Squark–anti-squark pair production: the electroweak contributions

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$$P\ P \longrightarrow \tilde{Q}^a\ \tilde{Q}^{a*} \quad @\ \mathcal{O}(\alpha_s^2\alpha)$$

- Motivations
- Closer look into (diagonal) squark–anti-squark production
  - Squark pair production at parton level
  - Renormalization
  - IR divergences
- Numerical Analysis
- Conclusions

Motivations

. . . Already given in Maike's talk:

- Minimal Supersymmetric Standard Model:
  - Theoretically consistent extension of SM
  - phenomenologically promising
  - it allows quantitatitive predicitions.
- Large Hadron Collider:
  - it will probe SUSY and MSSM . . .
  - . . . via direct production of colored particles

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Some features of the process $PP \to \tilde{Q}^a \tilde{Q}^{a*}$

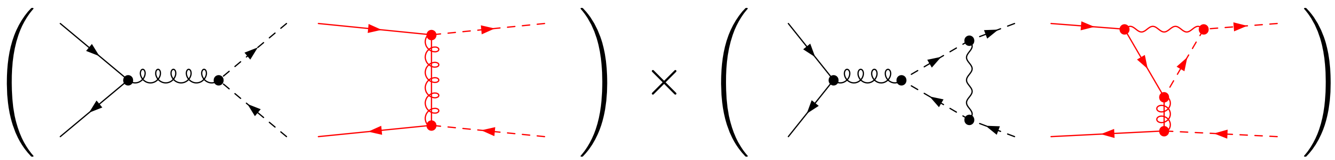
- it is an "LHC" process
- it is foreseen by the MSSM
- it leads to the direct production of colored particles
- its $\mathcal{O}(\alpha_s^2 \alpha)$ corrections are missing

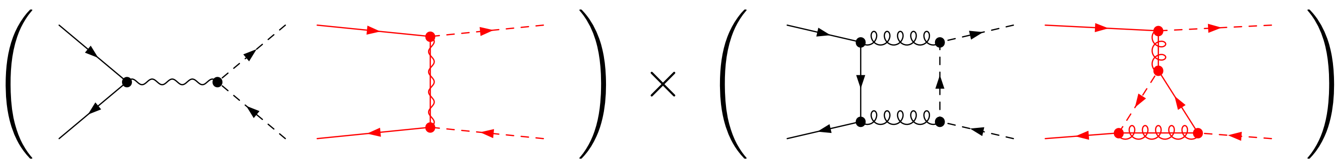
make our computation not unreasonable!

Diagonal squark P.P. at the parton level

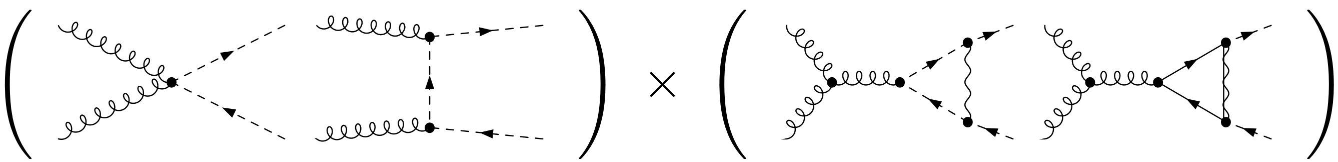
$\mathcal{O}(\alpha_s^2\alpha)$, virtual corrections

- $q\bar{q} \longrightarrow \tilde{Q}^a\tilde{Q}^{a*}$





- $gg \longrightarrow \tilde{Q}^a\tilde{Q}^{a*}$

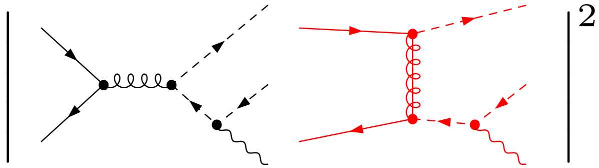


w.r.t. the stop case:
- New diagrams, full one loop QCD diagrams needed
- New interferences (that can be numerically important)
- New UV divergences
- Richer IR structure

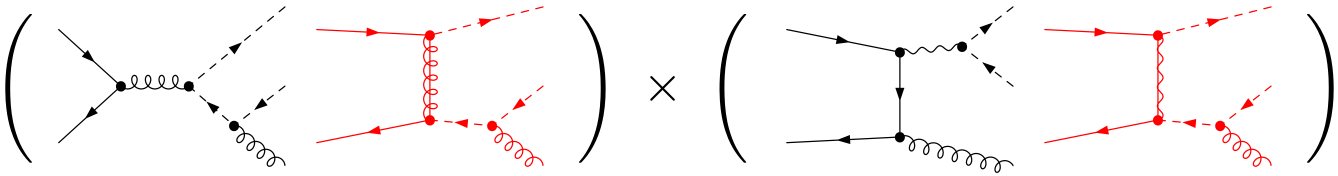
Diagonal squark P.P. at the parton level

$\mathcal{O}(\alpha_s^2\alpha)$, Real Corrections

- $q\bar{q} \longrightarrow \tilde{Q}^a\tilde{Q}^{a*}\gamma$



- $q\bar{q} \longrightarrow \tilde{Q}^a\tilde{Q}^{a*}g$



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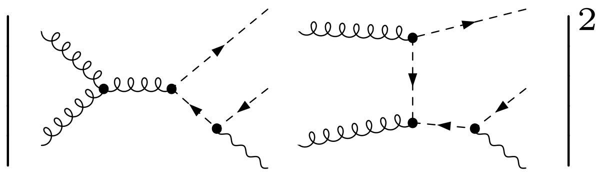
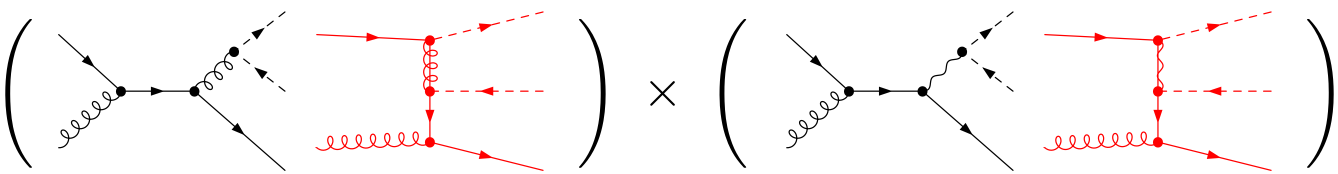


- $qg \longrightarrow \tilde{Q}^a\tilde{Q}^{a*}q$



R stands for trouble . . .

. . . Indeed:

- Renormalization
- Real radiation
- Resonances

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Renormalizing . . . What?

Problem: loop integrals are divergent.

but Divergences drops out after renormalization
$\hookrightarrow$ [reparametrization in term of physical quantities]

- Renormalization procedure, general strategy:
  - Find the sectors of the MSSM we have to renormalize
  - In each sector, find independent set of parameter and fields
  - Reparametrize them
  - Fix the new parameter set imposing (renormalization) conditions

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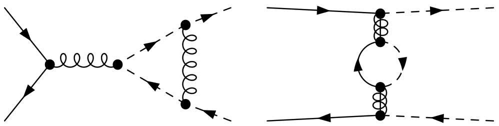
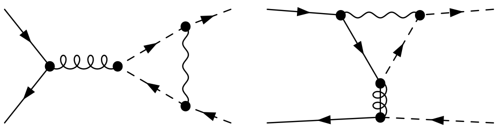
- Sectors to renormalize:



$\longrightarrow$ gluino, quark, squark sectors; α_s, $@\ \mathcal{O}(\alpha_s)$



$\longrightarrow$ quark, squark sector $@\ \mathcal{O}(\alpha)$

- We focus here on the renormalization of the squark sector and of α_s

Renormalizing the strong coupling

$$\mathcal{L}_{\text{strong}} = -\frac{1}{4}G^A_{\mu\nu}G^{A,\mu\nu} + g_s T^A \bar{\Psi}_u \gamma^\mu \Psi_u G^A_\mu$$
$$- \sqrt{2}\hat{g}_s \left[T^A \bar{\Psi}^A_{\tilde{g}}\, \omega_- \,\Psi_u \Phi^*_{\tilde{u},L} + h.c.\right] + \ \ldots$$

one field two couplings to reparametrize:

$$G^A_\mu \to \left(1 + \frac{\delta Z_G}{2}\right) G^A_\mu, \qquad g_s \to g_s + \delta g_s, \qquad \hat{g}_s \to \hat{g}_s + \delta\hat{g}_s$$

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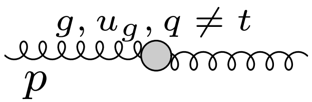
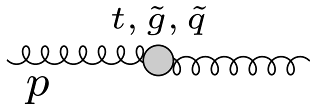
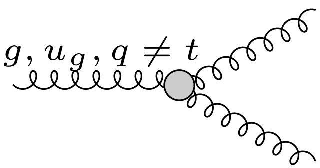
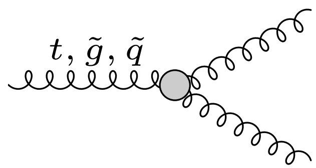
- Definition of δg_s, δZ_G:

$\frac{\partial}{\partial p^2}$ 

$|\Delta$ $+$ $\frac{\partial}{\partial p^2}$ 

$|\Delta - \ln(M^2/\mu^2)$ $\stackrel{!}{=} 0$



$|\Delta$ $+$ 

$|\Delta - \ln(M^2/\mu^2)$ $\stackrel{!}{=} 0$

- Divergences within DREG, $\Delta = 2/\epsilon - \gamma + \ln 4\pi$
- $\overline{\text{MS}}$ (DREG + UV poles subtraction) if light particles in loops
- Zero momentum subtraction scheme if heavy particles in loops
  - $\hookrightarrow$ SM-like running of g_s

Renormalizing the strong coupling

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- Definition of $\delta\hat{g}_s$:
  - Owing to SUSY should be a dependent parameter

    $\hookrightarrow \hat{g}_s = g_s; \qquad \delta\hat{g}_s = \delta g_s$
  - But DREG spoils SUSY @ NLO [Beenakker *et al.*'96,98]

    $\hookrightarrow$ SUSY restored setting: $\hat{g}_s = g_s + g_s\frac{\alpha_s}{3\pi}; \qquad \delta\hat{g}_s = \delta g_s$
  - The mismatch between g_s and $\hat{g}_s$ is reabsorbed into $\delta\hat{g}_s$:

    $\hookrightarrow \hat{g}_s = g_s; \qquad \delta\hat{g}_s = \delta g_s + g_s\frac{\alpha_s}{3\pi}$

Renormalizing the squark sector, I

$$\mathcal{L}_{\text{squark}} = \sum_{\tilde q=\tilde t,\tilde b} \left\{ \left(\partial_\mu \Phi^*_{\tilde q_L}, \partial_\mu \Phi^*_{\tilde q_R}\right) \begin{pmatrix} \partial^\mu \Phi_{\tilde q_L} \\ \partial^\mu \Phi_{\tilde q_R} \end{pmatrix} - \left(\Phi^*_{\tilde q_L}, \Phi^*_{\tilde q_R}\right) \mathbf{M}_{\tilde q} \begin{pmatrix} \Phi_{\tilde q_L} \\ \Phi_{\tilde q_R} \end{pmatrix} \right\}$$

where:

$$\mathbf{M}_{\tilde q} = \begin{pmatrix} {M_L}^2 + {m_q}^2 + M_Z^2 \cos 2\beta (T_q^3 - e_q \sin^2\theta_W) & m_q (A_{\tilde q} - \mu\lambda_{\tilde q}) \\ m_q (A_{\tilde q} - \mu\lambda_{\tilde q}) & {M_{\tilde q,R}}^2 + {m_q}^2 + e_q M_Z^2 \cos 2\beta \sin^2\theta_W \end{pmatrix}.$$

[(SU(2) invariance) $\Rightarrow M_{\tilde t,L} = M_{\tilde b,L} = M_L$]

[$\lambda_{\tilde t} = 1/\tan\beta$ & $\lambda_{\tilde b} = \tan\beta$]

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5 independent parameters: $M_L, M_{\tilde{t},R}, M_{\tilde{t},R}, A_{\tilde{b}}, A_{\tilde{t}}$

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after diagonalization $\begin{pmatrix} \Phi_{\tilde q_1} \\ \Phi_{\tilde q_2} \end{pmatrix} = \begin{pmatrix} c_{\theta\tilde q} & s_{\theta\tilde q} \\ -s_{\theta\tilde q} & c_{\theta\tilde q} \end{pmatrix} \begin{pmatrix} \Phi_{\tilde q_L} \\ \Phi_{\tilde q_R} \end{pmatrix}$

$$\mathbf{M}_{\tilde q} = \begin{pmatrix} c^2_{\theta\tilde q} m^2_{\tilde q,1} + s^2_{\theta\tilde q} m^2_{\tilde q,2} & c_{\theta\tilde q} s_{\theta\tilde q} (m^2_{\tilde q,1} - m^2_{\tilde q,2}) \\ c_{\theta\tilde q} s_{\theta\tilde q} (m^2_{\tilde q,1} - m^2_{\tilde q,2}) & c^2_{\theta\tilde q} m^2_{\tilde q,2} + s^2_{\theta\tilde q} m^2_{\tilde q,1} \end{pmatrix}$$

SO one can choose different independent set:

- $\{m_{\tilde t,1},\ m_{\tilde t,2},\ m_{\tilde b,2},\ \theta_{\tilde t},\ \theta_{\tilde b}\}$
- $\{m_{\tilde t,1},\ m_{\tilde t,2},\ m_{\tilde b,2},\ \theta_{\tilde t},\ A_b\}$
- $\ldots$

Renormalizing the squark sector, II

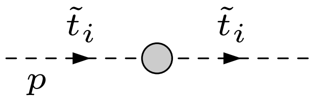
"m_b $\overline{\text{DR}}$ scheme": $\{m_{\tilde{t},1},\ m_{\tilde{t},2},\ m_{\tilde{b},2},\ \theta_{\tilde{t}},\ A_{\tilde{b}}\ \}$

<u>Stop sector</u>

reparametrizing:

$$m^2_{\tilde{t},i} \longrightarrow m^2_{\tilde{t},i} + \delta m^2_{\tilde{t},i};\quad \theta_{\tilde{t}} \longrightarrow \theta_{\tilde{t}} + \delta\theta_{\tilde{t}};\quad \begin{pmatrix} \Phi_{\tilde{t},1} \\ \Phi_{\tilde{t},2} \end{pmatrix} \rightarrow \left[1 + \frac{1}{2}\begin{pmatrix} \delta Z^{\tilde{t}}_{1,1} & \delta Z^{\tilde{t}}_{1,2} \\ \delta Z^{\tilde{t}}_{2,1} & \delta Z^{\tilde{t}}_{2,2} \end{pmatrix}\right]\begin{pmatrix} \Phi_{\tilde{t},1} \\ \Phi_{\tilde{t},2} \end{pmatrix}$$

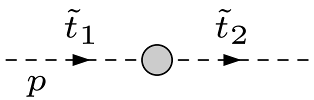
- $\delta m^2_{\tilde{t},i}$ and $\delta Z^{\tilde{t}}_{i,i}$ are fixed in the *on-shell* scheme



$$= \frac{i}{p^2 - m^2_{\tilde{t},i}} + \frac{i}{p^2 - m^2_{\tilde{t},i}}\,\hat{\Sigma}_{i,i}(p^2)\,\frac{i}{p^2 - m^2_{\tilde{t},i}} \stackrel{!}{=} \frac{i}{p^2 - m^2_{\tilde{t},i}} \qquad p^2 \longrightarrow m^2_{\tilde{t},i};$$

$$[\hat{\Sigma}_{i,i} = \Sigma_{i,i} + (p^2 - m^2_{\tilde{t},i})\delta Z^{\tilde{t}}_{i,i} - \delta m^2_{\tilde{t},i}\,]$$

- $\delta Z^{\tilde{t}}_{i,j}$ and $\delta\theta_{\tilde{t}}$ imposing no squark mixing on-shell:



$\stackrel{!}{=}\ 0$ if $p^2 \longrightarrow m^2_{\tilde{t},1}$ or $p^2 \longrightarrow m^2_{\tilde{t},2}$

- $\delta A_{\tilde{t}}$ dependent, function of $\delta\mu$, $\delta\tan\beta$, δm_t $\hookrightarrow$ [on shell scheme]

Renormalizing the squark sector, II

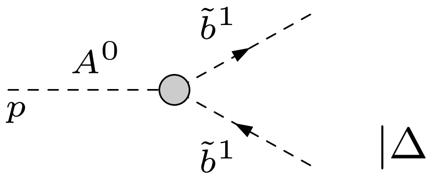
"m_b $\overline{\text{DR}}$ scheme": $\{m_{\tilde{t},1},\ m_{\tilde{t},2},\ m_{\tilde{b},2},\ \theta_{\tilde{t}},\ A_{\tilde{b}}\ \}$

Sbottom sector

reparametrizing:

$$m^2_{\tilde{b},2} \to m^2_{\tilde{b},2} + \delta m^2_{\tilde{b},2};\quad A_{\tilde{b}} \to A_{\tilde{b}} + \delta A_{\tilde{b}};\quad \begin{pmatrix} \Phi_{\tilde{b},1} \\ \Phi_{\tilde{b},2} \end{pmatrix} \to \left[1 + \frac{1}{2} \begin{pmatrix} \delta Z^{\tilde{b}}_{1,1} & \delta Z^{\tilde{b}}_{1,2} \\ \delta Z^{\tilde{b}}_{2,1} & \delta Z^{\tilde{b}}_{2,2} \end{pmatrix} \right] \begin{pmatrix} \Phi_{\tilde{b},1} \\ \Phi_{\tilde{b},2} \end{pmatrix}$$

- $\delta m^2_{\tilde{b},2}$ and $\delta Z^{\tilde{b}}_{i,j}$ fixed as in the $\tilde{t}$ case.
- $\delta A_{\tilde{b}}$ defined imposing:



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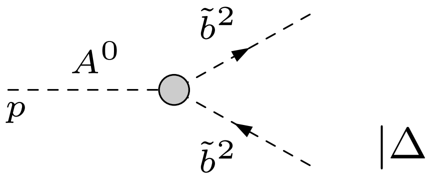


$\stackrel{!}{=} 0$ when $p = 0$

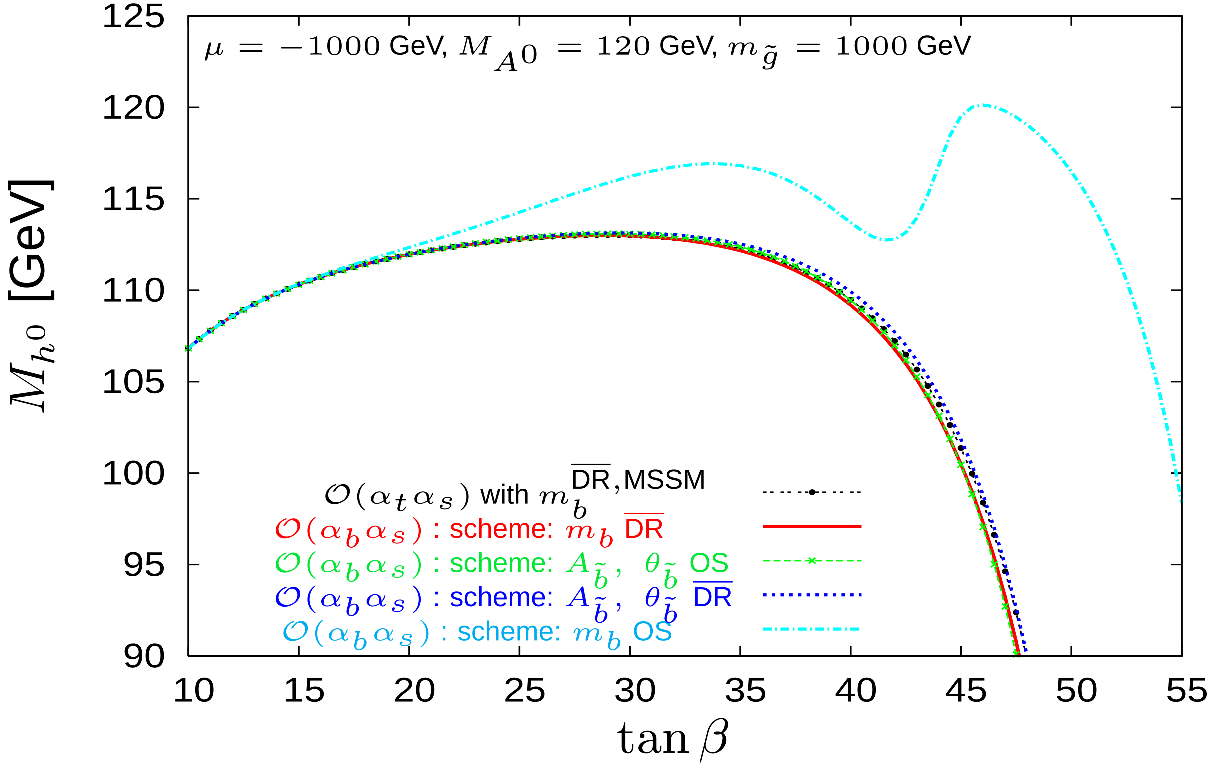
  - Divergences wthin DRED; $\Delta = 2/\epsilon - \gamma + \ln 4\pi$
  - $\overline{\text{DR}}$ scheme (DRED + UV poles subtraction)

- $\delta\theta_{\tilde{b}}$ dependent, function of $\delta\mu$, $\delta \tan\beta$, δm_t, δm_b $\hookrightarrow$ [fixed via the $\overline{\text{DR}}$ prescription]

- Difference beetween stops and sbottoms . . . Why?

Renormalizing the squark sector, II

A good reason: M_{h^0} including dominant two loops contribution:



[Heinemeyer, Hollik, Rzehak, Weiglein '05]

- m_b OS scheme: sbottoms treated as stops
- $A_{\tilde{b}}$ dependent and: $\frac{\delta A_{\tilde{b}}}{A_{\tilde{b}}} \sim \frac{\delta m_b}{m_b} \frac{A_{\tilde{b}} - \mu \tan\beta}{A_{\tilde{b}}} \sim \alpha_s \frac{m_{\tilde{g}}}{m_b} \frac{A_{\tilde{b}} - \mu \tan\beta}{A_{\tilde{b}}}$

  $\hookrightarrow$ $\delta A_b \sim A_b$ if $\tan\beta$ big ($\delta A_{\tilde{b}} \sim 3A_{\tilde{b}}$ if $\tan\beta = 50$)

  $\hookrightarrow$ m_b OS scheme not reliable.

R stands for trouble . . .

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- Renormalization
- Real radiation
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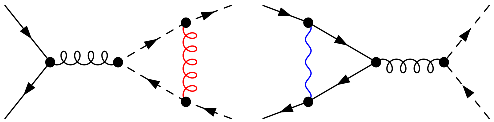
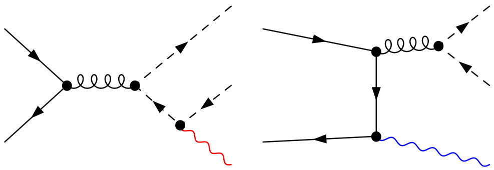
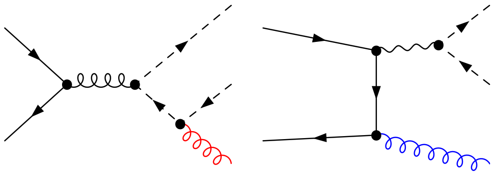
IR & Collinear Divergences



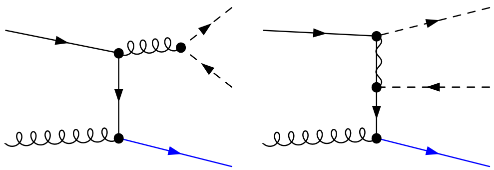
$\ldots \leadsto$ IR & Collinear singularities in $q\bar{q} \to \tilde{Q}^a \tilde{Q}^{a*}$



$\ldots \leadsto$ IR & Collinear singularities in $q\bar{q} \to \tilde{Q}^a \tilde{Q}^{a*} \gamma$



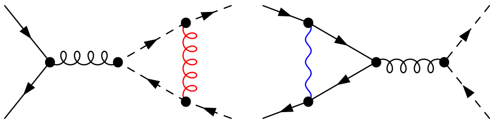
$\ldots \leadsto$ IR & Collinear singularities in $q\bar{q} \to \tilde{Q}^a \tilde{Q}^{a*} g$



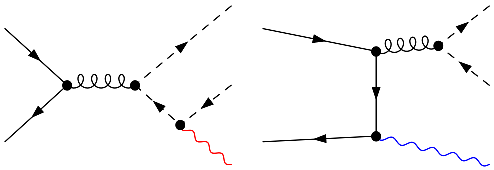
$\ldots \leadsto$ Collinear singularities in $qg \to \tilde{Q}^a \tilde{Q}^{a*} q$

- Two classes of singularities enter virtual corrections :
  - Infrared $\to$ exchange of γ (g) between external particles
  - Collinear $\to$ massless external particle splitting into two massless particles

IR & Collinear Divergences



$\ldots \rightsquigarrow$ IR & Collinear singularities in $q\bar{q} \to \tilde{Q}^a \tilde{Q}^{a*}$



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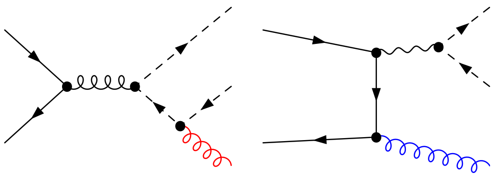

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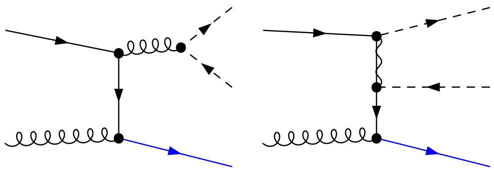
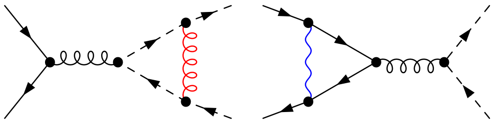


$\ldots \rightsquigarrow$ Collinear singularities in $qg \to \tilde{Q}^a \tilde{Q}^{a*} q$

- For real emission processes two kinds of singularities as well:
  - Infrared $\to$ low-energy γ (g) emitted from an external particle
  - Collinear $\to$ massless particle emitted collinearly from the initial states
- They appear in the integration over the phase space

IR & Collinear Divergences



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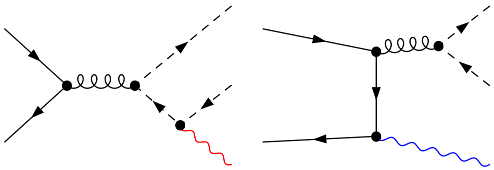

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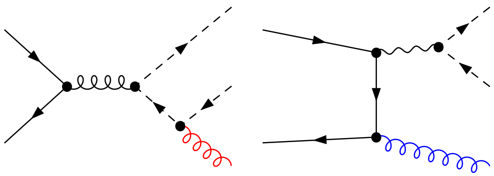

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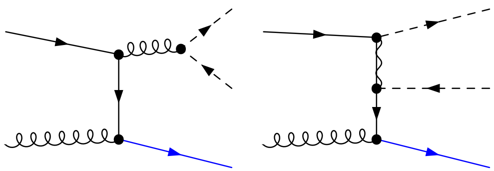

. . . $\rightsquigarrow$ Collinear singularities in $qg \to \tilde{Q}^a \tilde{Q}^{a*} q$ +

PDF factorization =

[KLN theorem] IR & Collinear Safe

Mass singularities in real life

Problem: how to regularize and extract IR & collinear singularities in real emission processes.

- Introduction of m_γ, m_g and m_q regularizes IR & collinear singularities

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But QCD enters in $q\bar{q} \to \tilde{Q}\tilde{Q}$

- no gluon as external state
- IR structure QED-like

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. . . After performing the color algebra properly †

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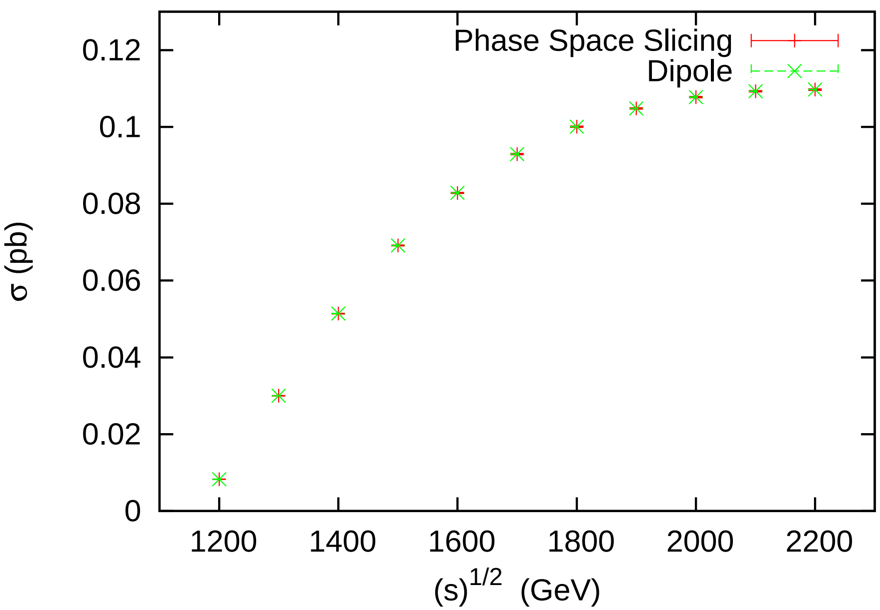
- Two methods † to extract singularities from γ (g) phase space integration:
  - Phase Space Slicing
  - Dipole Subtraction

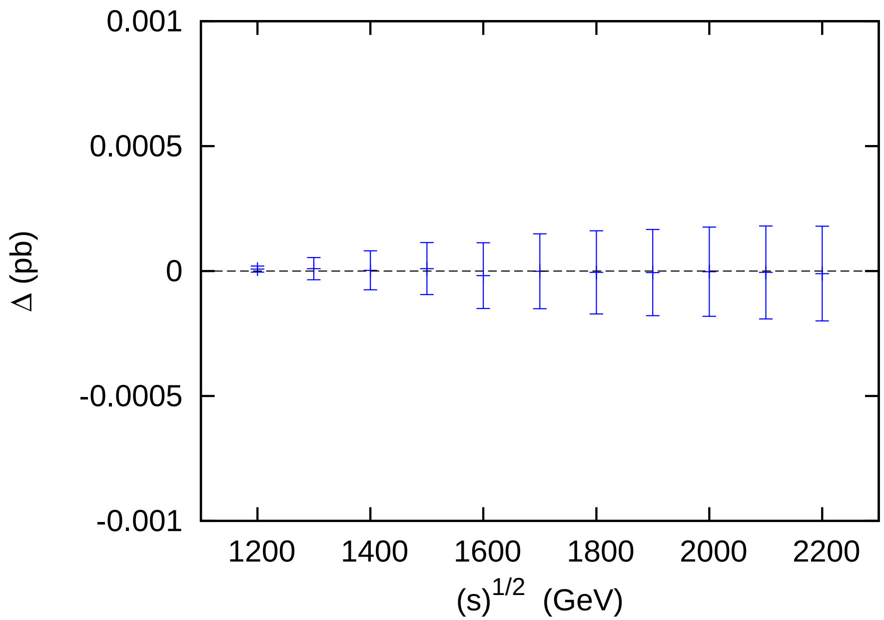
Singular contributions known analitically and numerics involve regular functions.

† more on this upon request

Mass singularities in real life

- Slicing & Subtraction
  - Two completely different approaches to the problem
  - Their comparison is a non trivial check for IR treatment
- Result of the comparison for the process $u\overline{u} \to \tilde{u}^L \tilde{u}^{L*} \gamma$



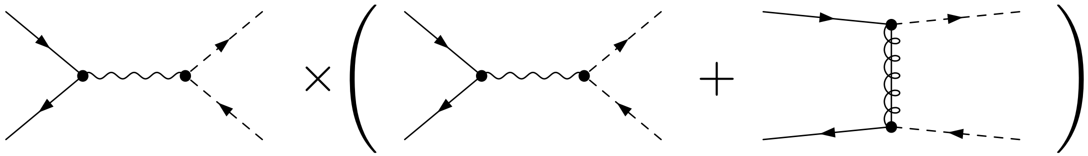


$\Delta = ($Dipole – Slicing$)$, point SPS1a′

Numerical Results

General Informations

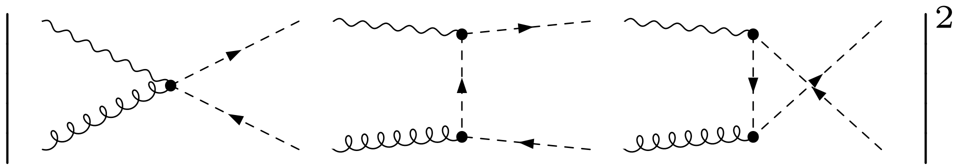
- The $\mathcal{O}(\alpha_s\alpha + \alpha^2)$ contributions



have been included

$\hookrightarrow$ their importance already known [Bornhauser, Drees, Dreiner & Kim, '07]

- The partonic process $g\gamma \to \tilde{Q}^a\tilde{Q}^{a*}$:



has been included

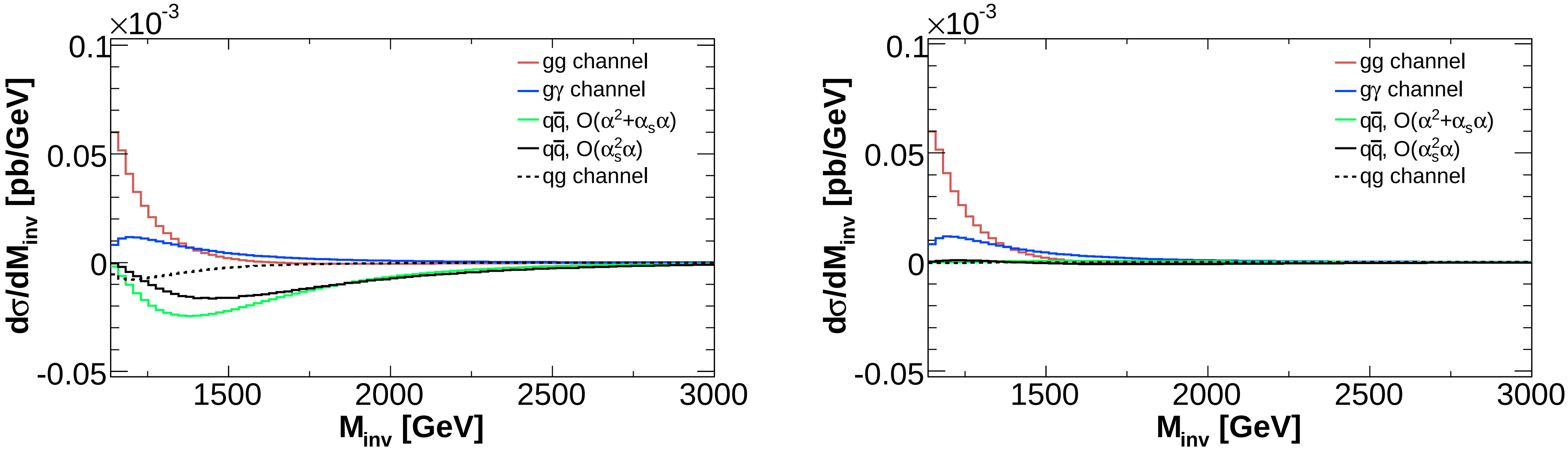
$\hookrightarrow$ Photon induced $\mathcal{O}(\alpha_s\alpha)$ process

$\hookrightarrow$ included in the stop case, found to be important [Hollik, Kollar & Trenkel, '07]

Dependence on the flavour

Invariant mass distribution, $\left[\delta = \frac{\mathcal{O}^{\text{NLO}} - \mathcal{O}^{\text{LO}}}{\mathcal{O}^{\text{NLO}}}\right]$

$\tilde{u}^L$ $\tilde{c}^L$



- Different behaviour of $q\bar{q} \to \tilde{Q}^a \tilde{Q}^{a*}$ with q & $\tilde{Q}$ in the same SU(2) doublet
  - $\hookrightarrow$ $\tilde{c}$ case: They are suppressed by charm PDF
  - $\hookrightarrow$ $\tilde{u}$ case: Sizable and negative

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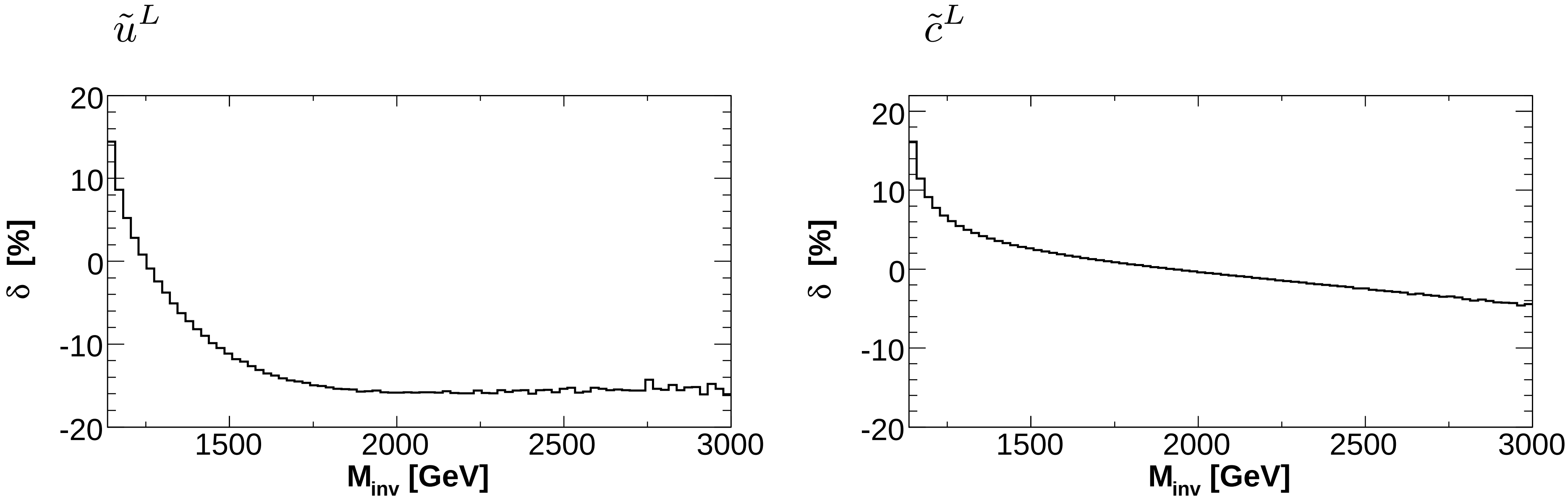

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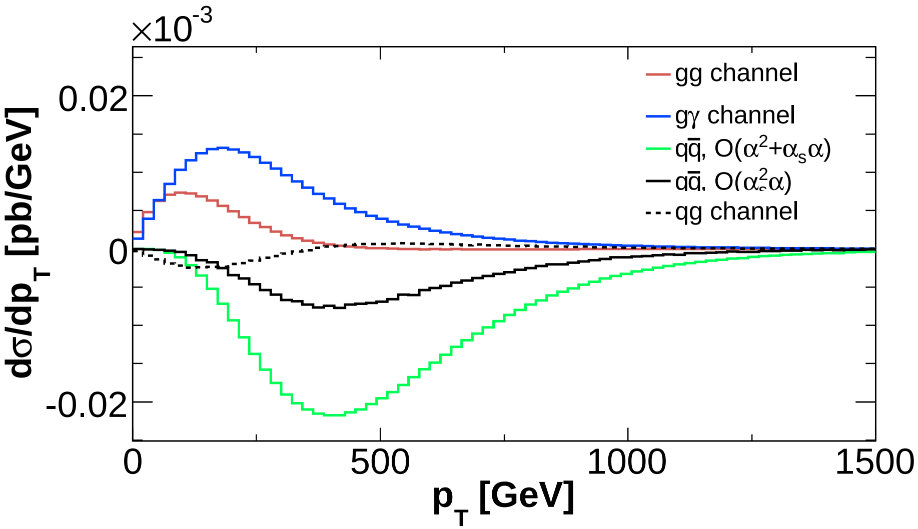
  - $\hookrightarrow$ $\tilde{c}$ case: They are suppressed by charm PDF
  - $\hookrightarrow$ $\tilde{u}$ case: Sizable and negative
- key role of the $q\bar{q}$ channels:
  - $\hookrightarrow$ $\tilde{u}, \tilde{c}$ case: lead the corrections in the high M_{inv} region
  - $\hookrightarrow$ $\tilde{u}$ case: render EW corrections sizable (& negative) for $M_{\text{inv}} > 1300$ GeV

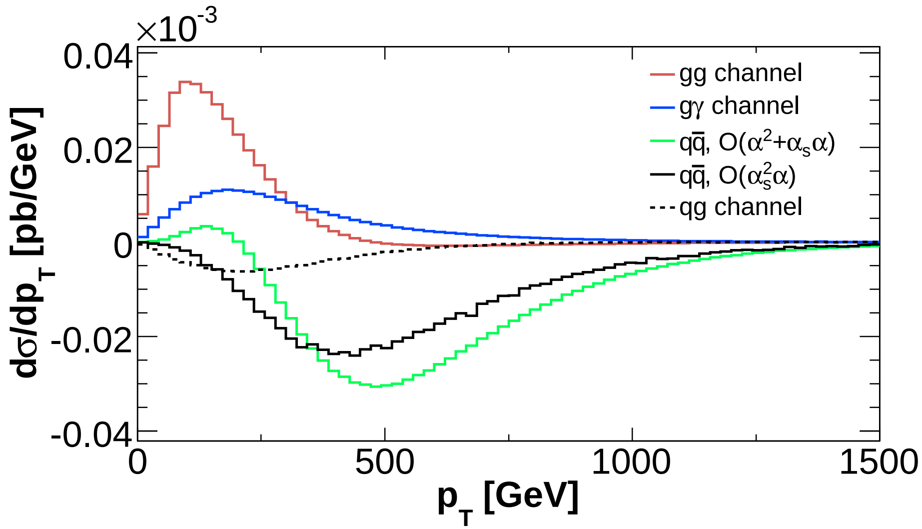
Dependence on the chirality

Transverse Momentum distribution, $\left[\delta = \frac{\mathcal{O}^{\mathrm{NLO}} - \mathcal{O}^{\mathrm{LO}}}{\mathcal{O}^{\mathrm{NLO}}}\right]$

$\tilde{u}^R$, point SPS1a′



$\tilde{u}^L$, point SPS1a′



- $g\gamma$ channel chirality-independent, the others more important in the $\tilde{u}^L$ case
- $\mathcal{O}(\alpha_s^2\alpha)$ & $\mathcal{O}(\alpha_s\alpha + \alpha^2)$ comparable (at least in the left handed case)
- $q\bar{q}$ @ $\mathcal{O}(\alpha_s\alpha + \alpha^2)$ different behaviour in the low p_T region
  $\hookrightarrow$ MSSM is chiral $\Rightarrow d\ \tilde{u}\ \chi^{\pm}$ vertex is chiral dependent

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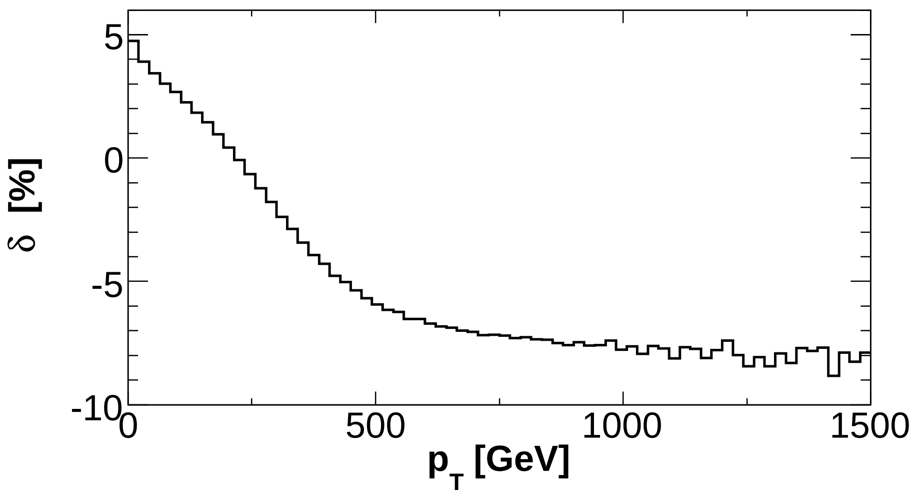

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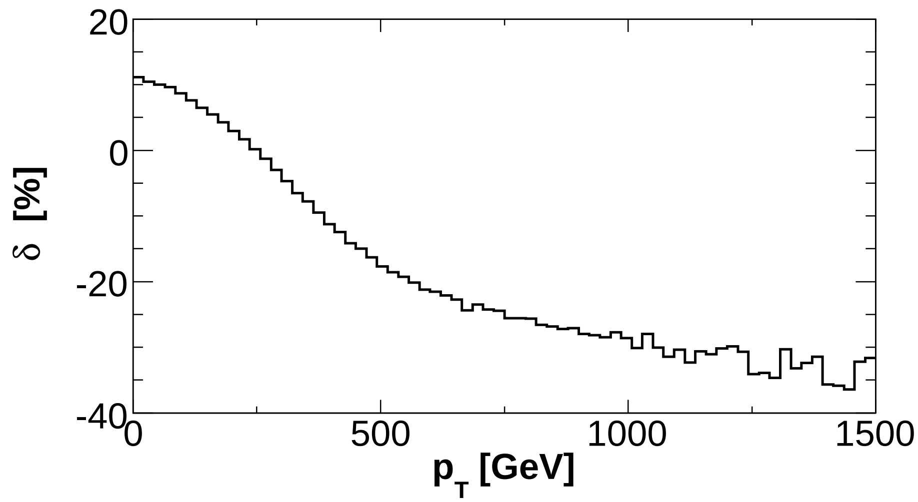

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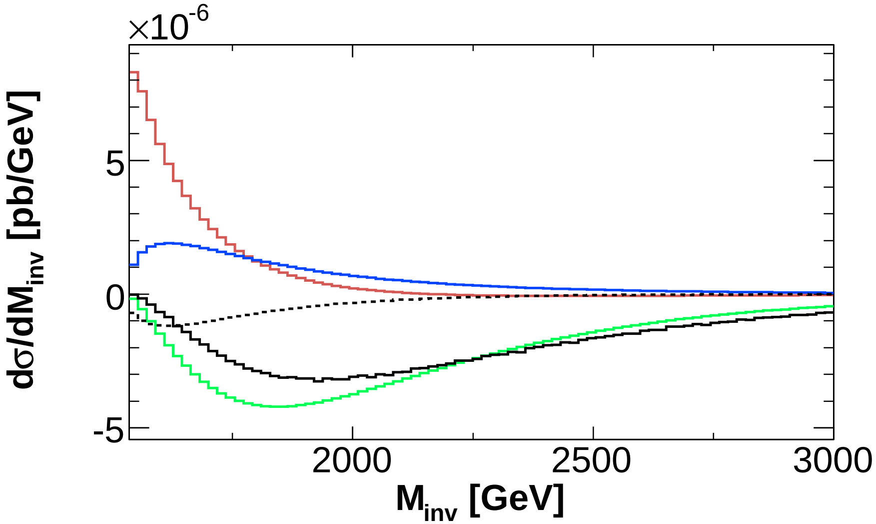
- NLO EW corrections important (at least in the left handed case).

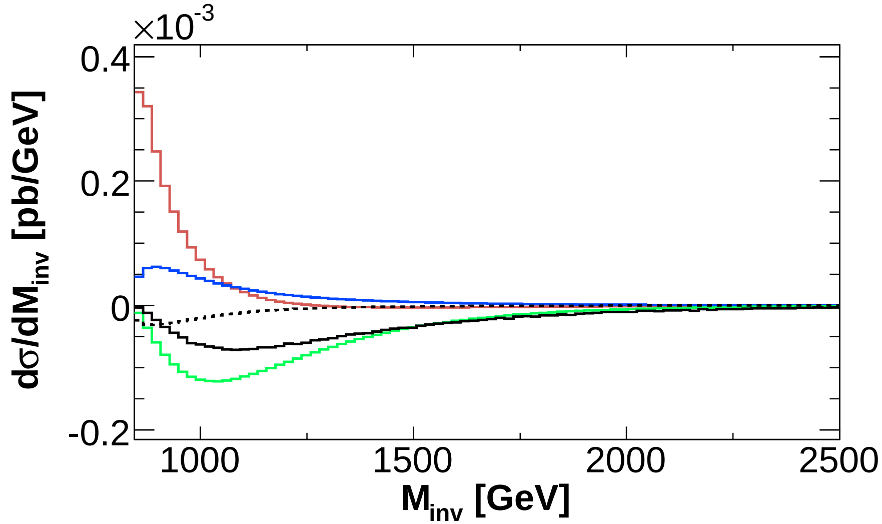
Dependence on the Scenario

Invariant Mass distribution for $\tilde{u}^L$ production

point SU1, $m_{\tilde{u},L} = 766$ GeV



point SU4, $m_{\tilde{u},L} = 420$ GeV



- The size of the contributions decrease as the squark mass increases
- Corrections dominated by $\left\{ \begin{array}{l} gg, \quad g\gamma \\ q\,\bar{q} \end{array} \right.$ channels in the $\left\{ \begin{array}{l} \text{low} \\ \text{high} \end{array} \right.$ M_{inv} region

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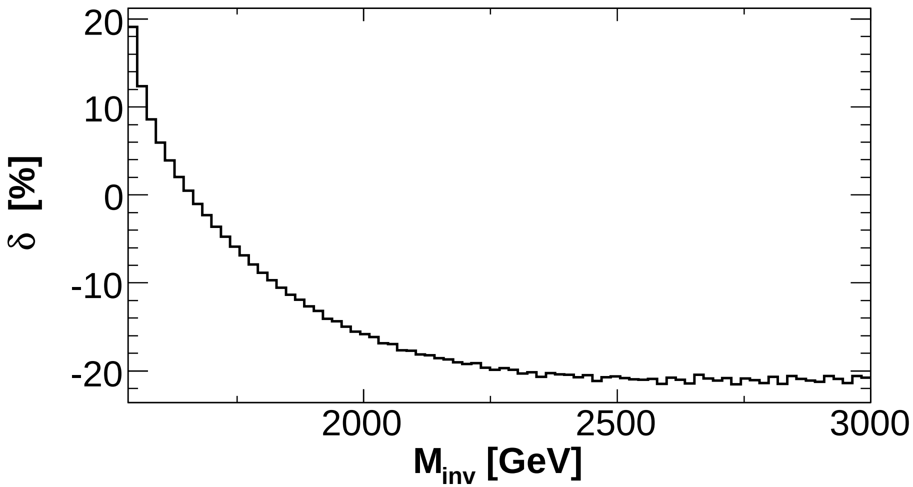

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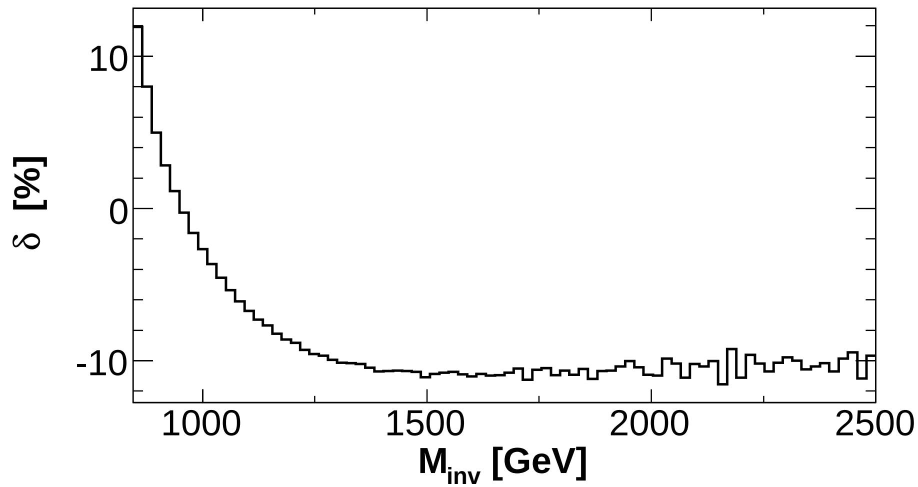

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- NLO EW corrections sizable, increasing as the squark mass increases

Conclusions & Outlook

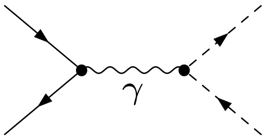
- $P\,P \rightarrow \tilde{Q}^a\,\tilde{Q}^{a*}$ important process in the hunting for SUSY
- $\mathcal{O}(\alpha_s^2\alpha)$ corrections to this process available for arbitrary $\tilde{Q}^a\,\tilde{Q}^{a*}$ pair:
  - depend on flavour, chirality and mass of the squarks,
  - sizable, at least in the distributions . . .
  - . . . Not trivial to compute.

Road map:

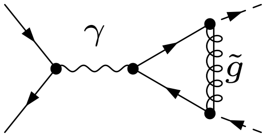
- Compute the $\mathcal{O}(\alpha_s^2\alpha)$ corrections to other processes of production of SUSY colored particle:
  - gluino pair production $\longrightarrow$ completed
  - squark-squark pair production $\longrightarrow$ next project
  - non diagonal squark–anti-squark production $\longrightarrow$ on the wish list
- Merge EW corrections with the NLO QCD ones

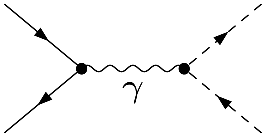
Backup Slides

Renormalization . . . How to get a number out of ∞

tree level 

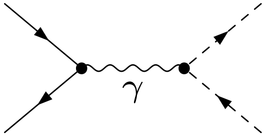
$= -\frac{32\pi}{9S}\alpha\; \bar{v}(k_{\bar{q}})\gamma^{\mu} k_{\tilde{u}} u(k_q)$

one loop (Dim. Reg.) 

$= \frac{2}{3}\frac{\alpha_s}{\pi}\Delta$ 

$+$ finite

Renormalization ... How to get a number out of ∞

tree level 

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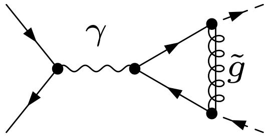
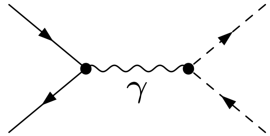
one loop (Dim. Reg.) 

$= \frac{2}{3}\frac{\alpha_s}{\pi}\Delta$ 

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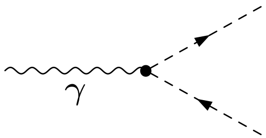
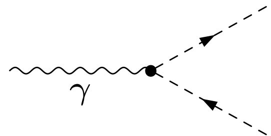
BUT parameters are not connected to real world! ... reparametrizing via:

$$\alpha \to (1+\delta Z_e)\alpha; \quad \Phi_{\tilde{q}} \to \left(1+\frac{\delta Z_{\tilde{Q}}}{2}\right)\Phi_{\tilde{u}}; \quad \begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} \to \left[1+\frac{1}{2}\begin{pmatrix} \delta Z_{AA} & \delta Z_{AZ} \\ \delta Z_{ZA} & \delta Z_{ZZ} \end{pmatrix}\right]\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix}$$



$$\longrightarrow \left(1+\delta Z_e+\delta Z_{\tilde{Q}}+\frac{\delta Z_{AA}}{2}+\frac{(3-4s_W^2)}{16c_W s_W}\frac{\delta Z_{ZA}}{2}\right)$$


connecting α and the fields to physical quantities (more on this later)

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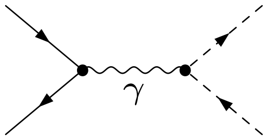
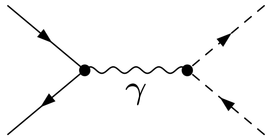
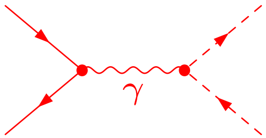
tree level 

$\longrightarrow$ 

$-\frac{2}{3}\frac{\alpha_s}{\pi}\Delta$ 

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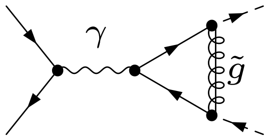
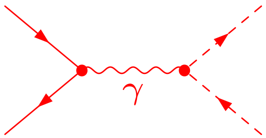
one loop (Dim. Reg.) 

$= \frac{2}{3}\frac{\alpha_s}{\pi}\Delta$ 

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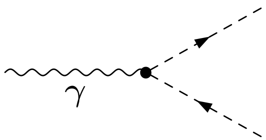

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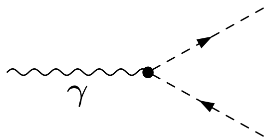

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$$\delta Z_e = \delta Z_{AZ} = \delta Z_{ZZ} = 0, \qquad \delta Z_{\tilde{Q}} = -\frac{2}{3}\frac{\alpha_s}{\pi}\Delta$$

SO $\mathcal{O}(\alpha_s)$ corrections are finite!

Phase Space Slicing & Dipole

Consider the process $q \ \bar{q} \ \rightarrow \ \tilde{Q}^a \ \tilde{Q}^{a*} \ \gamma$

$$\sigma_{q\bar{q}\rightarrow\tilde{Q}^a\tilde{Q}^{a*}\gamma} = \int \qquad d\Phi_3 \ \ |\mathcal{M}|^2$$

$d\phi_3$ = phase space measure

Phase Space Slicing & Dipole

Consider the process $q \; \bar{q} \; \rightarrow \; \tilde{Q}^a \; \tilde{Q}^{a*} \; \gamma$

$$\sigma_{q\bar{q}\rightarrow\tilde{Q}^a\tilde{Q}^{a*}\gamma} = \int_{\substack{E_\gamma > \Delta E \\ \theta_{q\gamma},\, \theta_{\bar{q}\gamma} < \Delta\theta}} d\Phi_3 \; |\mathcal{M}|^2 + \boxed{\int_{\text{singular region}} d\Phi_3 \; |\mathcal{M}|^2}$$

computed in eikonal approx.

$d\phi_3$ = phase space measure
$\theta_{i\gamma}$ = angle between γ and i
E_γ = energy of γ

- Phase Space Slicing. The photon phase space is divided into two parts introducing cuts:
  - regular region integrated numerically
  - singular region eikonal approximation after mass regularization
  - Remarks:
    - + Intutive method
    - – Cuts have to be small (eikonal approximation) . . .
    - – . . . But not too much (numerical instabilities)

Phase Space Slicing & Dipole

Consider the process $q \ \bar{q} \ \rightarrow \ \tilde{Q}^{a} \ \tilde{Q}^{a*} \ \gamma$

$$\sigma_{q\bar{q}\rightarrow\tilde{Q}^{a}\tilde{Q}^{a*}\gamma} = \int d\Phi_3 \left[\, |\mathcal{M}|^2 - |\mathcal{M}_{\text{sub}}|^2 \, \right] + \boxed{\int d\Phi_3 |\mathcal{M}_{\text{sub}}|^2}$$

exactly computed

$d\phi_3 \ =$ phase space measure

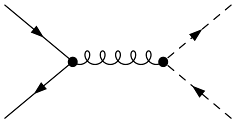
- Subtraction method. Add and subtract a function $\mathcal{M}_{\text{sub}}$ such that
  $i)$ $\mathcal{M}_{\text{sub}}$ and $\mathcal{M}$ have same singularity structure
  $ii)$ $\mathcal{M}_{\text{sub}}$ easy enough to be analitically computed
  - $(|\mathcal{M}_{\text{sub}}|^2 - |\mathcal{M}|^2)$ is regular and evaluated numerically
  - $\int d\Phi_3 |\mathcal{M}_{\text{sub}}|^2$ exactly evaluated (after mass regularization)
  - Remarks:
    + All numerics involve regular functions
    + No cut off are needed
    + leads to more precise results

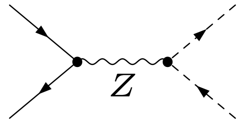
Gluon emission & color algebra

Caveat Gluon carries charge $\Rightarrow$ color correlation after gluon emission.

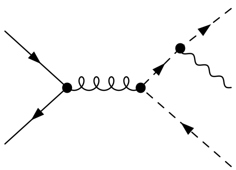
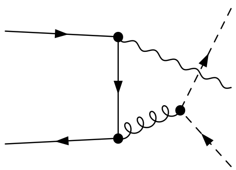
$\hookrightarrow$ Color algebra has to be considered when extracting singularities

consider the colored amplitudes

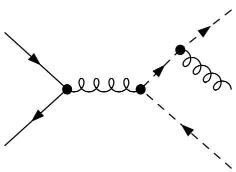
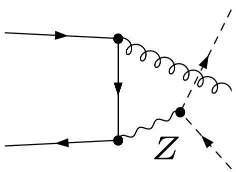
$\mathcal{M}_g^{c_q\ c_{\bar q}\ c_{\tilde Q}\ c_{\tilde Q^*}} =$ 

$\mathcal{M}_Z^{c_q\ c_{\bar q}\ c_{\tilde Q}\ c_{\tilde Q^*}} =$ 

so in the soft limit we have:

 $\times$ 

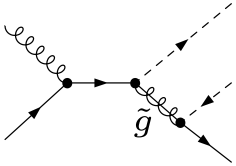
$$\sim \sum_c \quad e_q \quad e_{\tilde Q} \quad \mathcal{M}_g^{c_q\ c_{\bar q}\ c_{\tilde Q}\ c_{\tilde Q^*}} \left(\mathcal{M}_g^{c_q\ c_{\bar q}\ c_{\tilde Q}\ c_{\tilde Q^*}}\right)^*$$

 $\times$ 

$$\sim \sum_{c,b} \quad t^A_{b_q c_q}\ t^A_{c_{\tilde Q} b_{\tilde Q}} \mathcal{M}_g^{c_q\ c_{\bar q}\ c_{\tilde Q}\ c_{\tilde Q^*}} \left(\mathcal{M}_Z^{b_q\ c_{\bar q}\ b_{\tilde Q}\ c_{\tilde Q^*}}\right)^*$$

$\hookrightarrow$ In case of g emission amplitudes with different color structure interfere.

Resonances



$$= \frac{F(\bar{s})}{\bar{s} - m_{\tilde{g}}^2} \qquad \bar{s} = (p_q + p_{\tilde{Q}^*})^2$$

- when $m_{\tilde{g}} > m_{\tilde{Q}}$ we have $\bar{s} = m_{\tilde{g}} \Rightarrow$ singularity!
- Reasonable questions:
  - Where is it from?

    if $m_{\tilde{g}} > m_{\tilde{Q}}$ then $\tilde{g}$ is unstable

    $\hookrightarrow$ unconsistent with perturbation theory (that assumes stable particles)
  - How to cope?

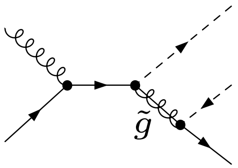
    Build a consistent perturbative field theory with stable particles only

    $\hookrightarrow$ done for a toy model [Veltmann '63]

    $\hookrightarrow$ the propagators of the (virtual) unstable particle are Dyson resummed:

    $\frac{1}{p^2 - m^2} \rightarrow \frac{1}{p^2 - \mu^2}$ $\qquad$ μ^2 is complex so no singularity!

Resonances



$$= \frac{F(\bar{s})}{\bar{s}-m_{\tilde{g}}^2} \longrightarrow \frac{F(\bar{s})}{\bar{s}-m_{\tilde{g}}^2+im_{\tilde{g}}\Gamma_{\tilde{g}}} \qquad \bar{s} = (p_q + p_{\tilde{Q}^*})^2$$

- when $m_{\tilde{g}} > m_{\tilde{Q}}$ we have $\bar{s} = m_{\tilde{g}} \Rightarrow$ singularity!
- Reasonable questions:
  - Where is it from?

    if $m_{\tilde{g}} > m_{\tilde{Q}}$ then $\tilde{g}$ is unstable

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    $\frac{1}{p^2-m^2} \longrightarrow \frac{1}{p^2-\mu^2}$ $\qquad \mu^2$ is complex so no singularity!
- Naive regularization:

$$(\bar{s}-m_{\tilde{g}}^2) \rightarrow (\bar{s}-\mu_{\tilde{g}}^2) = (\bar{s}-m_{\tilde{g}}^2+im_{\tilde{g}}\,\Gamma_{\tilde{g}}) \qquad @\ \mathcal{O}(\alpha_s)$$

Singularity $\rightarrow$ Resonance