Exploring the Standard Candle
The many faces of $B \rightarrow X_s \gamma$ decay

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References (2004-2005)

- MN: *"Renormalization-Group Improved Calculation of the $B \to X_s \gamma$ Branching Ratio"* [hep-ph/0408179] → EPJC
- MN: *"Advanced Predictions for Moments of the $B \to X_s \gamma$ Photon Spectrum"* [hep-ph/0506245] → PRD
- B. Lange, MN, G. Paz: *"Theory of Charmless Inclusive B Decays and the Extraction of $|V_{ub}|$"* [hep-ph/0504071] → PRD
- B. Lange, MN, G. Paz: *"A Two-Loop Relation between Inclusive Radiative and Semileptonic B-Decay Spectra"* [hep-ph/0508178] → JHEP
- T. Becher, MN: *"Toward a NNLO Prediction for the $B \to X_s \gamma$ Decay Rate with a Cut on Photon Energy: I. Two-Loop Result for the Soft Function"* [hep-ph/0512208]

Exploring the Standard Candle

The many faces of $B \to X_s \gamma$ decay

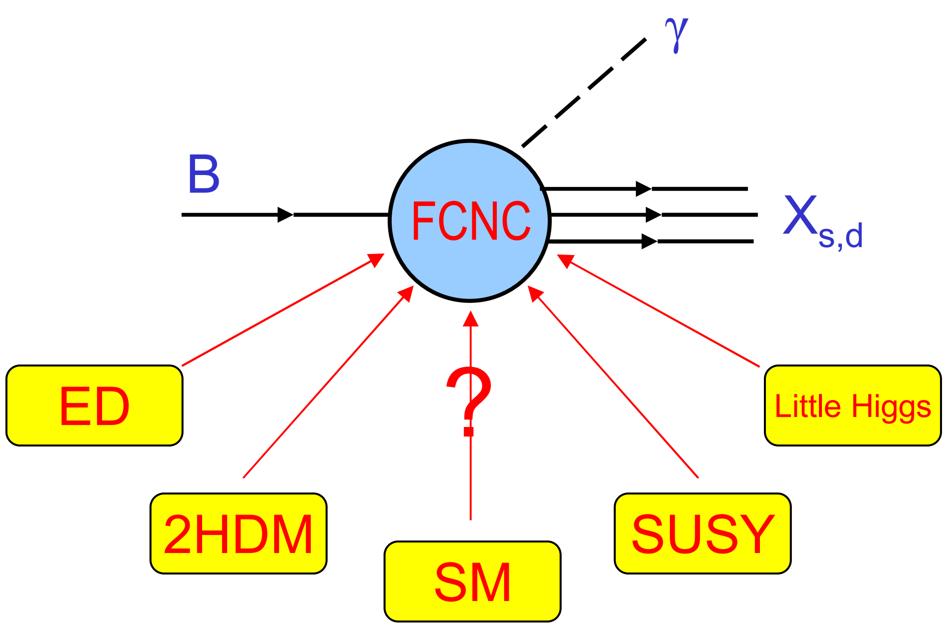
Introduction
Soft-collinear factorization
Scale separation
Results:
- $\Gamma(B \to X_s \gamma)$, New Physics, b-quark mass
- Extraction of $|V_{ub}|$

Introduction

Inclusive and exclusive
$B \to X_s \gamma$ decays,
Cut on photon energy

Rare radiative decays $B \longrightarrow X\gamma$

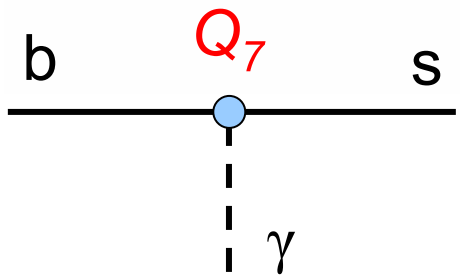
- Prototype FCNC processes



Inclusive decay rate

$$\Gamma(B \to X_s\gamma) = \frac{G_F^2\alpha}{32\pi^4}\,|V_{tb}V_{ts}^*|^2\,m_b^5\,C_{7\gamma}^2$$

$$\times\left\{1 + O(\alpha_s) + O(\alpha_s^2) + O\left(\frac{\Lambda_{\rm QCD}^2}{m_b^2}\right) + \ldots\right\}$$

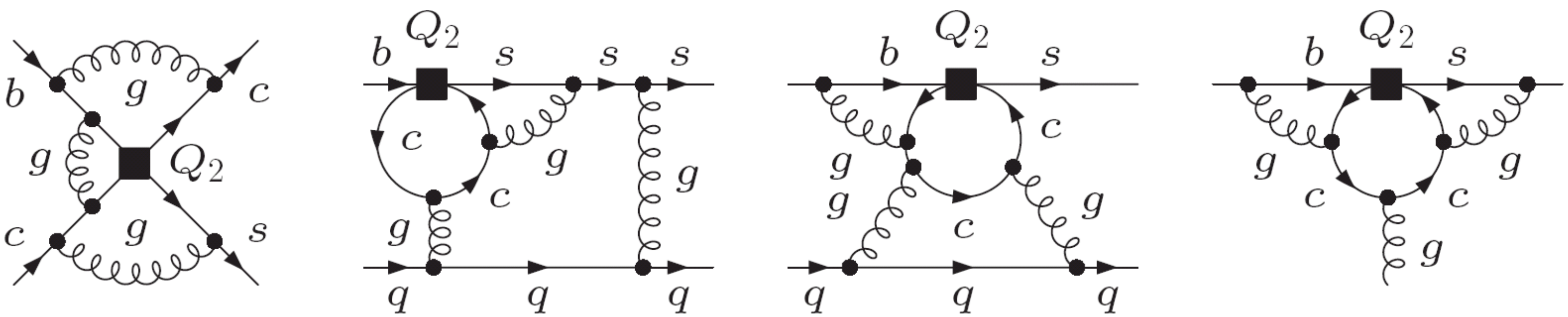


Calculable using OPE

- NLO calculation completed (1991-2001)
- NNLO calculation in progress (2003-…)

Inclusive decay rate at NNLO

- Effective weak Hamiltonian at NNLO
  - 3-loop (☺) and 4-loop (¿) anomalous dimensions
  - 2-loop (☺) and 3-loop (¿) matching conditions

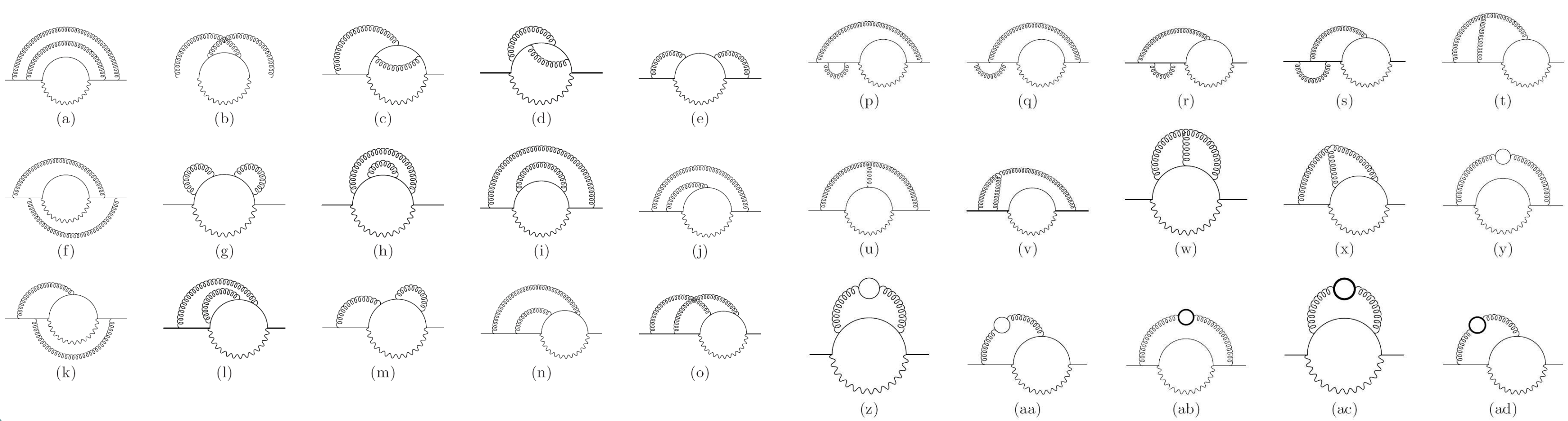


[Gorbahn, Haisch: hep-ph/0411071
Gorbahn, Haisch, Misiak: hep-ph/0504194]

Inclusive decay rate at NNLO

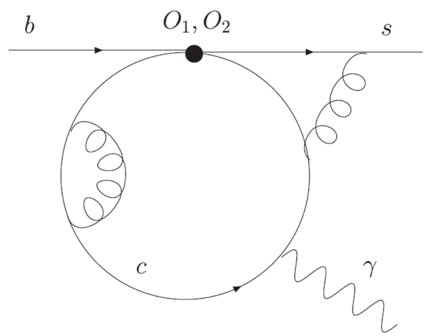
- Matrix elements at NNLO
  - 2-loop matrix elements of dipole operators (☺)

[Melnikov, Mitov: hep-ph/0505097
Blokland, Czarnecki, Misiak, Slusarczyk, Tkachov: hep-ph/0506055]



Inclusive decay rate at NNLO

- Matrix elements at NNLO
  - 3-loop (☻) penguin matrix elements



[Bieri, Greub, Steinhauser: hep-ph/0302051
Asatrian, Greub, Hovhannisyan, Hurth,
Poghosyan: hep-ph/0505068]

- One of the hardest calculations in QFT, with many people devoting significant resources

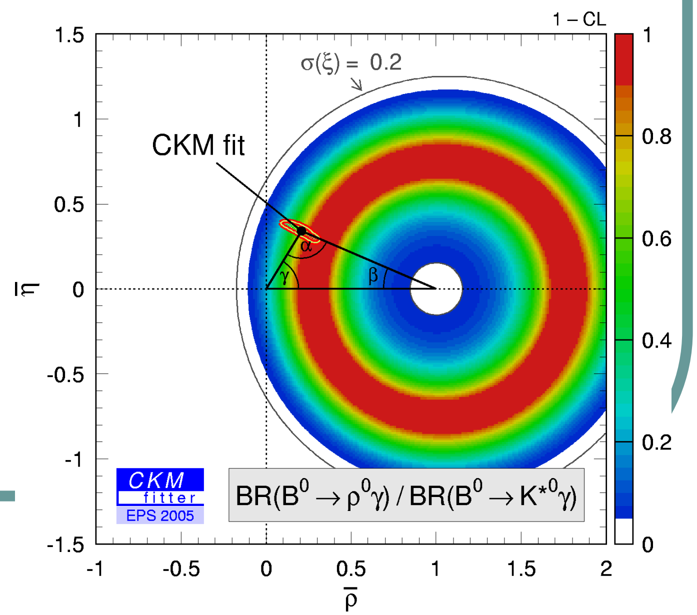
Exclusive decay rates

- QCD factorization formula

$$\langle V\gamma(\epsilon)|Q_i|\bar{B}\rangle = \left[F^{B\to V}(0)\,T_i^I + \int_0^1 d\xi\,dv\,T_i^{II}(\xi,v)\,\Phi_B(\xi)\,\Phi_V(v)\right]\cdot\epsilon$$

[Beneke, Feldmann, Seidel: hep-ph/0106067
Bosch, Buchalla: hep-ph/0106081
Proof: Becher, Hill, MN: hep-ph/0503263]

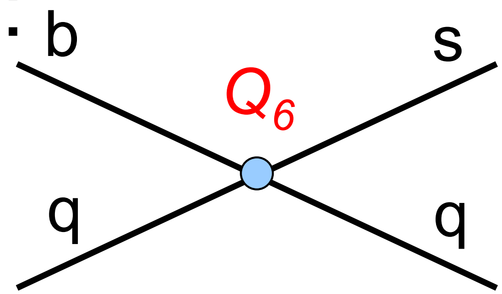
- Hadronic uncertainties
- Useful constraint on UT from $\Gamma(B\to\rho\gamma)/\Gamma(B\to K^*\gamma)$ $\to |V_{td}/V_{ts}|$



Other interesting observables

- Exclusive isospin asymmetry:

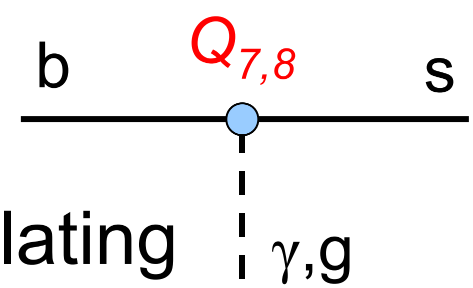
$$\Delta_{0-} \equiv \frac{\Gamma(\bar{B}^0 \to \bar{K}^{*0}\gamma) - \Gamma(B^- \to K^{*-}\gamma)}{\Gamma(\bar{B}^0 \to \bar{K}^{*0}\gamma) + \Gamma(B^- \to K^{*-}\gamma)}$$



  - Sensitive probe of new contributions to QCD penguin operators

    [Kagan, MN: hep-ph/0110078]

- Inclusive CP asymmetry:



  - Sensitive probe of new, CP-violating contributions to dipole operators

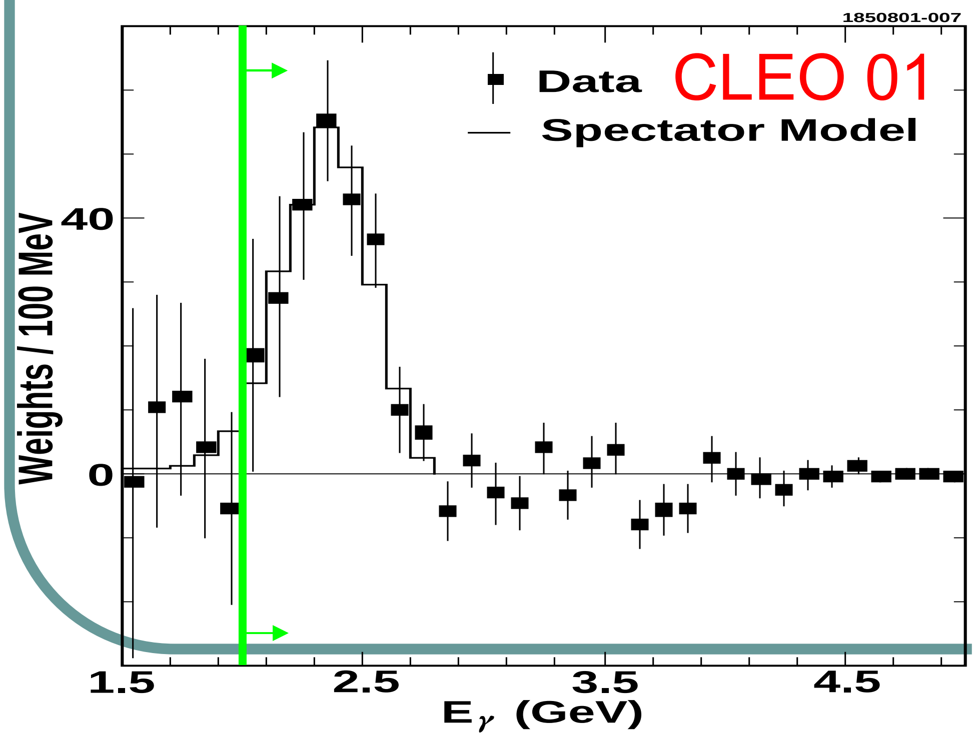
    [Kagan, MN: hep-ph/9803368]

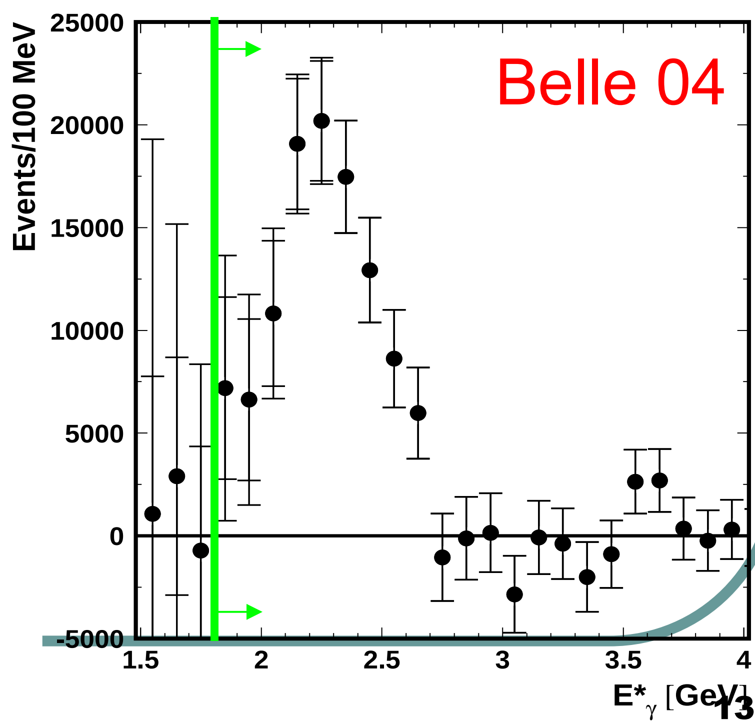
Partially inclusive decay rate

- Total decay rate cannot be measured for various reasons
  - Soft-photon divergence (need combination with $\Gamma(B \to X_s g)$)
  - Need to suppress background from $B \to \psi X_s$ followed by $\psi \to X\gamma$ [Misiak: hep-ph/0002007]
  - Experimental signature is high-energy photon (otherwise huge background)

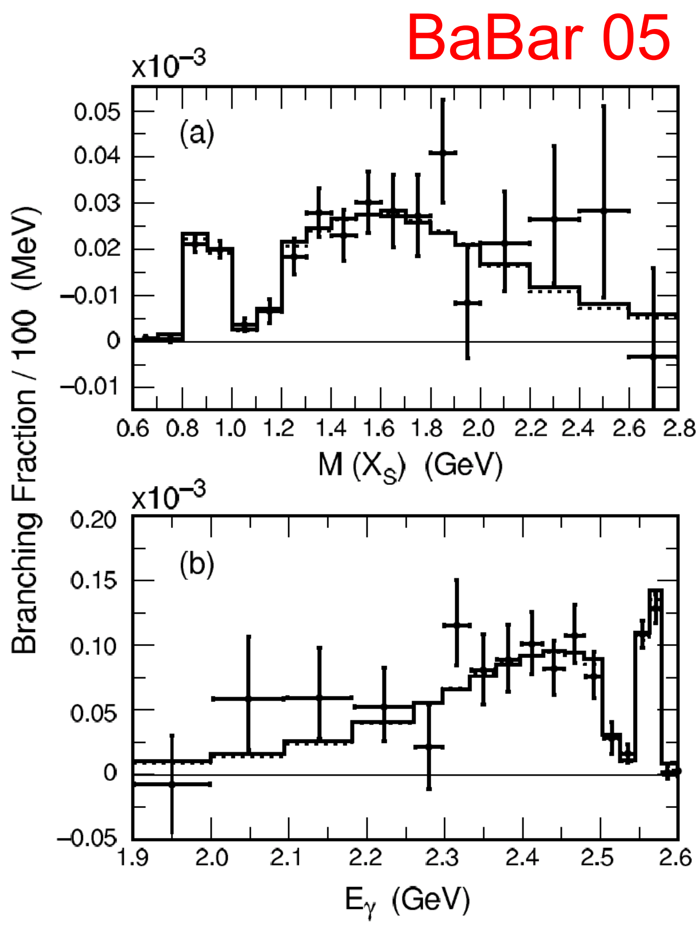
Partially inclusive decay rate

- Restriction to high-energy part of photon spectrum: $E_\gamma > E_0 = 1.8$ GeV or larger (in B-meson rest frame)





Partially inclusive decay rate



- Presence of the photon-energy cut leads to significant complications in the theoretical analysis!

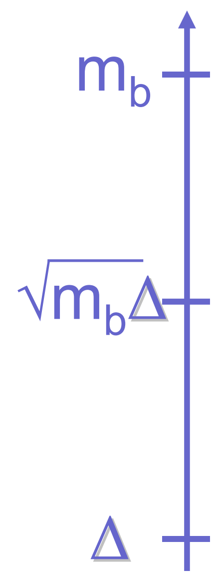
[MN: hep-ph/0408179]

Soft-collinear factorization

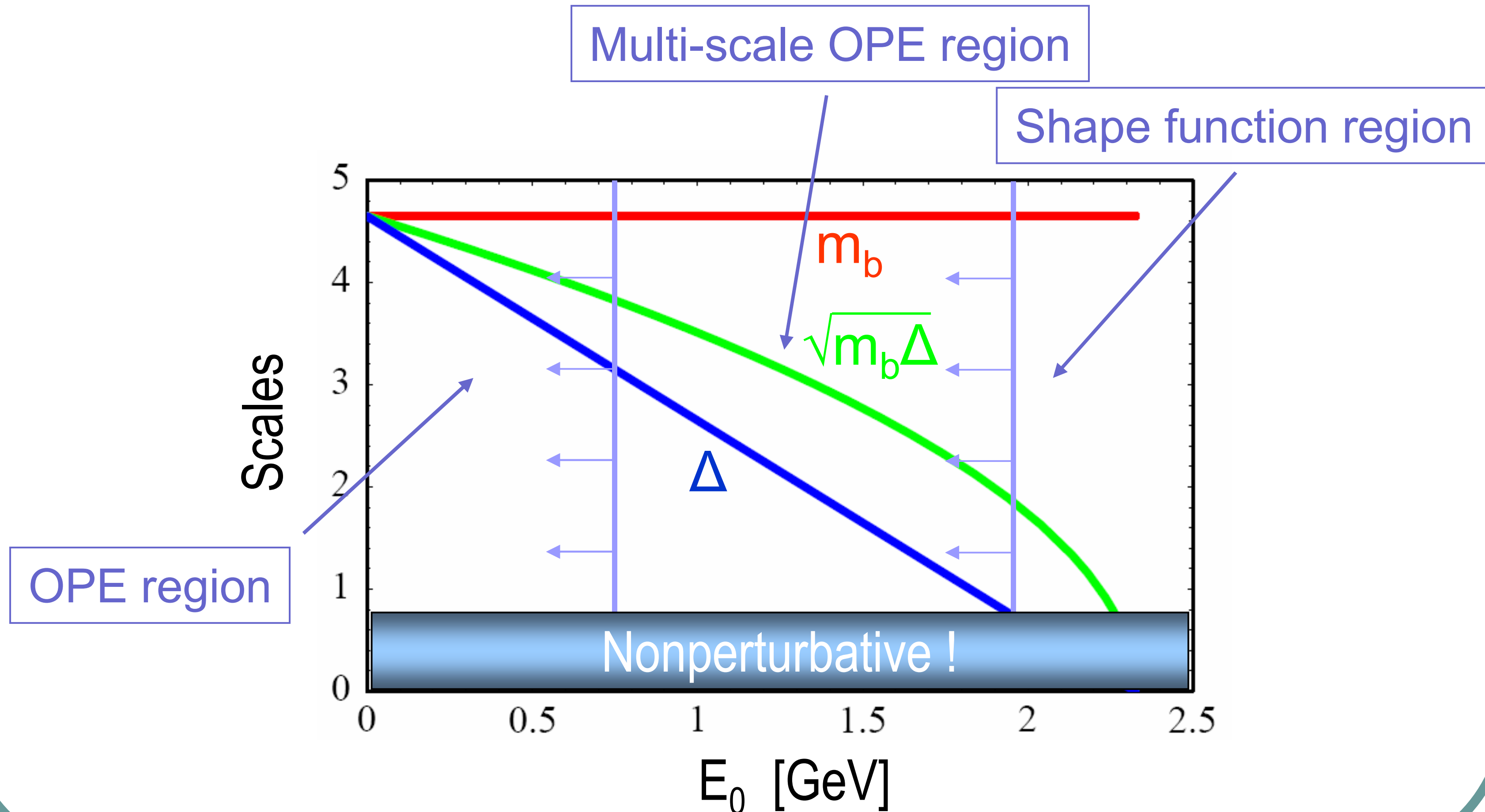
Different scales,
Factorization formula,
Non-local matrix elements

Relevance of different scales

- Presence of photon-energy cut introduces new scale $\Delta = m_b - 2E_0$
- Must distinguish:
  - Hard scale m_b: quantum corrections to effective weak-interaction vertices
  - Intermediate scale $\sqrt{m_b\Delta}$: invariant mass of final-state hadronic jet
  - Soft scale Δ: scale at which internal structure of B-meson is probed

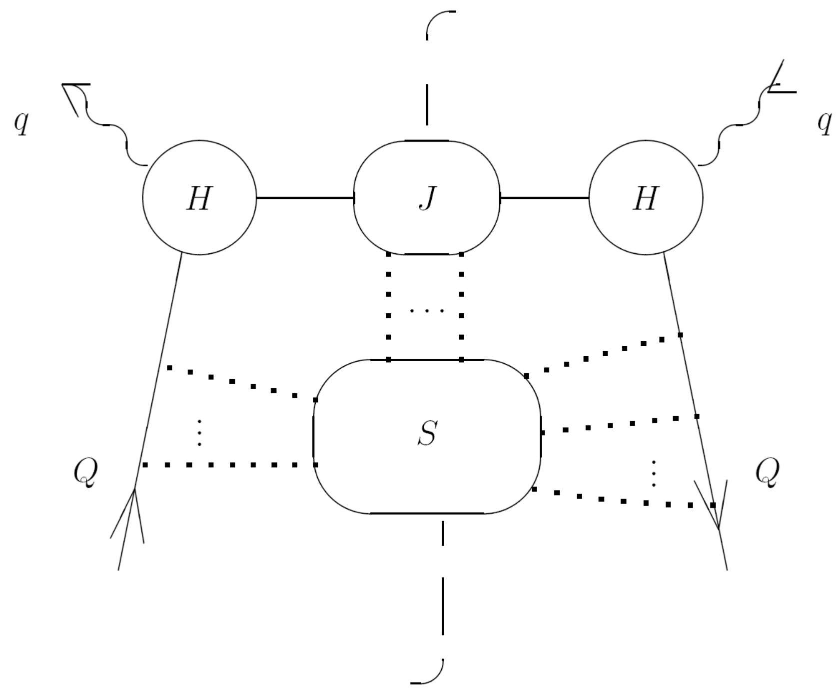


Relevance of different scales



Factorization theorem

- At leading power in Λ_{QCD}/m_b, the decay rate factorizes into a convolution of three objects:



$$\frac{d\Gamma}{dE_\gamma} = \frac{G_F^2\alpha}{2\pi^4}\,|V_{tb}V_{ts}^*|^2\, m_b^2\, E_\gamma^3$$

$$\times\, |H_\gamma(\mu)|^2 \int_0^{M_B-2E_\gamma} d\hat{\omega}\, m_b\, J(m_b(M_B-2E_\gamma-\hat{\omega}),\mu)\, S(\hat{\omega},\mu) + \ldots$$

[MN: hep-ph/9312311
Korchemsky, Sterman: hep-ph/9407344
Proof: Bauer, Pirjol, Stewart: hep-ph/0109045]

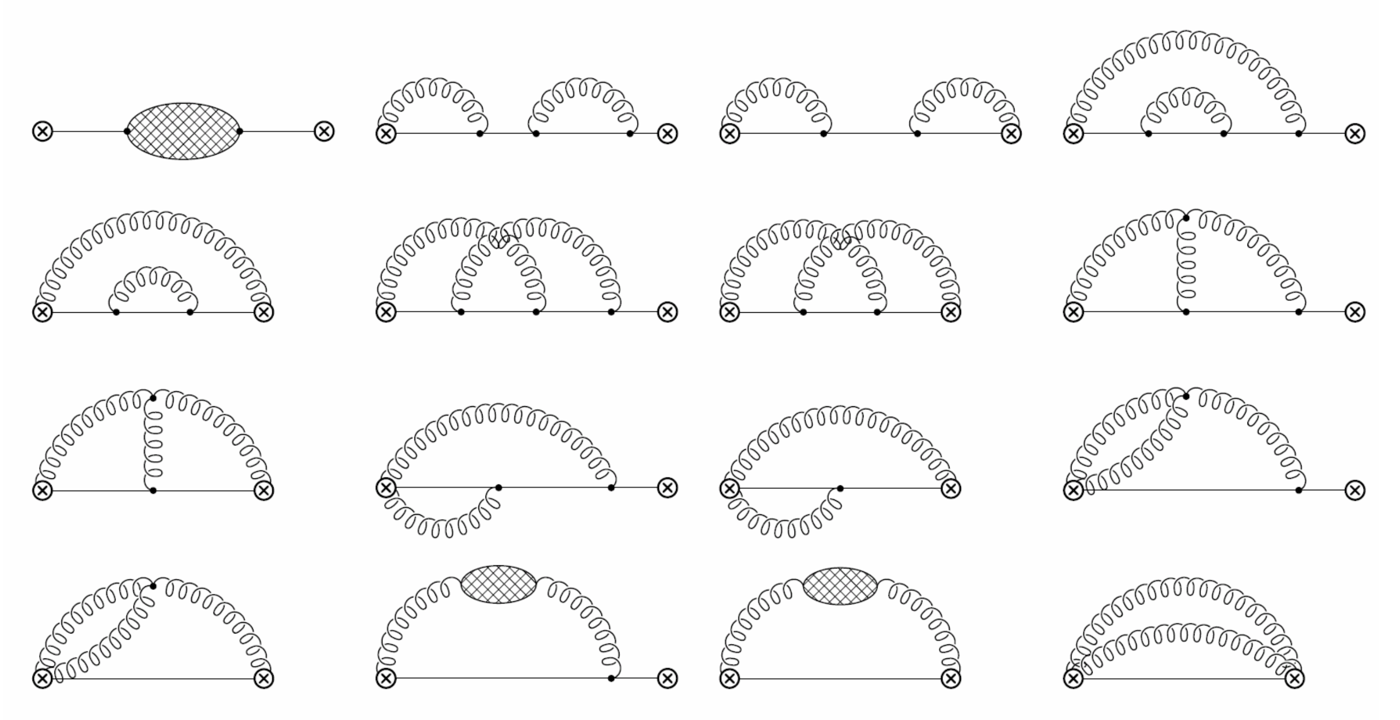
Non-local matrix elements

- Jet function $J(p^2,\mu)$:
  - Fourier transform of hard-collinear quark propagator dressed by Wilson line
    perturbative quantity
- Shape function $S(\hat{\omega},\mu)$:
  - Fourier transform of forward matrix element $\langle B|\bar{h}_v(0)[0,x]h_v(x)|B\rangle$ at light-like separation
    nonperturbative object (parton distribution)

Non-local matrix elements

$$J(p^2,\mu) = \delta(p^2)\left[1+\frac{C_F\alpha_s(\mu)}{4\pi}(7-\pi^2)\right] + \frac{C_F\alpha_s(\mu)}{4\pi}\left[\frac{1}{p^2}\left(4\ln\frac{p^2}{\mu^2}-3\right)\right]_*^{[\mu^2]}$$

$+\ O(\alpha_s^2)$ [Becher, MN: in preparation]



Hadronic physics

- Shape function integrals can be written as contour integrals in the complex ω-plane:

$$\int_{-\bar{\Lambda}}^{\Delta} d\omega\, S(\omega,\mu)\, f(\omega) \propto \oint_{|\omega|=\Delta} d\omega\, f(\omega)\, \langle \bar{B}(v)|\, \bar{h}_v \frac{1}{\omega + in\cdot D + i\epsilon}\, h_v\, |\bar{B}(v)\rangle$$

- For sufficiently large Δ, right-hand side can expand in *local* operators
- Expansion in $(\Lambda_{QCD}/\Delta)^n$ and $\alpha_s(\Delta)$
  → not a heavy-quark expansion

Scale separation

Evolution equations,

Exact solutions

Evolution equations

- Use RGEs to disentangle contributions associated with:
  - Hard scale $\mu_h \sim m_b$
  - Intermediate scale $\mu_i \sim \sqrt{m_b \Delta}$
  - Soft scale $\mu_0 \sim \Delta$
- Set $\mu=\mu_i$ in factorization formula and evolve hard and soft functions to the hard and soft scales, respectively

$$\frac{d\Gamma}{dE_\gamma} = \frac{G_F^2 \alpha}{2\pi^4} |V_{tb} V_{ts}^*|^2 \, m_b^2 \, E_\gamma^3$$
$$\times \, |H_\gamma(\mu)|^2 \int_0^{M_B - 2E_\gamma} d\hat{\omega} \, m_b \, J(m_b(M_B - 2E_\gamma - \hat{\omega}), \mu) \, S(\hat{\omega}, \mu) + \ldots$$

Evolution equations

$$\frac{d}{d\ln\mu} H_\gamma(\mu) = \left[-\Gamma_c(\alpha_s)\,\ln\frac{\mu}{m_b} + \gamma_J(\alpha_s)\right] H_\gamma(\mu)$$

Sudakov logarithms

$$\frac{d}{d\ln\mu} S(\hat\omega,\mu) = \left[2\Gamma_c(\alpha_s)\,\ln\frac{\mu}{\hat\omega} - 2\gamma(\alpha_s)\right] S(\hat\omega,\mu) + 2\Gamma_c \int_0^{\hat\omega} d\hat\omega'\,\frac{S(\hat\omega',\mu) - S(\hat\omega,\mu)}{\hat\omega - \hat\omega'}$$

- Anomalous dimensions:

$$\Gamma_c(\alpha_s) = 0.424\alpha_s + 0.271\alpha_s^2 + 0.216\alpha_s^3 + \dots$$

[Korchemsky, Radyushkin: 1987
Moch, Vermaseren, Vogt: hep-ph/0403192]

$$\gamma_J(\alpha_s) = -0.531\alpha_s - 0.200\alpha_s^2 + \dots$$

[MN: hep-ph/0408179]

$$\gamma(\alpha_s) = -0.212\alpha_s - 0.389\alpha_s^2 + \dots$$

[Korchemsky, Marchesini: hep-ph/9210281
Gardi: hep-ph/0501257]

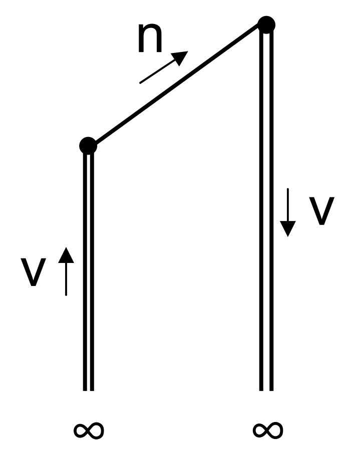
Exact solutions

$$|H_\gamma(\mu_i)|^2 = U_1(\mu_h,\mu_i)\,|H_\gamma(\mu_h)|^2$$

$$S(\hat{\omega},\mu_i) = U_2(\mu_i,\mu_0)\,\frac{e^{-\gamma_E\eta}}{\Gamma(\eta)}\int_0^{\hat{\omega}} d\hat{\omega}'\,\frac{S(\hat{\omega}',\mu_0)}{\mu_0^\eta(\hat{\omega}-\hat{\omega}')^{1-\eta}}$$

- With:

$$\boxed{\eta = 2\int_{\mu_0}^{\mu_i}\frac{d\mu}{\mu}\,\Gamma_c[\alpha_s(\mu)]}$$



Wonderful formula

- Master formula for the partial rate:

$$\Gamma \sim H(\mu_h) \; * \; U(\mu_h,\mu_i) \; * \; J(\mu_i) \; * \; U(\mu_i,\mu_0) \; * \; M(\mu_0)$$

QCD ⟶ SCET ⟶ RG evolution ⟶ HQET ⟶ RG evolution ⟶ Local OPE

$\mu_h \sim m_b$
$\mu_i \sim \sqrt{m_b \Delta}$
$\mu_0 \sim \Delta$

Perturbation theory

Hadronic physics

Wonderful formula

- Perform triple integral exactly: [MN: hep-ph/0506245]

$$\Gamma_{\rm OPE}(\Delta) = \frac{G_F^2\alpha}{32\pi^4}\,|V_{tb}V_{ts}^*|^2\, m_b^3\,\overline{m}_b^2(\mu_h)\,|H_\gamma(\mu_h)|^2\,U_1(\mu_h,\mu_i)\,U_2(\mu_i,\mu_0)$$
$$\times\quad \widetilde{j}\Big(\ln\frac{m_b\mu_0}{\mu_i^2}+\partial_\eta,\mu_i\Big)\,\widetilde{s}\,(\partial_\eta,\mu_0)\,\frac{e^{-\gamma_E\eta}}{\Gamma(1+\eta)}\left(\frac{\Delta}{\mu_0}\right)^\eta\left[1-\frac{\eta(1-\eta)}{6}\,\frac{\mu_\pi^2}{\Delta^2}+\dots\right]$$

Explicit dependence on Δ

Scales: $\mu_h \sim m_b$
$\mu_i \sim \sqrt{m_b\Delta}$
$\mu_0 \sim \Delta$

Kinetic-energy parameter

Wonderful formula

- The jet and soft functions, $\widetilde{j}$ and $\widetilde{s}$, are polynomials of their arguments and are derived from integrals of the original jet and shape functions (J and S): [MN: hep-ph/0506245]

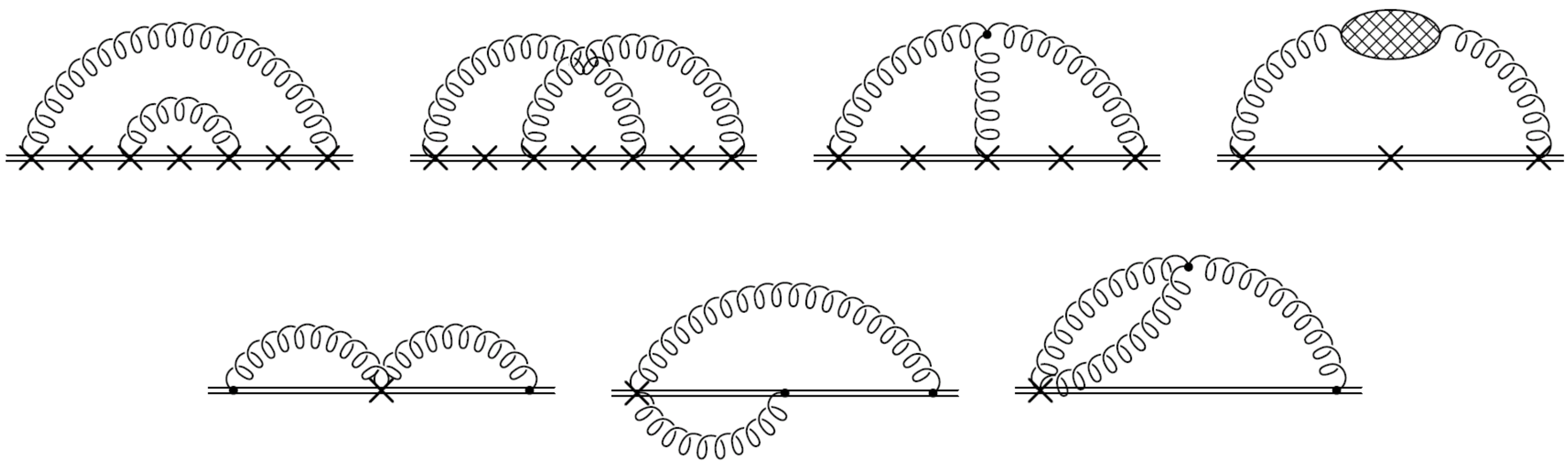
$$\widetilde{s}(L,\mu) = 1 + \left(-0.873 - 0.424L - 0.424L^2\right)\alpha_s$$
$$+ \left(\boxed{-0.603} + 0.750L + 0.471L^2 + 0.368L^3 + 0.090L^4\right)\alpha_s^2 + \dots$$
$$\widetilde{j}(L,\mu) = 1 + \left(0.045 - 0.318L + 0.212L^2\right)\alpha_s$$
$$+ \left(\dots + 0.145L + 0.301L^2 - 0.114L^3 + 0.023L^4\right)\alpha_s^2 + \dots$$

[Becher, MN: hep-ph/0512208]

work in progress

Wonderful formula

- Two-loop calculation of soft function:



[Becher, MN: hep-ph/0512208]

Results

Partial decay rate,

Implications for New Physics,

Determination of m_b

Partial $B \to X_s \gamma$ branching ratio

- Theoretical calculation with a cut at $E_0 = 1.8\text{GeV}$ (NLO):

$$Br(1.8\text{GeV}) = (3.30 \pm 0.33[\text{pert}] \pm 0.17[\text{pars}]) \cdot 10^{-4}$$

[MN: hep-ph/0408179]

  - Significant reduction of perturbative error expected at NNLO

- Experiment (Belle 2004):

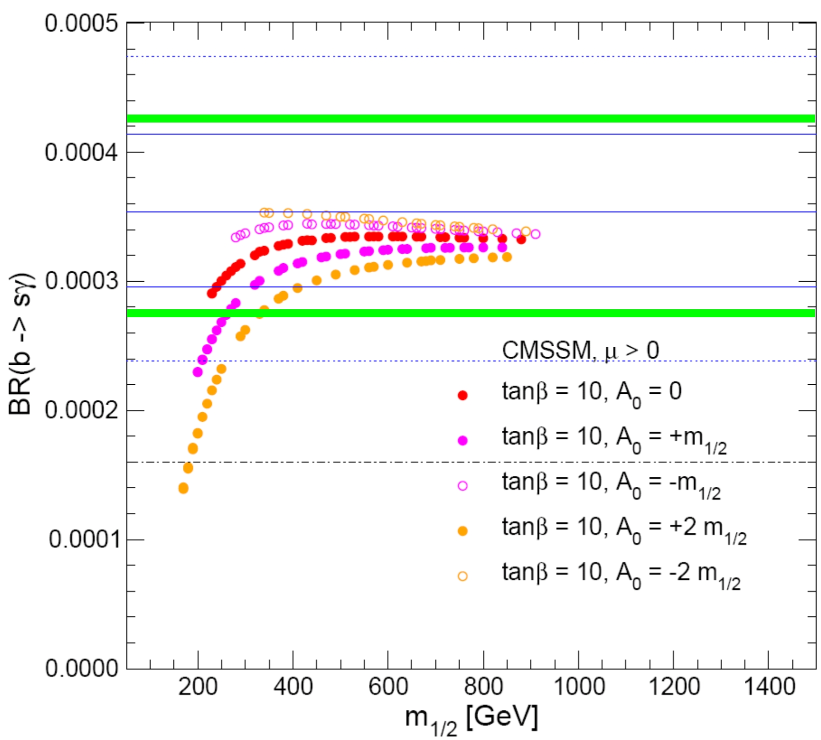
$$Br(1.8\text{GeV}) = (3.38 \pm 0.30[\text{stat}] \pm 0.28[\text{syst}]) \cdot 10^{-4}$$

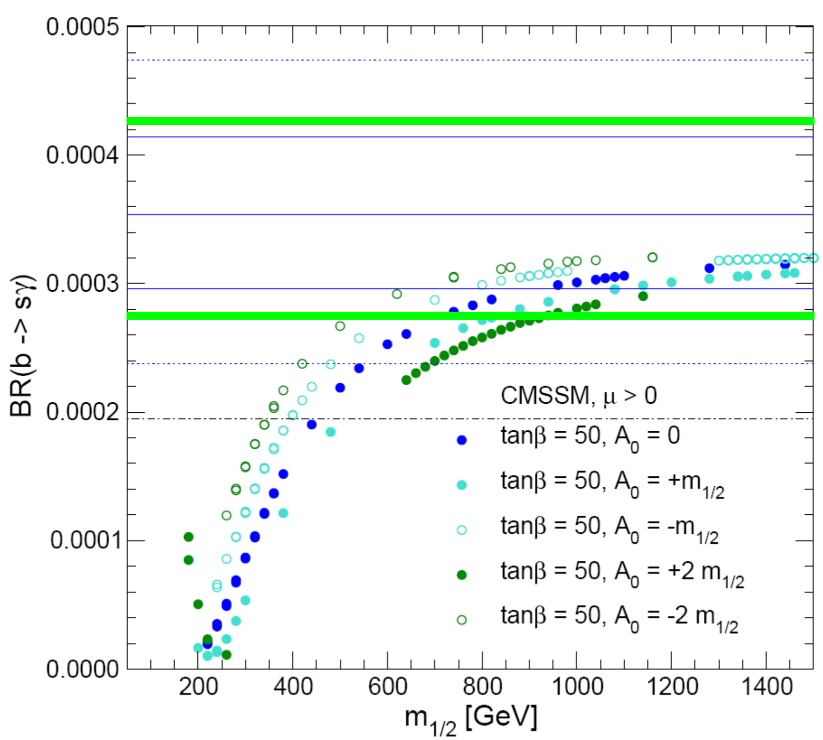
Implications for New Physics

- Increased theory errors and improved agreement with experiment weaken constraints on parameter space of New Physics models!
- E.g., type-II two-Higgs doublet model:
  - m(H+) > 200 GeV (at 95% CL)
    (compared with previous bound of 500 GeV)

Implications for New Physics

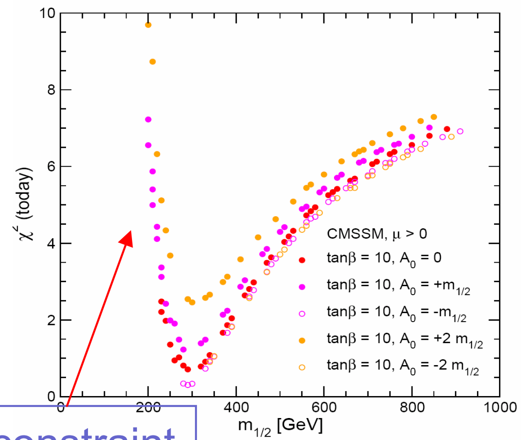
- Bounds on CMSSM parameters:

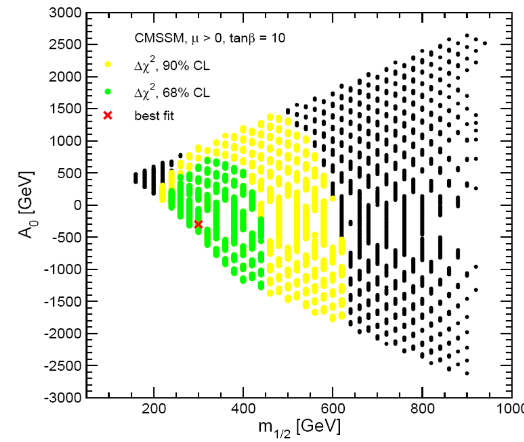




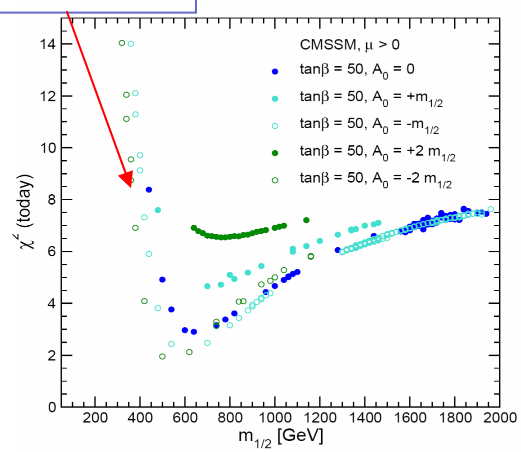
[Ellis, Heinemeyer, Olive, Weiglein: hep-ph/0411216]

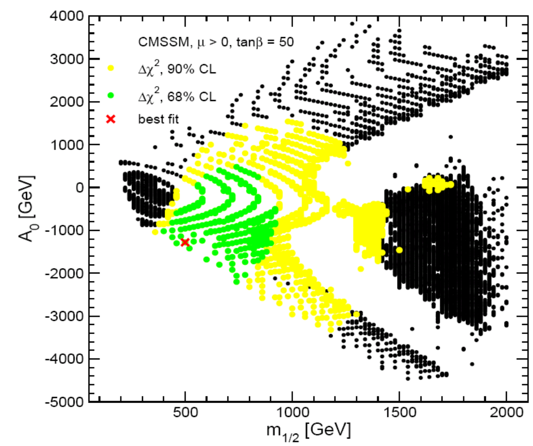
Implications for New Physics





$B \to X_s \gamma$ constraint





[Ellis, Heinemeyer, Olive, Weiglein: hep-ph/0411216]

Moments of the photon spectrum

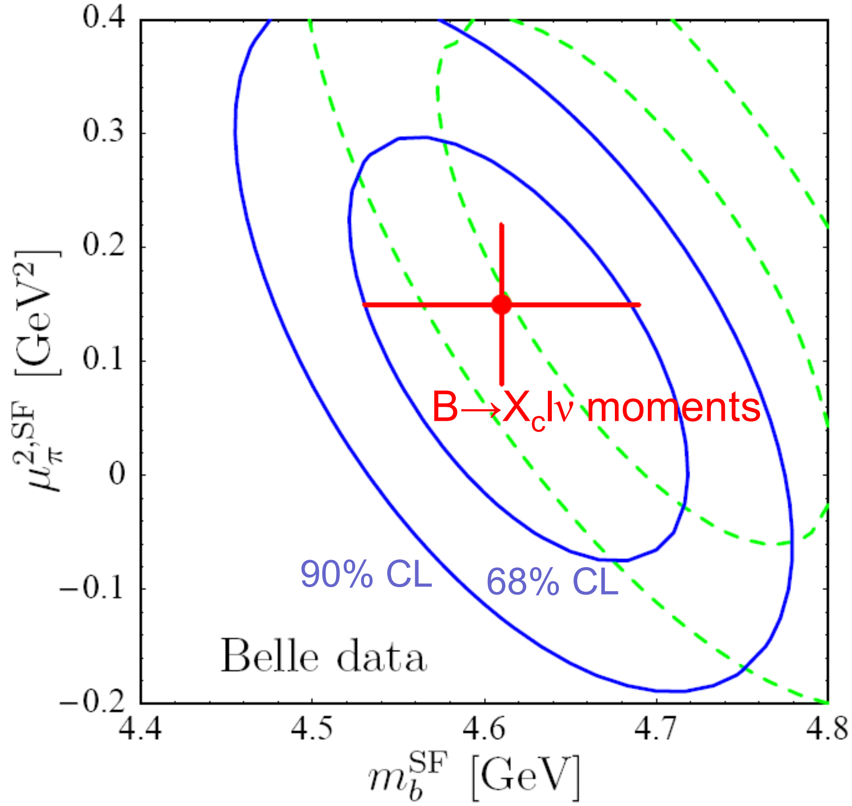
- Marvelous QCD laboratory
- Extraction of heavy-quark parameters (m_b,μ_π^2) with exquisite precision
- NNLO accuracy already achieved for $\langle E_\gamma \rangle$ and $\langle E_\gamma^2 \rangle$ - $\langle E_\gamma \rangle^2$:
  - Full two-loop corrections (+ 3-loop running)
  - Same accuracy for leading power corrections $\sim(\Lambda_{QCD}/\Delta)^2$; fixed-order results for $1/m_b$ terms

Predictions for moments

	$O(1)$	$O(1/m_b)$	$O(1/m_b^2)$
Perturbation Theory	Complete resummation at NNLO	α_s^2	α_s^2
Hadronic Parameters	m_b, μ_π^2	μ_π^2 ρ_D^3, ρ_{LS}^3	ρ_D^3, ρ_{LS}^3

Fit to Belle data (E_0 = 1.8 GeV)

[MN: hep-ph/0506245]



(parameters defined in shape-function scheme)

- Fit results: **!**

$m_b = (4.62 \pm 0.10_{exp} \pm 0.03_{th})$ GeV

$\mu_\pi^2 = (0.11 \pm 0.19_{exp} \pm 0.08_{th})$ GeV^2

- Combined results ($B \to X_s\gamma$ and $B \to X_c l\nu$):

$m_b = (4.61 \pm 0.06)$ GeV

$\mu_\pi^2 = (0.15 \pm 0.07)$ GeV^2

Fit to Belle data (E_0 = 1.8 GeV)

- Theoretically, the most precise extraction of m_b to date (0.7% accuracy!)
- Translation to $\overline{\text{MS}}$ scheme (2 loops):

$$\overline{m}_b(\overline{m}_b) = (4.23 \pm 0.05)\ \text{GeV}$$

- Compare with extractions from Υ spectrum (sum rules and lattice) :

$$\overline{m}_b(\overline{m}_b) = (4.20 \pm 0.09)\ \text{GeV}$$

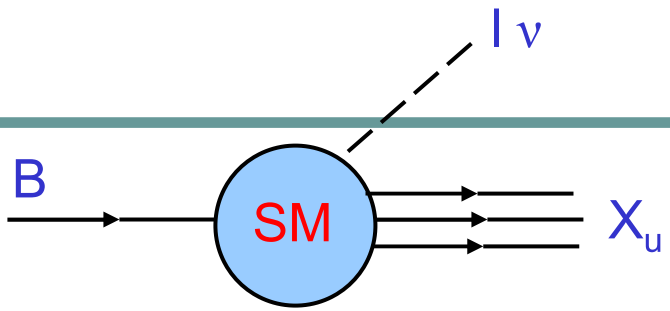
[Corcella, Hoang: hep-ph/0212297]

$$= (4.2 \pm 0.1 \pm 0.1)\ \text{GeV}$$

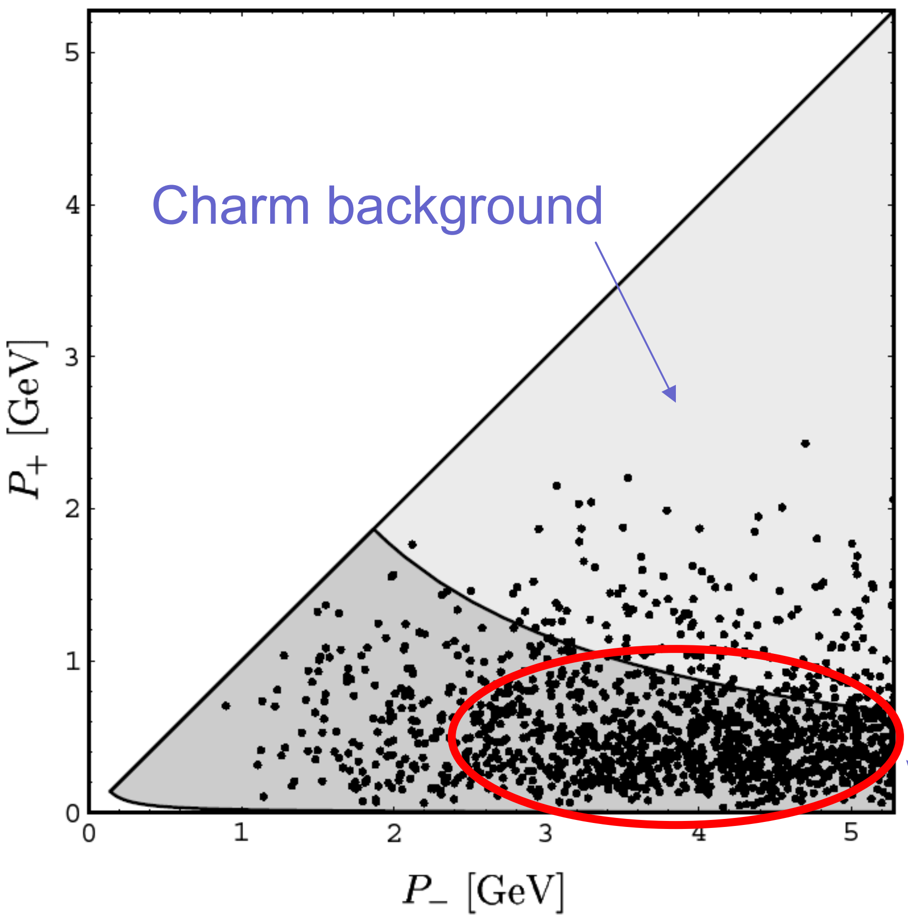
[Lattice compilation: Sachrajda for PDG]

$|V_{ub}|$ from $B \to X_u l \, \nu$ Decay

Factorization for inclusive semileptonic decay spectra



Inclusive semileptonic decays



- Factorization theorem analogous to $B \to X_s \gamma$
- Hadronic phase space is most transparent in the variables $P_{\mp} = E_X \pm P_X$
- In practice, $\Delta = P_+ - \bar{\Lambda}$ is always of order Λ_{QCD} for cuts eliminating the charm background

Shape-function region

Strategy

- Exploit universality of shape function
- Extract shape function in $B \to X_s \gamma$ (fit to photon spectrum), then predict arbitrary distributions in $B \to X_u l\, \nu$ decay [Lange, MN, Paz: hep-ph/0504071]
- Functional form of fitting function constrained by model-independent moment relations
  - Knowledge of m_b and μ_π^2 helps!
- Variant: construct "shape-function independent relations" between spectra (equivalent)

[Lange, MN, Paz: hep-ph/0508178]

Results for various cuts

	m_b [GeV]	4.50	4.55	4.60	4.65	4.70	Theory Error
$M_X \le M_D$ Eff = 84%	a Functional Form	9.5 1.4%	8.8 1.1%	8.2 0.8%	7.7 0.5%	7.3 0.4%	7%
$M_X \le 1.7\,\text{GeV}$ Eff = 75%	a Functional Form	12.5 2.9%	11.5 2.6%	10.5 2.2%	9.7 1.9%	8.9 1.6%	7%
$M_X \le 1.7\,\text{GeV}$ $q^2 \ge 8\,\text{GeV}^2$ 35%	a Functional Form	10.3 2.0%	9.8 1.7%	9.3 1.5%	9.0 1.4%	8.7 1.4%	10%
$q^2 \ge (M_B - M_D)^2$ Eff = 18%	a Functional Form	11.4 5.0%	11.1 4.4%	10.9 4.0%	10.8 3.6%	10.6 3.2%	15%
$P_+ \le M_D^2/M_B$ Eff = 65%	a Functional Form	16.7 5.3%	15.0 4.8%	13.6 4.4%	12.2 4.0%	11.1 3.6%	7%
$E_l \ge 2.2\,\text{GeV}$ Eff = 11%	a Functional Form	22.6 16.2%	21.0 13.1%	19.7 11.0%	18.5 9.3%	17.4 7.9%	19%

Rate $\Gamma \sim (m_b)^a$

[Lange, MN, Paz: hep-ph/0504071]

Results for various cuts

- Different determinations now consistent:

	nominal f_u	$\lvert V_{ub}\rvert \times 10^3$
$*$CLEO [79] $E_e > 2.1\,\mathrm{GeV}$	0.19	$4.02 \pm 0.47 \pm 0.35$
$*$BABAR [82] $E_e,\ s_{\mathrm{h}}^{\mathrm{max}}$	0.19	$4.06 \pm 0.27 \pm 0.36$
$*$BABAR [81] $E_e > 2.0\,\mathrm{GeV}$	0.26	$4.23 \pm 0.27 \pm 0.31$
$*$BELLE [80] $E_e > 1.9\,\mathrm{GeV}$	0.34	$4.82 \pm 0.45 \pm 0.31$
$*$BABAR [86] M_X/q^2	0.34	$4.76 \pm 0.34 \pm 0.32$
$*$BELLE [87] M_X/q^2	0.34	$4.38 \pm 0.46 \pm 0.30$
BELLE [85] M_X/q^2	0.34	$4.68 \pm 0.37 \pm 0.32$
BELLE [85] $P_+ < 0.66\,\mathrm{GeV}$	0.57	$4.14 \pm 0.35 \pm 0.29$
$*$BELLE [85] $M_X < 1.7\,\mathrm{GeV}$	0.66	$4.08 \pm 0.27 \pm 0.25$
Average of $*$		$\boxed{4.38 \pm 0.19 \pm 0.27}$
$\chi^2 = 5.9/6$, CL=0.43		

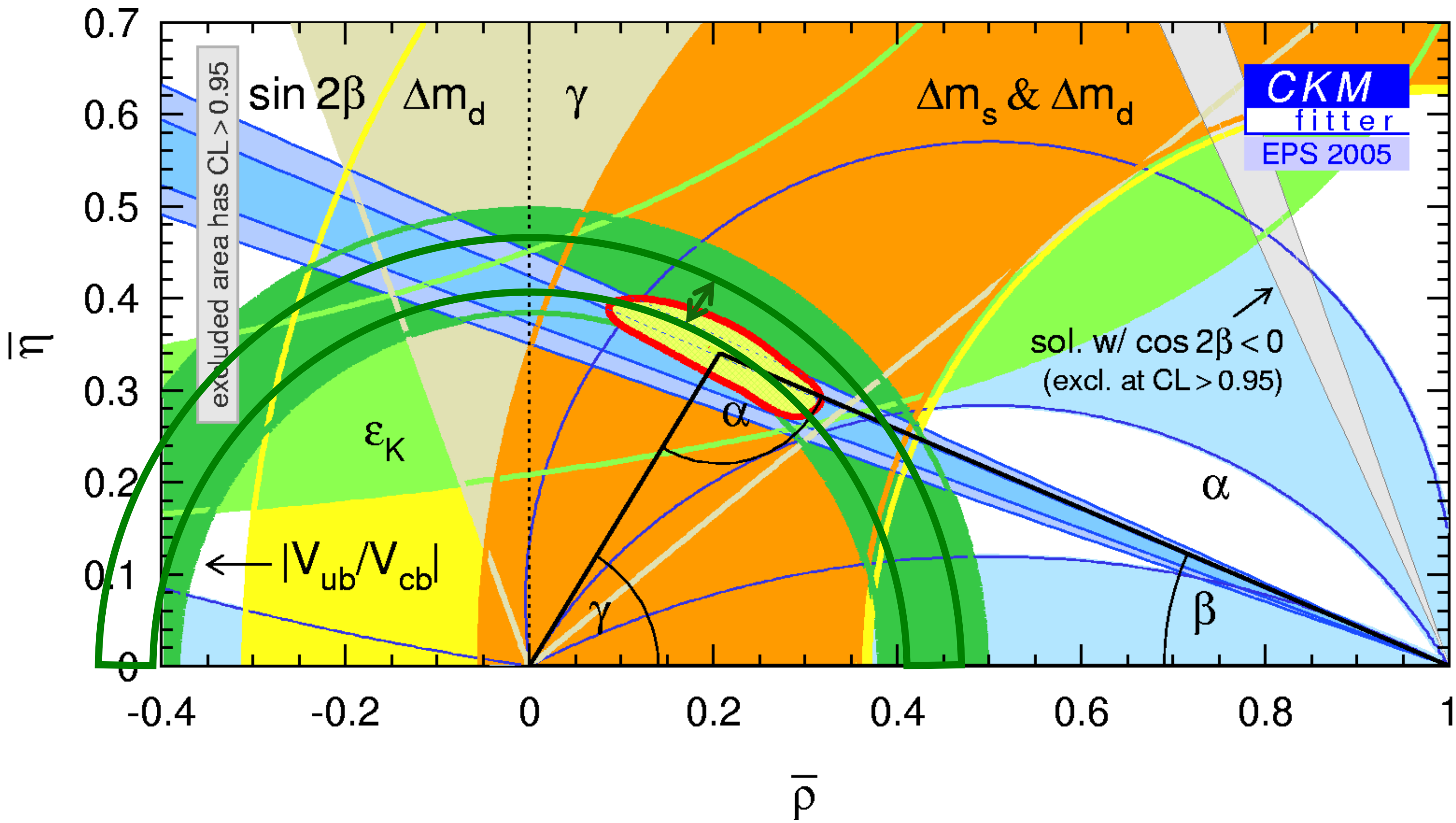
Combined result

- Theory error on $|V_{ub}|$ is 5-10% for several different cuts (10% now conservative – seemed unrealistic only a few years ago)
- Average of different extractions gives $|V_{ub}|$ with a *total* error of 7%:

$$|V_{ub}| = (4.38 \pm 0.33) \cdot 10^{-3}$$

- Needed to match the precision of sin2β

Impact of precise $|V_{ub}|$



Shape-function free relations

- Example for $P_+ = E_X - P_X$ spectrum:

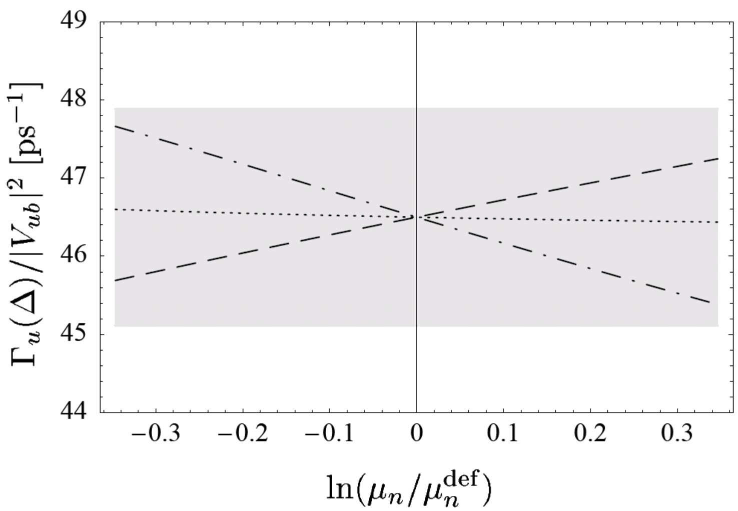
$$\Gamma_u(\Delta) = \underbrace{\int_0^\Delta dP_+ \frac{d\Gamma_u}{dP_+}}_{\text{exp. input}} = |V_{ub}|^2 \int_0^\Delta dP_+ \underbrace{W(\Delta, P_+)}_{\text{theory}} \underbrace{\frac{1}{\Gamma_s(E_*)} \frac{d\Gamma_s}{dP_+}}_{\text{exp. input}}$$

  - Weight function is perturbatively calculable and *independent* of soft scale
  - Small hadronic uncertainties enter at order $1/m_b$ only

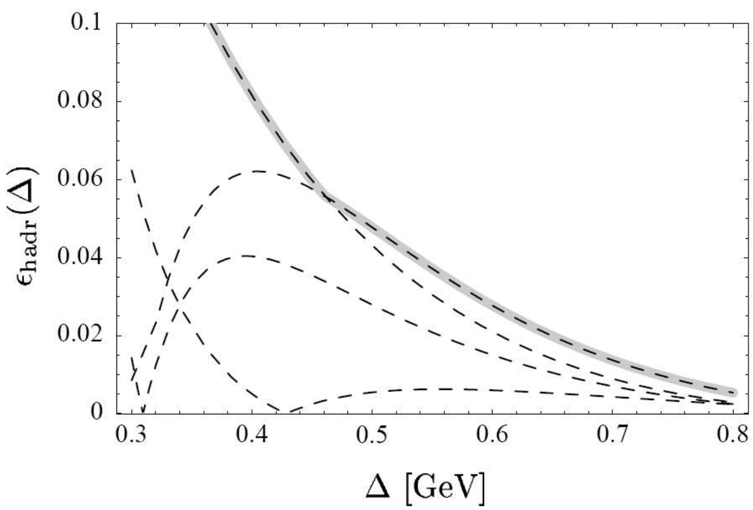
[Lange, MN, Paz: hep-ph/0508178]

Shape-function free relations

- Perturbative stability:



- Hadronic uncertainty:



$$\begin{aligned}\Gamma_u(0.65\,\text{GeV}) &= (46.5 \pm 1.4\,_{[\text{pert}]} \pm 1.8\,_{[\text{hadr}]} \pm 1.8\,_{[m_b]} \pm 0.8\,_{[\text{pars}]} \pm 2.8\,_{[\text{norm}]})\,|V_{ub}|^2\,\text{ps}^{-1} \\ &= (46.5 \pm 4.1)\,|V_{ub}|^2\,\text{ps}^{-1}\,,\end{aligned}$$

Provides extraction of $|V_{ub}|$ with 4.4% theoretical uncertainty!

Conclusions

Summary

- $B \to X_s \gamma$ decay remains one of the most sensitive probes of New Physics
- Strong motivation for NNLO calculation in Standard Model (many people involved!)
- Important to disentangle hard and soft effects using factorization and resummation

Summary

- Many applications besides New Physics searches:
  - Most precise determination of b-quark mass
  - Most precise determination of $|V_{ub}|$

$$\overline{m}_b(\overline{m}_b) = (4.23 \pm 0.05)\ \mathrm{GeV}$$

$$|V_{ub}| = (4.38 \pm 0.33)\cdot 10^{-3}$$

Substantial advances in a challenging field !