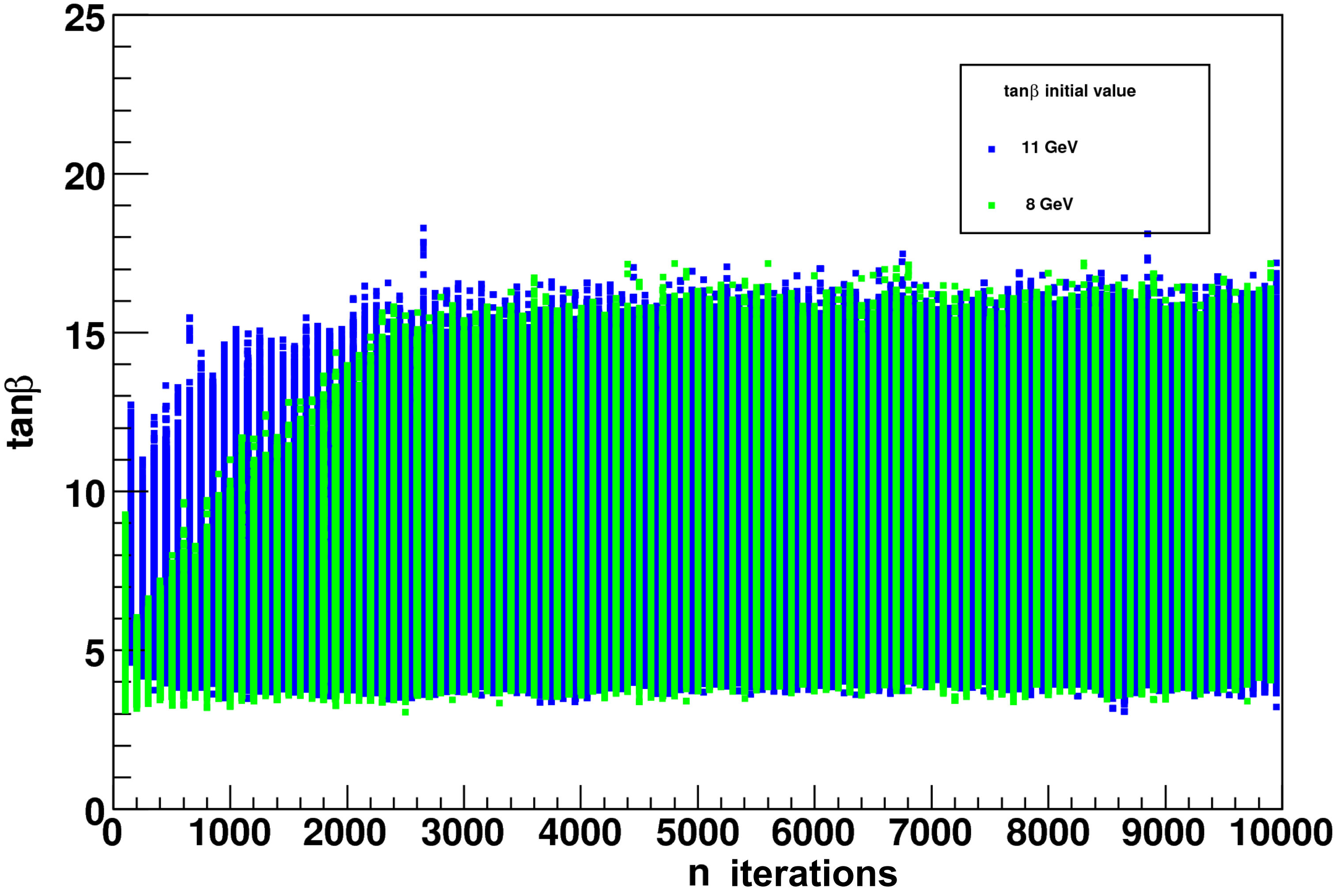
Note different scales!

SPS1a: M_0 = 100 GeV, $M_{1/2}$ = 250 GeV



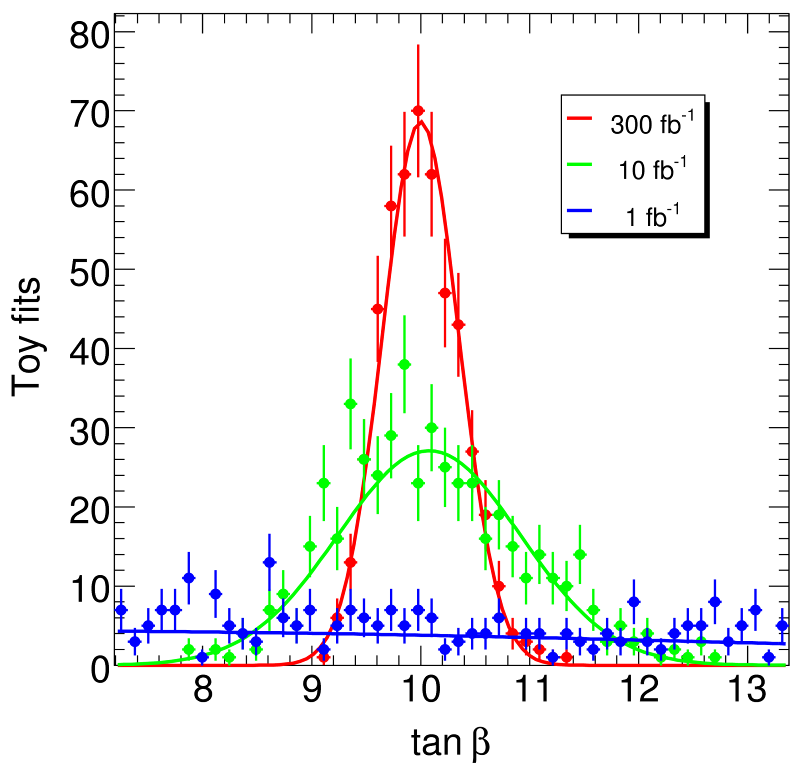
Caveat

Markov chains need sufficient number of iterations to settle down, i. e. results become independent of start values



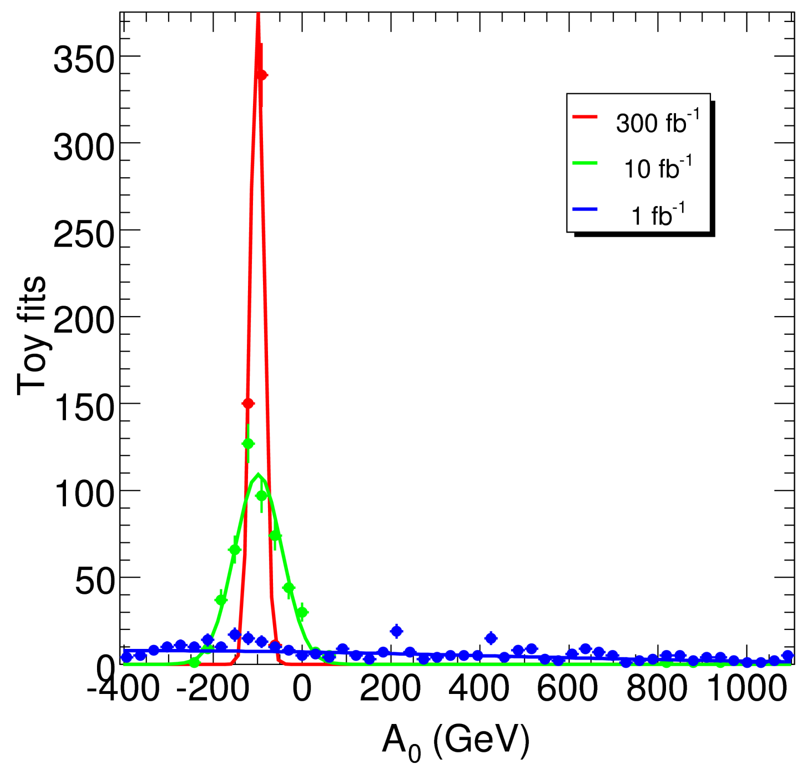
mSUGRA parameter uncertainties from LHC

mSUGRA parameter distributions for toy fits:



Luminosity	Uncertainty
1 fb-1	3.7 (37 %)
10 fb-1	0.8 (8 %)
300 fb-1	0.4 (4 %)

SPS1a: $\tan\beta = 10$

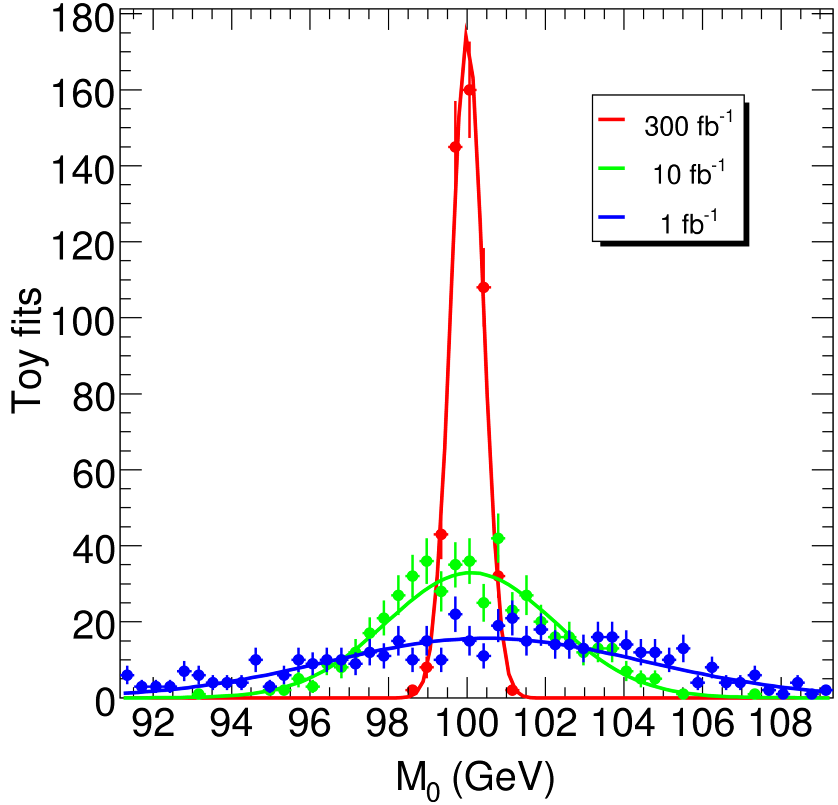


Luminosity	Uncertainty
1 fb-1	742 (742 %)
10 fb-1	53 (53 %)
300 fb-1	11 (11 %)

SPS1a: A_0 = -100 GeV

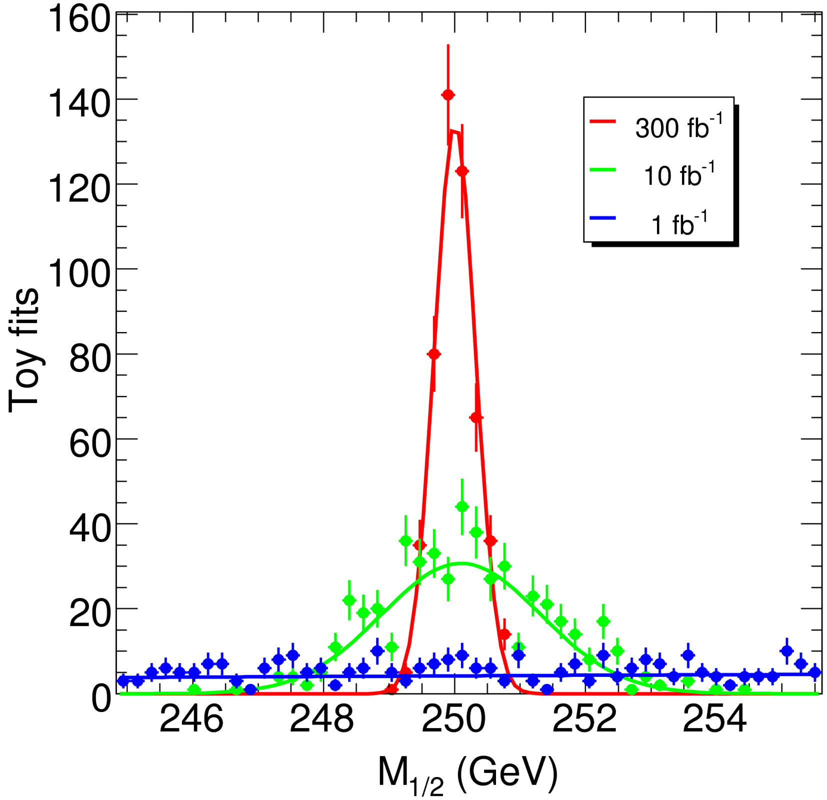
mSUGRA parameter uncertainties from LHC

mSUGRA parameter distributions for toy fits:



Luminosity	Uncertainty
1 fb-1	4.2 (4.2 %)
10 fb-1	2.1 (2.1 %)
300 fb-1	0.39 (0.4 %)

SPS1a: M_0 = 100 GeV



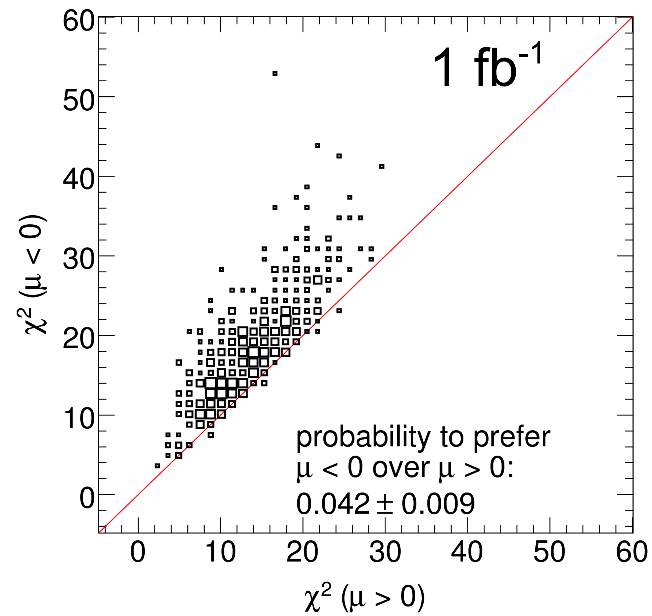
Luminosity	Uncertainty
1 fb-1	6.7 (2.7 %)
10 fb-1	1.2 (0.5 %)
300 fb-1	0.30 (0.1 %)

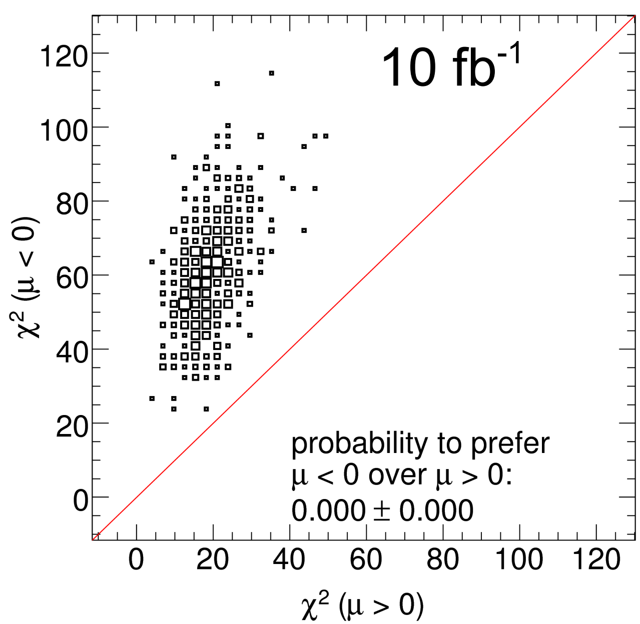
SPS1a: $M_{1/2}$ = 250 GeV

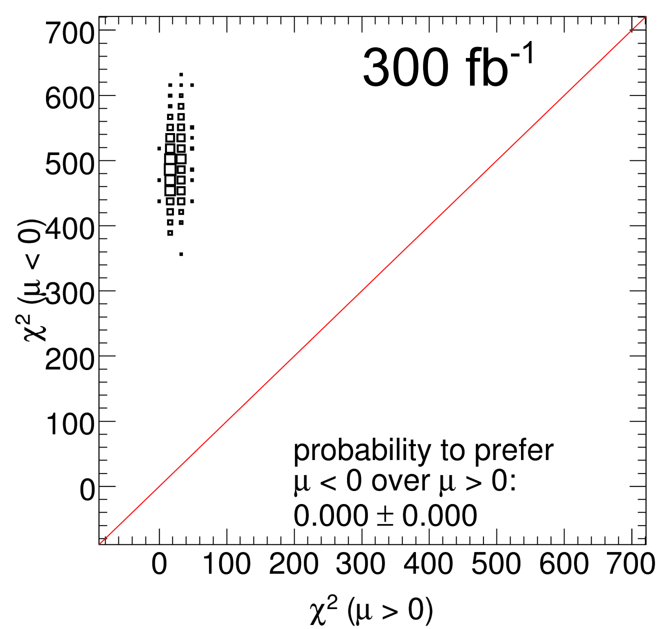
$\mu>0$ vs. $\mu<0$ from LHC "measurements"

Performed two fits (with $\mu>0$ and $\mu<0$) for every toy data set smeared around best fit values and compared χ^2 values

SPS1a: $\mu>0$







Combination LHC+LE

	Luminosity	Uncertainty LHC	Uncertainty LHC+LE
$\tan\beta$	1 fb-1	3.7 (41 %)	2.5 (25 %)
	10 fb-1	0.8 (8 %)	0.8 (8 %)
	300 fb-1	0.4 (4 %)	0.3 (3 %)

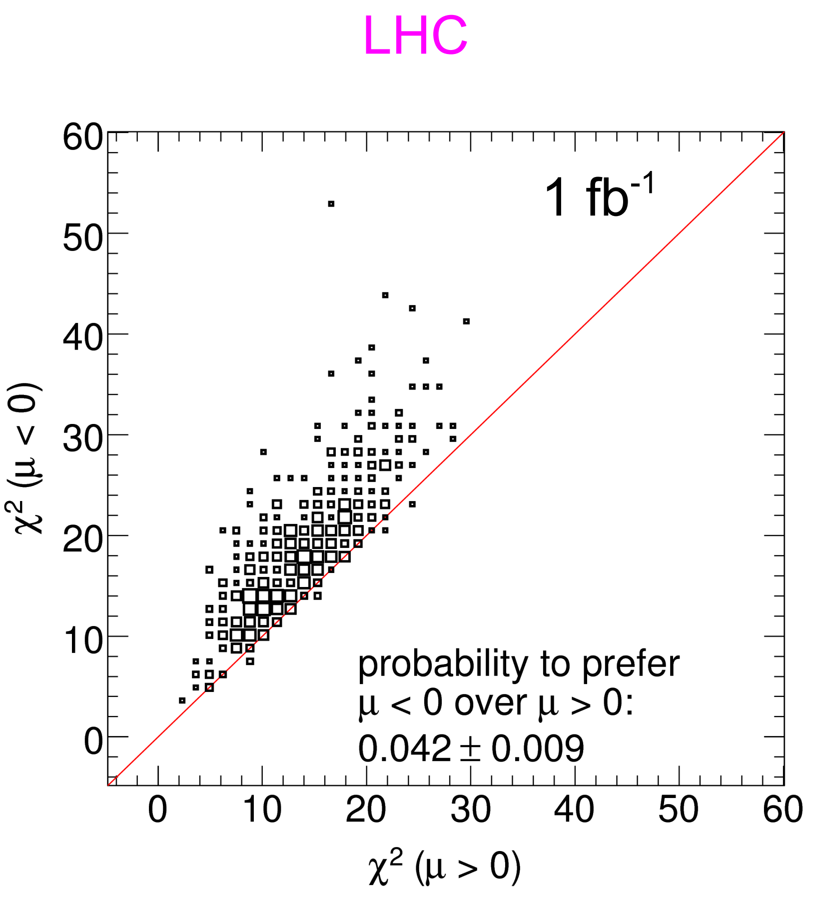
	Luminosity	Uncertainty LHC	Uncertainty LHC+LE
A_0 (GeV)	1 fb-1	742 (742 %)	169 (169 %)
	10 fb-1	53 (53 %)	48 (48 %)
	300 fb-1	11 (11 %)	12 (12 %)

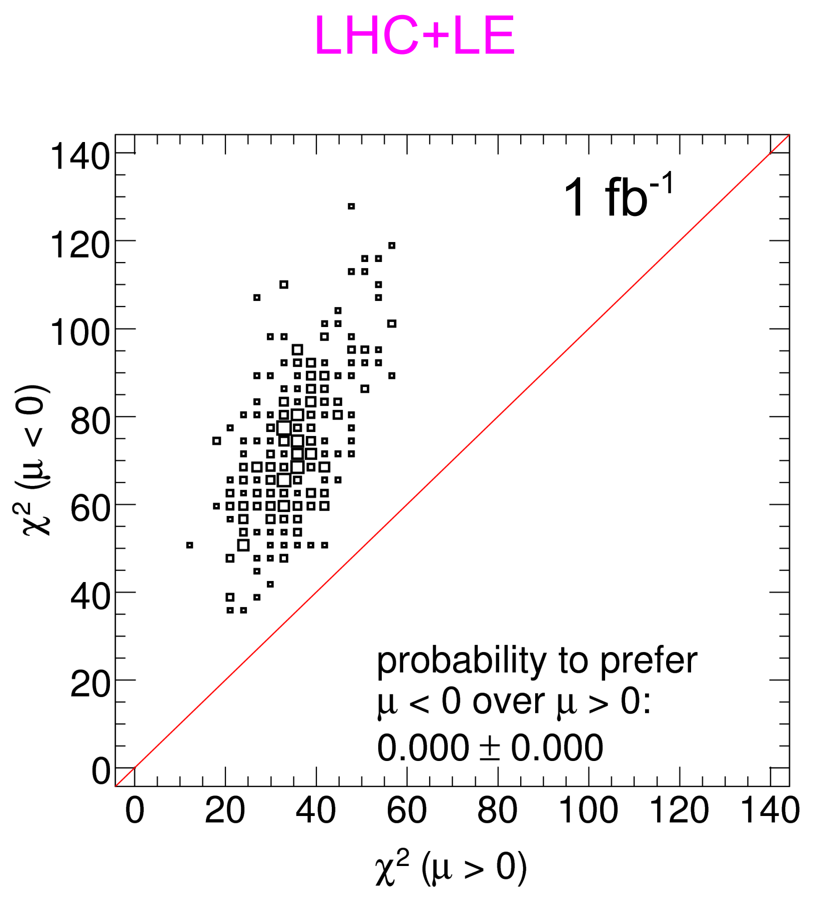
	Luminosity	Uncertainty LHC	Uncertainty LHC+LE
M_0 (GeV)	1 fb-1	4.2 (4.2 %)	3.3 (3.3 %)
	10 fb-1	2.1 (2.1 %)	1.9 (1.9 %)
	300 fb-1	0.39 (0.4 %)	0.44 (0.4 %)

	Luminosity	Uncertainty LHC	Uncertainty LHC+LE
$M_{1/2}$ (GeV)	1 fb-1	6.7 (2.7 %)	4.9 (2.0 %)
	10 fb-1	1.2 (0.5 %)	1.1 (0.4 %)
	300 fb-1	0.30 (0.1 %)	0.32 (0.1 %)

LE observables set to nominal SPS1a values for this combination

$\mu>0$ vs. $\mu<0$ from LHC+LE





MSSM18 fit to LHC300+LE

So far we always assumed certain SUSY breaking mechanism

Can we also fit more general models to LHC+LE observables?

YES, WE CAN!

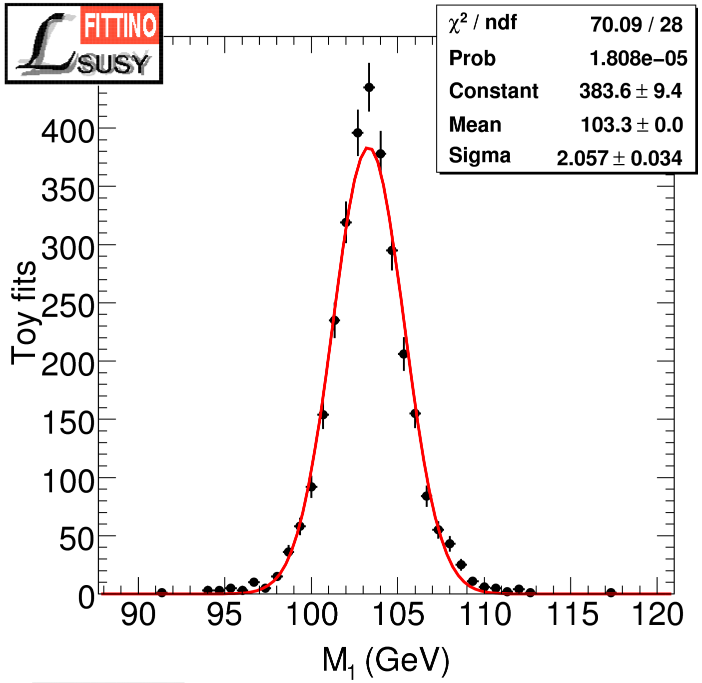
Assumptions:

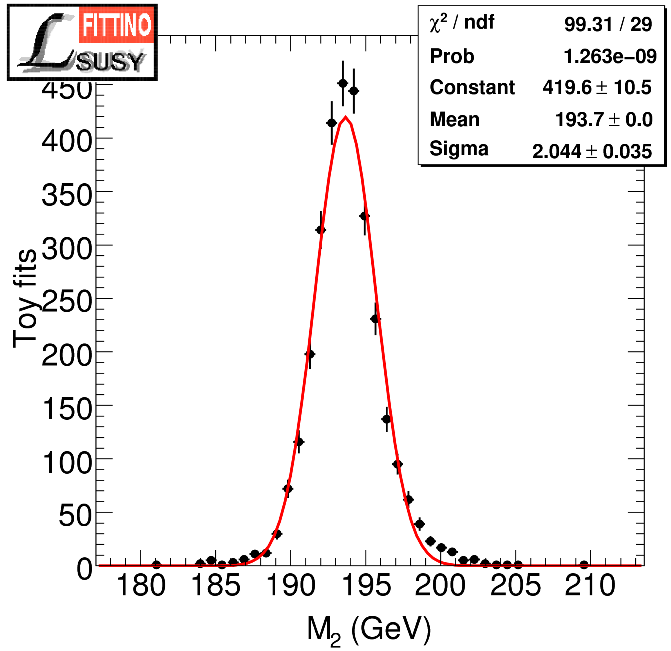
- No CP violation (all phases = 0)
- No mixing between generations
- No mixing within the first two generations
- Universality of same-type sfermion mass parameters in first two generations

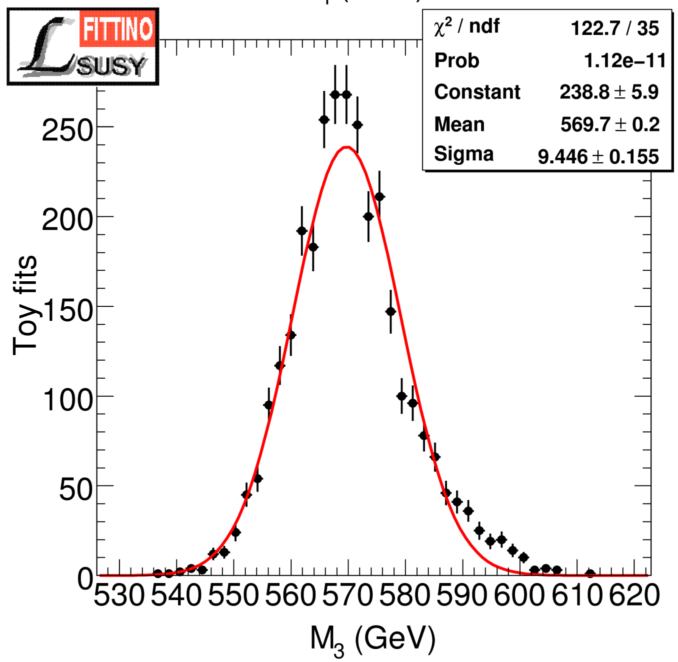
→ MSSM18

MSSM18 fit to LHC300+LE

Excerpts from toy fit parameter distributions...

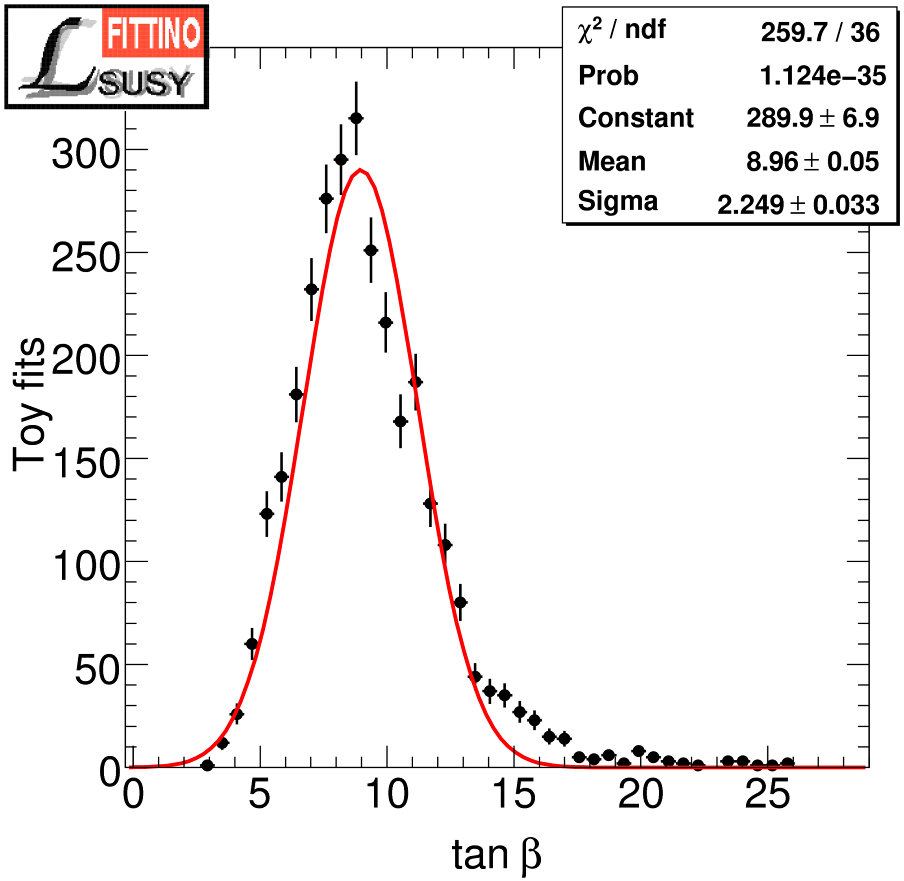


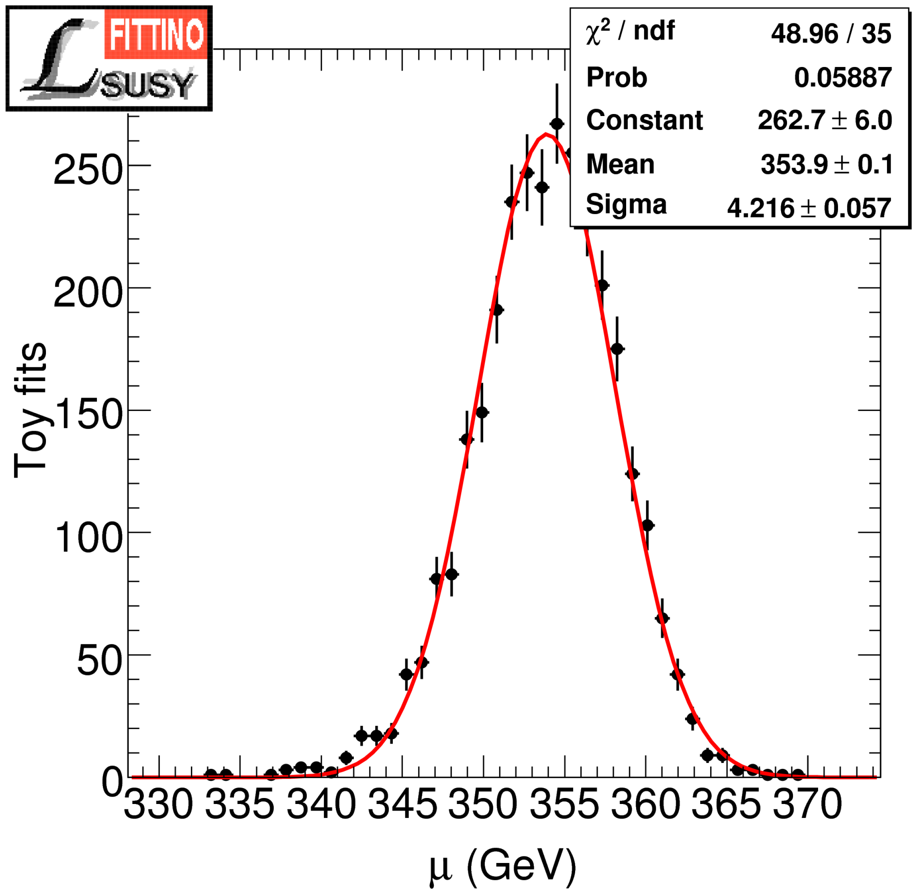




MSSM18 fit to LHC300+LE

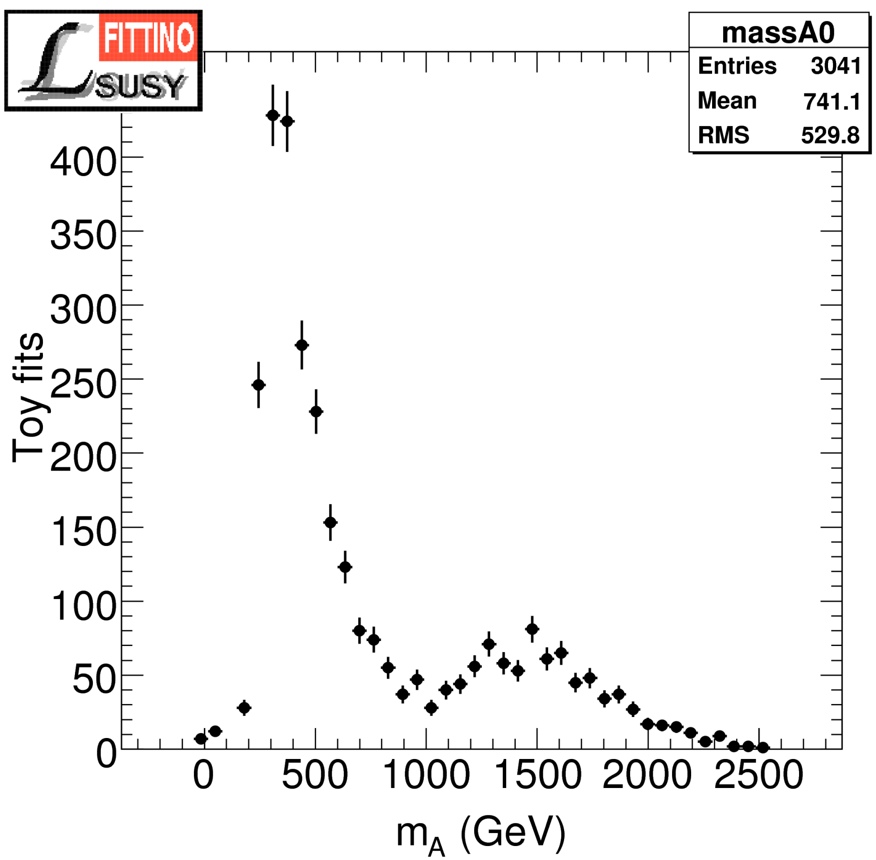
Excerpts from toy fit parameter distributions...

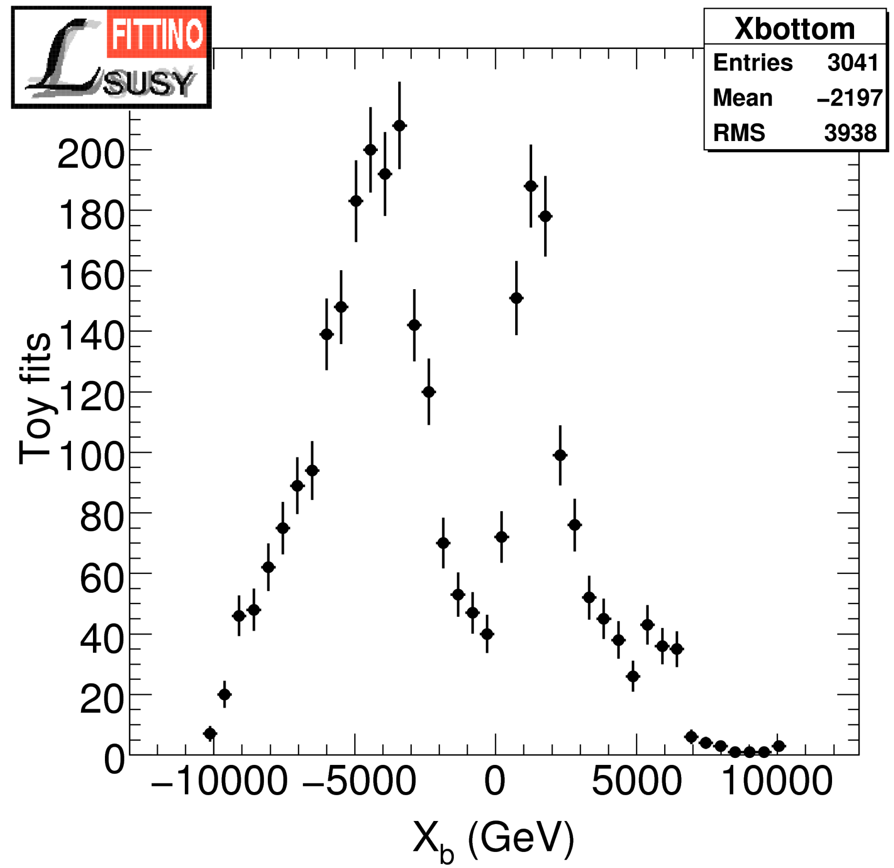




MSSM18 fit to LHC300+LE

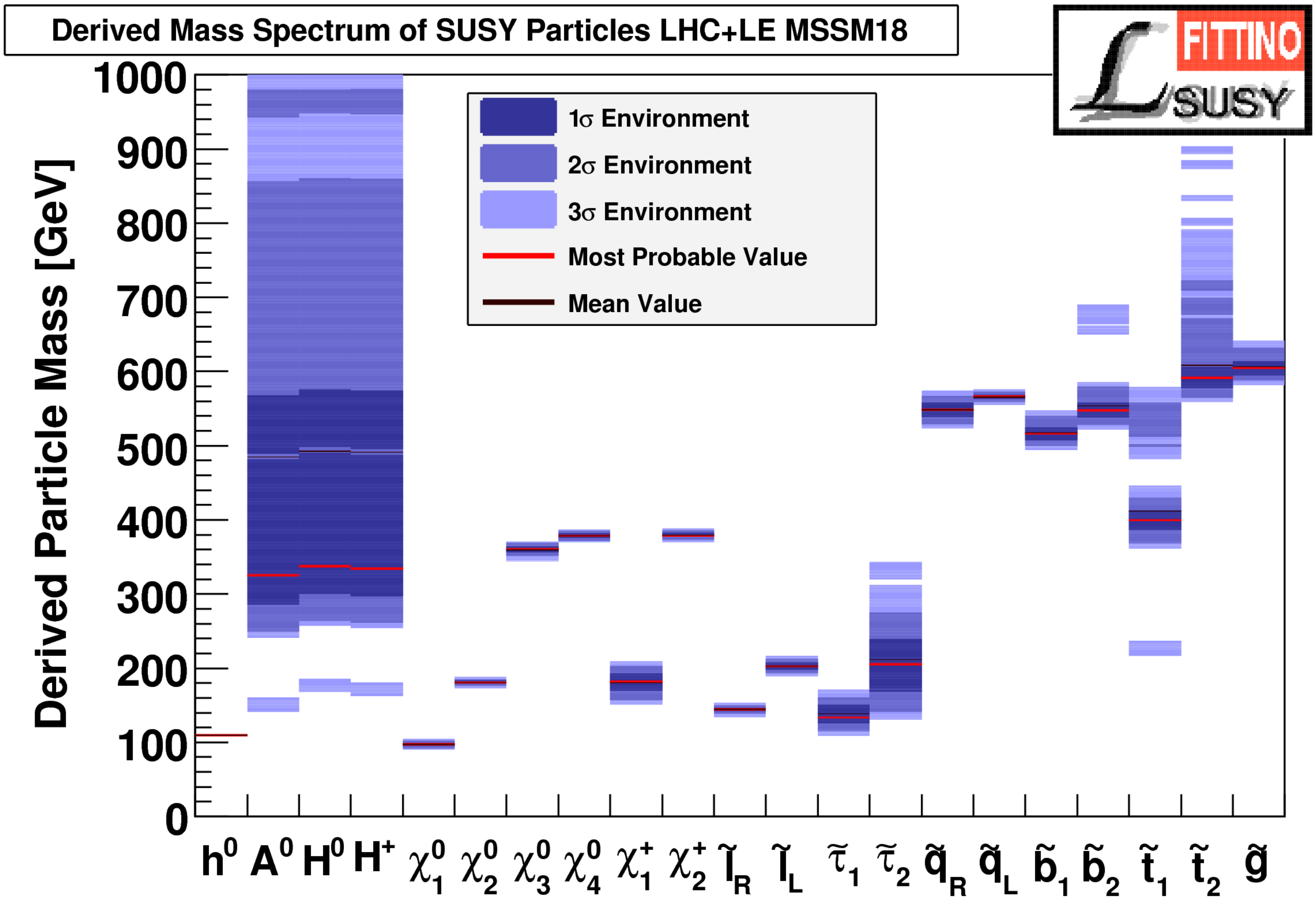
Excerpts from toy fit parameter distributions...



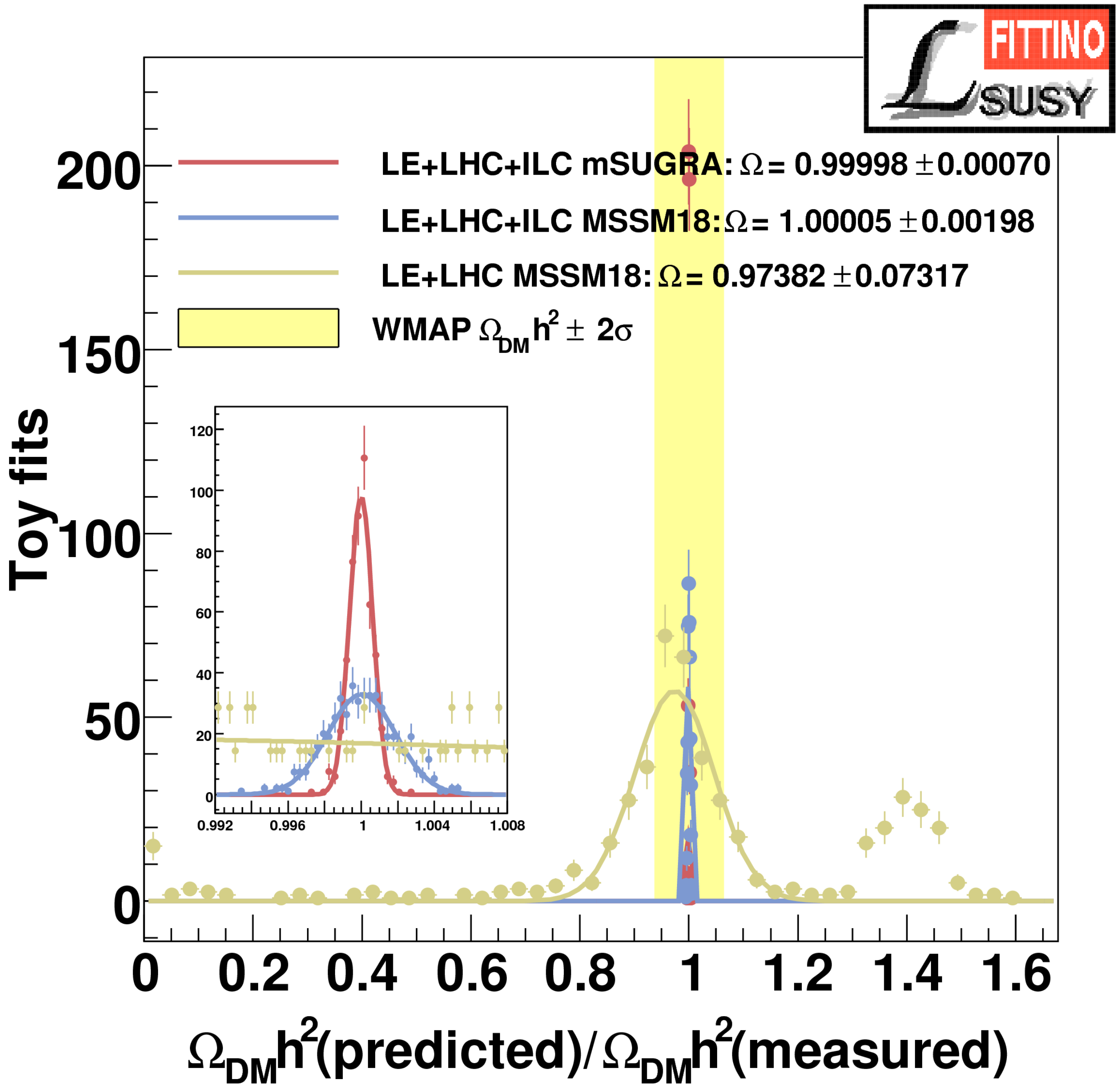


$$X_{\rm b} = A_{\rm b} - \mu \tan \beta$$

Mass spectrum from MSSM18 fit (LHC300+LE)



Relic density



Summary

- Discovery of new physics at the LHC might be the "easy" part (if Nature is not too nasty)
- Pinning down the underlying model might be more demanding
- LE measurements favour SUSY masses ≤ 1 TeV
- LE measurements might still provide valuable constraints for SUSY in the first phase of the LHC
- LHC results pretty powerful in constrained SUSY models
- LHC results might exhibit ambiguities in more general SUSY models → ILC
- Might, might, might ... LHC will hopefully end speculations soon!