Automatic calculation of one-loop amplitudes

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in collaboration with

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Motivation

- LHC is a proton-proton collider, and the physical events to be studied are collision events;
- The events are related to physical quantities in a statistical manner via distributions;
- physics at LHC demands precise qualitative knowledge about signals and backgrounds;
- Monte Carlo programs are a preferred tools to crystallize such knowledge;
- multi-leg hard processes need to be included in these. Many interesting signals (Higgs production) include decaying heavy particles.
- NLO corrections have to be included
  - to reduce scale dependence;
  - to get better description of shapes of distributions;
- several groups of researchers are dealing with the problem of calculating multi-leg processes at NLO.

Motivation

Backgrounds

- $pp \to VV + j$ Dittmaier,Kallweit,Uwer; Campbell,Ellis,Zanderighi
- $pp \to t\bar{t}b\bar{b}$ Bredenstein,Denner,Dittmaier,Pozzorini
- $pp \to VVV$ ZZZ:Lazopoulos,Melnikov,Petriello; WWZ:Hankele,Zeppenfeld;
  VVV: Binoth,Ossola,Papadopoulos,Pittau
- $pp \to VV + 2j$ VBF: Jäger,Oleari,Zeppenfeld; Bozzi
- $pp \to t\bar{t}Z$ Lazopoulos,Melnikov,Petriello
- $pp \to t\bar{t} + j$ Dittmaier,Uwer,Weinzierl
- $pp \to W + 3j$ Ellis,Melnikov,Zanderighi
  Berger,Bern,Dixon,Febres Cordero,Forde,Gleisberg,Ita,Kosower,Maître

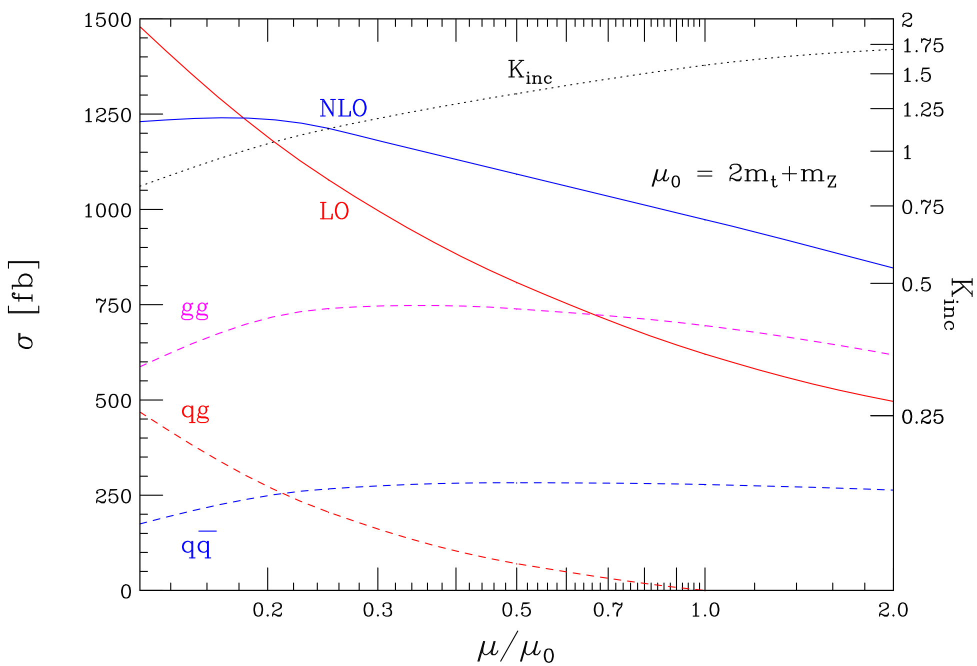
Signals

- $pp \to H + 2j$ Campbell,Ellis,Zanderighi; Ciccolini,Denner,Dittmaier
- $pp \to H + t\bar{t}$ Beenakker,Dittmaier,Krämer,Plümer,Spira,Zerwas;
  Dawson,Jackson,Reina,Wackeroth

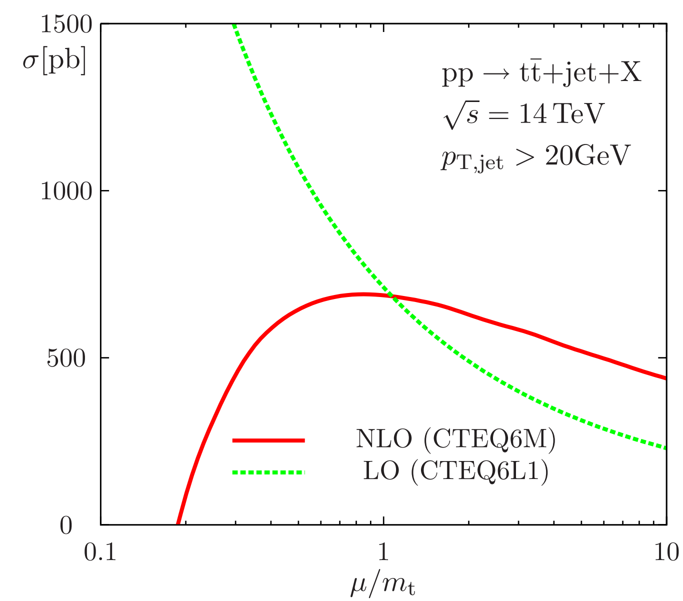
Motivation

Scale dependence ($\mu = \mu_R = \mu_F$)

$pp \to t\bar{t}Z$ Lazopoulos,Melnikov,Petriello



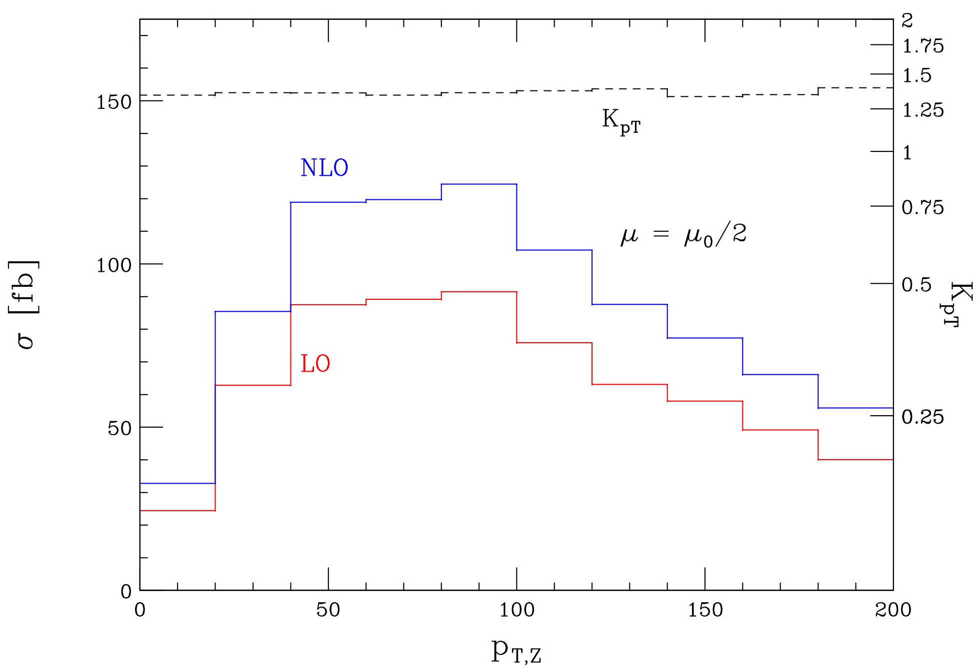
$pp \to t\bar{t} + j$ Dittmaier,Uwer,Weinzierl



Motivation

Shape p_T-distribution

$pp \to t\bar{t}Z$ Lazopoulos,Melnikov,Petriello



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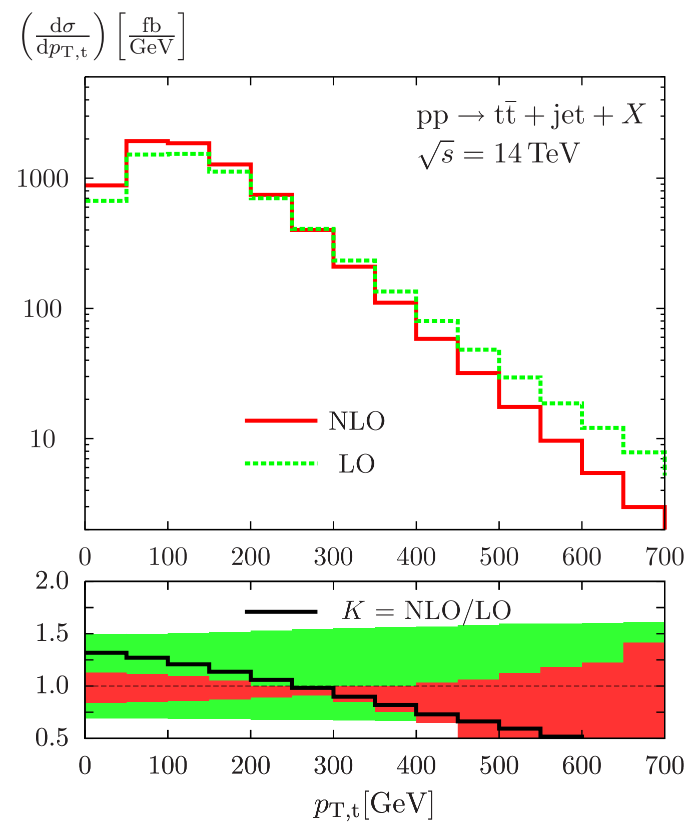

Motivation

- so far, mostly dedicated studies applying several computational techniques;
- LO calculations (including partonic phase-space generation) have been completely automatized: HELAC, ALPGEN, MadGraph, Amegic++, GRACE, ...;
- we want to do the same with NLO calculations

  Czakon,Dragiottis,Garzelli,Ossola,Pittau,Papadopoulos,Worek,AvH

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- we want to do the same with NLO calculations
  HELAC Czakon,Dragiottis,Garzelli,Ossola,Pittau,Papadopoulos,Worek,AvH
- and we are not the only ones:
  ROCKET Ellis,Giele,Kunszt,Melnikov,Zanderighi
  BLACKHAT/SHERPA Berger,Bern,Dixon,Febres Cordero,Forde,Gleisberg Ita,Kosower,Maître
- one of the bottlenecks is the evaluation of the virtual, one-loop, contribution.
  Automation also by:
  GOLEM Binoth,Guffanti,Guillet,Heinrich,Karg,Kauer,Reiter,Reuter
  D-dim Unitarity Lazopoulos

Ingredients for the calculations

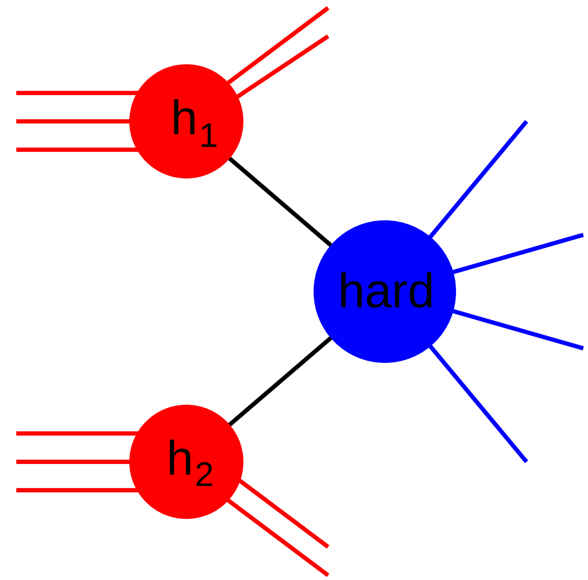
The mathematical framework of calculations in elementary particle physics is Quantum Field Theory. Two important ingredients in the calculations related to LHC physics are:

<u>Factorization</u>

$$d\sigma(\,h_1(p_1)h_2(p_2) \to X\,) =$$

$$\sum_{k,l}\int dx_1\,dx_2\,f_{1,k}(x_1,\mu_F)f_{2,l}(x_2,\mu_F)$$

$$\times\,d\sigma_{\text{hard}}(\,\phi_k(x_1p_1)\phi_l(x_2p_2) \to X;\,\mu_F\,)$$



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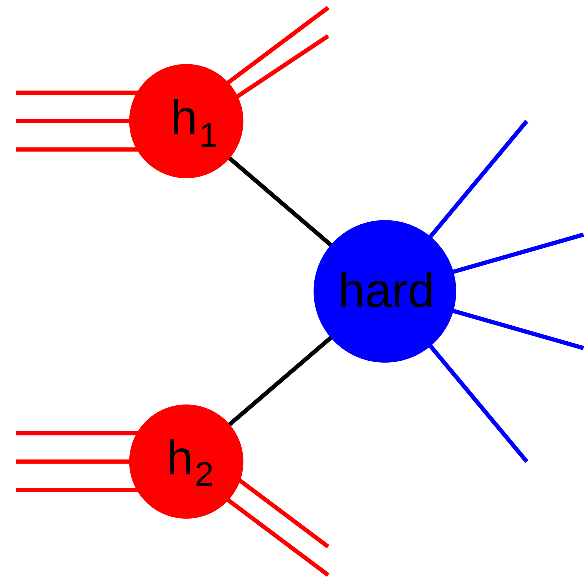
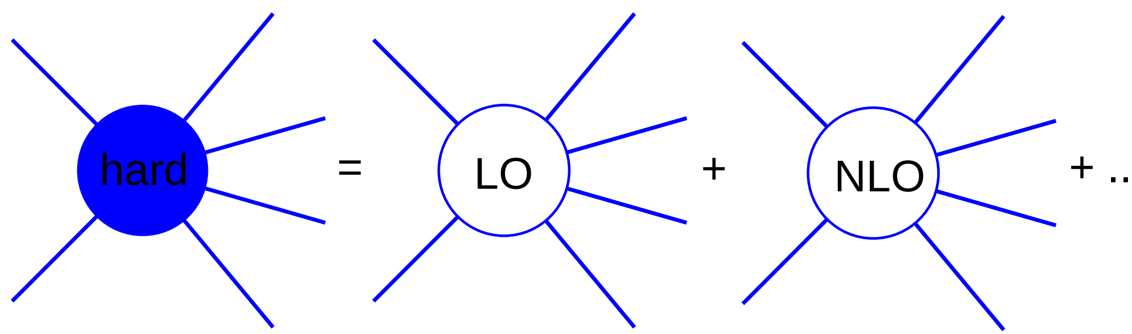


<u>Perturbation theory</u>

$$d\sigma_{\text{hard}} = d\sigma^{(0)}_{\text{hard}} + \alpha\,d\sigma^{(1)}_{\text{hard}} + \ldots$$



Ingredients for the calculations

NLO cross sections

- one order higher in perturbation theory: one more loop or one more leg (squared);
- IR-divergence of integral over phase space for which the extra leg is unobserved cancels against IR-divergence of loop integral KLN.

$$\langle O\rangle^{\mathrm{LO}} = \int d\Phi_n \, |\mathcal{M}_n^{(0)}|^2 \, O_n^{\mathrm{LO}}$$

$$\langle O\rangle^{\mathrm{NLO}} = \int d\Phi_n \left[2\Re\left(\mathcal{M}_n^{(0)}\mathcal{M}_n^{(1)}\right) + \mathcal{C}_n + \int d\Phi_1 \, \mathcal{A}_{n+1} \right] O_n^{\mathrm{LO}} + \int d\Phi_{n+1} \left[\, |\mathcal{M}_{n+1}^{(0)}|^2 \, O_{n+1}^{\mathrm{NLO}} - \mathcal{A}_{n+1} O_n^{\mathrm{LO}} \, \right]$$

Eg. dipole subtraction Catani, Seymour ’97

Ingredients for the calculations

Monte Carlo integration

$$\langle O \rangle = \int d\Phi_n(P;\{p\}_n)\,|\mathcal{M}_n(\{p\}_n)|^2\,O_n(\{p\}_n)$$

In practice, PS integration has to be, and *can* be, done by Monte Carlo.

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Helicity amplitudes

$$\langle O \rangle = \int d\Phi_n(P;\{p\}_n) \sum_{\{\lambda\}_n} |\mathcal{M}_n(\{p\}_n, \{\lambda\}_n)|^2\, O_n(\{p\}_n)$$

Avoid proliferation of terms from algebra, and perform the square and sum over helicities numerically, the latter maybe even by MC.

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Color treatment

$$\langle O\rangle = \int d\Phi_n(P;\{p\}_n)\sum_{\{\lambda\}_n}\sum_{\{a\}_n}|\mathcal{M}_n(\{p\}_n,\{\lambda\}_n,\{a\}_n)|^2\,O_n(\{p\}_n)$$

$$\mathcal{M}_n(\{p\}_n,\{\lambda\}_n,\{a\}_n) = \sum_{\text{perm}}\mathcal{C}(\{a\}_n)\,\mathcal{A}_n(\{p\}_n,\{\lambda\}_n)$$

Ingredients for the calculations

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Perform sum over colors numerically, maybe even by MC
Draggiotis,Kleiss,Papadopoulos ’98; Caravaglios,Mangano,Moretti,Pittau ’99.

Aim

We want to design a program to evaluate $\mathcal{M}_n^{(1)}(\{p\}_n, \{\lambda\}_n, \{a\}_n)$ as functions of its input as efficiently as possible.

The program should be highly automatic.

Philosophy

We are not particularly interested in algebraic/analytic expressions.

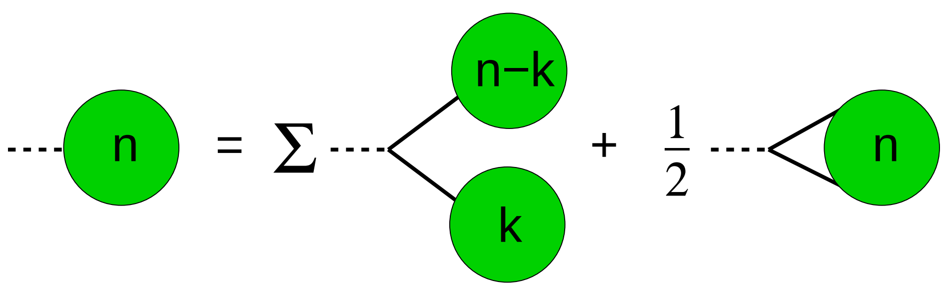
Amplitude calculation

LSZ-formula: amplitude = connected Green function with external propagators replaced by spinors/polarization vectors.

Dyson-Schwinger equation (=field theory): for the connected Green functions (for scalar ϕ^3-theory)

$$-i(p^2-m^2)G_{n+1}(p,p_1,\ldots,p_n) =$$
$$g\int dp_b\delta(p-p_a-p_b)\Big[\sum_{\{j\}} G_{k+1}(p_a,p_{j_1},\ldots,p_{j_k})\,G_{n-k+1}(p_b,p_{j_{k+1}},\ldots,p_{j_n})$$
$$+\frac{1}{2}G_{n+2}(p_a,p_b,p_1,\ldots,p_n)\Big]$$

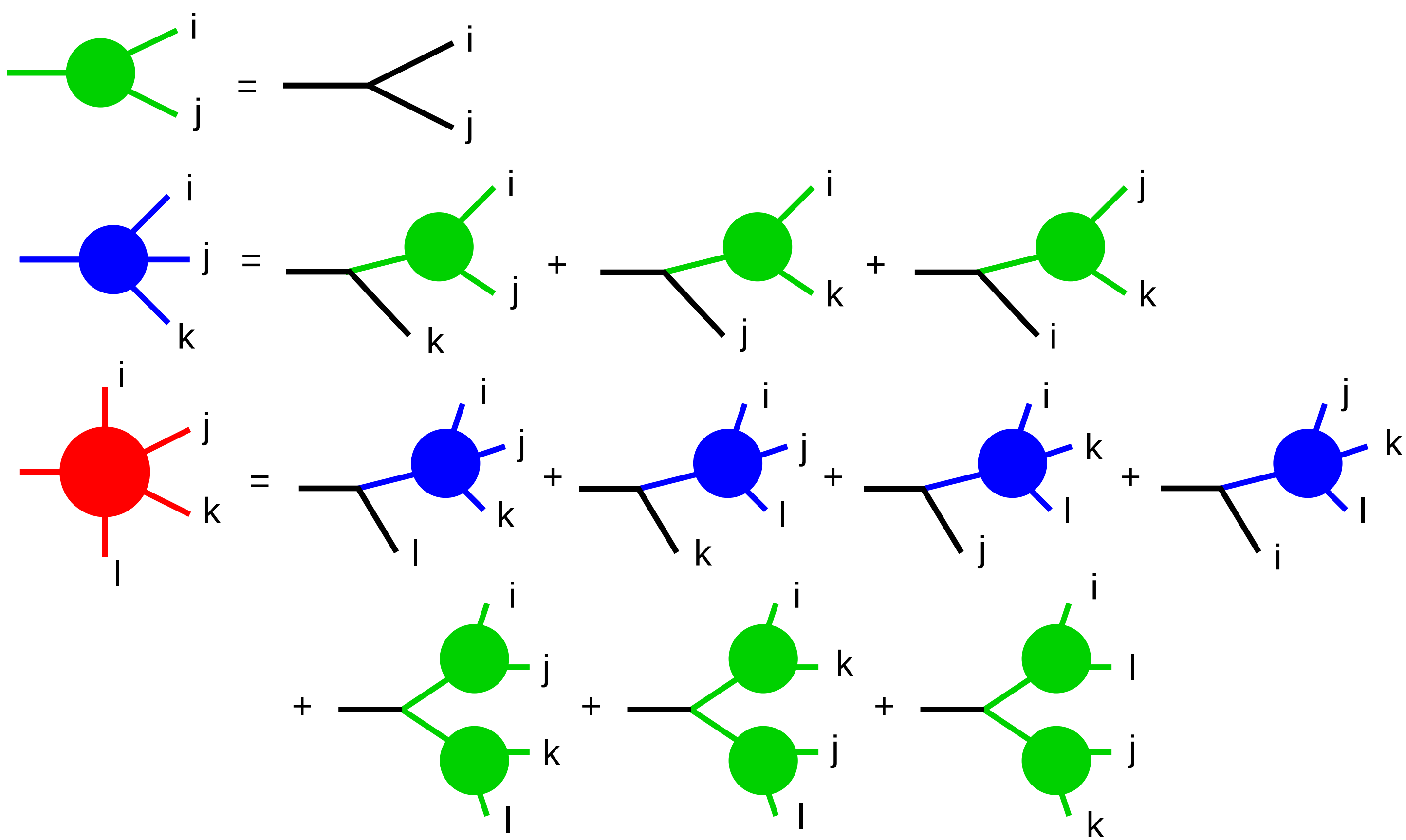
Replace external propagators 1 to n by spinors/polarization vectors
$\rightarrow$ off-shell currents.



Analytic solution:
sum of Feynman graphs.

Calculation of tree-level amplitudes

Dyson-Schwinger approach: Calculate off-shell currents instead of graphs.



Berends,Giele '88; Caravaglios,Moretti '95

- Efficient: $O(n!)$ for graphs to $O(3^n)$, $n =$ number of exteral legs.
- Straightforward to automatize.

One-loop amplitude with O_{ssola}P_{apadopoulos}P_{ittau}

Identify a set of n_{tot} denominators and write

$$\mathcal{M}^{(1)} = \sum_{I\subset\{0,1,2,\dots,n_{\text{tot}}-1\}} \int d^{\text{Dim}}q \, \frac{N_I(q)}{\prod_{i\in I} D_i} \quad , \quad D_i = (q+p_i)^2 - m_i^2$$

One-loop amplitude with OPP

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For $\text{Dim} = 4$ one can understand that

$$\frac{N(q)}{D_0 D_1 \cdots D_{n-1}} = \sum_{i_1,i_2,i_3,i_4} \frac{N_{i_1 i_2 i_3 i_4}(q)}{D_{i_1} D_{i_2} D_{i_3} D_{i_4}} \quad , \quad N_{i_1 i_2 i_3 i_4}(q) \text{ polynomial}$$

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Can we even write

$$\frac{N(q)}{D_0 D_1 \cdots D_{n-1}} \overset{?}{=}$$

$$\sum_{i_1,i_2,i_3,i_4} \frac{d(i_1,i_2,i_3,i_4)}{D_{i_1} D_{i_2} D_{i_3} D_{i_4}} + \sum_{i_1,i_2,i_3} \frac{c(i_1,i_2,i_3)}{D_{i_1} D_{i_2} D_{i_3}} + \sum_{i_1,i_2} \frac{b(i_1,i_2)}{D_{i_1} D_{i_2}} + \sum_{i_1} \frac{a(i_1)}{D_{i_1}} + P$$

No.

One-loop amplitude with OPP

$$
\frac{N(q)}{D_0 D_1 \cdots D_{n-1}} = \sum_{i_1,i_2,i_3,i_4} \frac{d(i_1,i_2,i_3,i_4) + \tilde{d}(q;i_1,i_2,i_3,i_4)}{D_{i_1} D_{i_2} D_{i_3} D_{i_4}}
+ \sum_{i_1,i_2,i_3} \frac{c(i_1,i_2,i_3) + \tilde{c}(q;i_1,i_2,i_3)}{D_{i_1} D_{i_2} D_{i_3}}
+ \sum_{i_1,i_2} \frac{b(i_1,i_2) + \tilde{b}(q;i_1,i_2)}{D_{i_1} D_{i_2}} + \sum_{i_1} \frac{a(i_1) + \tilde{a}(q;i_1)}{D_{i_1}} + \tilde{P}(q)
$$

- $\tilde{d}, \tilde{c}, \tilde{b}, \tilde{a}$ are polynomials in q with few coefficients (1,6,8,4);
- $\tilde{P}$ is zero in renormalizable gauge;
- terms with $\tilde{d}, \tilde{c}, \tilde{b}, \tilde{a}$ integrate to zero.

One-loop amplitude with OPP

$$\frac{N(q)}{D_0 D_1 \cdots D_{n-1}} = \sum_{i_1,i_2,i_3,i_4} \frac{d(i_1,i_2,i_3,i_4) + \tilde{d}(q;i_1,i_2,i_3,i_4)}{D_{i_1} D_{i_2} D_{i_3} D_{i_4}}$$

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$$\mathcal{M}^{(1)} = \sum_{i_1,i_2,i_3,i_4} \int \frac{d^{\text{Dim}}q \; d(i_1,i_2,i_3,i_4)}{D_{i_1} D_{i_2} D_{i_3} D_{i_4}} + \sum_{i_1,i_2,i_3} \int \frac{d^{\text{Dim}}q \; c(i_1,i_2,i_3)}{D_{i_1} D_{i_2} D_{i_3}}$$

$$+ \sum_{i_1,i_2} \int \frac{d^{\text{Dim}}q \; b(i_1,i_2)}{D_{i_1} D_{i_2}} + \sum_{i_1} \int \frac{d^{\text{Dim}}q \; a(i_1)}{D_{i_1}} + \text{rational terms} + O(\text{Dim} - 4)$$

One-loop amplitude with OPP

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$$+ \sum_{i_1,i_2} \int \frac{d^{\text{Dim}}q\ b(i_1,i_2)}{D_{i_1}D_{i_2}} + \sum_{i_1} \int \frac{d^{\text{Dim}}q\ a(i_1)}{D_{i_1}} + \text{rational terms} + O(\text{Dim}-4)$$

- universal set of scalar-functions can be coded once and for all eg. QCDloop Ellis,Zanderighi, OneLOop;
- coefficients d, c, b, a can be determined in 4 dimensions.
- to NLO we are not interested in $O(\text{Dim}-4)$.
- rational terms can be written in terms of
  - simple universal integrals with already determined coefficients (R_1, coming from denominators for $\text{Dim} \neq 4$),
  - plus a finite renormalization, with extra Feynman rules Draggiotis, Garzelli,Papadopoulos,Pittau (R_2, coming from numerator for $\text{Dim} \neq 4$).

One-loop amplitude with OPP

For all q:

$$N(q) = \sum_{i_1,i_2,i_3,i_4} \left[\, d(i_1,i_2,i_3,i_4) + \tilde{d}(q;i_1,i_2,i_3,i_4) \,\right] \prod_{j\neq i_1,i_2,i_3,i_4} D_j$$
$$+ \sum_{i_1,i_2,i_3} \left[\, c(i_1,i_2,i_3) + \tilde{c}(q;i_1,i_2,i_3) \,\right] \prod_{j\neq i_1,i_2,i_3} D_j$$
$$+ \sum_{i_1,i_2} \left[\, b(i_1,i_2) + \tilde{b}(q;i_1,i_2) \,\right] \prod_{j\neq i_1,i_2} D_j$$
$$+ \sum_{i} \left[\, a(i) + \tilde{a}(q;i) \,\right] \prod_{j\neq i} D_j$$

$$D_j = (q+p_j)^2 - m_j^2$$

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$$+ \sum_{i} \left[\, a(i) + \tilde{a}(q;i)\,\right] \prod_{j\neq i} D_j \qquad D_j = (q+p_j)^2 - m_j^2$$

Choose $q = q_0$ such that $D_{i_1} = D_{i_2} = D_{i_3} = D_{i_4} = 0$:

$$N(q_0) = \left[\, d(i_1,i_2,i_3,i_4) + \tilde{d}(q_0;i_1,i_2,i_3,i_4)\,\right] \prod_{j\neq i_1,i_2,i_3,i_4} D_j$$

There are exactly 2 such q_0, enough to determine $d, \tilde{d}$. So by using values of q such that denominators are zero, the equation triangularizes.

One-loop amplitude with OPP

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- CutTools Pittau solves this system given $N(q)$ as input.
- final problem to be adressed is how to evaluate $N(q)$.

Evaluation of the numerator

- Need to evaluate $N(q)$ at values of q for which at least one $D_j = 0$;
- for such q, $N(q)$ only contains contributions from Feynman graphs containing *at least* the zero-denominators; graphs not containing these denominators do not contribute;

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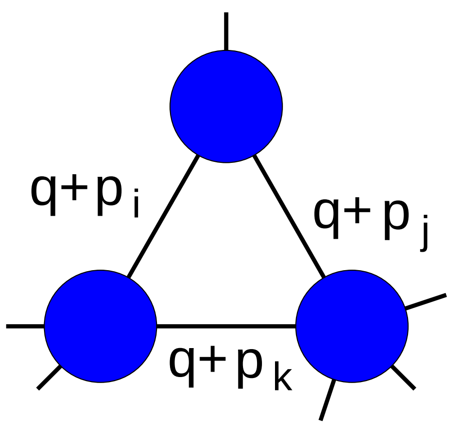

- $D_i = (q + p_i)^2 - m_i^2$ etc.;
- the external momenta into the blobs, and thus the external particles into the blobs, are determined by $p_j - p_i$, $p_k - p_j$, $p_i - p_k$;
- o.s.-currents without q already calculated;
- the blobs are tree-like.

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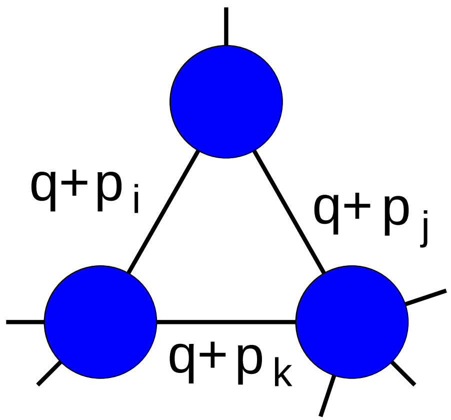

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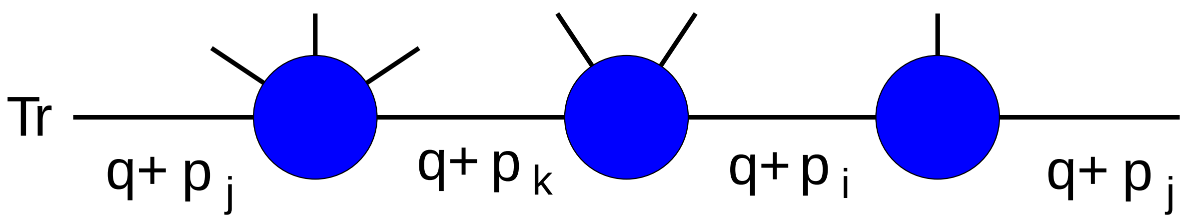
- o.s.-currents without q already calculated;
- the blobs are tree-like.

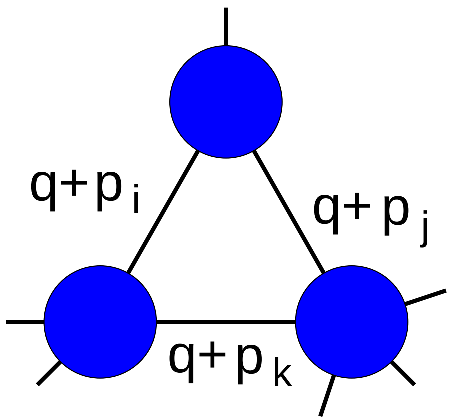
We can use the tree-level machinery to calculate the one-loop integrand.



Evaluation of the numerator

- Need to evaluate $N(q)$ at values of q for which at least one $D_j = 0$;
- for such q, $N(q)$ only contains contributions from Feynman graphs containing *at least* the zero-denominators; graphs not containing these denominators do not contribute;

Suppose q is such that $D_i = D_j = D_k = 0$:



- $D_i = (q + p_i)^2 - m_i^2$ etc.;
- the external momenta into the blobs, and thus the external particles into the blobs, are determined by $p_j - p_i$, $p_k - p_j$, $p_i - p_k$;
- o.s.-currents without q already calculated;
- the blobs are tree-like.

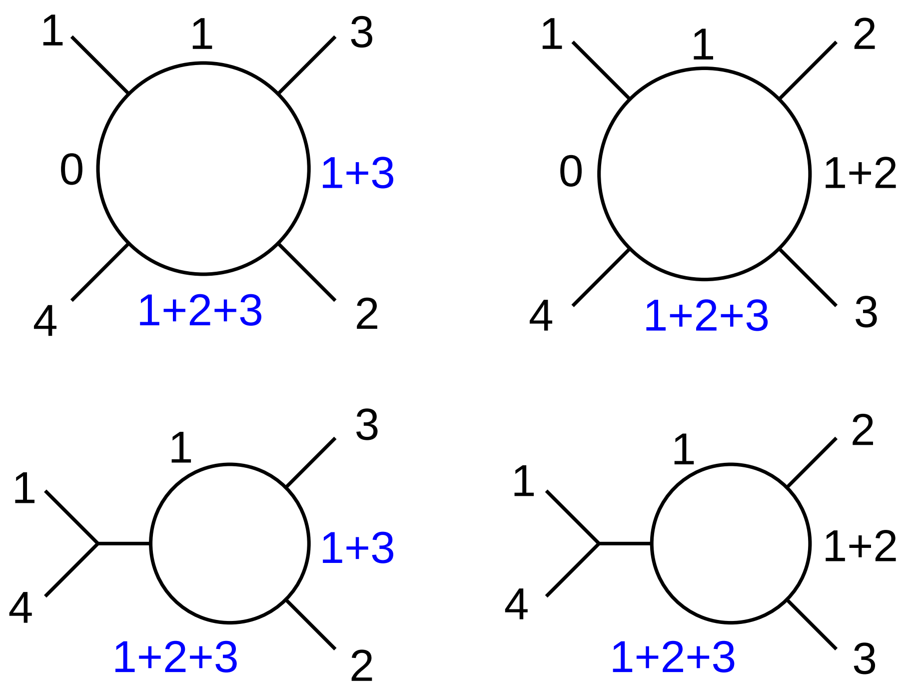
We can use the tree-level machinery to calculate the one-loop integrand.

Analogous to “unitarity-cut method” for ordered amplitudes.

Bern,Dixon,Dunbar,Kosower ’94; Bern,Dixon,Kosower ’97; Britto,Cachazo,Feng ’04

Evaluation of the numerator

Suppose q is such that $(q + p_1^{\text{ext}} + p_3^{\text{ext}})^2 = (q + p_1^{\text{ext}} + p_2^{\text{ext}} + p_3^{\text{ext}})^2 = 0$:

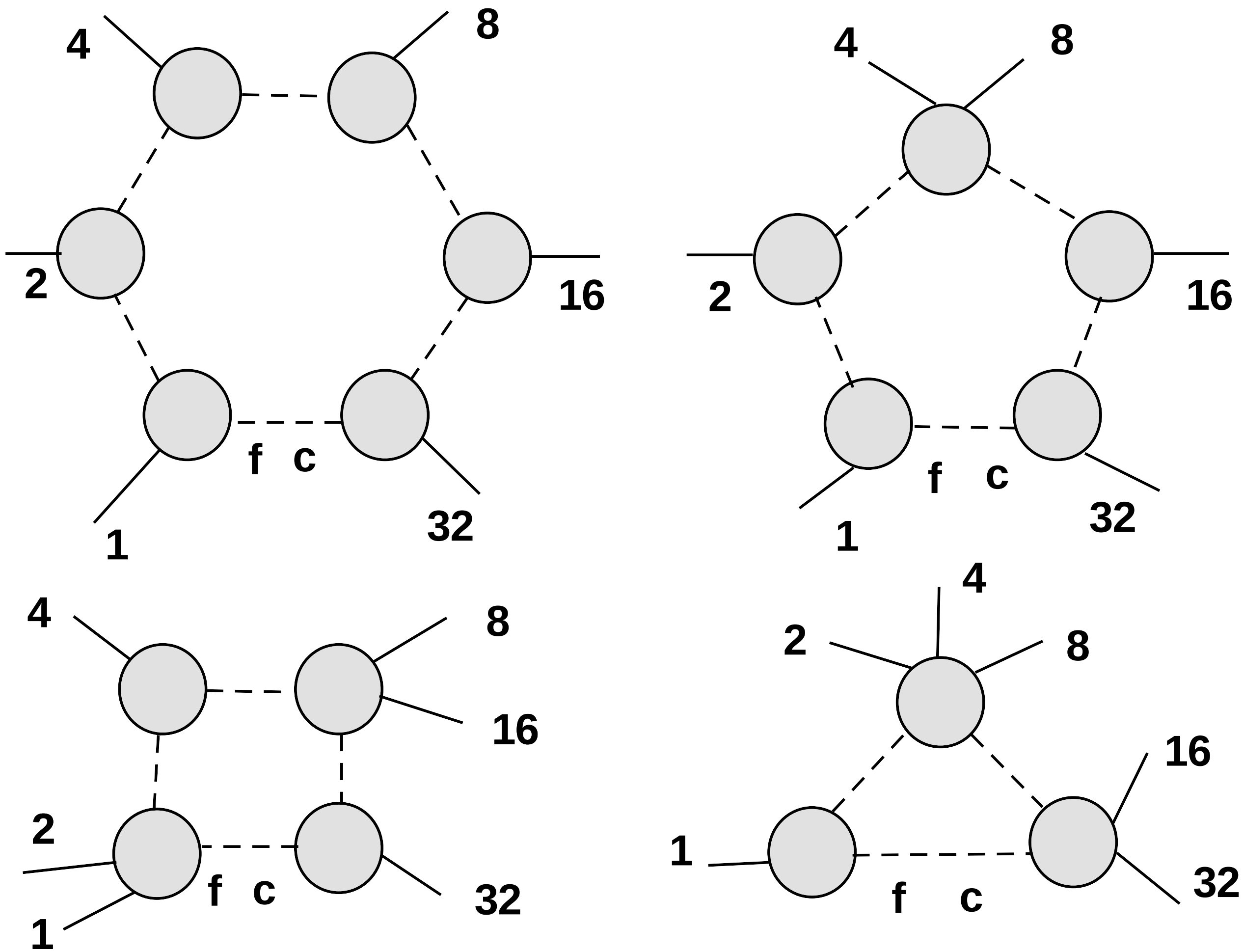


- the two left graphs contribute, the two on the right do not;
- upper two graphs are not equivalent, the lower two graphs *are* equivalent after loop integration;
- straightforward calculation of tree-level blobs leads to double-counting!.

- Need to return to graphs at the level of loops with external currents;
- can be extracted from the list of ‘DS-vertices’, as a rooted tree;
- unwanted graphs can be identified by simple algorithm;
- rooted tree-structure factorizes final calculation.

Evaluation of the numerator

Alternative: go through all denominator structures explicitly, keep tree-level blobs independent of q:



Summary

- NLO precision is needed for LHC;
- preferably obtained with the help of automatic tools;
- OPP is a good method to automatize the calculation of the one-loop amplitude, neccesary for the virtual part in the NLO contribution;
- HELAC in combination with CutTools is able so far to deal with 6-leg one-loop amplitudes, eg $\mathrm{pp} \to \mathrm{t\bar{t}\,b\bar{b}},\ \mathrm{pp} \to W^+W^-\,\mathrm{b\bar{b}},\ \mathrm{pp} \to \mathrm{b\bar{b}b\bar{b}}$, $\mathrm{pp} \to \mathrm{V ggg},\ \mathrm{pp} \to \mathrm{t\bar{t}gg}$.