NNLO corrections to charmless non-leptonic B decays

Tobias Huber
Institut für Theoretische Physik E,
RWTH Aachen

In collaboration with Martin Beneke and Xin-Qiang Li

Durham, June 4th, 2009

Outline

- Introduction, (quark) flavour in the SM
- Theoretical framework, motivation for NNLO QCDF
- Technical details of two-loop calculation
- Results and outlook

Flavour Physics in the SM

- Yukawa terms in the Standard Model

$$\mathcal{L}_{Yuk} = -\lambda^E_{IJ}\,\overline{L}_I\,\Phi\,E_J - \lambda^D_{IJ}\,\overline{Q}_I\,\Phi\,D_J - \lambda^U_{IJ}\,\overline{Q}_I\,\widetilde{\Phi}\,U_J + \mathrm{h.c.}$$

- Unitary rotations in family space and SSB
  - diagonalise the Yukawa matrices
  - yields Dirac mass terms for quarks and charged leptons
  - coupling of fermions to the Higgs
- However, terms in the Lagrangian density of charged current weak interaction cannot be diagonalised simultaneously

$$\mathcal{L}_{CC} = \frac{g_2}{2\sqrt{2}}\left(J^+_\mu\,W^{+\mu} + J^-_\mu\,W^{-\mu}\right)\;,$$

$$J^+_\mu = (\bar{u}d')_{V-A} + (\bar{c}s')_{V-A} + (\bar{t}b')_{V-A} + \sum_{l=e,\mu,\tau}(\bar{\nu}_l\,l)_{V-A}$$

Flavour Physics in the SM

- After phase redefinitions, mass eigenstates (d, s, b) and weak eigenstates (d', s', b') are connected via the CKM matrix *[Cabibbo'63; Kobayashi,Maskawa'73]*

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \underbrace{\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}}_{V_{CKM}} \cdot \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

- V_{CKM}: Unitary in the SM $\Rightarrow$ no FCNC processes at tree level in the SM (GIM mechanism) *[Glashow,Iliopoulos,Maiani'70]*

- With N generations of quarks, physical content of V_{CKM} consists of

$$\frac{1}{2} N(N-1) \qquad \text{Euler angles}$$

$$\frac{1}{2}(N-1)(N-2) \qquad \text{phases.}$$

Flavour Physics in the SM

- CKM phases are the only known source of CP violation in the SM
  $\Rightarrow N > 3$ necessary for CP violation, but not sufficient. Need also

$$(m_t^2 - m_c^2)\,(m_t^2 - m_u^2)\,(m_c^2 - m_u^2)\,(m_b^2 - m_s^2)\,(m_b^2 - m_d^2)\,(m_s^2 - m_d^2) \times J_{CP} \neq 0$$

with the Jarlskog invariant

$$J_{CP} = 2\,A_\Delta = |Im(V_{i\alpha}\,V_{j\beta}\,V_{i\beta}^*\,V_{j\alpha}^*)| \quad (i \neq j\,,\,\alpha \neq \beta)$$

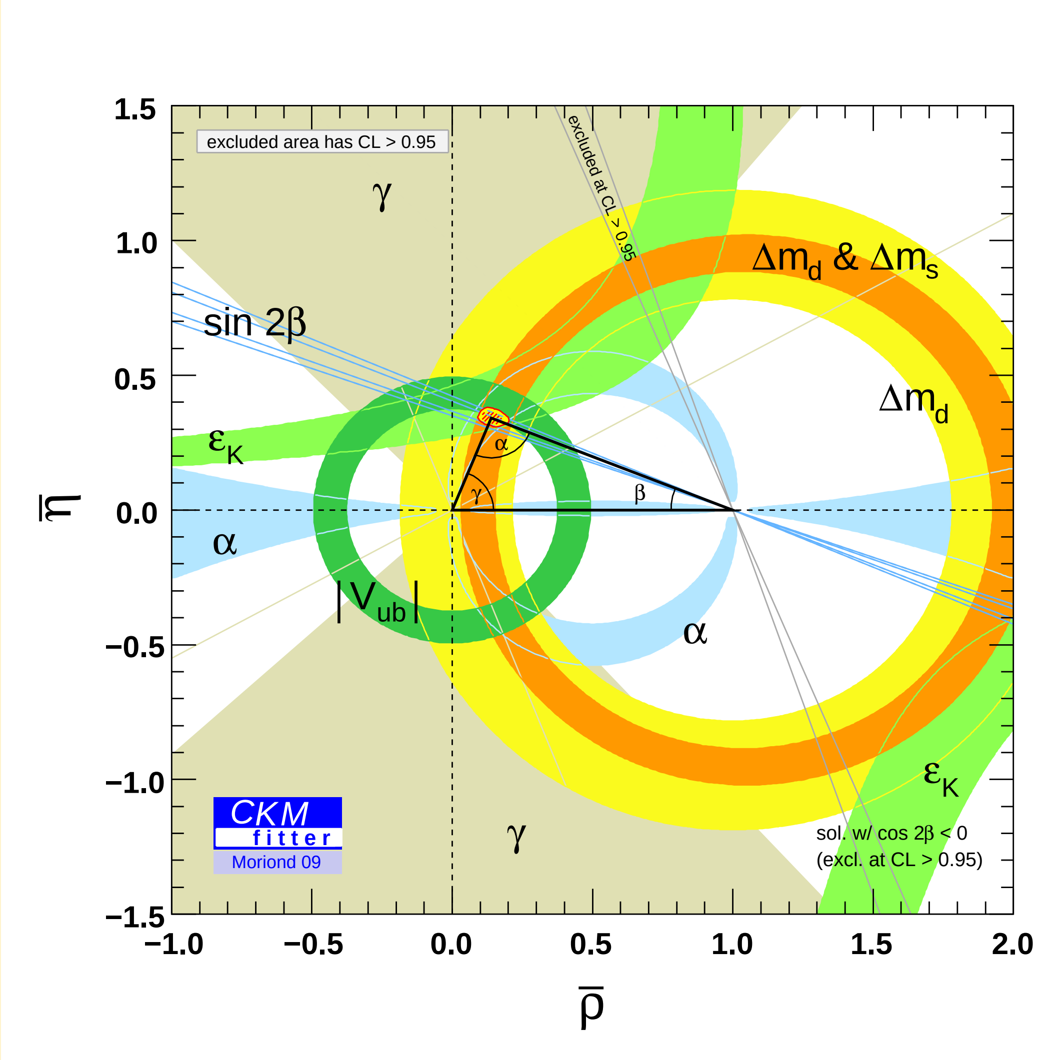
- Wolfenstein parameterization: Expansion in $\lambda \approx 0.226$ *[Wolfenstein'83]*

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & \lambda^3 A\,(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & \lambda^2 A \\ \lambda^3 A\,(1-\rho-i\eta) & -\lambda^2 A & 1 \end{pmatrix} + \mathcal{O}\left(\lambda^4\right)\ .$$

- From unitarity conditions: Construct "The" Unitarity Triangle

Flavour Physics in the SM

- $|V_{ud}|$: β decay,
  decay of charged π
- $|V_{us}|$: K, hyperon, τ decays
- $|V_{cb}|, |V_{ub}|$: inclusive and
  exclusive $\bar{B} \to X_{c,u}\ell\nu$
- Hadronic B decays important
  for extracting UT quantities
- α: time dep. CP asym. in
  $B \to \rho\rho$, $B \to \rho\pi$, $B \to \pi\pi$
- β: from $B \to J/\psi K_S$ (tree),
  $B \to \phi K_S$ (penguin)
- γ: from $B \to DK$



Flavour Physics in the SM

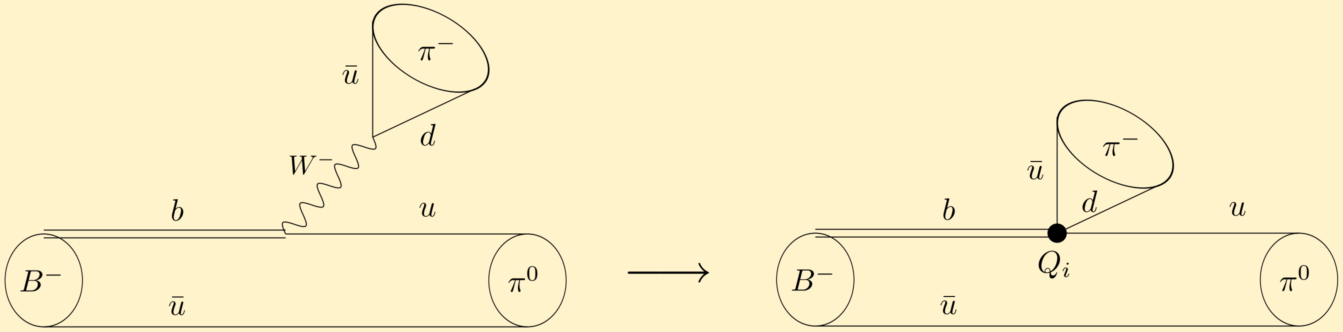
- CKM mechanism of quark flavour transitions well established

  [Cabibbo'63; Kobayashi,Maskawa'73; Nobel Prize 2008]

- Quark flavour sector still an active field of research, era of precision physics
  - Quantify its amount of CP violation
    $\Rightarrow$ Baryon asymmetry of the Universe
  - Indirect search for new physics (NP). Smoking guns:
    * β_s
    * $\Delta A_{\mathrm{CP}}(\pi K)$
  - Many observables: branching ratios, CP asymmetries, polarisations, ...

- Need precision in theory predictions and experimental measurements to disentangle NP from SM background

Effective theory for B decays



- M_W, M_Z, $m_t \gg m_b$: integrate out heavy gauge bosons and t-quark

- Effective Hamiltonian: *[Buras,Buchalla,Lautenbacher'96; Chetyrkin,Misiak,Münz'98]*

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left[C_1 Q_1^p + C_2 Q_2^p + \sum_{k=3}^{6} C_k Q_k + C_8 Q_8 \right] + \text{h.c.}$$

$$Q_1^p = (\bar{d}_L \gamma^\mu T^a p_L)(\bar{p}_L \gamma_\mu T^a b_L) \qquad Q_4 = (\bar{d}_L \gamma^\mu T^a b_L) \sum_q (\bar{q} \gamma_\mu T^a q) \qquad Q_8 = -\frac{g_s}{16\pi^2} m_b \, \bar{d}_L \, \sigma_{\mu\nu} G^{\mu\nu} b_R$$

$$Q_2^p = (\bar{d}_L \gamma^\mu p_L)(\bar{p}_L \gamma_\mu b_L) \qquad Q_5 = (\bar{d}_L \gamma^\mu \gamma^\nu \gamma^\rho b_L) \sum_q (\bar{q} \gamma_\mu \gamma_\nu \gamma_\rho q)$$

$$Q_3 = (\bar{d}_L \gamma^\mu b_L) \sum_q (\bar{q} \gamma_\mu q) \qquad Q_6 = (\bar{d}_L \gamma^\mu \gamma^\nu \gamma^\rho T^a b_L) \sum_q (\bar{q} \gamma_\mu \gamma_\nu \gamma_\rho T^a q) \qquad \lambda_p = V_{pb} V_{pd}^*$$

Effective theory for B decays

- To be supplemented by evanescent operators, e. g.

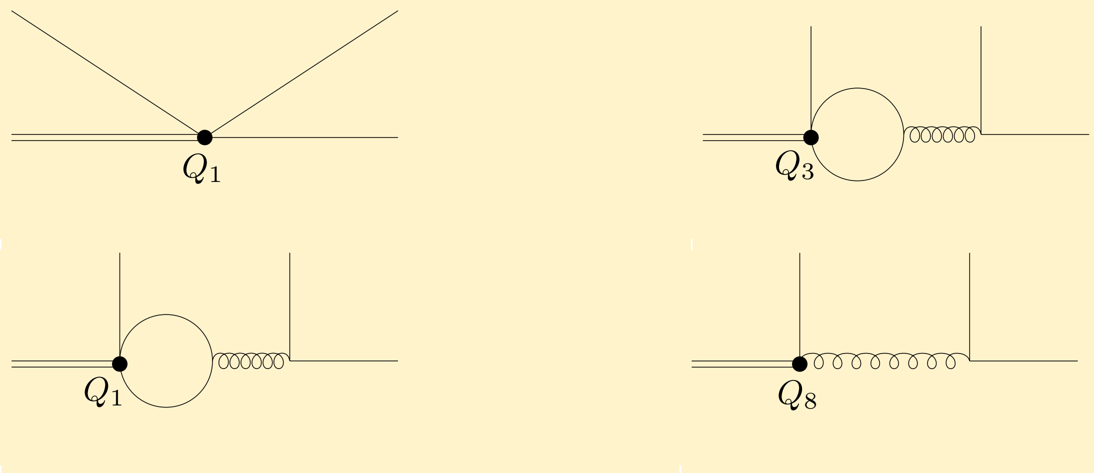
$$E_2^u = (\bar{d}_L \gamma^\mu \gamma^\nu \gamma^\rho u_L)(\bar{u}_L \gamma_\mu \gamma_\nu \gamma_\rho b_L) - 16\, Q_2^u$$

  - Vanishes in 4 dim. due to $\gamma^\mu \gamma^\nu \gamma^\rho = g^{\mu\nu}\gamma^\rho - g^{\mu\rho}\gamma^\nu + g^{\nu\rho}\gamma^\mu + i\epsilon^{\mu\nu\rho\sigma}\gamma_\sigma\gamma_5$
  - Non-vanishing in D dimensions
  - Required to make the system closed under renormalisation

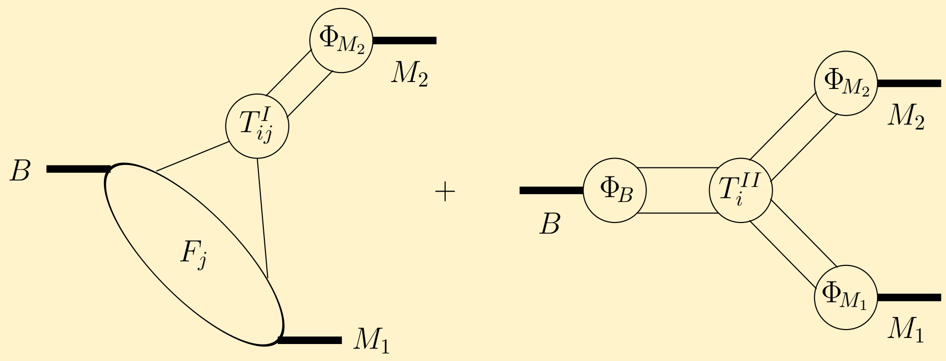
- Convenient resummation of large logarithms $L \equiv \ln(\frac{\mu_W}{\mu_b})$ via RG techniques

  LO: $\mathcal{O}(\alpha_s^n L^n)$ NLO: $\mathcal{O}(\alpha_s^n L^{n-1})$ NNLO: $\mathcal{O}(\alpha_s^n L^{n-2})$

- Can use naïvely anticommuting γ_5 in CMM basis



QCD factorisation



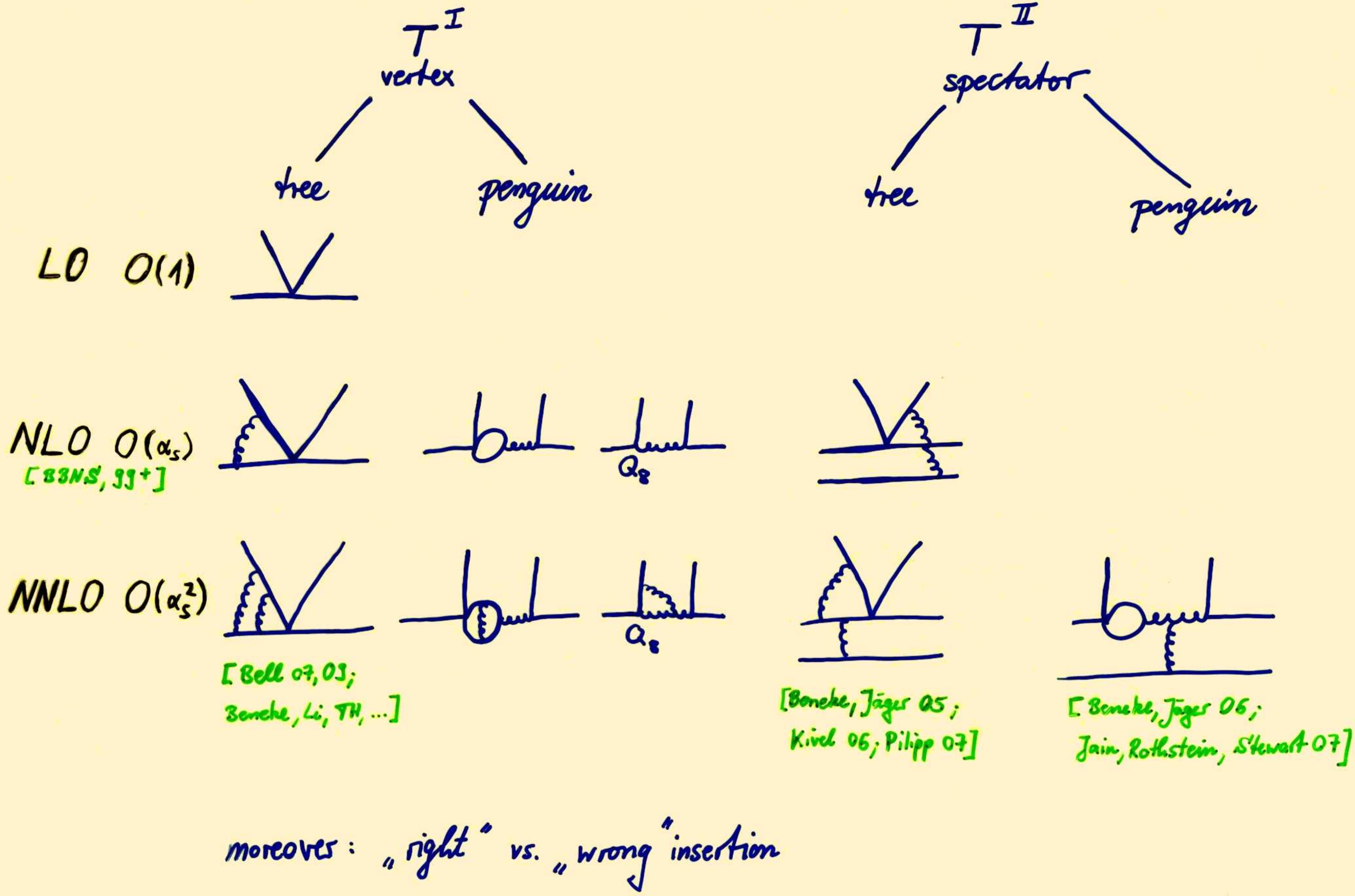
- Theoretical description of non-leptonic B decays difficult due to complicated QCD effects in the purely hadronic final state
- Simplification in the limit $m_b \gg \Lambda_{\rm QCD}$ *[Beneke,Buchalla,Neubert,Sachrajda'99-'04]*

$$\langle M_1 M_2 | Q_i | \bar{B} \rangle \simeq m_B^2 \, F_+^{B\to M_1}(0) \, f_{M_2} \int_0^1 du \; T_i^I(u) \, \phi_{M_2}(u)$$

$$+ f_B \, f_{M_1} \, f_{M_2} \int_0^1 d\omega dv du \; T_i^{II}(\omega, v, u) \, \phi_B(\omega) \, \phi_{M_1}(v) \, \phi_{M_2}(u)$$

- $T^{I,II}$: Hard scattering kernels, perturbatively calculable. $T^{II} = \mathcal{O}(\alpha_s)$
- F_+ : $B \to M$ form factor
  f_i : decay constants
  ϕ_i : light-cone distribution amplitudes

QCD factorisation



Motivation for NNLO

- Phenomenologically relevant
  - Strong phases start at $\mathcal{O}(\alpha_s)$
    * Direct CP asymmetries known to lowest order only
    * Large (scale) uncertainties
    * NNLO is therefore only the first correction
  - C/T seems to be too small
    * Large cancellation in LO + NLO
    * Particularly sensitive to NNLO
- Conceptual and systematic aspects
  - Verification of factorisation at NNLO
    * Does factorisation hold at all?
  - QCDF: systematic framework for computing perturbative corrections

    $\Rightarrow$ Let's do!

QCDF vs. experiment

- Branching ratios and CP asymmetries

$$\mathcal{B}(\bar{B} \to \bar{f}) = \frac{1}{2}\left[\mathcal{B}(\bar{B} \to \bar{f}) + \mathcal{B}(B \to f)\right]$$

$$\mathcal{A}_{CP} = \frac{\mathcal{B}(\bar{B}^0 \to \bar{f}) - \mathcal{B}(B^0 \to f)}{\mathcal{B}(\bar{B}^0 \to \bar{f}) + \mathcal{B}(B^0 \to f)}$$

QCDF

$$\mathcal{B}(B^- \to \pi^-\pi^0) = (5.5 \pm 1.0) \times 10^{-6}$$

$$\mathcal{B}(\bar{B}^0 \to \pi^+\pi^-) = (5.0 \pm 1.2) \times 10^{-6}$$

$$\mathcal{B}(\bar{B}^0 \to \pi^0\pi^0) = (0.73 \pm 0.54) \times 10^{-6}$$

[Beneke, Jäger'05]

$$\mathcal{B}(\bar{B}^0 \to \rho^0\rho^0) = (0.9 \pm 1.4) \times 10^{-6}$$

[Beneke, Rohrer, Yang'06]

$$\mathcal{A}_{CP}(\bar{B}^0 \to \pi^+\pi^-) = -0.065 \pm 0.135$$

$$\mathcal{A}_{CP}(\bar{B}^0 \to \pi^0\pi^0) = 0.451 \pm 0.592$$

[Beneke, Neubert'03]

Experiment

$$\mathcal{B}(B^- \to \pi^-\pi^0) = (5.7 \pm 0.5) \times 10^{-6}$$

$$\mathcal{B}(\bar{B}^0 \to \pi^+\pi^-) = (5.13 \pm 0.24) \times 10^{-6}$$

$$\mathcal{B}(\bar{B}^0 \to \pi^0\pi^0) = (1.62 \pm 0.31) \times 10^{-6}$$

$$\mathcal{B}(\bar{B}^0 \to \rho^0\rho^0) = (1.1 \pm 0.4) \times 10^{-6}$$

$$\mathcal{A}_{CP}(\bar{B}^0 \to \pi^+\pi^-) = 0.38 \pm 0.07$$

$$\mathcal{A}_{CP}(\bar{B}^0 \to \pi^0\pi^0) = 0.48 \pm 0.30$$

[PDG'08]

- Q: Does NNLO QCDF tend toward the right direction?

QCD factorisation

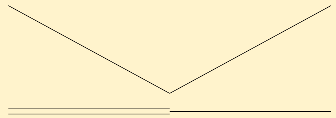
- Alternative representation of matrix elements *[Beneke,Neubert'03]*

$$\sqrt{2}\,\langle\pi^-\pi^0|\,\mathcal{H}_{eff}\,|B^-\rangle = \lambda_u\big[\alpha_1(\pi\pi)+\alpha_2(\pi\pi)\big]\;A_{\pi\pi}$$

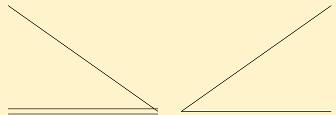
$$\langle\pi^+\pi^-|\,\mathcal{H}_{eff}\,|\bar{B}^0\rangle = \big\{\lambda_u\big[\alpha_1(\pi\pi)+\alpha_4^u(\pi\pi)\big]+\lambda_c\,\alpha_4^c(\pi\pi)\big\}\;A_{\pi\pi}$$

$$-\,\langle\pi^0\pi^0|\,\mathcal{H}_{eff}\,|\bar{B}^0\rangle = \big\{\lambda_u\big[\alpha_2(\pi\pi)-\alpha_4^u(\pi\pi)\big]-\lambda_c\,\alpha_4^c(\pi\pi)\big\}\;A_{\pi\pi}$$

- α_1 : colour-allowed tree amplitude, "right insertion"



- α_2 : colour-suppressed tree amplitude, "wrong insertion"



- $\alpha_4^{u/c}$: Penguin amplitudes

$$\alpha_1(\pi\pi) = 1.015 + [0.025+0.012i]_V + [??+0.027i]_{VV} - \left[\tfrac{r_{\rm sp}}{0.485}\right]\big\{[0.020]_{\rm LO}+[0.034+0.029i]_{HV}+[0.012]_{\rm tw3}\big\}$$

$$= 0.975^{+0.034}_{-0.072} + (0.010^{+0.025}_{-0.051})i$$

$$\alpha_2(\pi\pi) = 0.184 - [0.153+0.077i]_V + [??-0.049i]_{VV} + \left[\tfrac{r_{\rm sp}}{0.485}\right]\big\{[0.122]_{\rm LO}+[0.050+0.053i]_{HV}+[0.071]_{\rm tw3}\big\}$$

$$= 0.275^{+0.228}_{-0.135} + (-0.073^{+0.115}_{-0.082})i$$

[Beneke,Buchalla,Neubert,Sachrajda'99,'01]
[Beneke,Neubert'03; Beneke,Jäger'05,'06; Kivel'06; Pilipp'07; Bell'07]
[Hill,Becher,Lee,Neubert'04; Becher,Hill'04; Kirilin'05; Beneke,Yang'05]

- Goal: $\mathcal{O}(\alpha_s^2)$ vertex corrections to α_1 and α_2 $\Leftrightarrow$ 2-loop matrix elements of Q_1, Q_2

SCET operator basis

Right insertion

$$O_1 = \left[\bar{\chi}\,\frac{\not{n}_-}{2}(1-\gamma_5)\chi\right]\left[\bar{\xi}\,\not{n}_+(1-\gamma_5)h_v\right]$$

$$O_2 = \left[\bar{\chi}\,\frac{\not{n}_-}{2}(1-\gamma_5)\gamma_\perp^\alpha\gamma_\perp^\beta\chi\right]\left[\bar{\xi}\,\not{n}_+(1-\gamma_5)\gamma^\perp_\beta\gamma^\perp_\alpha h_v\right]$$

$$O_3 = \left[\bar{\chi}\,\frac{\not{n}_-}{2}(1-\gamma_5)\gamma_\perp^\alpha\gamma_\perp^\beta\gamma_\perp^\gamma\gamma_\perp^\delta\chi\right]\left[\bar{\xi}\,\not{n}_+(1-\gamma_5)\gamma^\perp_\delta\gamma^\perp_\gamma\gamma^\perp_\beta\gamma^\perp_\alpha h_v\right]$$

In addition, for a massive final state

$$O_1' = \left[\bar{\chi}\,\frac{\not{n}_-}{2}(1-\gamma_5)\chi\right]\left[\bar{\xi}\,\not{n}_+(1+\gamma_5)h_v\right]$$

$$O_2' = \left[\bar{\chi}\,\frac{\not{n}_-}{2}(1-\gamma_5)\gamma_\perp^\alpha\gamma_\perp^\beta\chi\right]\left[\bar{\xi}\,\not{n}_+(1+\gamma_5)\gamma^\perp_\alpha\gamma^\perp_\beta h_v\right]$$

$$O_3' = \left[\bar{\chi}\,\frac{\not{n}_-}{2}(1-\gamma_5)\gamma_\perp^\alpha\gamma_\perp^\beta\gamma_\perp^\gamma\gamma_\perp^\delta\chi\right]\left[\bar{\xi}\,\not{n}_+(1+\gamma_5)\gamma^\perp_\alpha\gamma^\perp_\beta\gamma^\perp_\gamma\gamma^\perp_\delta h_v\right]$$

Wrong insertion (only massless final state)

$$\tilde{O}_1 = \left[\bar{\xi}\,\gamma_\perp^\alpha(1-\gamma_5)\chi\right]\left[\bar{\chi}(1+\gamma_5)\gamma^\perp_\alpha h_v\right]$$

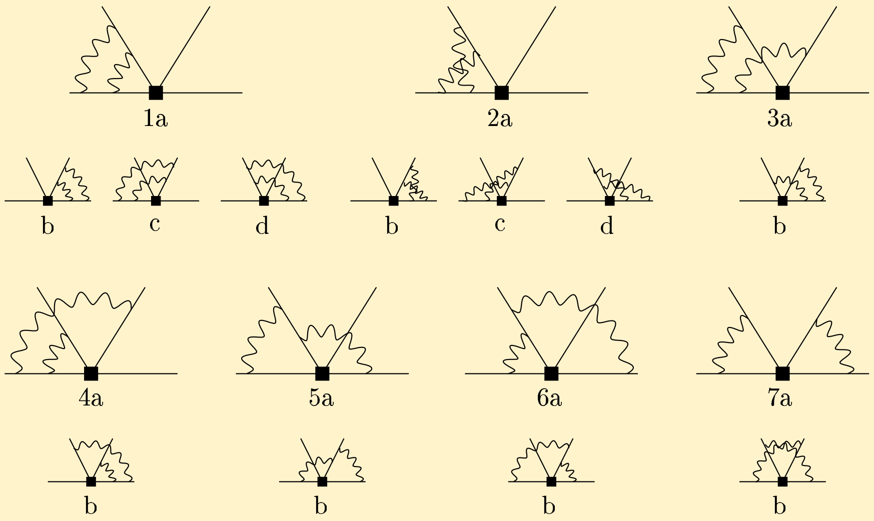
$$\tilde{O}_2 = \left[\bar{\xi}\,\gamma_\perp^\alpha\gamma_\perp^\beta\gamma_\perp^\gamma(1-\gamma_5)\chi\right]\left[\bar{\chi}(1+\gamma_5)\gamma^\perp_\alpha\gamma^\perp_\gamma\gamma^\perp_\beta h_v\right]$$

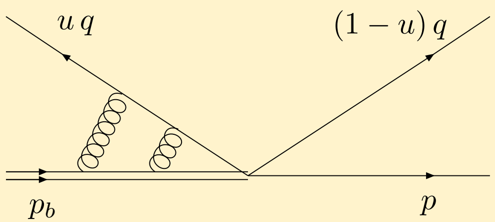
$$\tilde{O}_3 = \left[\bar{\xi}\,\gamma_\perp^\alpha\gamma_\perp^\beta\gamma_\perp^\gamma\gamma_\perp^\delta\gamma_\perp^\epsilon(1-\gamma_5)\chi\right]\left[\bar{\chi}(1+\gamma_5)\gamma^\perp_\alpha\gamma^\perp_\epsilon\gamma^\perp_\delta\gamma^\perp_\gamma\gamma^\perp_\beta h_v\right]$$

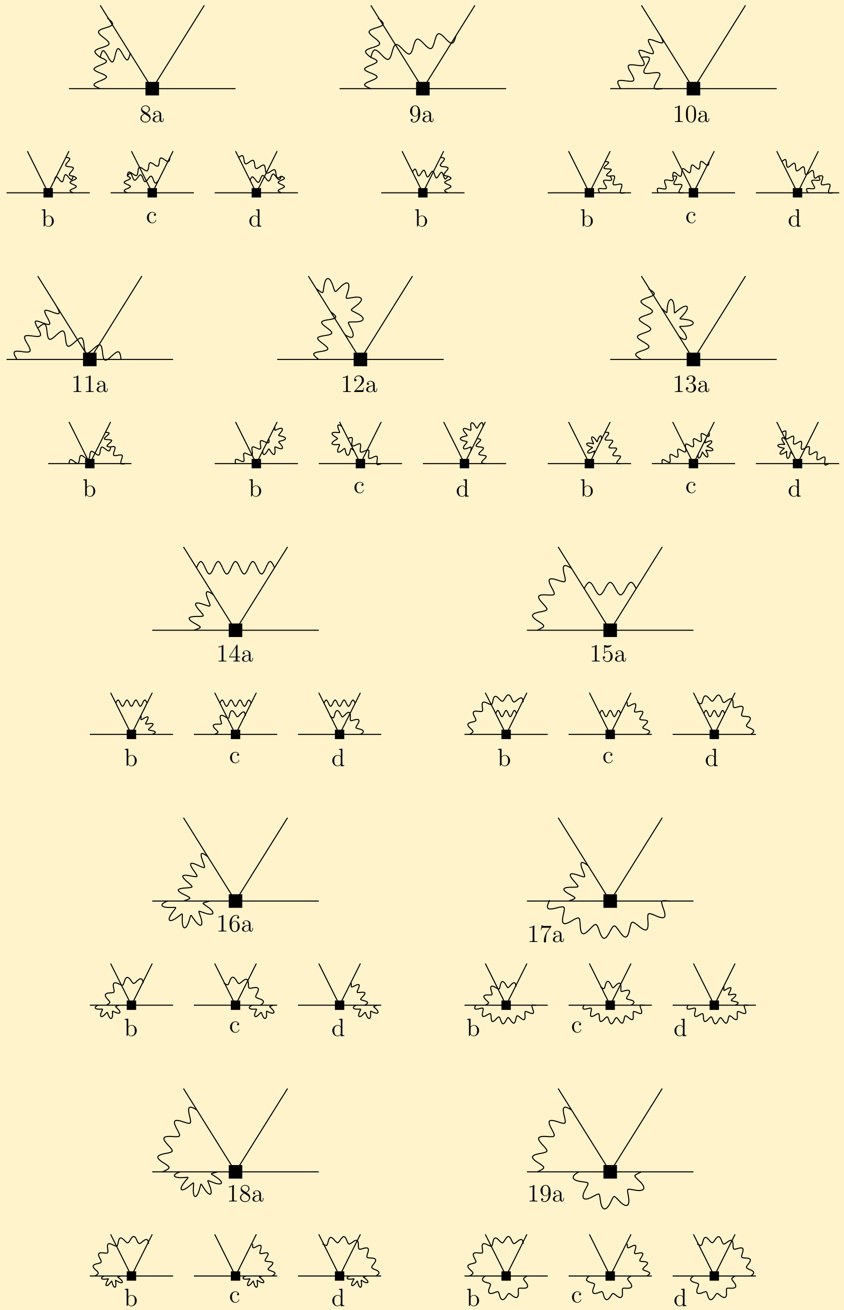
All operators with indices 2 and 3 are evanescent. Moreover: Fierz$(\tilde{O}_1) = O_1$ in $D = 4$.

Two-loop diagrams

- Non-factorizable two-loop diagrams for non-leptonic B-decays *[Beneke,Buchalla,Neubert,Sachrajda'00]*







- Kinematics: $p_b^2 = m_b^2$, $q^2 = 0$, $p^2 = 0$ or $p^2 = m_c^2$

Reduction methods

- Work in dimensional regularisation with $D = 4 - 2\epsilon$, to regulate UV and IR divergences. Poles up to $1/\epsilon^4$.
- Elimination of tensor structure via a Passarino-Veltman ansatz *[Passarino,Veltman'79]*
- Yields scalar integrals with irreducible scalar products in the numerator, e. g.

$$I = \int \frac{d^D k}{(2\pi)^D} \int \frac{d^D l}{(2\pi)^D} \frac{l^2}{[(k+p_b)^2 - m_b^2]\,[(l+p_b)^2 - m_b^2]\,(k+uq)^2\,(l+uq)^2\,k^2\,(l-k)^2}$$

- Integration-by-parts (IBP) identities, 8 per diagram *[Tkachov'81; Chetyrkin,Tkachov'81]*

$$\int \frac{d^D k}{(2\pi)^D} \int \frac{d^D l}{(2\pi)^D} \frac{\partial}{\partial a^\mu} \left[b^\mu\, f(k,l,p_i) \right] = 0\,; \qquad a^\mu = k^\mu,\, l^\mu\,; \qquad b^\mu = k^\mu,\, l^\mu,\, p_i^\mu$$

Reduction methods

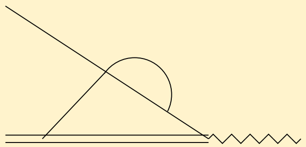
- Lorentz-Invarianz (LI) identities, 1 per diagram *[Gehrmann,Remiddi'99]*

$$\int \frac{d^D k}{(2\pi)^D} \int \frac{d^D l}{(2\pi)^D} \, \delta\epsilon^\mu_\nu \left[\sum_i p_i^\nu \frac{\partial}{\partial p_i^\mu} \right] f(k,l,p_j) = 0$$

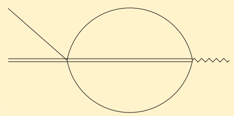
- Solve system of equations by means of Laporta algorithm

[Laporta'01; Anastasiou,Lazopoulos'04; Smirnov'08]

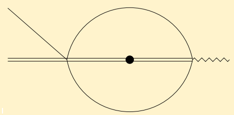
- Obtain scalar integrals as a linear combination of master integrals



$$= \frac{(8-3D)(7uD-8D-24u+28)}{3(D-4)^2\, m_b^4\, u^3}$$

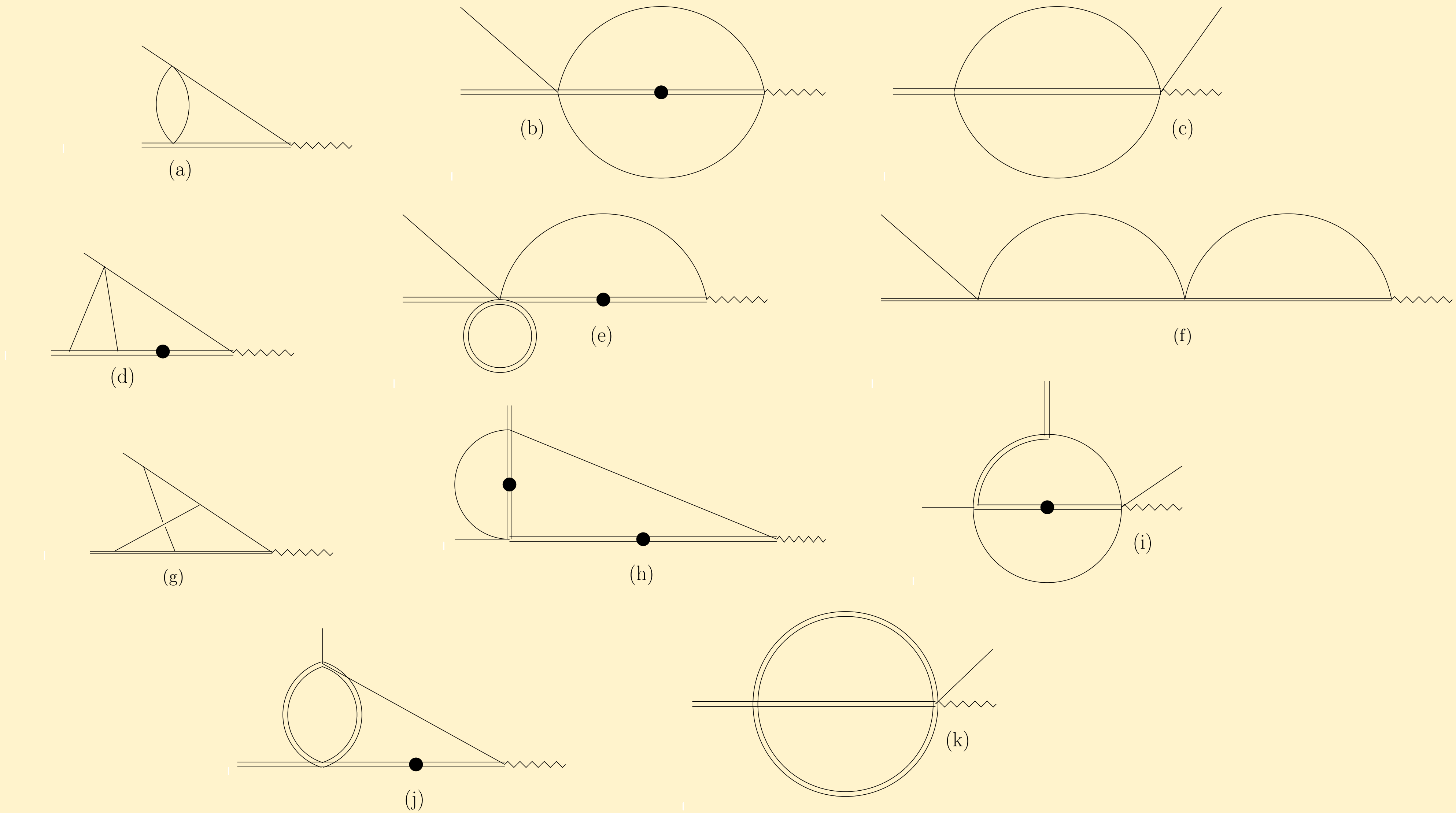


$$- \frac{2[u^2(D-4)+(16D-56)(1-u)]}{3(D-4)^2\, m_b^2\, u^3}$$



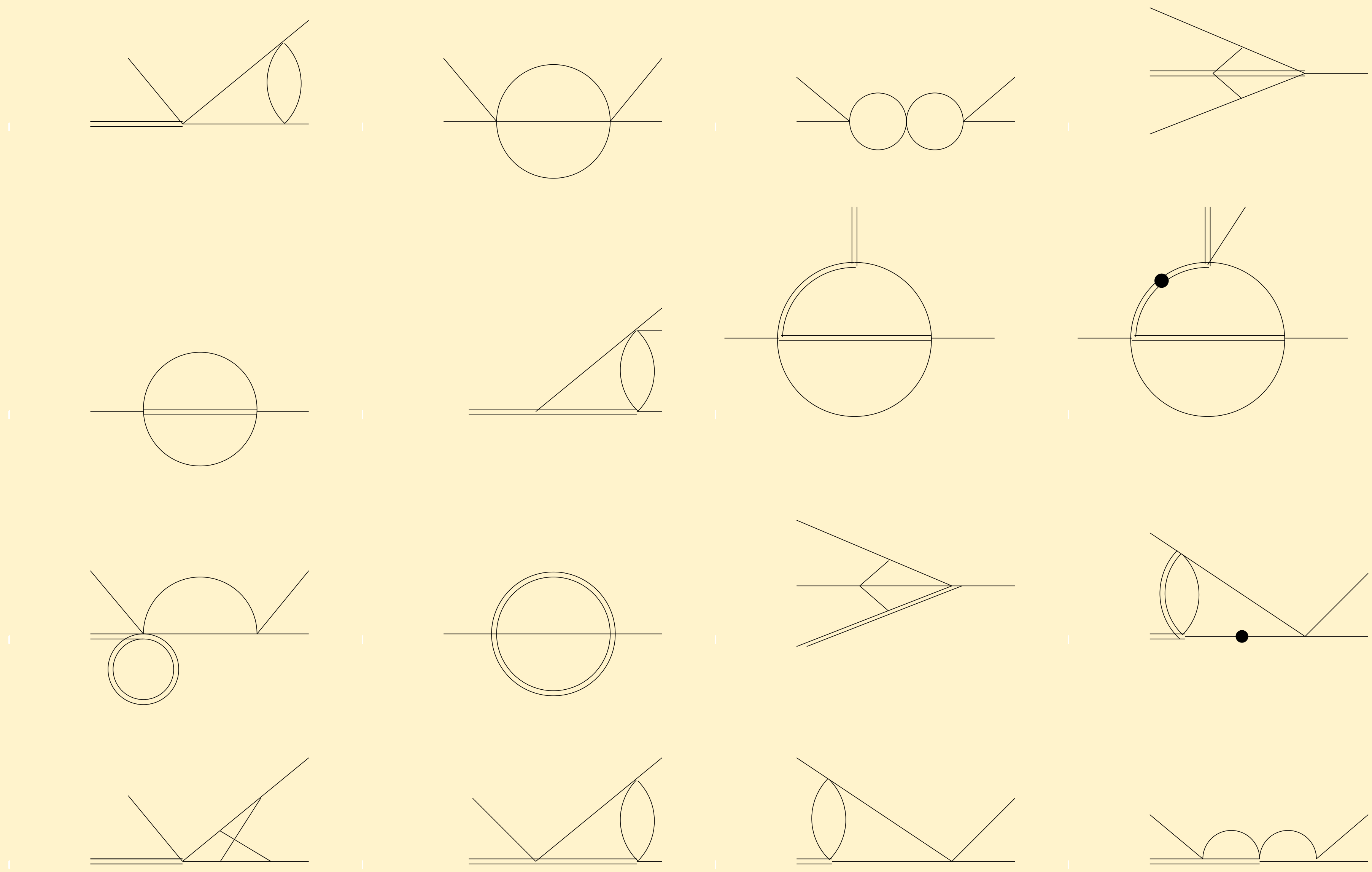
- Reduction is carried out for $m_c = 0$ $(B \to \pi\pi)$ and $m_c \neq 0$ $(B \to D\pi)$

Master integrals I ($m_c = 0$)

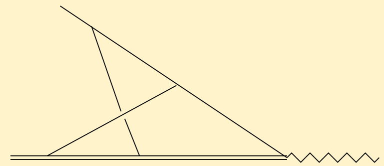


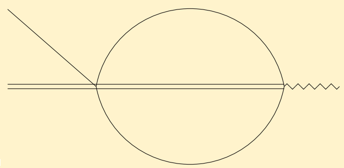
- Double lines are massive, single lines are massless
- Dots on lines denote squared propagators

Master integrals II ($m_c = 0$)



Master Integrals

- Reduction yields 42 master integrals for $m_c = 0$. For finite m_c, this roughly doubles.
- Poles up to $1/\epsilon^4$. Analytic calculation of coefficient functions for $m_c = 0$. Harmonic polylogarithms up to weight 4 of argument u or $1-u$. *[Remiddi,Vermaseren'99]*
- Several calculations in agreement *[Bell'07; Bonciani,Ferroglia'08; Asatrian,Greub,Pecjak'08; Beneke,Li,TH'08]*
- Most difficult master integral:  *[TH'09]*
  - Possesses a three-fold Mellin-Barnes integral at $u = 1$
- Applied techniques
  - Hypergeometric functions



$$= \frac{(m_b^2)^{1-2\epsilon}}{(4\pi)^{4-2\epsilon}} \frac{\Gamma^2(1-\epsilon)\Gamma(\epsilon)\Gamma(2\,\epsilon-1)}{\Gamma(2-\epsilon)} \, {}_2F_1(\epsilon, 2\,\epsilon-1\,;\,2-\epsilon\,;\,1-u)$$

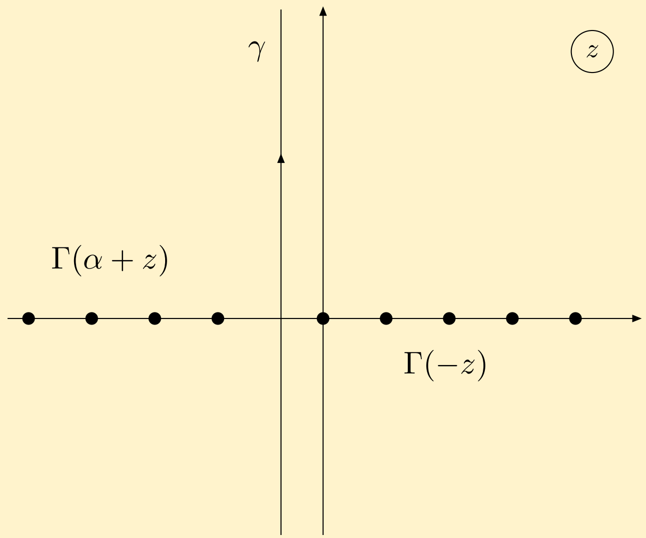
    * ϵ-expansion: XSummer (Form), `HypExp` (Mathematica) *[Moch,Uwer'05; Maitre,TH'05,'07]*

Master Integrals (cont'd.)

- Applied techniques (cont'd.)

  – Mellin-Barnes representation *[Smirnov'99; Tausk'99]*

$$\frac{1}{(A_1 + A_2)^\alpha} = \int_\gamma \frac{dz}{2\pi i}\, A_1^z\, A_2^{-\alpha-z}\, \frac{\Gamma(-z)\,\Gamma(\alpha+z)}{\Gamma(\alpha)}$$



  * partially automated
  * Numerical cross checks possible

  [Czakon'05; Gluza,Kajda,Riemann'07]

  – Differential equations *[Kotikov'91; Remiddi'97]*

$$\frac{\partial}{\partial u}\mathrm{MI}_i(u) = f(u,\epsilon)\,\mathrm{MI}_i(u) + \sum_{j\neq i} g_j(u,\epsilon)\,\mathrm{MI}_j(u)$$

  * Requires result of Laporta reduction.
  * Boundary condition in $u = 0$ or $u = 1$ from Mellin-Barnes representation

Master formula, right insertion

- Master formula for the hard scattering kernel (right insertion)

$$
\begin{aligned}
T_i^{(1)} &= A_{i1}^{(1),nf} + Z_{ij}^{(1)}\,A_{j1}^{(0)} \\
T_i^{(2)} &= A_{i1}^{(2),nf} + Z_{ij}^{(1)}\,A_{j1}^{(1)} + Z_{ij}^{(2)}\,A_{j1}^{(0)} \\
&\quad + Z_{\alpha}^{(1)}\,A_{i1}^{(1),nf} + (-i)\,\delta_m^{(1)}\,A_{i1}^{\prime(1),nf} \\
&\quad + T_i^{(1)}\big[\xi_{45}^{(1)} - C_{FF}^{(1)} - Z_J^{(1)} - Z_{BL}^{(1)} + Z_{ext}^{(1)}\big] \\
&\quad - \sum_{b>1} H_{ib}^{(1)}\,Y_{b1}^{(1)}
\end{aligned}
$$

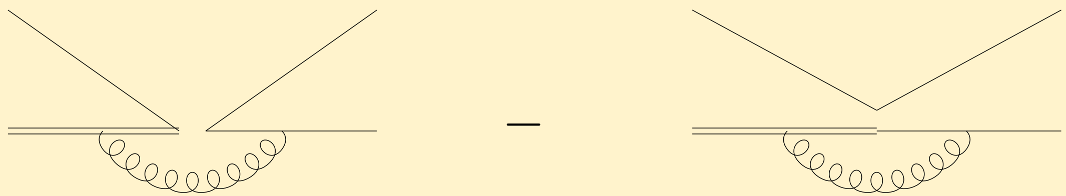
- Important check: $T_i^{(1)}$ and $T_i^{(2)}$ free of poles ✓
- Higher order ϵ terms in $T_i^{(1)}$ required for $T_i^{(2)}$
- $A_{j1}^{(0)}$ and $A_{j1}^{(1)}$ include ME of evanescent operators
- Terms like $T_i^{(1)}\,Z_{BL}^{(1)}$ contain a convolution
- $Y_{b1}^{(1)}$: renormalization constants of the SCET operator basis

Master formula, wrong insertion

- Master formula for the hard scattering kernel (wrong insertion)

$$\widetilde{T}_i^{(1)} = \widetilde{A}_{i1}^{(1),nf} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(0)} + \underbrace{\widetilde{A}_{i1}^{(1),f} - A^{(1),f}\,\widetilde{A}_{i1}^{(0)}}_{\mathcal{O}(\epsilon)} - \underbrace{[\widetilde{Y}_{11}^{(1)} - Y_{11}^{(1)}]\,\widetilde{A}_{i1}^{(0)}}_{\mathcal{O}(\epsilon)}$$

$$\begin{aligned}\widetilde{T}_i^{(2)} = {} & \widetilde{A}_{i1}^{(2),nf} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(1)} + Z_{ij}^{(2)}\,\widetilde{A}_{j1}^{(0)} + Z_{\alpha}^{(1)}\,\widetilde{A}_{i1}^{(1),nf} \\ & + (-i)\,\delta_m^{(1)}\,\widetilde{A}_{i1}'^{(1),nf} + (Z_{ext}^{(1)} + \xi_{45}^{(1)})\,[\widetilde{A}_{i1}^{(1),nf} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(0)}] \\ & -\widetilde{T}_i^{(1)}\,[C_{FF}^{(1)} + \widetilde{Y}_{11}^{(1)}] - \sum_{b>1}\widetilde{H}_{ib}^{(1)}\,\widetilde{Y}_{b1}^{(1)} \\ & +[\widetilde{A}_{i1}^{(2),f} - A^{(2),f}\,\widetilde{A}_{i1}^{(0)}] + (-i)\,\delta_m^{(1)}\,[\widetilde{A}_{i1}'^{(1),f} - A'^{(1),f}\,\widetilde{A}_{i1}^{(0)}] \\ & +(Z_{\alpha}^{(1)} + Z_{ext}^{(1)} + \xi_{45}^{(1)})[\widetilde{A}_{i1}^{(1),f} - A^{(1),f}\,\widetilde{A}_{i1}^{(0)}] \\ & -C_{FF}^{(1)}\,\widetilde{A}_{i1}^{(0)}[\widetilde{Y}_{11}^{(1)} - Y_{11}^{(1)}] - [\widetilde{Y}_{11}^{(2)} - Y_{11}^{(2)}]\,\widetilde{A}_{i1}^{(0)}\end{aligned}$$



Results

- Topological tree amplitude to NNLO (right insertion)

$$\alpha_1(M_1 M_2) = C_2 + \frac{\alpha_s}{4\pi}\frac{C_F}{2N_c}\left\{C_1\,V^{(1)} + \frac{\alpha_s}{4\pi}\left[C_1\,V_1^{(2)} + C_2\,V_2^{(2)}\right] + \mathcal{O}(\alpha_s^2)\right\} + \ldots$$

$$\frac{C_F}{2N_c}V_i^{(j)} = \int_0^1 du\,T_i^{(j)}(u)\,\phi_{M_2}(u)$$

$$\phi_{M_2}(u) = 6u(1-u)\left[1 + \sum_{n=1}^{\infty} a_n^{M_2}\,C_n^{(3/2)}(2u-1)\right]$$

- We obtain at $\mu = m_b$ (numbers still preliminary!)

$$V^{(1)} = (-22.500 - 9.425\,i) + (5.500 - 9.425\,i)\,a_1^{M_2} + (-1.050)\,a_2^{M_2}$$

$$V_1^{(2)} = (-178.39 - 349.44\,i) + (641.65 - 119.36i)\,a_1^{M_2} + (-85.39 - 62.63\,i)\,a_2^{M_2}$$

$$V_2^{(2)} = (322.19 + 320.94\,i) + (-212.97 + 154.41\,i)\,a_1^{M_2} + (3.8146 - 34.0626\,i)\,a_2^{M_2}$$

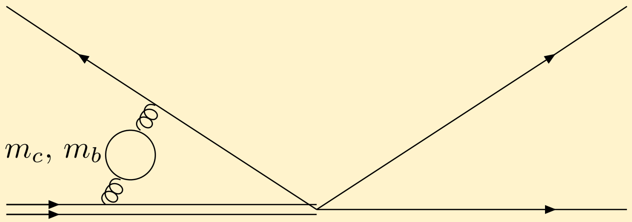
$$\alpha_1(\pi\pi) = [1.008]_{|V^{(0)}} + [0.022 + 0.009i]_{|V^{(1)}} + [0.026 + 0.028i]_{|V^{(2)}} + \ldots$$

Results

- We obtain at $\mu = m_b$ (numbers still preliminary!)

$$
\begin{aligned}
V^{(1)} &= (-22.500 - 9.425\, i) + (5.500 - 9.425\, i)\, a_1^{M_2} + (-1.050)\, a_2^{M_2} \\
V_1^{(2)} &= (-178.39 - 349.44\, i) + (641.65 - 119.36 i)\, a_1^{M_2} + (-85.39 - 62.63\, i)\, a_2^{M_2} \\
V_2^{(2)} &= (322.19 + 320.94\, i) + (-212.97 + 154.41\, i)\, a_1^{M_2} + (3.8146 - 34.0626\, i)\, a_2^{M_2} \\
\alpha_1(\pi\pi) &= [1.008]_{|V^{(0)}} + [0.022 + 0.009i]_{|V^{(1)}} + [0.026 + 0.028i]_{|V^{(2)}} + \ldots
\end{aligned}
$$

- Have expressions for $V_i^{(j)}$ completely analytically, including charm mass dependence



Factorisation test

Recall

$$\sqrt{2}\,\langle\pi^-\pi^0|\,\mathcal{H}_{eff}\,|B^-\rangle \;=\; \lambda_u\big[\alpha_1(\pi\pi)+\alpha_2(\pi\pi)\big]\,A_{\pi\pi}$$

$$A_{\pi\pi} \;=\; i\,\tfrac{G_F}{\sqrt{2}}\,m_B^2 F^{B\to\pi}(0)$$

$$R \;\equiv\; \frac{\Gamma(B^-\to\pi^-\pi^0)}{d\Gamma(\bar{B}^0\to\pi^+\ell^-\bar{\nu})/dq^2\big|_{q^2=0}} = 3\pi^2 f_\pi^2\,|V_{ud}|^2\,|\alpha_1(\pi\pi)+\alpha_2(\pi\pi)|^2$$

- Dependence on form factor and V_{ub} drops out of the ratio
- Depending on the input, can extract information on α_2, $|V_{ub}|\,F^{B\to\pi}(0)$, or λ_B

Outlook

To do

- Two-loop color suppressed amplitude (wrong insertion) of vertex correction
- Comparison with results of G. Bell *[Bell'09]*
- Massive final state ($B \to D\pi$)
- Penguin amplitudes

$$\alpha_4^u(\pi\pi) = -0.029 - [0.002 + 0.001i]_V + [0.003 - 0.013i]_P + [?? \; + \; ?? \; i]_{\mathcal{O}(\alpha_s^2)}$$

$$+ \left[\frac{r_{\rm sp}}{0.485}\right] \left\{[0.001]_{\rm LO} + [0.001 + 0.000i]_{HV+HP} + [0.001]_{\rm tw3}\right\} = -0.024^{+0.004}_{-0.002} + (-0.012^{+0.003}_{-0.002})i$$

$$\alpha_4^c(\pi\pi) = -0.029 - [0.002 + 0.001i]_V - [0.001 + 0.007i]_P + [?? \; + \; ?? \; i]_{\mathcal{O}(\alpha_s^2)}$$

$$+ \left[\frac{r_{\rm sp}}{0.485}\right] \left\{[0.001]_{\rm LO} + [0.001 + 0.001i]_{HV+HP} + [0.001]_{\rm tw3}\right\} = -0.028^{+0.005}_{-0.003} + (-0.006^{+0.003}_{-0.002})i$$

[Beneke,Buchalla,Neubert,Sachrajda'99,'01; Beneke,Neubert'03; Beneke,Jäger'05,'06; Kivel'06; Pilipp'07; Bell'07]

[Hill,Becher,Lee,Neubert'04; Becher,Hill'04; Kirilin'05; Beneke,Yang'05]

- Complete phenomenological analysis

Backup slides

Some definitions

$$
\begin{aligned}
A_{\pi\pi} &= i\,\frac{G_F}{\sqrt{2}}\,m_B^2 F_+^{B\to\pi}(0) f_\pi \\
r_{\rm sp} &= \frac{9 f_\pi \hat{f}_B}{m_b \lambda_B F_+^{B\to\pi}(0)} \\
\lambda_B^{-1} &= \int_0^\infty \frac{d\omega}{\omega}\,\phi_B(\omega,\mu)
\end{aligned}
$$