QED at high intensity and high energy

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- arXiv:0903.415 and papers to appear soon.
- With Chris Harvey, Tom Heinzl (Plymouth), Mattias Marklund (Umeå).

Outline

- Motivation – energy and intensity.
- Background fields in QED.
  - Background generated by coherent states.
  - ‘Furry picture’ and calculations.
- Intensity effects: nonlinear Compton scattering.
  - Background driven processes.
  - Intensity effects and observables.
- High intensity, high energy: noncommutative effects.
  - High energy QED + backgrounds.
  - Noncommutative corrections to intensity effects.

Motivation

1. LHC is coming. Higgs? New physics – SUSY?

- Further off, ILC and CLIC.

2. Optical lasers probe 'high intensity, low energy' QED.

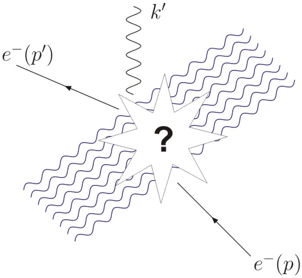
- Laser fields currently at 10^{22} W/cm^2 (Vulcan) .
- ELI and HiPER will reach 10^{25} W/cm^2 .

- High intensity $\rightarrow$ 'new' physics: birefringence, pair production, i.e. effects which do not occur in vacuum.

Part 1– background fields in QED

High intensity → background fields

- High intensity: very large numbers of photons present.



- How many:
  - incoming laser photons?
  - interactions with $e^-(p)$?

- Look for approximations which allow us to calculate.

High intensity → background fields

- Large photon numbers $\implies$ treat the laser classically.
- Laser → classical background field, $a_\mu(x)$. Kibble 1964
- Essentially, neglect depletion of the beam.

Bialynicki–Birula, 1973

- Approach from a scattering perspective.

1. Start with asymptotic states. Generate background field.
2. Relate to $A_\mu \to A_\mu + a_\mu$ shift.

Coherent asymptotic states

- Model laser asymptotically by coherent state of photons.

$$|\,C\,\rangle = \exp\int\frac{\mathrm{d}^3k}{(2\pi)^3}\;C^\mu(\boldsymbol{k})\hat{a}^\dagger_\mu(\boldsymbol{k})\,|\,0\,\rangle$$

- $C^\mu(\boldsymbol{k})$: spread of polarisations and momenta (in beam).
- These are 'most classical' states (minimal uncertainty).
- Consider asymptotic states of scattered particles ('in', 'out') and coherent states C,

$$\langle\,\text{out}; C\,|\ldots|\,\text{in}; C\,\rangle$$

Coherent states in quantum mechanics

- Quantum harmonic oscillator: $H = \omega(\hat{a}^\dagger \hat{a} + \frac{1}{2})$.
- Vacuum state: $\hat{a}|\,0\,\rangle = 0$. Vacuum wavefunction

$$\hat{a}\,\psi_0(x) \propto \left[x + i\omega^{-1}\hat{p}\right]\psi_0(x) = 0 \implies \psi_0(x) = \exp\left(-\frac{\omega}{2}x^2\right)$$

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- Coherent state: $|\,c\,\rangle = \exp(c\,\hat{a}^\dagger)|\,0\,\rangle$. Wavefunction:

$$\psi_c(x) = \exp\big(-\frac{\omega}{2}(x-c)^2\big)$$

- This is a (config. space) translation of the vacuum state:

$$|\,c\,\rangle = \mathbb{T}_c|\,0\,\rangle\ .$$

Scattering between coherent states

- Represent our coherent states as translations:

$$\langle\, \text{out}; C \,| = \langle\, \text{out}\, | \mathbb{T}_C^\dagger \;, \qquad |\, \text{in}; C\,\rangle = \mathbb{T}_C |\, \text{in}\,\rangle \;.$$

- Now calculate S–matrix elements: $\mathbb{S} = \mathcal{T} e^{-i \int \mathrm{d}t \;\, H_I(t)}$

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- Shift in interaction Hamiltonian (only):
- $A_\mu \to A_\mu + a_\mu$, a classical background field.

- a_μ = on–shell Fourier transform of $C_\mu(\boldsymbol{k})$.

Coherence $\longrightarrow$ background field

- Coherent state $\leftrightarrow$ interactions with background field:
- Interaction vertex: $\overline{\psi}\not{A}\psi \;\; \rightarrow \;\; \overline{\psi}\not{A}\psi + \overline{\psi}\not{a}\psi$

- S–matrix elements from amputated Feynman diagrams generated by:

$$\text{Amp.} \int \mathcal{D}(A,\psi)\; \langle \psi \ldots A \ldots \rangle \; \mathrm{e}^{i\int \mathcal{L}_a}$$

$$\mathcal{L}_a = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}(i\not{\partial} - m)\psi - e\overline{\psi}\not{a}\psi - e\overline{\psi}\not{A}\psi$$

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- Compare: asymptotic vacuum → $i\epsilon$ prescription

Weinberg, QFT Vol 1.

- The 'shifted vacuum' gives $i\epsilon$ and interactions.

The Furry picture

- Take above action as a starting point.
- Background fields ↔ Furry picture. Furry, PRD 81 (1951)
- Choose 'free'–'interacting' split in Hamiltonian.
- Compare interaction picture (use Lagrangian for clarity)

$$\mathcal{L}_a = \underbrace{-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}(i\partial\!\!\!/ - m)\psi}_{\text{free}} \underbrace{- e\overline{\psi}a\!\!\!/\psi - e\overline{\psi}A\!\!\!/\psi}_{\text{interacting}} \ .$$

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- Interactions with the background → 'free' Hamiltonian.

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- Canonical quantisation:
  - Free (‘bound’) states see background field.
  - New commutation relations between modes.
  - Canonical transform of interaction picture fields.
  - New charge conjugation relations.
- Continue working with S–matrix and Feynman diagrams.

Calculating ‘without’ the Furry picture

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- Propagators $\leftarrow$ inverse of quadratic terms.

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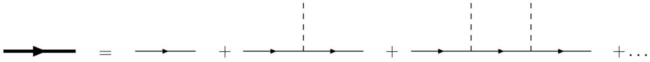

- Background ‘dresses’ free propagator.
- Other effects?

Calculating 'without' the Furry picture

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- That's all! Only vertex is $\overline{\psi}A\!\!\!/\psi$.
- To calculate S–matrix elements:

1. Write down usual Feynman diagrams, but with dressed fermion lines.
2. Amputate external photon legs as normal.
3. Amputating external fermions → Volkov wavefunctions.

External legs: Volkov wavefunctions

- Amputating free propagator → free spinor wavefunctions:

$$\exp(-ip.x)\, u_p \;, \qquad \not{p}\, u_p = m u_p$$

- Amputating dressed propagator → Volkov wavefunctions.
- Solutions of Dirac eqn. in background a_μ. Volkov, 1935
- Assume throughout that $a_\mu \equiv a_\mu(k.x)$, a plane wave

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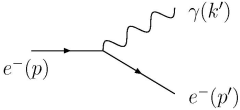
- Amputating dressed propagator → Volkov wavefunctions.
- Solutions of Dirac eqn. in background a_μ. Volkov, 1935
- Assume throughout that $a_\mu \equiv a_\mu(k.x)$, a plane wave:

$$e^{-ip.x} \exp\left(\frac{1}{2ik.p} \int^{k.x} 2e\, a.p - e^2 a^2 \right) \left[\mathbb{1} + \frac{e}{2k.p} \not{k}\not{a} \right] u_p \,,$$

- S–matrix elements not supported on usual momentum conserving delta functions.

Part 2– Intensity effects: nonlinear Compton scattering.

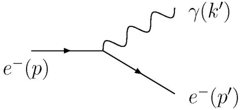
Spontaneous photon emission



- Spontaneous (real) photon emission.
- Cannot occur in vacuum because of momentum conservation:

$$p_\mu \neq k'_\mu + p'_\mu \ .$$

Spontaneous photon emission



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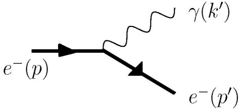

- Can happen in the presence of a background field.
- Background → source of extra energy needed to put the photon on–shell.
- ‘Nonlinear Compton scattering’.

Nonlinear Compton scattering

- Our laser background: circularly polarised plane wave:

$$a^\mu(x) = a_1^\mu \cos k\cdot x + a_2^\mu \sin k\cdot x \ .$$

- $a_i\cdot a_j = -|a|^2\delta_{ij} < 0$.
- $k_\mu k^\mu = 0$, beam direction and frequency, $k_\mu \sim \omega(1,0,0,1)$.
- We define the dimensionless intensity parameter

T.H., A.I, Opt. Comm. 09

$$a_0 = \frac{e|a|}{m} \ ,$$ parameterises all background effects.

- $a_0 \approx 20$ (FZD, Vulcan), $a_0 \approx 10^3 \to 10^4$ (ELI, HiPER).

The S–matrix and cross section

- Head on collision between e^- and laser: $p_\mu \to k'_\mu + p'_\mu$
- S–matrix element, summed over spins and polarisations:

Gol'dmann, Nikishov, Ritus, Narozhnyi 1964

$$\frac{1}{VT}\sum |S_{\mathsf{fi}}|^2 = \sum_{n=1}^{\infty} \mathcal{S}_n \, \delta^4\left(q_\mu + nk_\mu - q'_{\ \mu} - k'_{\ \mu}\right) \ .$$

- An ∞ sum of contributions with different kinematic support.
- Amplitudes $\mathcal{S}_n$ below.
- First examine kinematics $\Longleftarrow$ delta functions.

Kinematics of $q_\mu + nk_\mu = q'_\mu + k'_\mu$

- Electron 'quasi–momenta' q_μ, shifted by intensity effects

$$q_\mu := p_\mu + \frac{a_0^2 m^2}{2p.k}\, k_\mu \; .$$

- Implies 'effective mass': $q^2 = m^2(1 + a_0^2) \equiv m_*^2$

Sengupta 1952

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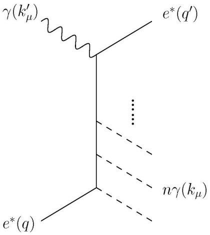
Sengupta 1952

- n^{th} contribution to scattering $(n = 1 \ldots \infty)$:



- Effective process.

1. Incoming heavy electron, $q^2 = m_*^2$,
2. Absorbs n laser 'quanta' k_μ,
3. Emits one scattered photon k'_μ.

Kinematics of $q_\mu + nk_\mu = q'_\mu + k'_\mu$

- These multi–photon 'harmonic processes' are the origin of the name 'nonlinear' Compton scattering.
- Mass shift not observed directly.
- Higher harmonic generation has been observed.

  Chen et. al., Nature 1998
- Quasi–momentum conservation $\Longrightarrow$ relations for on–shell momenta.
- Contributions from different harmonics are observable, even though q_μ, q'_μ not on–shell.

Scattering amplitude

- The S–matrix amplitudes have the form Narozhnyi 1964

$$\mathcal{S}_n = -2J_n^2(z) + a_0^2\left(1 + \frac{x^2}{2(1+x)}\right)\left(J_{n+1}^2(z) + J_{n-1}^2(z) - 2J_n^2(z)\right)$$

- $x = k.k'/k.p'$ and $z \equiv z(x)$.

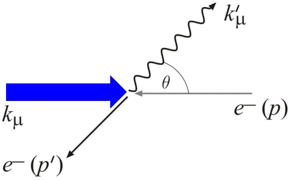
- Cross section independent of angle azimuthal to the beam. Due to:

1. Sum over polarisations,
2. Circular polarisation of beam.

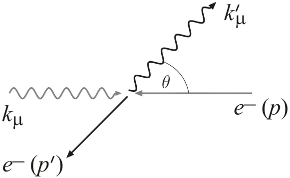
- Lab frame: $x \to$ scattered photon frequency/ scattering angle.

Cross section comparison

- Compare our cross section with ordinary Compton.

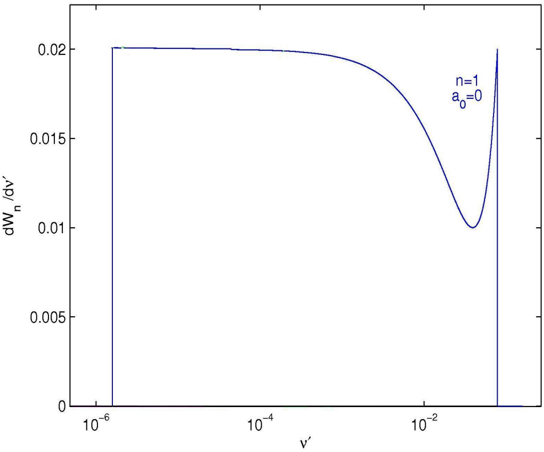


- Nonlinear Compton: many photons.
- $n = 1 \ldots \infty$ processes.
- High intensity, $a_0 > 1$.



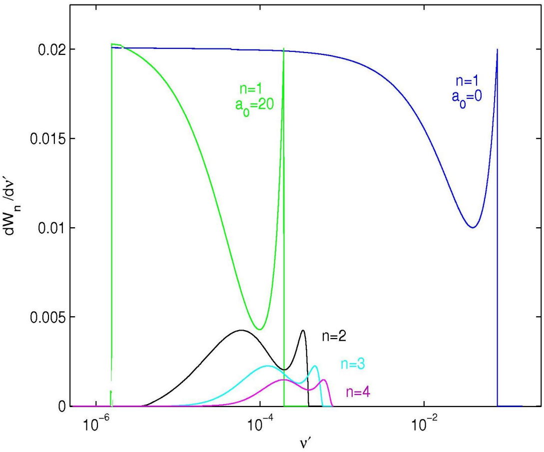
- Ordinary Compton: one photon.
- $n = 1$.
- Low intensity: $a_0 = 0$.

Non–linear Compton: cross section



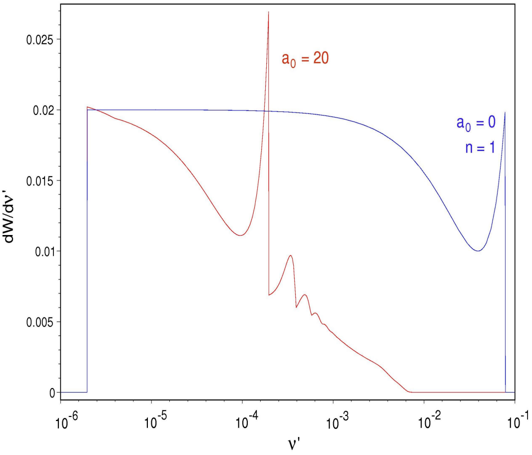
- Plot against $\nu' \equiv \omega'/m$.
- e^-: 40 MeV
  ω: 1eV
- Linear Compton in blue.
  $(n = 1,\ a_0 = 0)$

Non–linear Compton: cross section



- Plot against $\nu' \equiv \omega'/m$.
- e^-: 40 MeV
  ω: 1eV
- Linear Compton in blue.
  $(n = 1,\ a_0 = 0)$
- FZD: $a_0 \to 20$.
- Compton edge redshifted.
- Harmonics: $n = \mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}$.

Non–linear Compton: cross section



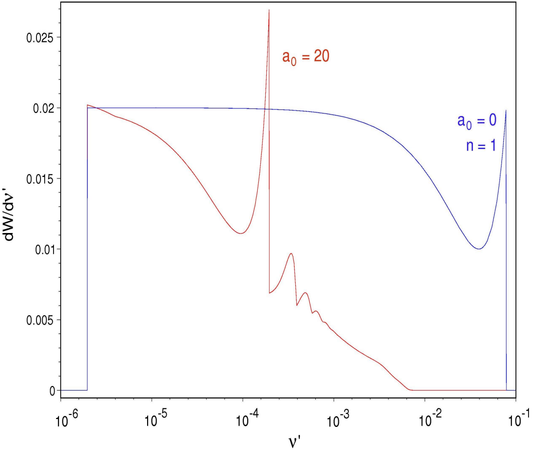
- Sum of first 50 harmonics.

  C.H., A.I., T.H. 2009

- Red shift of Compton edge.
- Higher harmonics:

  scattered photons can have higher energy.

Non–linear Compton: cross section



- Sum of first 50 harmonics.

  C.H., A.I., T.H. 2009
- Red shift of Compton edge.
- Higher harmonics:

  scattered photons can have higher energy.
- Many other effects (see paper).
- Non–linear edge is strongest signal.

Part 3– High intensity, high energy: noncommutative effects.

A source of new physics

- Chosen source of new physics: spacetime noncommutativity.
- $[x^\mu, x^\nu] = i\theta^{\mu\nu} \implies$ spacetime becomes 'fuzzy':

$$[x, y] = i\theta \implies \Delta x \Delta y \geq \frac{|\theta|}{2}\,, \qquad \text{a minimum area.}$$

- Originally proposed to deal with UV.
- UV/ IR mixing. Grosse, Wulkenhaar 2003, 2005
- New interest after reappearance in string theory.

NC QED

- Field theory: replace field products with Moyal–star products.

$$f(x) \star g(x) = f(x) \exp\left[\tfrac{i}{2}\overleftarrow{\partial}_\mu \theta^{\mu\nu} \overrightarrow{\partial}_\nu\right] g(x)\ .$$

- NC QED Lagrangian: Hayakawa, 1999

$$\mathcal{L} = -\tfrac{1}{4} F_{\mu\nu} \star F^{\mu\nu} + \overline{\psi} \star (i\not{D} - m) \star \psi$$

- Gauge group replaced by $\star$–gauge group:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ie[A_\mu \overset{\star}{,} A_\nu]$$

- Photon is self–interacting – $A^{\star 3}$, $A^{\star 4}$ vertices.

NC QED + background

- Introduce background: $A_\mu \to A_\mu + a_\mu$ in interactions.
- New terms: $\overline{\psi} \star \not{a} \star \psi$, c.f. QED, etc.
- Electron, photon propagators both dressed by background.
- New terms: $a \star A^{\star 3}$, etc: vertices

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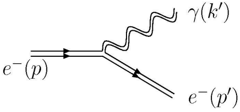


- Don't contribute to nonlinear Compton.
- Only vertex is $\overline{\psi} \star \not{A} \star \psi$.
- Pair production from crossing.

- Highlight some NC corrections to our scattering process.

External NC electron legs

- Amputated electron leg has same form as QED Volkov.
- But sees a background

Alvarez–Gaumé, Barbón, 2000

$$\tilde{a}_\mu(k.x) := a_\mu(k.x + k \wedge p)$$

- $k \wedge p := \frac{1}{2} k \cdot \theta \cdot p$, for p the electron momentum.
- Electrons see fields later/ earlier than commutative counterparts.
- Quasi–momenta as before: same mass shift $m \to m_*$.

External photon lines

- Amputating photon leg gives: T.H., A.I., M.M., to appear

$$e^{-ik'.x}\exp\left(\frac{1}{2ik.k'}\int^{k.x}2e\mathcal{A}.k'-e^2\mathcal{A}^2\right)\left[\delta^\mu_\nu+\frac{e}{2k.k'}(k^\mu\mathcal{A}_\nu-\mathcal{A}^\mu k_\nu)\right]\epsilon^\nu$$

- Very similar to electron Volkov solution.

- Scattered photon sees the background

$$\mathcal{A}_\mu(k.x) := a_\mu(k.x+k\wedge k') - a_\mu(k.x-k\wedge k')$$

- Photon is extended in NCQED. A dipole of length $k\cdot\theta\cdot k'$.

Alvarez–Gaumé, Barbón, 2000

NC corrections to cross section

$\longrightarrow$ Corrections to nonlinear Compton and pair production.

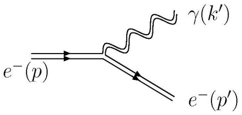
T.H., A.I., M.M., to appear

- Sensitive to lightlike noncommutativity: $k\wedge \sim \theta^{+\nu}$.

Aharony, Gomis, Mehan 2000

- Cross section now depends on azimuthal angle, due to preferred direction $\theta^{+\nu}$.

$\Longrightarrow$ Deviations from QED predictions.



- Nonlinear Compton: photon scattered in preferred directions.
- Pair production: pairs produced with preferred directions.

Size of NC effects

- Nonlinear Compton $\sim a_0^2 \sin^2(k \wedge k'_{\text{out}})$.
- Need laser at both high energy + intensity.
- Detection not likely with all–optical setup ($\omega \sim 1$ eV).

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- Pair production $\sim a_0^2 \sin^2(k \wedge p'_{\text{out}})$.

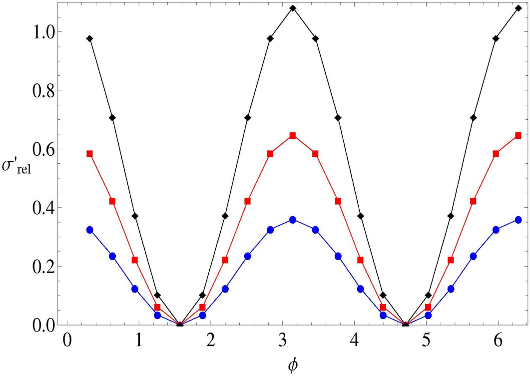
✓ Intense laser + high energy probe.

✓ High energy photons ← Compton back scattering .

- Measure % deviation from QED (relative cross section).

$$\frac{d\sigma_{\text{rel}}}{d\phi} := \left(\frac{d\sigma_{\text{NC}}}{d\phi} - \frac{d\sigma_{\text{QED}}}{d\phi}\right) \Big/ \frac{d\sigma_{\text{QED}}}{d\phi}$$

Example NC cross section



- Azimuthal dependence relative to QED.
- $a_0 = 10, 10^3, 10^4$ (Vulcan $\rightarrow$ ELI).
- Order 1% effect.
- Higher intensity increases sensitivity.

- NC scale $= 1.5$ TeV, $E_p = 500$ GeV, $\omega = 1$ TeV

Conclusions and prospects

- Interactions with background fields $\leftarrow$ coherent states.
- Calculations straightforward in LSZ picture.

$\longrightarrow$ Extend calculations beyond plane wave fields.
$\longrightarrow$ Gaussian profile, for example, to model laser pulse.
$\longrightarrow$ Loop corrections.

- Intensity effects observable at current laser facilities.

$\longrightarrow$ Comparison with theory – backgrounds, noise.

Conclusions and prospects

- NC can be detected with energetic probes.

$\longrightarrow$ High energy probe required: pair production.

- Intensity effects acquire azimuthal dependencies.
- Scattered particles produced with preference for certain directions.
- Increased intensity increases sensitivity to NC.