

Electroweak Precision Physics from LEP to LHC and ILC

W. HOLLIK



Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

INSTITUTE FOR PARTICLE PHYSICS PHENOMENOLOGY

DURHAM, 1 MAY 2009

Standard Model

- the symmetry group $SU(2) \times U(1) \times SU(3)_C$
- the principle of local gauge invariance
 - fermion – vector boson interaction
 - vector boson self-interaction
- Higgs mechanism and Yukawa interactions
 - masses $M_W, M_Z, m_{\text{fermion}}$

renormalizable quantum field theory

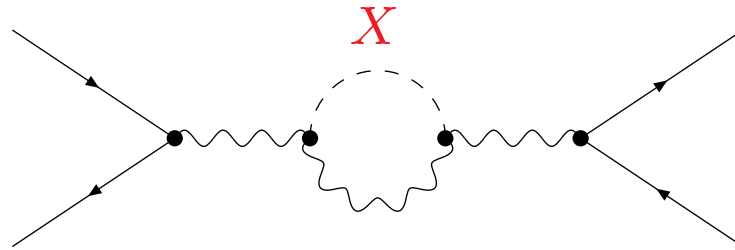
accurate theoretical predictions

- detect deviations → “new physics” ?
precise predictions required

Outline

- Electroweak precision observables – Standard Model
- Theory versus data
- Perspectives
- Extensions of the SM – Supersymmetry
- Outlook

electroweak precision tests through quantum loops



sensitivity to unknown particles (X)

X = Higgs [+ non-standard]

precision observables

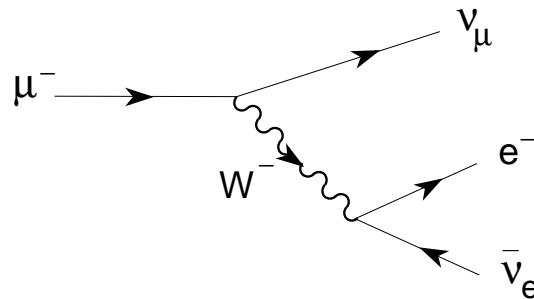
- μ lifetime: G_F
- Z observables: $M_Z, \Gamma_Z, g_V, g_A, \sin^2 \theta_{\text{eff}}, \dots$
- LEP 2, Tevatron: M_W, m_t
- low energies: $(g - 2)_\mu$

$M_W - M_Z$ correlation

Definition of Fermi constant G_F via muon lifetime:

$$\tau_\mu^{-1} = \frac{G_F^2 m_\mu^5}{192\pi^3} F\left(\frac{m_e^2}{m_\mu^2}\right) \left(1 + \frac{3}{5} \frac{m_\mu^2}{M_W^2}\right) (1 + \Delta q)$$

Δq : QED corrections in Fermi Model,



$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{M_W^2 \left(1 - M_W^2/M_Z^2\right)}$$

with loop contributions

$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{M_W^2 (1 - M_W^2/M_Z^2)} \cdot (1 + \Delta r)$$

Δr : quantum correction

$$\Delta r = \Delta r(m_t, M_H)$$

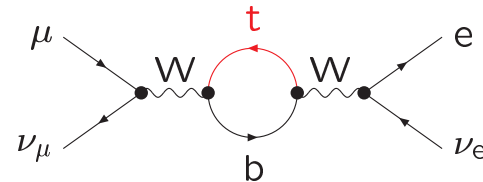
determines W mass

$$M_W = M_W(\alpha, G_F, M_Z, m_t, M_H)$$

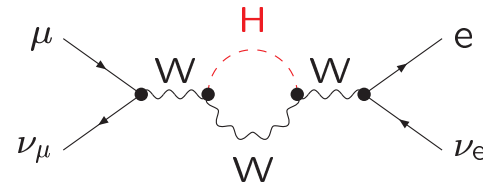
complete at 2-loop order

1-loop examples

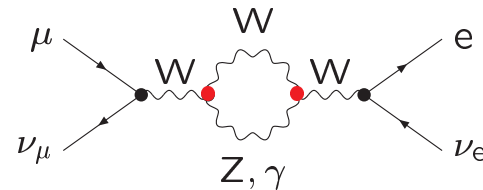
- top quark



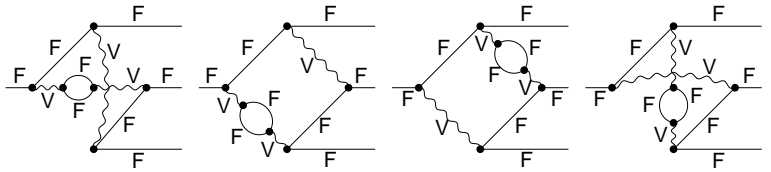
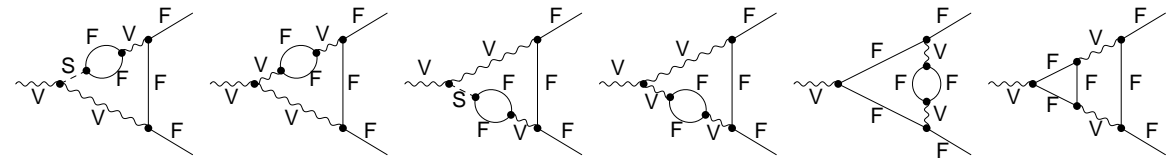
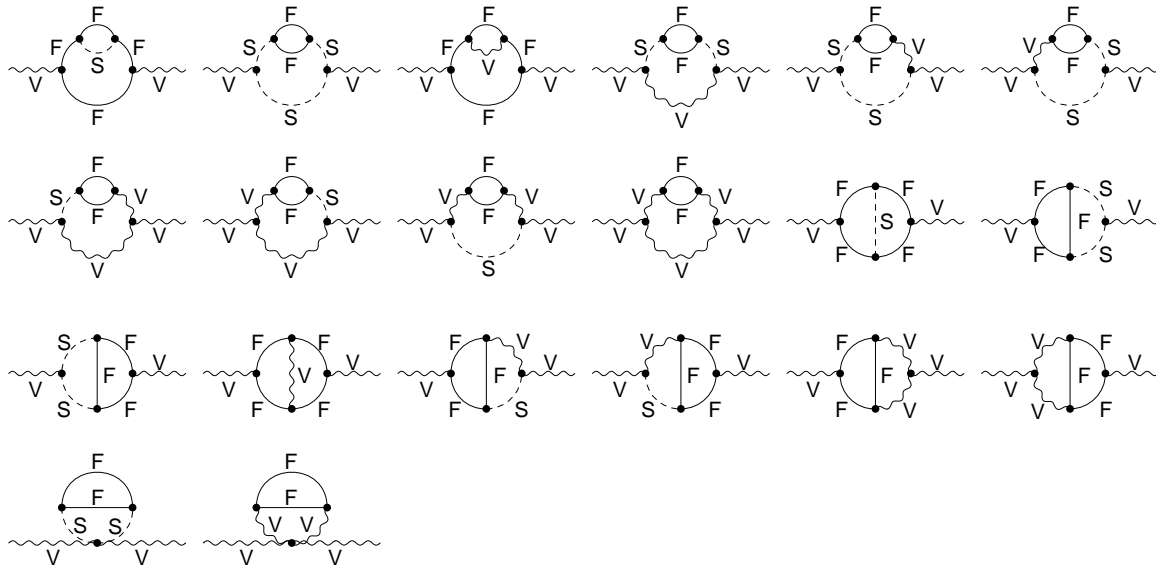
- Higgs boson



- gauge-boson self-couplings

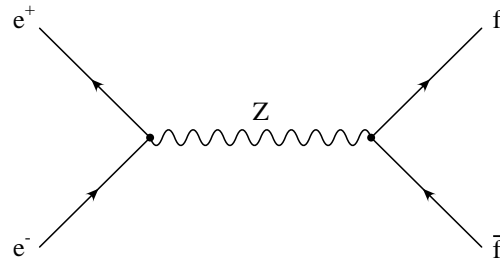
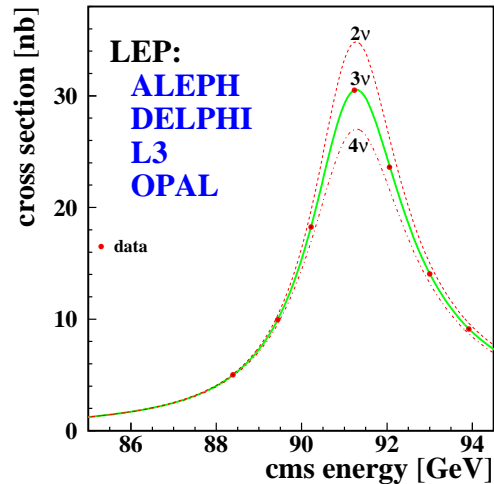


full structure of SM



2-loop examples

Z resonance

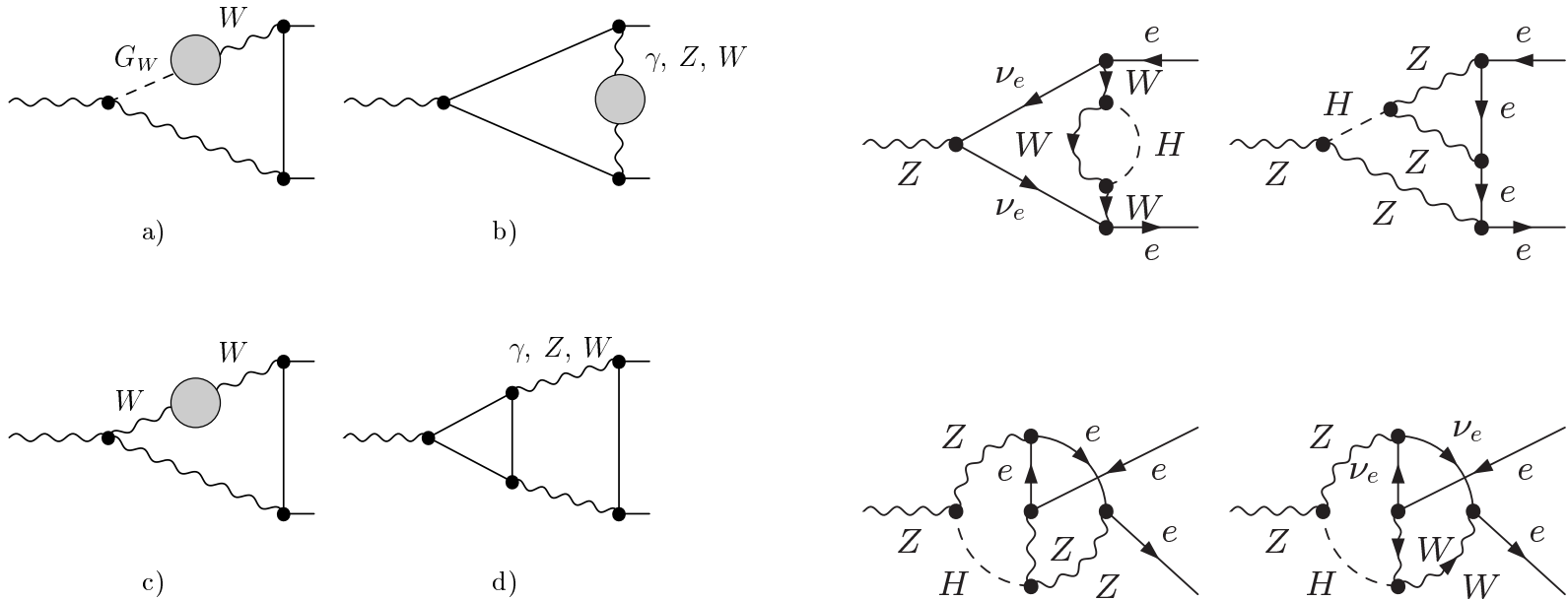


- effective Z boson couplings with higher-order $\Delta g_{V,A}$

$$g_V^f \rightarrow g_V^f + \Delta g_V^f, \quad g_A^f \rightarrow g_A^f + \Delta g_A^f$$

- effective ew mixing angle (for $f = e$):

$$\sin^2 \theta_{\text{eff}} = \frac{1}{4} \left(1 - \text{Re} \frac{g_V^e}{g_A^e} \right) = \kappa \cdot \left(1 - \frac{M_W^2}{M_Z^2} \right)$$



2-loop examples for Z couplings

complete 2-loop calculation available for $\sin^2 \theta_{\text{eff}}$

EW 2-loop calculations for Δr

Freitas, Hollik, Walter, Weiglein

Awramik, Czakon

Onishchenko, Veretin

EW 2-loop calculations for $\sin^2 \theta_{\text{eff}}$

Awramik, Czakon, Freitas, Weiglein

Awramik, Czakon, Freitas

Hollik, Meier, Uccirati

universal terms beyond 2-loop order (EW and QCD)

van der Bij, Chetyrkin, Faisst, Jikia, Seidensticker

Faisst, Kühn Seidensticker, Veretin

Boughezal, Tausk, van der Bij

Schröder, Steinhauser

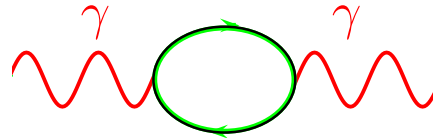
Chetyrkin, Faisst, Kühn

Chetyrkin, Faisst, Kühn, Maierhofer, Sturm

Boughezal, Czakon

charge renormalization $e + \delta e$ involves

photon vacuum polarization



$$\Pi^\gamma(M_Z^2) - \Pi^\gamma(0) \equiv \Delta\alpha \quad \rightarrow \quad \alpha(M_Z) = \frac{\alpha}{1 - \Delta\alpha}$$

$$\Delta\alpha = \Delta\alpha_{\text{lept}} + \Delta\alpha_{\text{had}},$$

$$\Delta\alpha_{\text{lept}} = 0.031498 \quad (3\text{-loop})$$

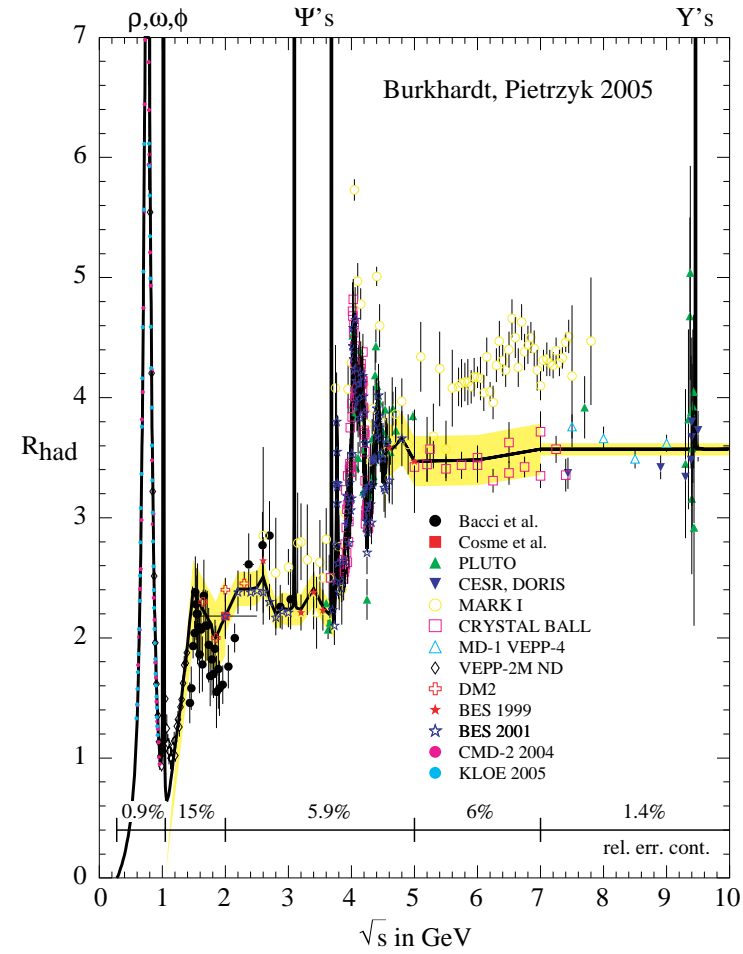
$$\Delta\alpha_{\text{had}} = 0.02758 \pm 0.00035$$

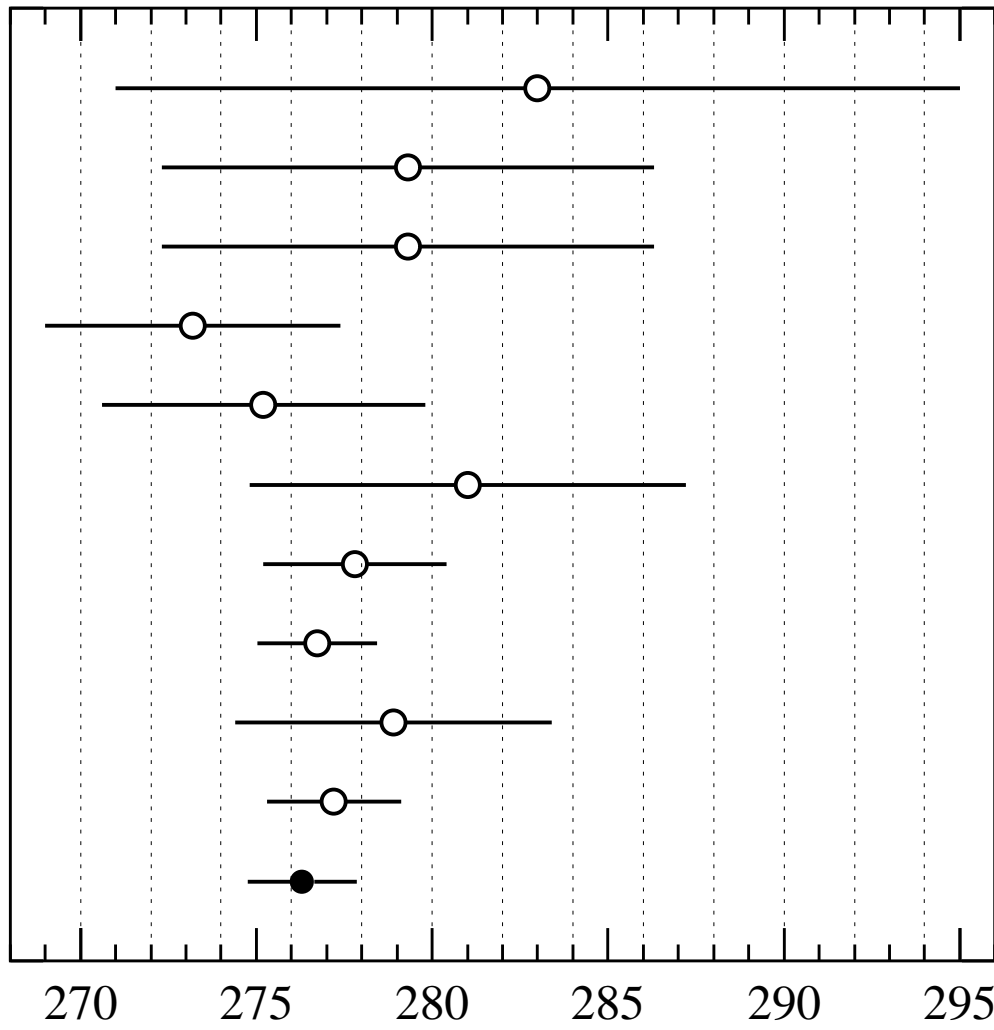
significant source of parametric uncertainty

$$\frac{\delta M_W}{M_W} \sim 0.23 \delta\Delta\alpha, \quad \frac{\delta \sin^2 \theta}{\sin^2 \theta} \sim 1.54 \delta\Delta\alpha$$

$$\Delta\alpha_{\text{had}} = -\frac{\alpha}{3\pi} M_Z^2 \operatorname{Re} \int_{4m_\pi^2}^{\infty} ds' \frac{R_{\text{had}}(s')}{s'(s' - M_Z^2 - i\epsilon)}$$

$$R_{\text{had}} = \frac{\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \mu^+\mu^-)}$$





- Lynn, Penso, Verzegnassi, '87
- Eidelman, Jegerlehner '95
- Burkhardt, Pietrzyk '95
- Martin, Zeppenfeld '95
- Swartz '96
- Aleman, Davier, Höcker '97
- Davier, Höcker '97
- Kühn, Steinhauser '98
- Groote et al. '98
- Erlar '98
- Davier, Höcker '98

$$\Delta\alpha_{\text{had}}(M_Z^2) \quad (\times 10^{-4})$$

input from experiments

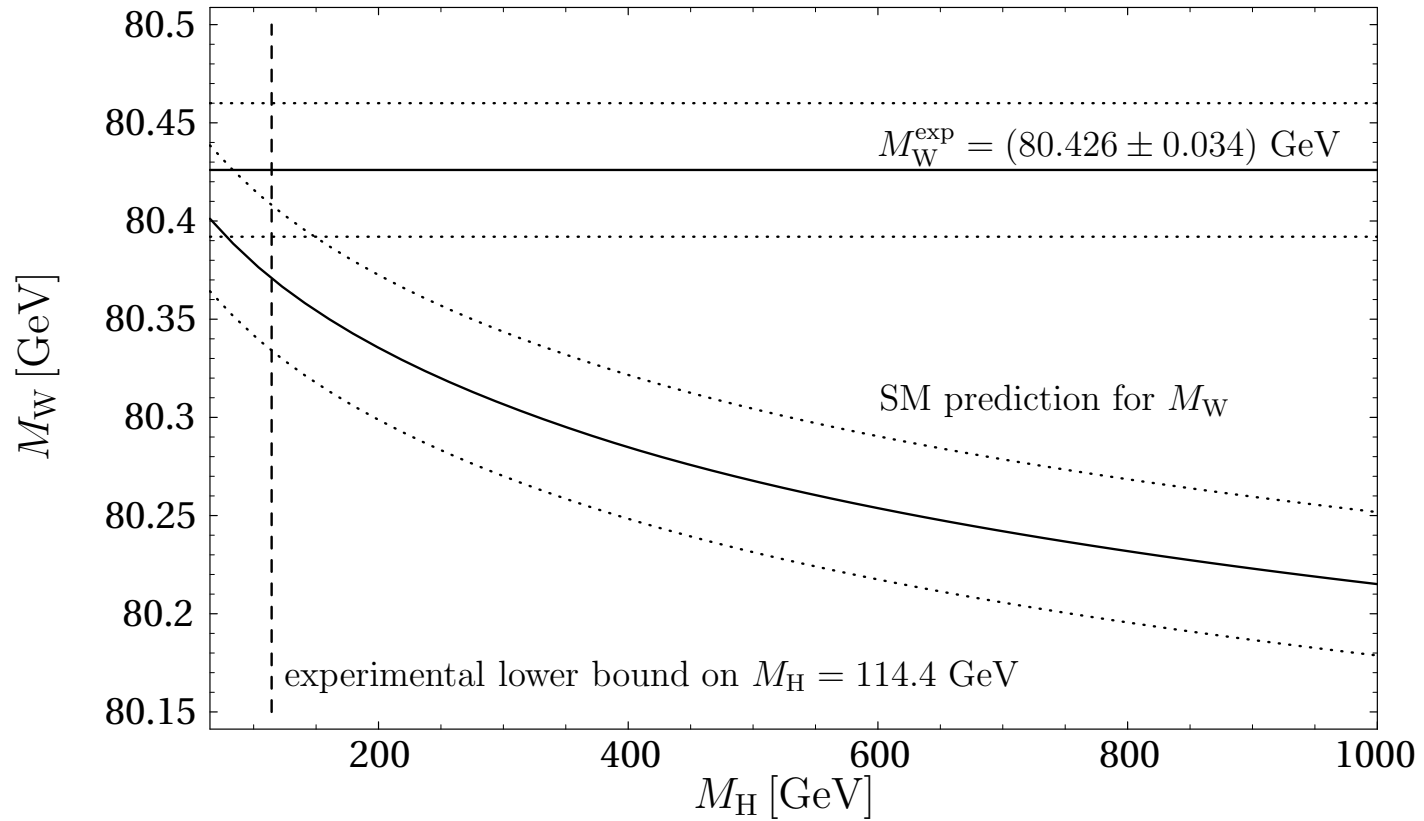
- **LEP1/SLC:** $e^+e^- \rightarrow Z \rightarrow f\bar{f}$
LEP1: $\sim 4 \times 10^6$ events/experiment
4 experiments (1989 – 1995)
- **LEP2:** $e^+e^- \rightarrow W^+W^-$
 $\mathcal{O}(10^4)$ W pairs (1996 – 2000)
- **Tevatron:** $q\bar{q}' \rightarrow W \rightarrow l\nu, q\bar{q}'$
($p\bar{p}$) $q\bar{q}' \rightarrow t\bar{t}, t \rightarrow W^+b \rightarrow \dots$
- **low-energy experiments** (μ decay, νN scattering, νe scattering, atomic parity violation, ...)

Theory versus Data

experimental results (selection)

M_Z [GeV]	$= 91.1875 \pm 0.0021$	0.002%
Γ_Z [GeV]	$= 2.4952 \pm 0.0023$	0.09%
$\sin^2 \theta_{\text{eff}}^{\text{lept}}$	$= 0.23148 \pm 0.00017$	0.07%
M_W [GeV]	$= 80.398 \pm 0.025$	0.04%
m_t [GeV]	$= 173.1 \pm 1.3$	0.75%
G_F [GeV ⁻²]	$= 1.16637(1)10^{-5}$	0.001%

quantum effects at least one order of magnitude larger than experimental uncertainties



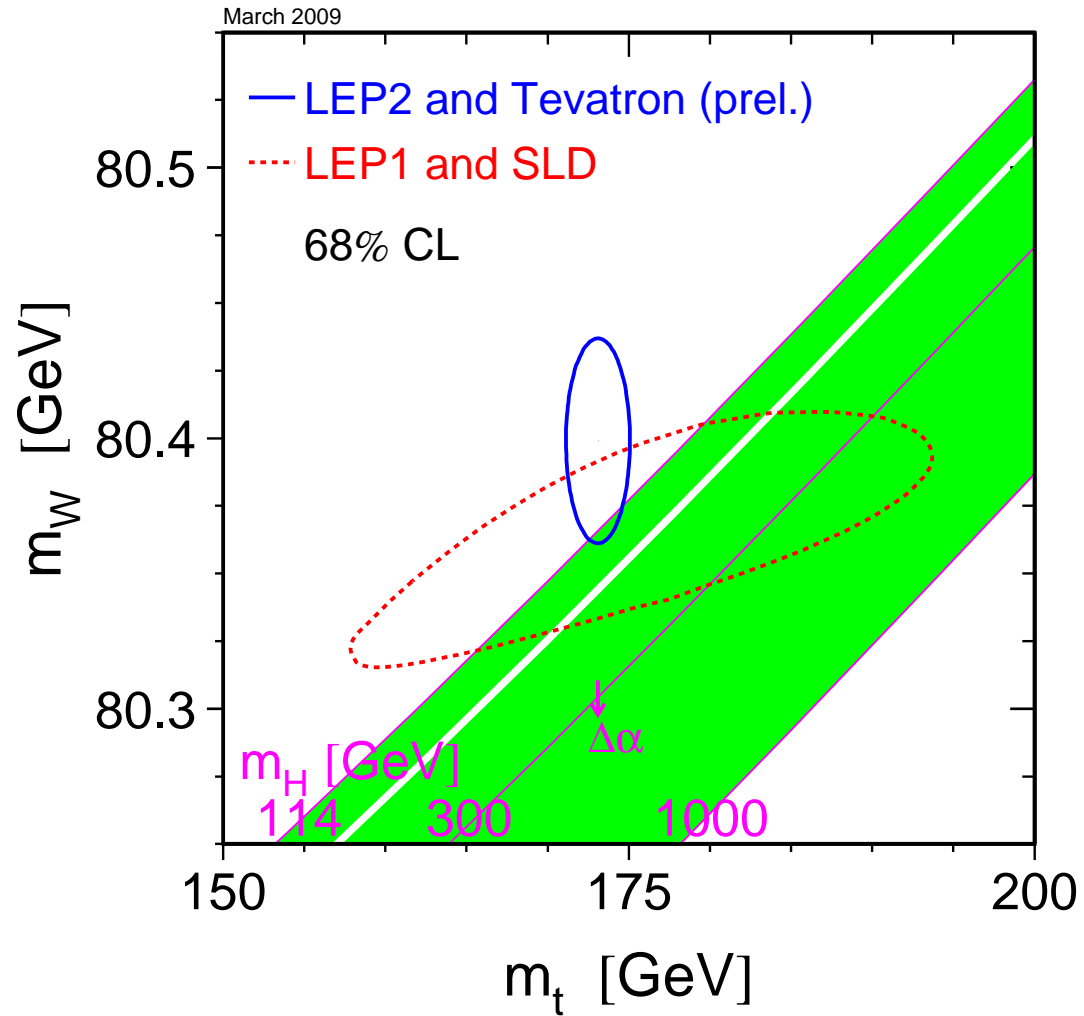
[Awramik, Czakon, Freitas, Weiglein]

$$m_t = 174.3 \pm 5.1 \text{ GeV}$$

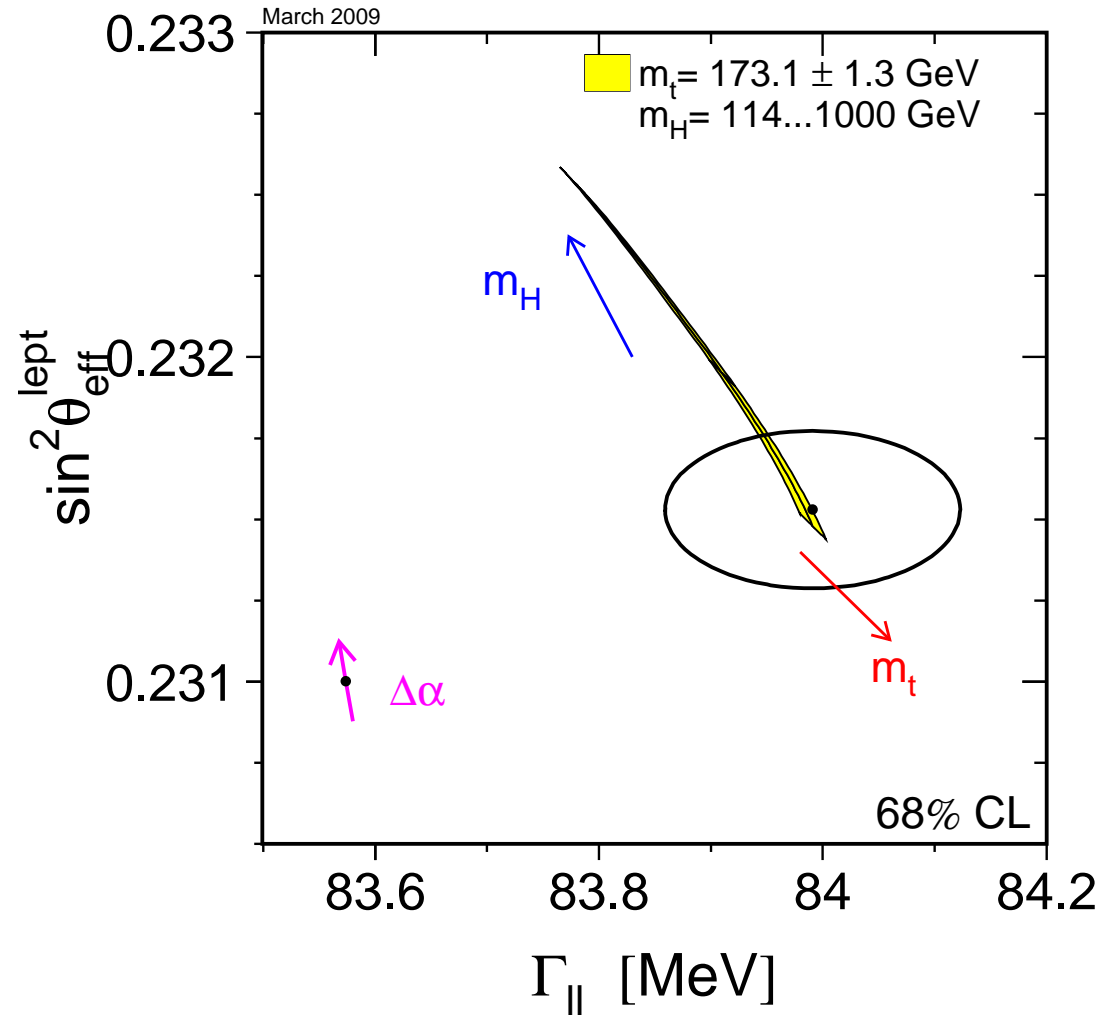
$$\delta M_W^{\text{theo}} \simeq 4 \text{ MeV}$$

$$\delta \sin^2 \theta_{\text{eff}}^{\text{theo}} \simeq 5 \cdot 10^{-5}$$

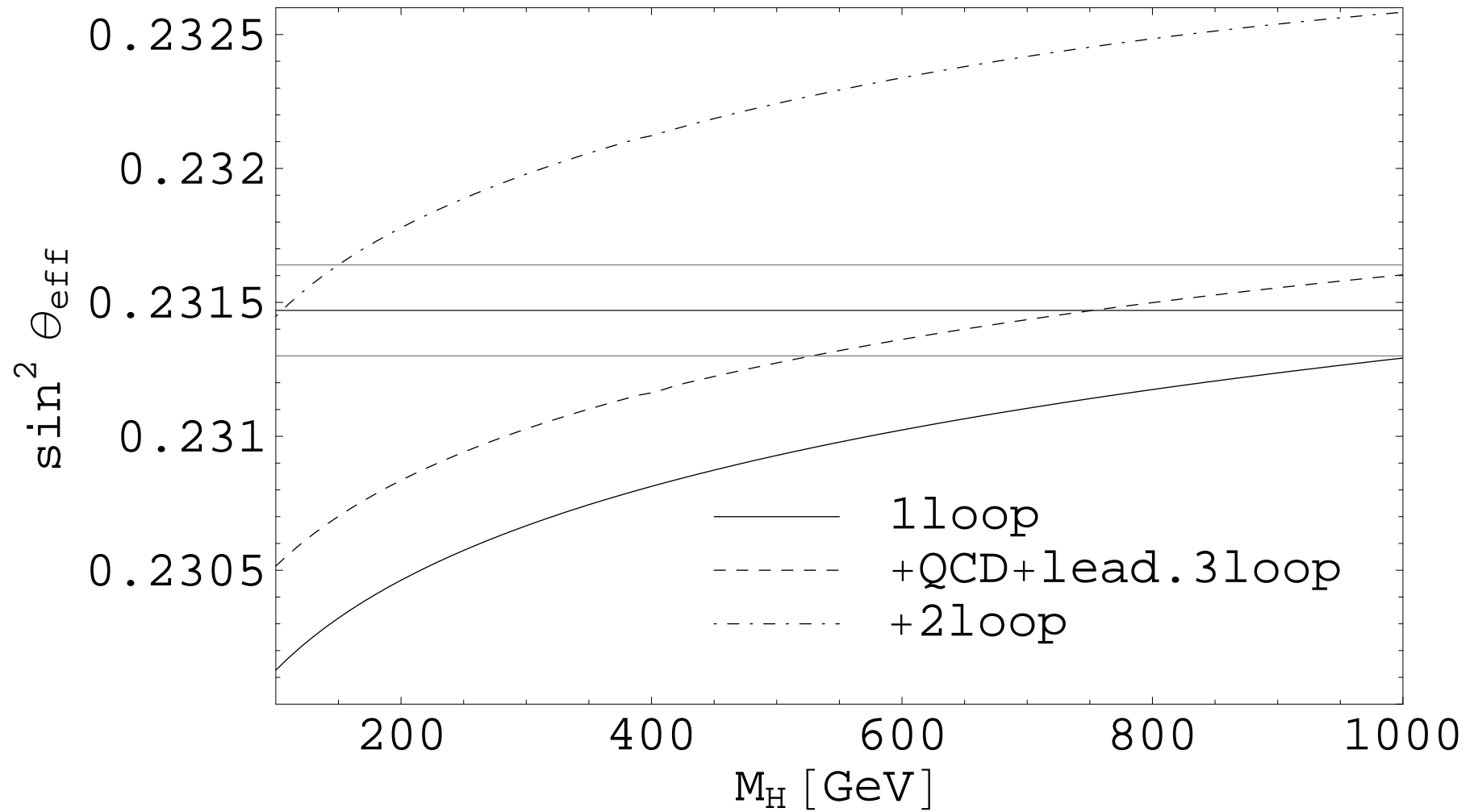
LEP Electroweak Working Group



LEP Electroweak Working Group



importance of two-loop calculations



ORGANISATION EUROPÉENNE POUR LA RECHERCHE NUCLÉAIRE
CERN EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

Z PHYSICS AT LEP 1

Edited by
Guido Altarelli, Ronald Kleiss and Claudio Verzegnassi

Volume 1: STANDARD PHYSICS

Co-ordinated and supervised by G. Altarelli

GENEVA
1989

development of precision

1990-1992

91.1904 ± 0.0065

1993-1994

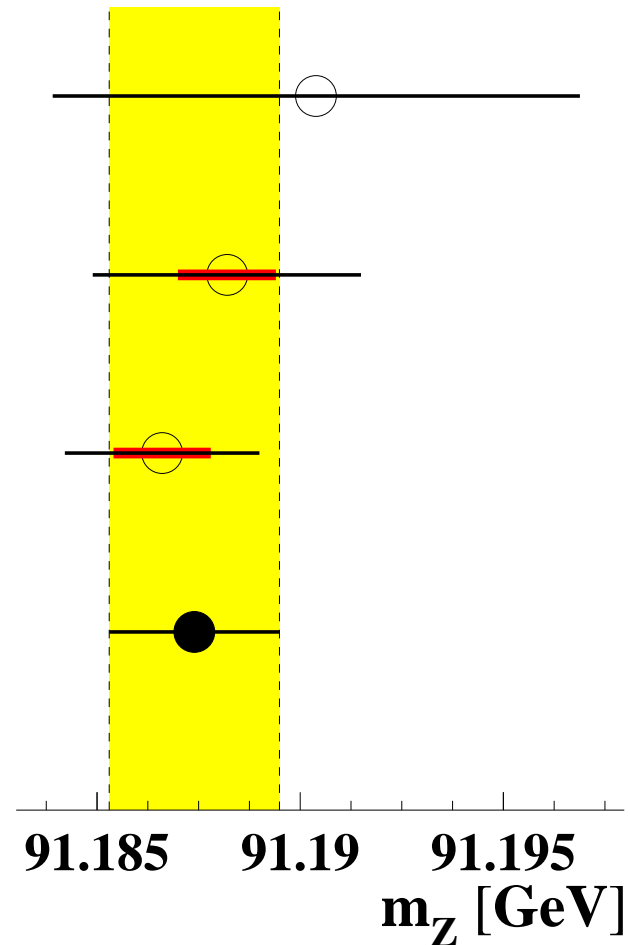
91.1882 ± 0.0033

1995

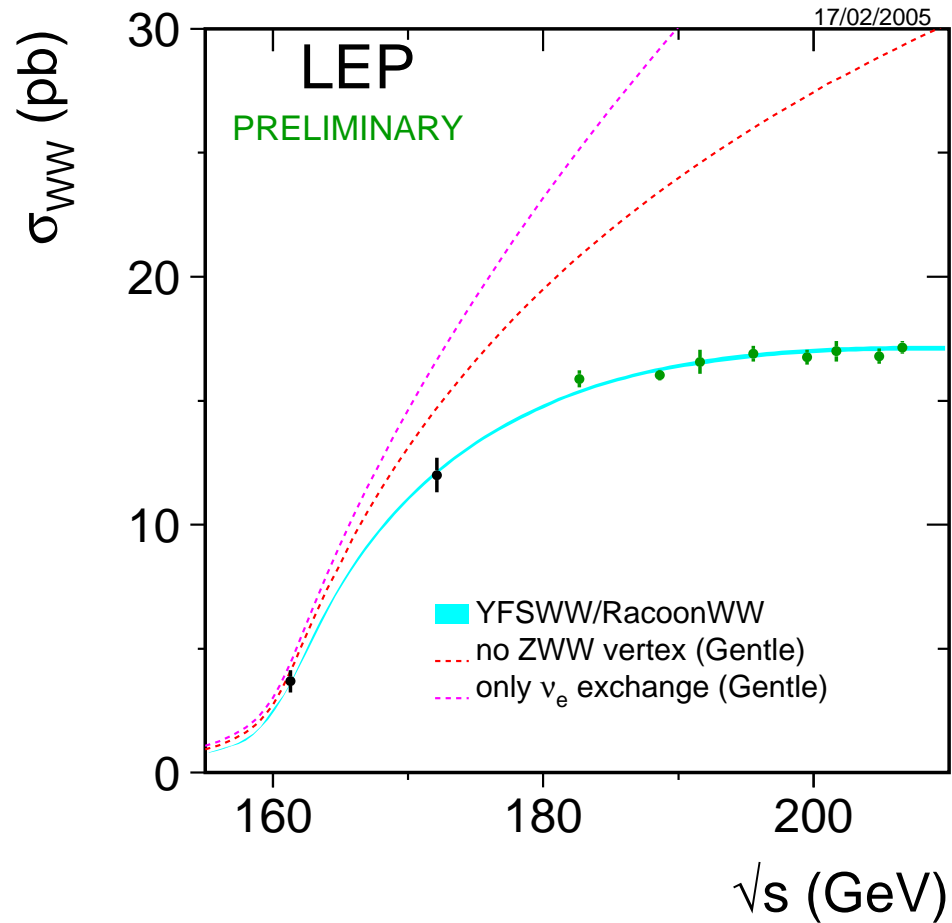
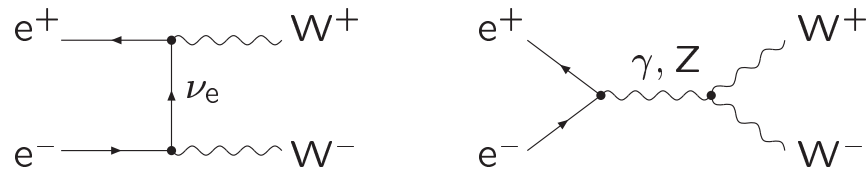
91.1866 ± 0.0024

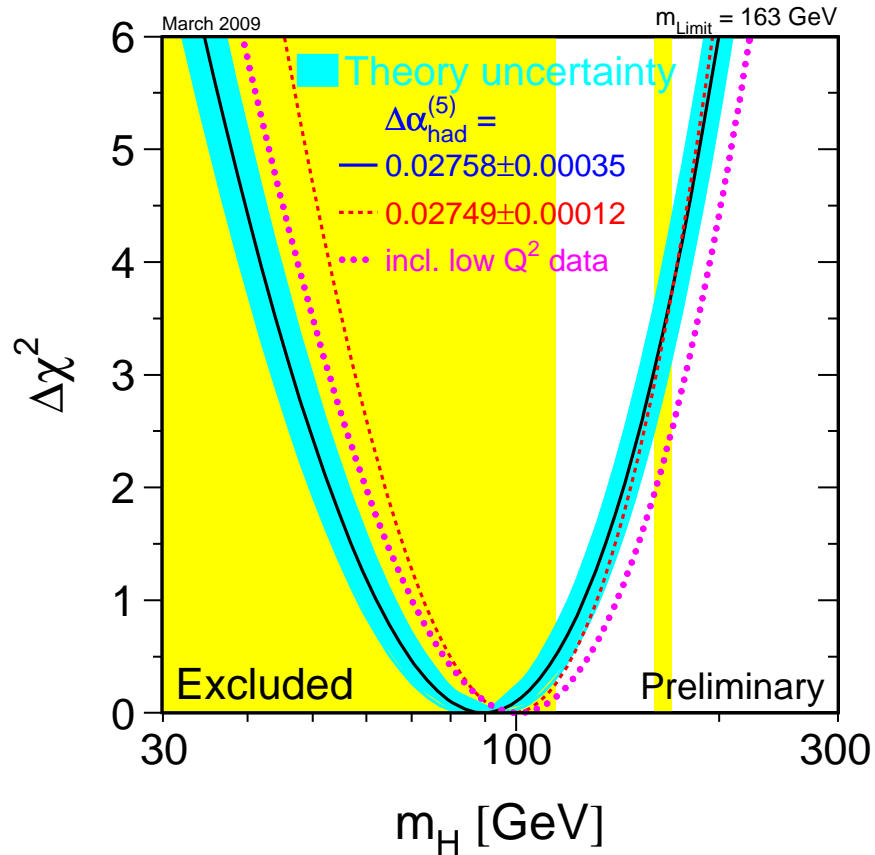
average

91.1874 ± 0.0021



W-pair production





blueband: theory uncertainty

“Precision Calculations
at the Z Resonance”

CERN 95-03

[Bardin, Hollik, Passarino (eds.)]

$$M_H < 163 \text{ GeV} \quad (95\% \text{C.L.})$$

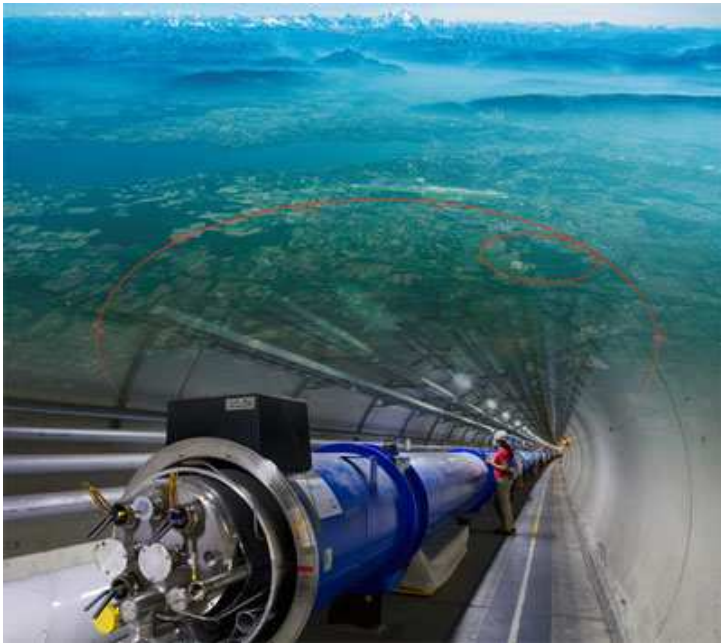
with renormalized probability for $M_H > 114 \text{ GeV}$:

$$M_H < 191 \text{ GeV} \quad (95\% \text{C.L.})$$

Perspectives

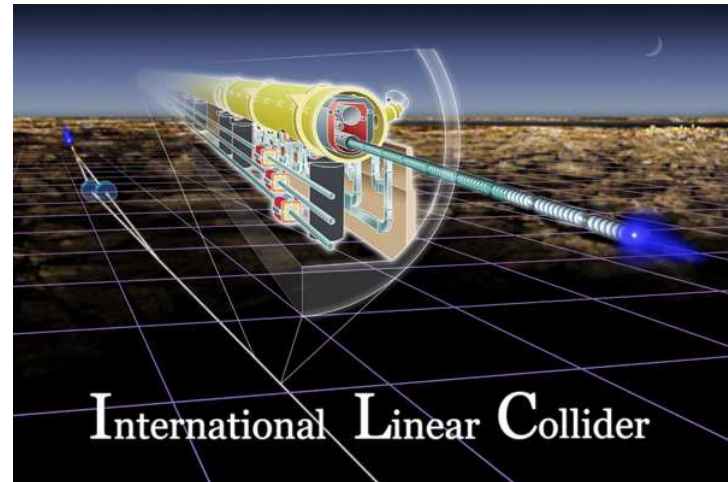
2008:

The Large Hadron Collider



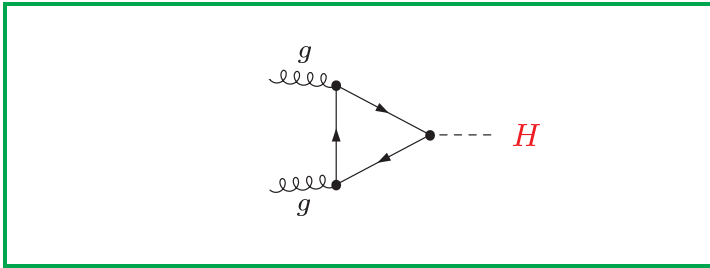
Future:

e^+e^- Linear Collider

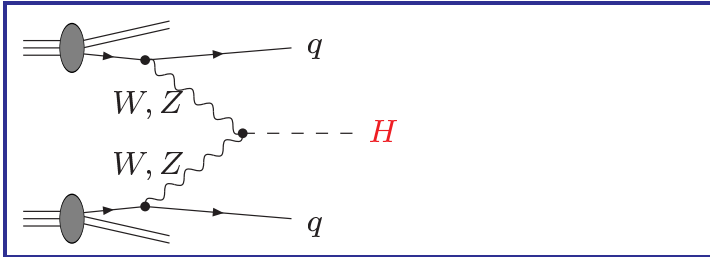


Higgs production at the LHC

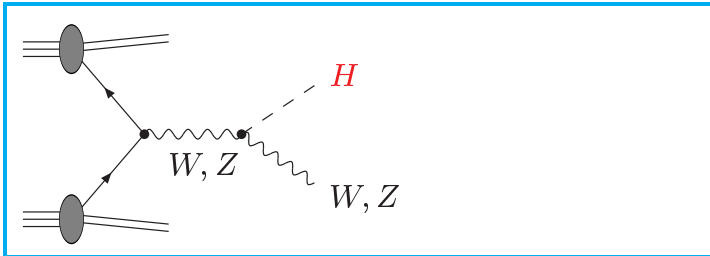
- gluon fusion, $gg \rightarrow H$



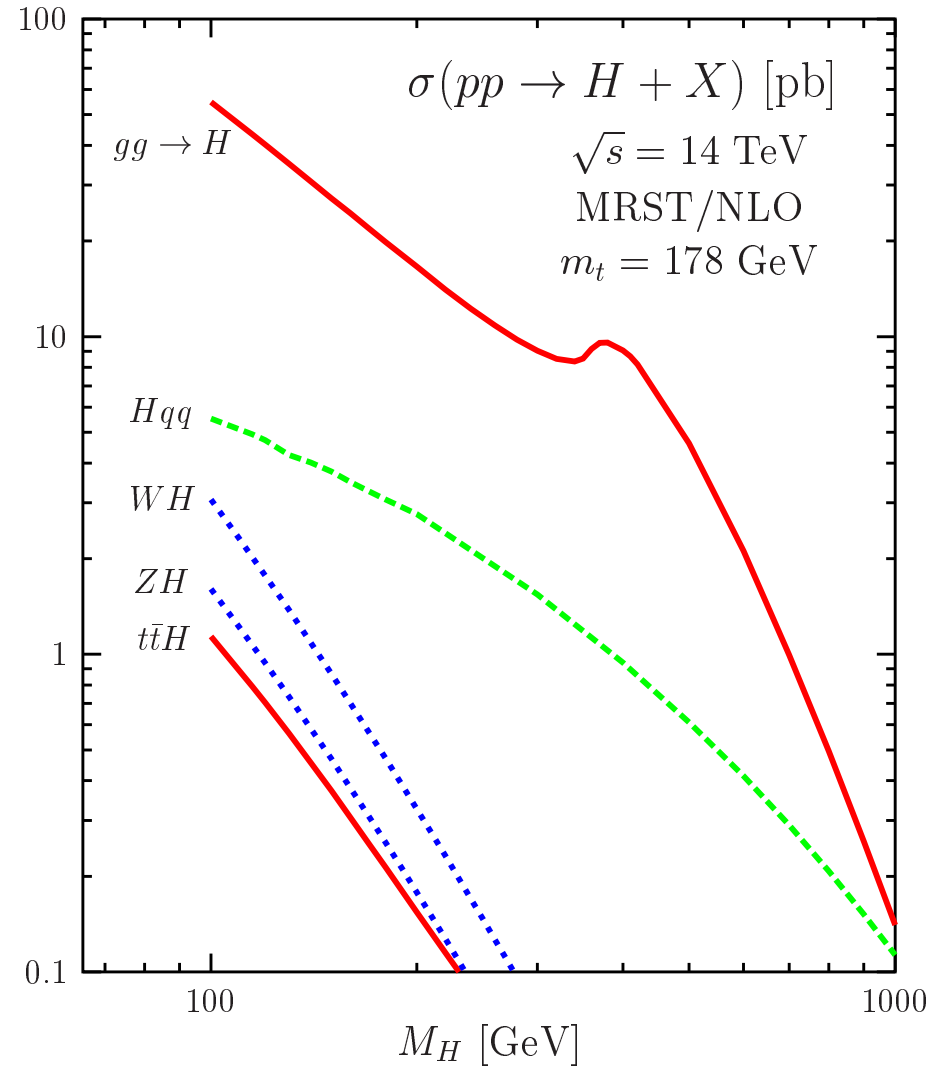
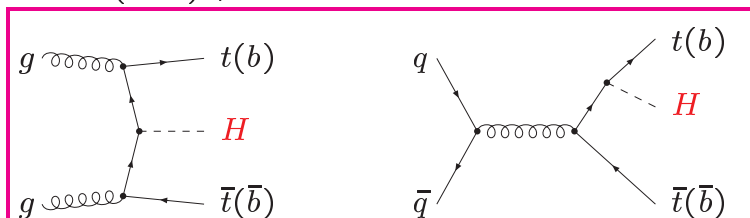
- vector boson fusion, $qq \rightarrow qqH$



- Higgs strahlung, $q\bar{q} \rightarrow VH$



- $t\bar{t}H$ ($b\bar{b}H$) production



(expected) experimental precision

error for	LEP/TeV	TeV/LHC	LC	LC/GigaZ
M_W [MeV]	25	15	15	7
$\sin^2 \theta_{\text{eff}}$	0.00017	0.00021		0.000013
m_{top} [GeV]	1.3	1	0.2	0.13
M_{Higgs} [GeV]	–	0.1	0.05	0.05

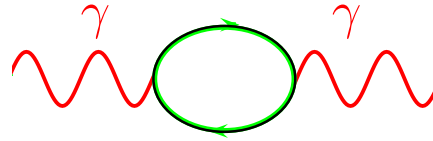
theory uncertainty:

$$\Delta M_W = 4 \text{ MeV}, \quad \Delta \sin^2 \theta_{\text{eff}} = 5 \cdot 10^{-5}$$

crucial parametric uncertainties for PO from

m_{top} and $\alpha_{\text{em}}(M_Z)$

photon vacuum polarization



$$\Pi^\gamma(M_Z^2) - \Pi^\gamma(0) \equiv \Delta\alpha \quad \rightarrow \quad \alpha(M_Z) = \frac{\alpha}{1 - \Delta\alpha}$$

$$\Delta\alpha_{\text{had}} = 0.02758 \pm 0.00035$$

$$0.02749 \pm 0.00012 \quad (\text{QCD based})$$

significant source of parametric uncertainty

$$\frac{\delta M_W}{M_W} \sim 0.23 \delta\Delta\alpha,$$

$$\frac{\delta \sin^2 \theta}{\sin^2 \theta} \sim 1.54 \delta\Delta\alpha$$

$$\delta M_W = 6 \text{ MeV}$$

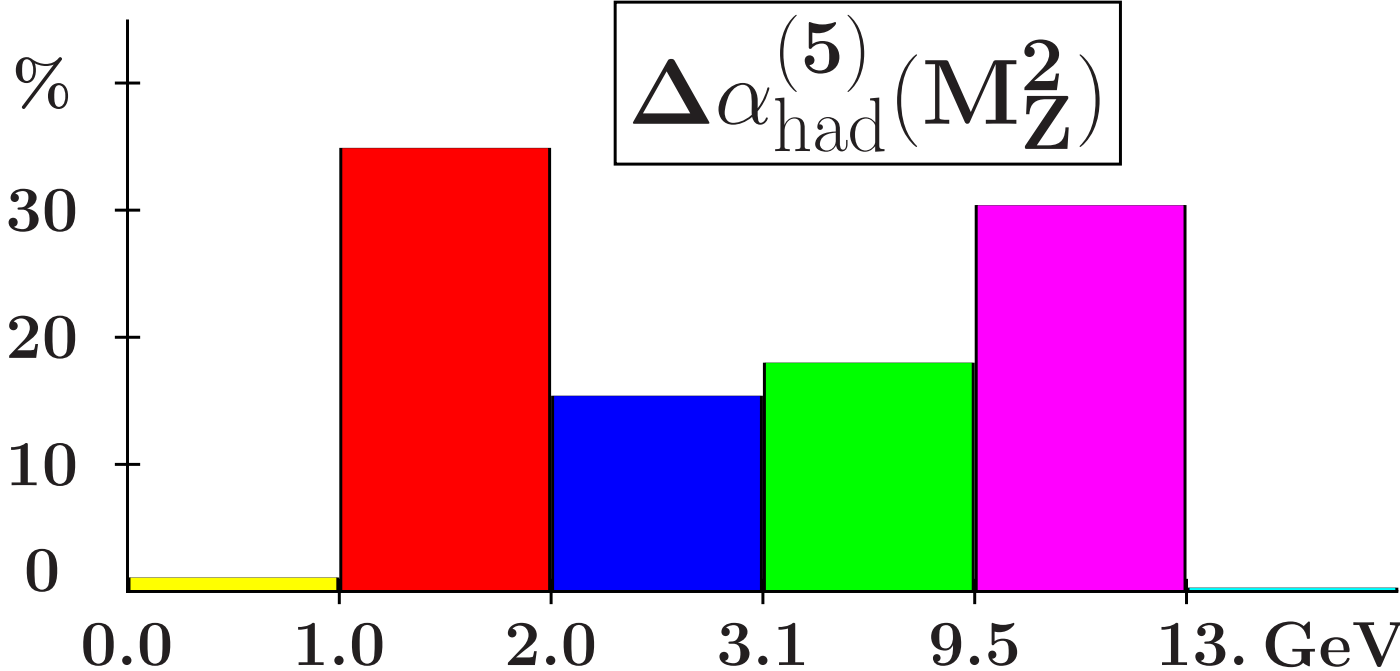
$$\delta \sin^2 \theta = 0.00012$$

$$\delta M_W = 2 \text{ MeV}$$

$$\delta \sin^2 \theta = 0.00004 \quad (\text{QCD based})$$

required for GigaZ precision: $\delta\Delta\alpha = 5 \cdot 10^{-5}$

distribution of uncertainties



[Jegerlehner]

M_W from Drell-Yan at the LHC

$$q\bar{q}' \rightarrow W^+ \rightarrow \ell^+ \nu_\ell$$

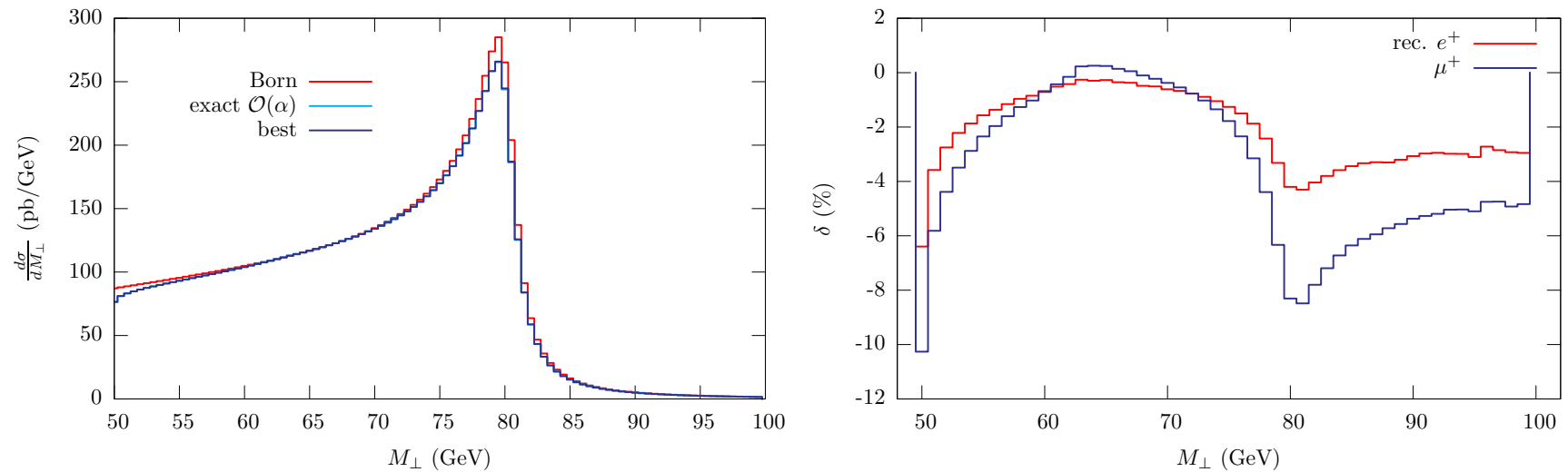


Fig. 1. Transverse mass distribution and $\mathcal{O}(\alpha)$ relative correction.

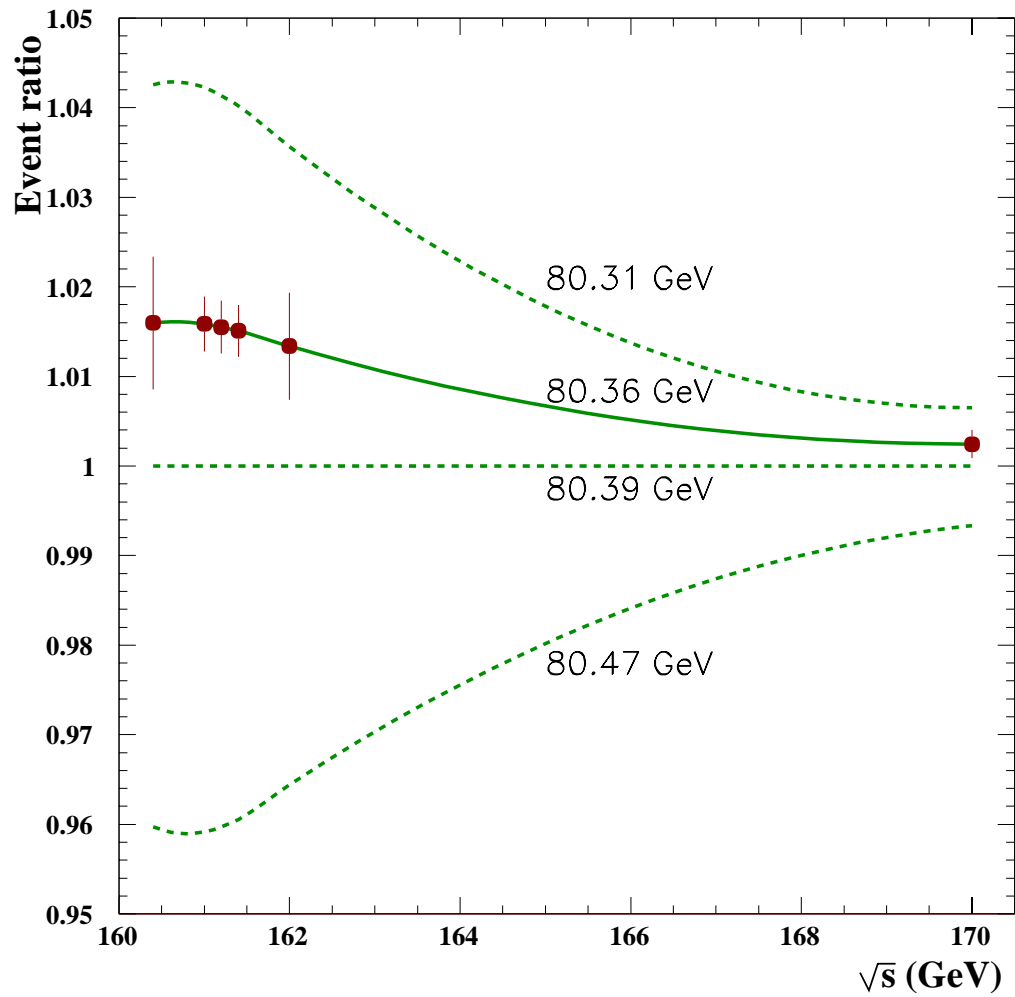
[Carloni Calame et al.]

[Baur, Wackerath]

[Dittmaier, Krämer]

[Arbuzov et al.]

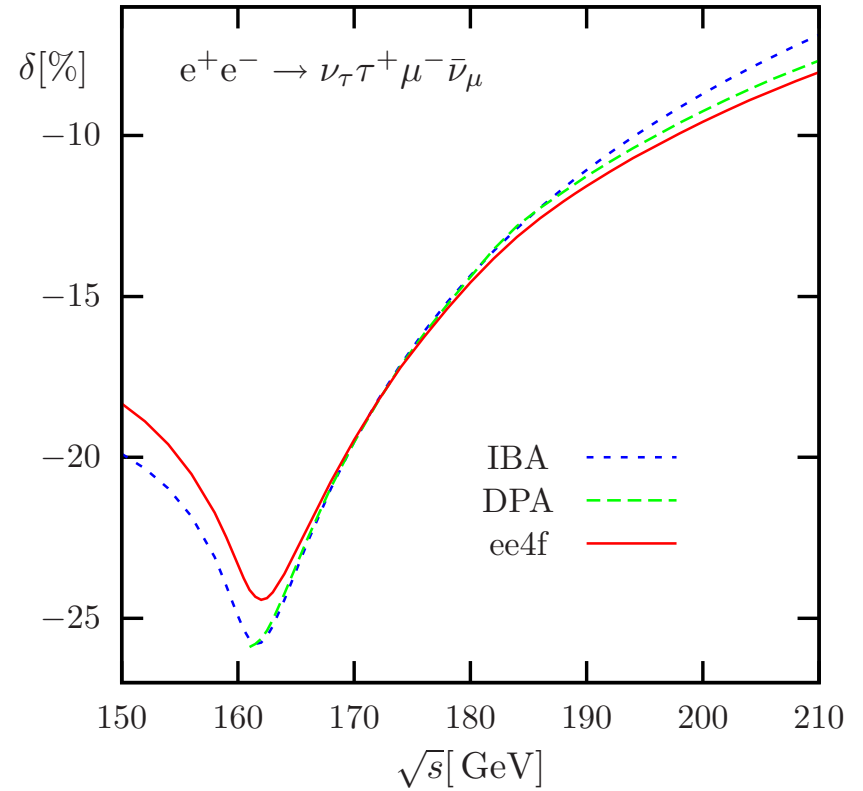
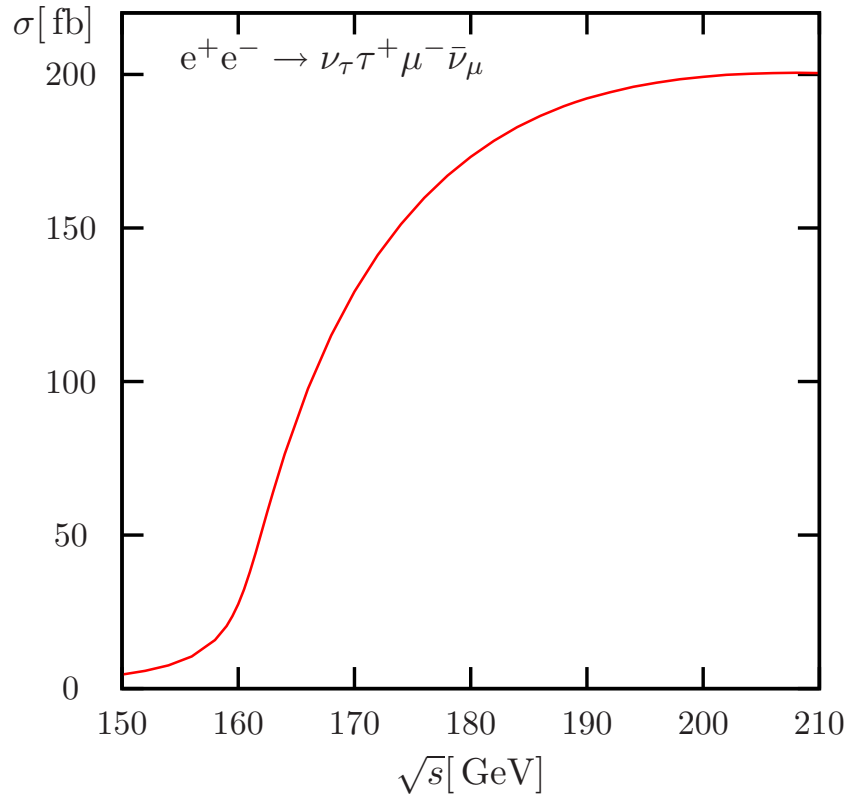
M_W from threshold scan in e^+e^- annihilation



TESLA Technical Design Report (DESY 2001)

M_W from threshold $e^+e^- \rightarrow WW \rightarrow 4f$

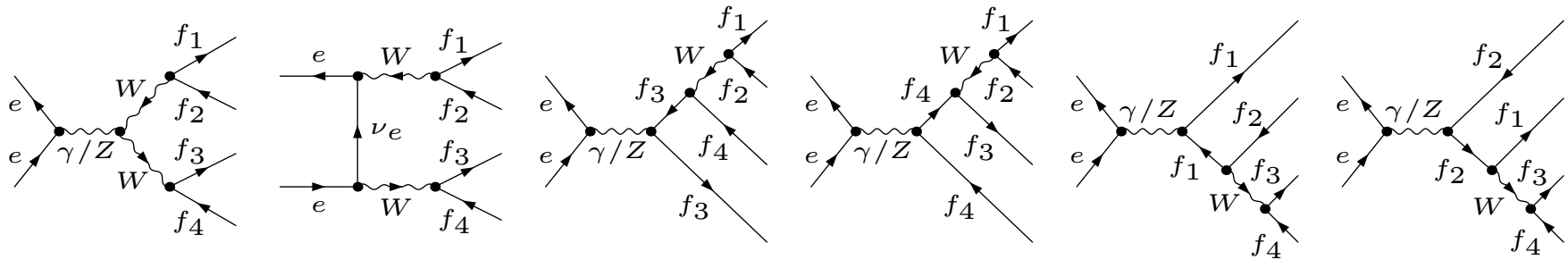
Denner, Dittmaier, Roth, Wieders '05



M_W from threshold $e^+e^- \rightarrow WW \rightarrow 4f$

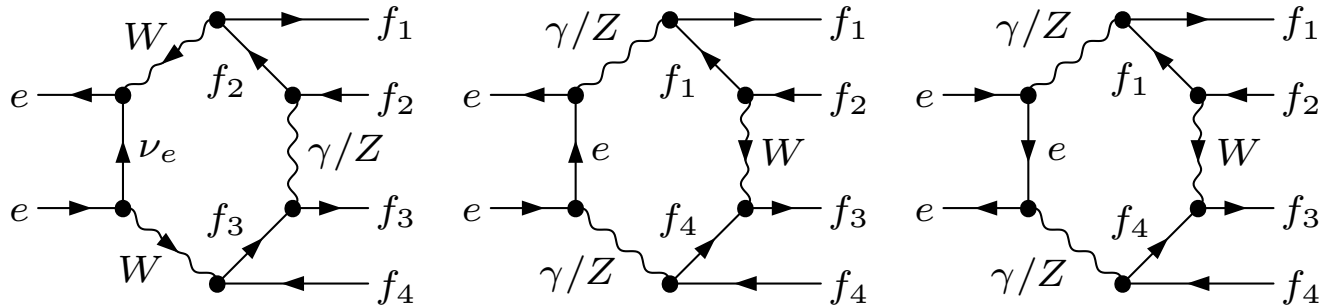
Some Feynman diagrams...

...for LO:



...for NLO: total number = $\mathcal{O}(1200)$

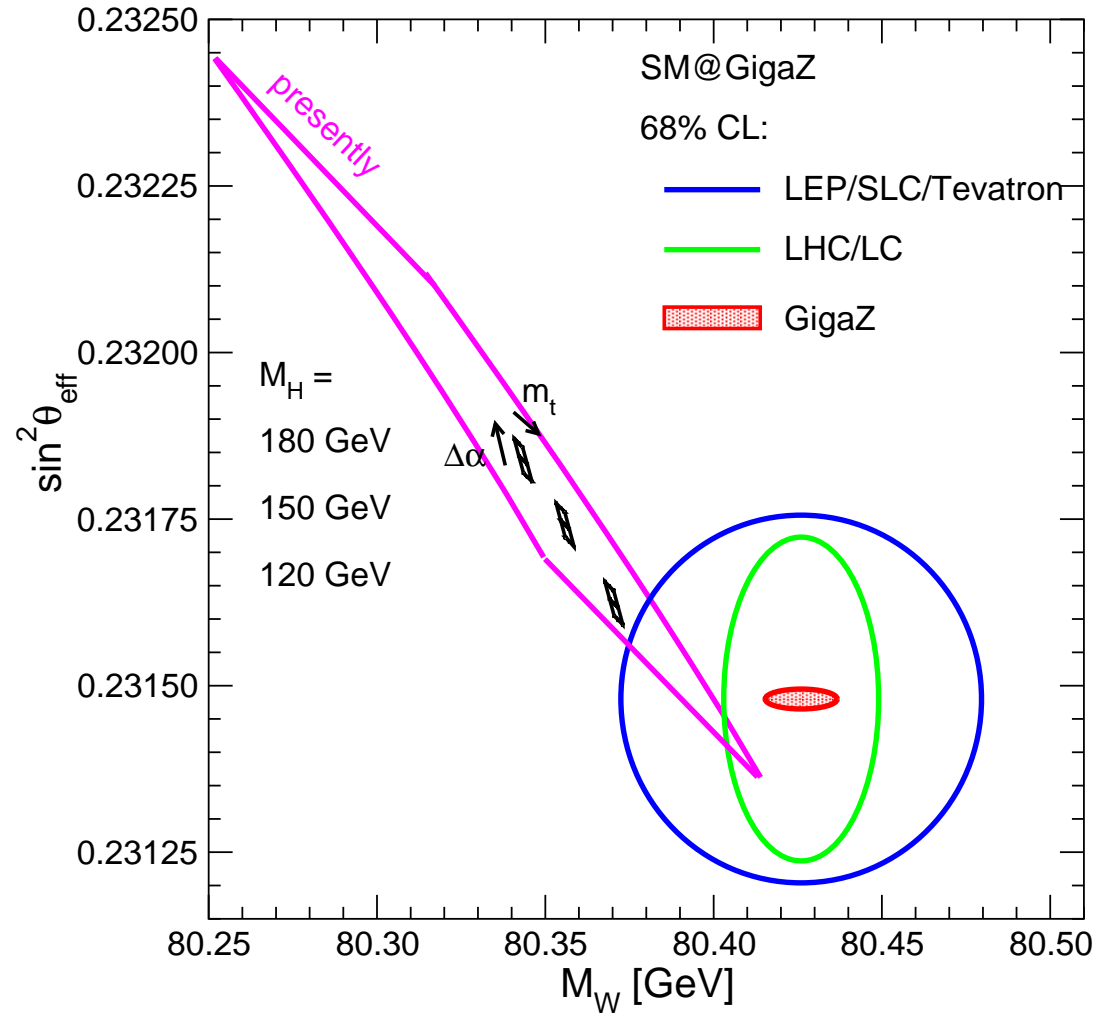
40 hexagons



+ graphs with reversed fermion-number flow in final state

+ 112 pentagons

+ 227 boxes ('tHF gauge) + many vertex and self-energy corrections

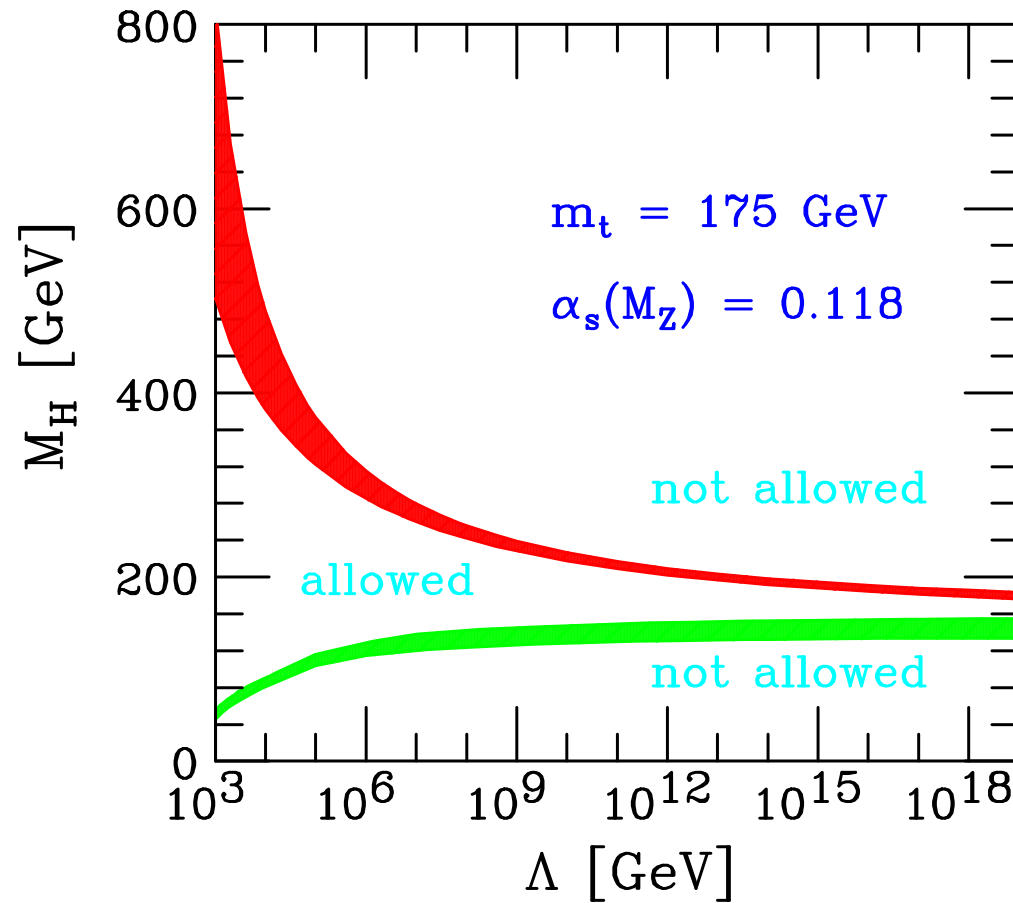


Theoretical bounds on Higgs boson mass from

- perturbativity \rightarrow upper bound
- unitarity \rightarrow upper bound
- triviality (Landau pole) \rightarrow upper bound
- vacuum stability \rightarrow lower bound

combined effects, RGE in two-loop order:

$$\frac{d\lambda}{dt} = \frac{1}{16\pi^2} (12\lambda^2 - 3g_t^4 + 6\lambda g_t^2 + \dots)$$



SM Higgs:

- λH^4 term ad hoc
- Higgs boson mass: free parameter $\sim \sqrt{\lambda}$
- no a-priori reason for a light Higgs boson

SUSY Standard Model avoids these questions

$$H_2 = \begin{pmatrix} H_2^+ \\ v_2 + H_2^0 \end{pmatrix}, \quad H_1 = \begin{pmatrix} v_1 + H_1^0 \\ H_1^- \end{pmatrix}$$

couples to u

couples to d

- SUSY gauge interaction $\rightarrow H^4$ terms
- self coupling remains weak

MSSM Higgs potential contains two Higgs doublets:

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.}) \\ + \underbrace{\frac{g'^2 + g^2}{8}}_{\text{gauge couplings, in contrast to SM}} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \underbrace{\frac{g^2}{2}}_{\text{gauge couplings, in contrast to SM}} |H_1 \bar{H}_2|^2$$

gauge couplings, in contrast to SM

Five physical states: h^0, H^0, A^0, H^\pm

Input parameters: $\tan \beta = \frac{v_2}{v_1}, M_A$

$\Rightarrow m_h, m_H, \text{mixing angle } \alpha, m_{H^\pm}$: no free parameters

coupling to vector bosons $V = W, Z$:

$$[h^0, H^0]VV = [\sin(\alpha - \beta), \cos(\alpha - \beta)] \cdot [\text{SM}]$$

coupling to fermions $(u \rightarrow H_2, d \rightarrow H_1)$:

$$[h^0, H^0]bb = \left[\frac{\sin \alpha}{\cos \beta}, \frac{\cos \alpha}{\cos \beta} \right] [\text{SM}], \quad [h^0, H^0]tt = \left[\frac{\cos \alpha}{\sin \beta}, \frac{\sin \alpha}{\sin \beta} \right] [\text{SM}]$$

$$A bb = \tan \beta [\text{SM}] \quad A tt = \cot \beta [\text{SM}]$$

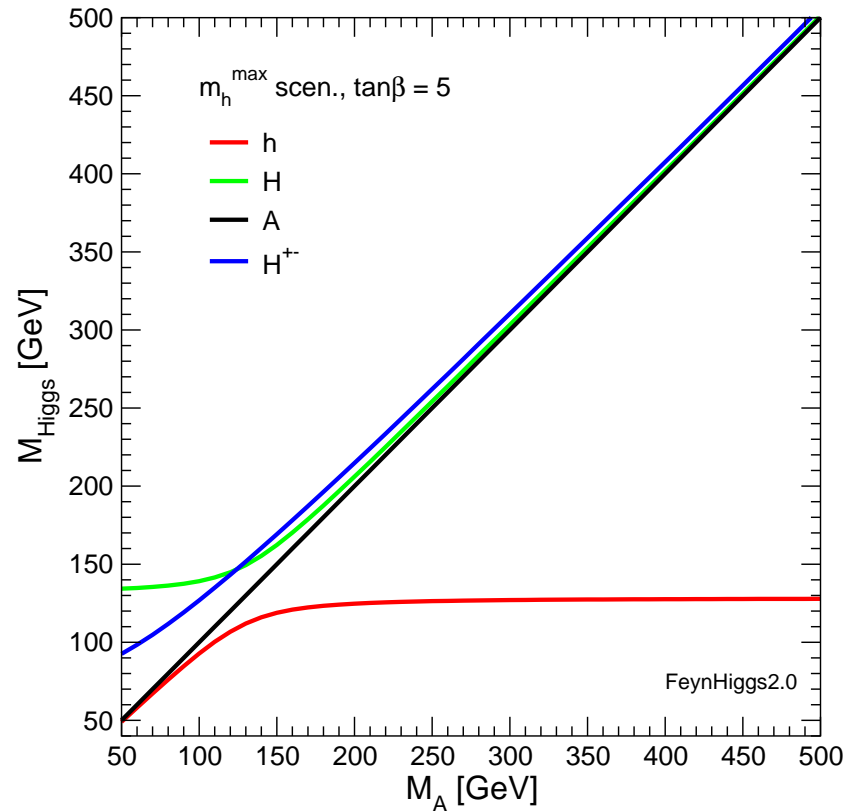
large M_A : h^0 is SM-like

H^0, A^0 have non-standard couplings

$m_H \sim m_A \sim m_{H^\pm} \gg m_Z, \alpha \rightarrow \beta - \frac{\pi}{2}$: 'decoupling regime'

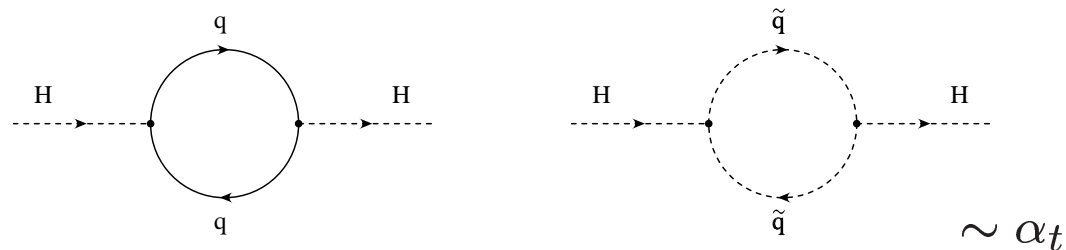
$m_h \rightarrow M_Z + \Delta m_h$ with large loop contributions Δm_h

Spectrum of Higgs bosons in the MSSM (example)



large M_A : h^0 like SM Higgs boson \sim decoupling regime

m_h^0 strongly influenced by quantum effects, e.g.



1-loop: complete

2-loop:

– QCD corrections $\sim \alpha_s \alpha_t, \alpha_s \alpha_b$

– Yukawa corrections $\sim \alpha_t^2$

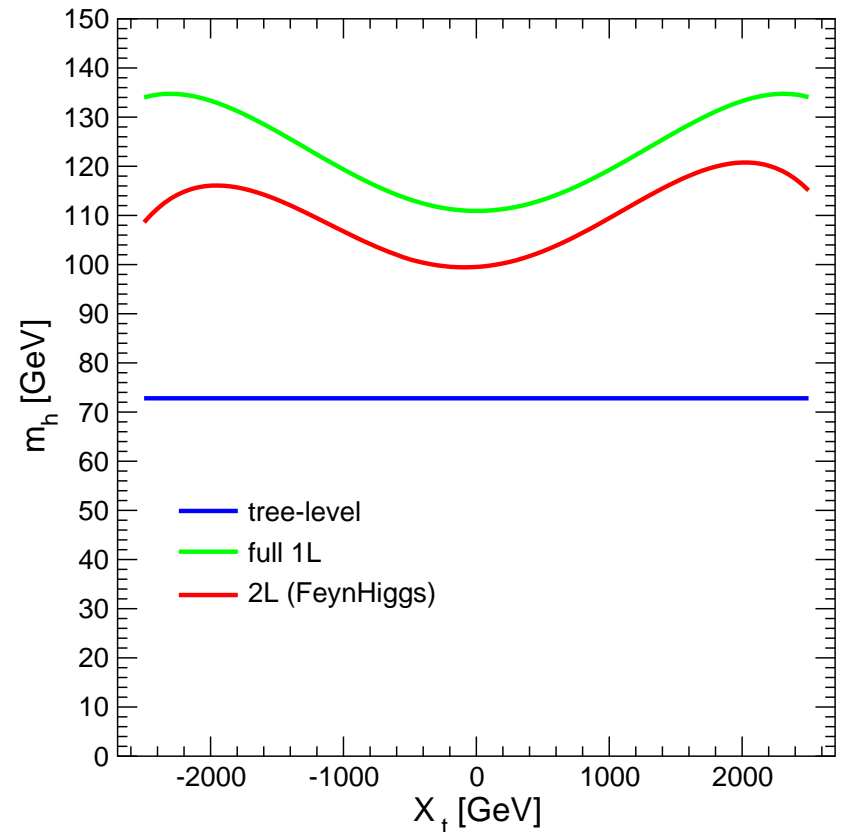
theoretical uncertainty:

$$\delta m_h \simeq 3\text{-}4 \text{ GeV}$$

[Degrassi, Heinemeyer, WH, Slavich,

Weiglein]

m_{h^0} prediction at different levels of accuracy:



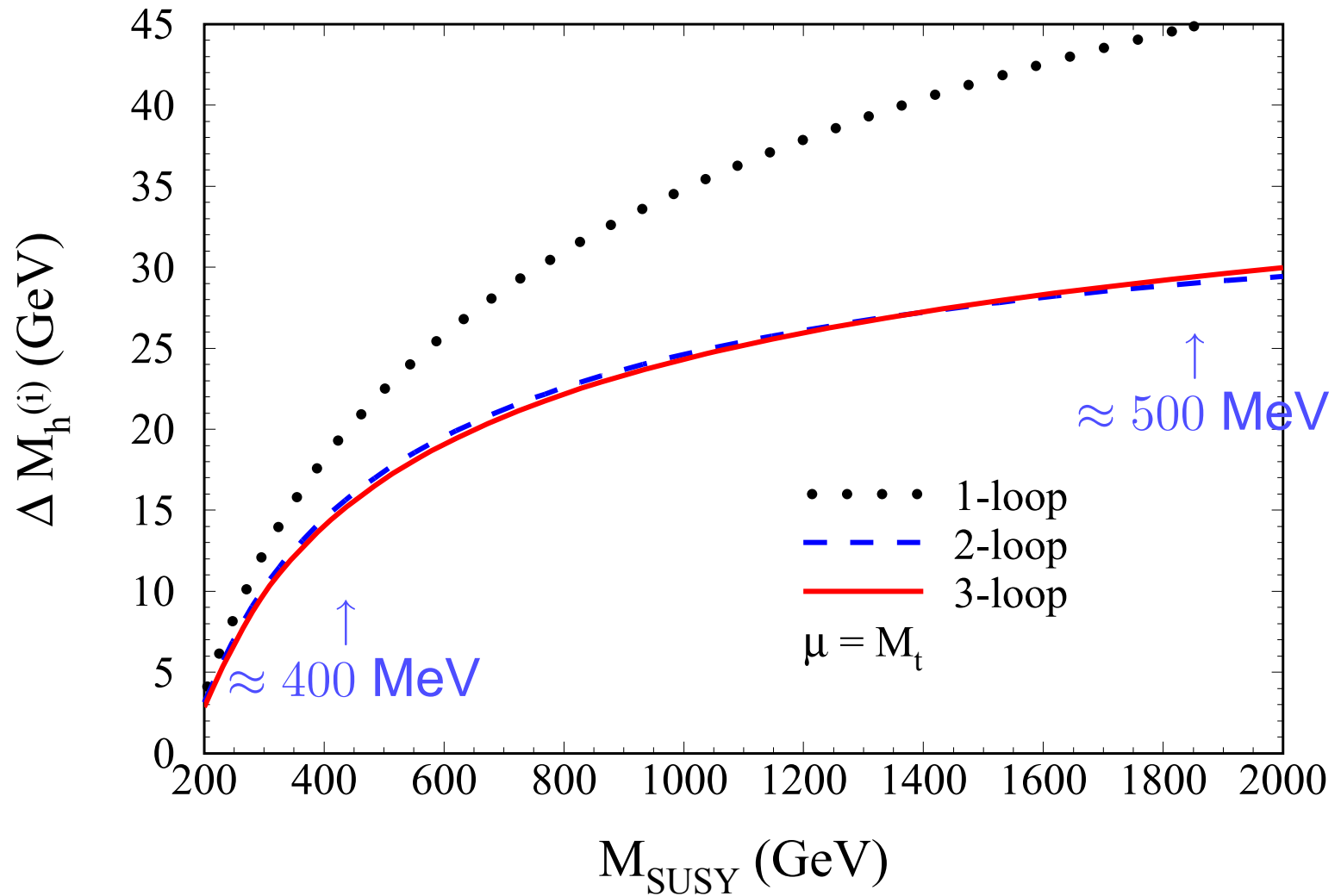
$\tan \beta = 3, \quad M_{\tilde{Q}} = M_A = 1 \text{ TeV}, \quad m_{\tilde{g}} = 800 \text{ GeV}$

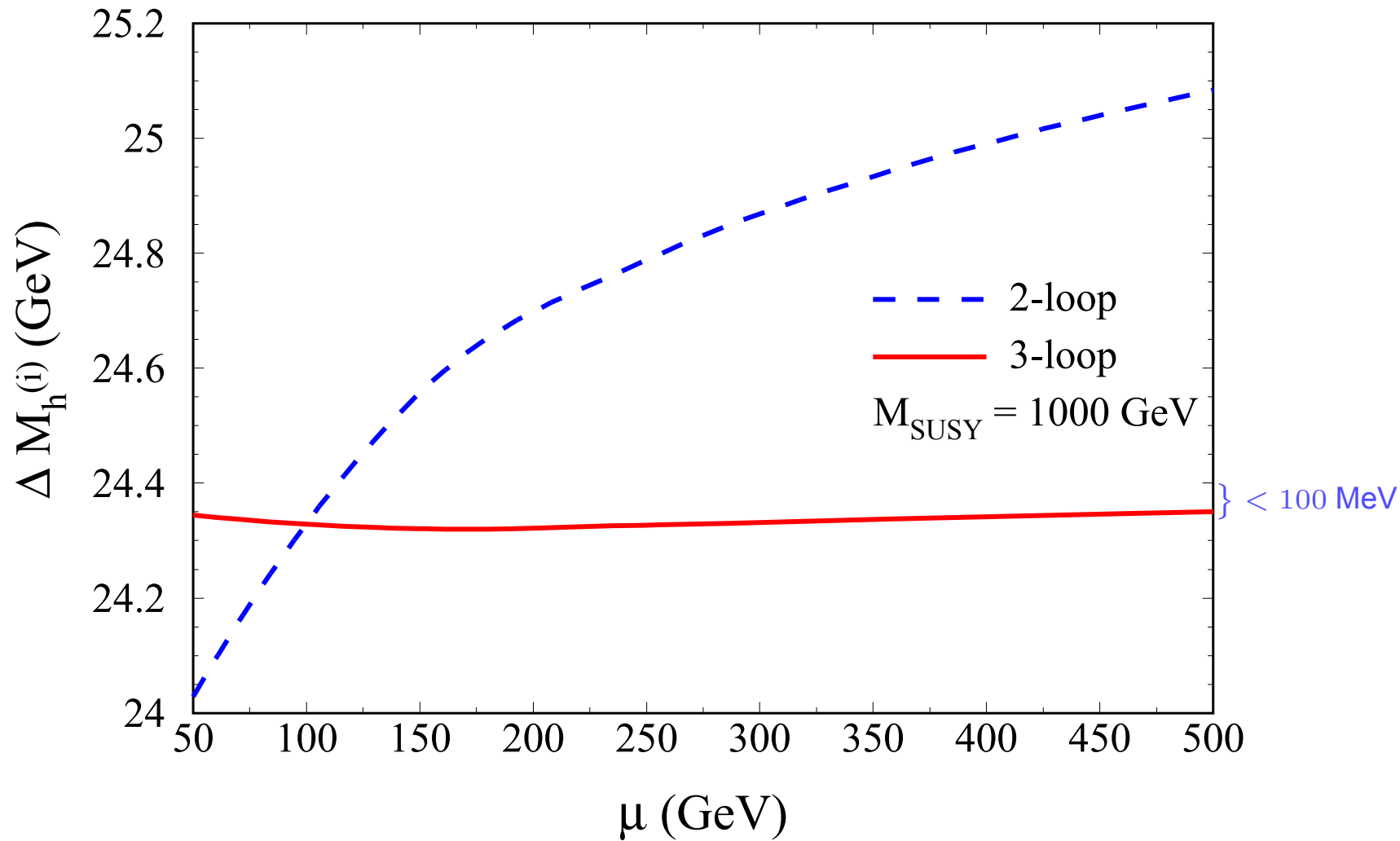
X_t : top-squark mixing parameter

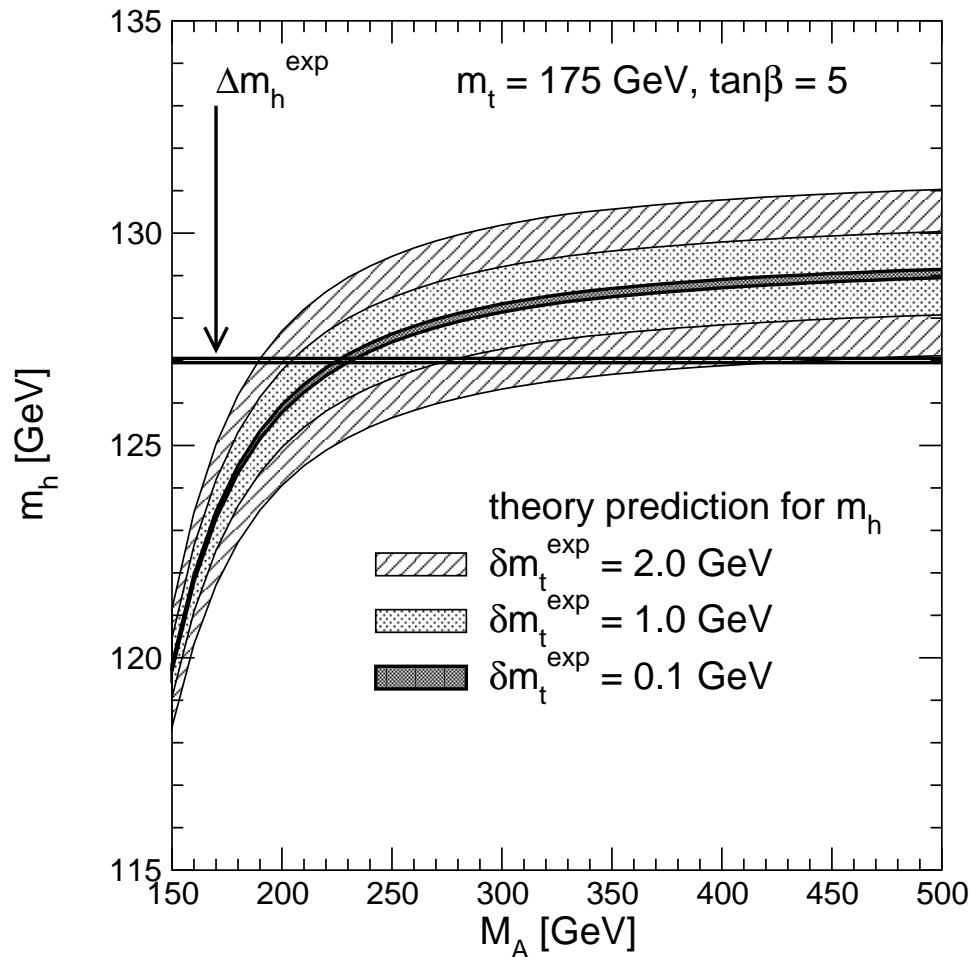
$$X_t = A_t - \mu \cot \beta$$

3-loop contributions to $M_h \sim \alpha_s^2 \alpha_t$

[Harlander, Kant, Mihaila, Steinhauser]







[Kraml et al.]

dependent on all SUSY particles and masses/mixings
through Higgs self-energies

masses and mixing of SUSY particles through soft-breaking

model parameters

- gaugino masses: M_1, M_2, M_3
- sfermion masses: $M_L, M_{\tilde{u}_R}, M_{\tilde{d}_R}$
for each doublet of squarks and sleptons
- trilinear coupling: $A_{\tilde{f}}$ for each \tilde{f}
→ L - R sfermion mixing
- supersymmetric Higgsino mass parameter: μ
- Higgs sector parameters: $M_A, \tan \beta = v_2/v_1$

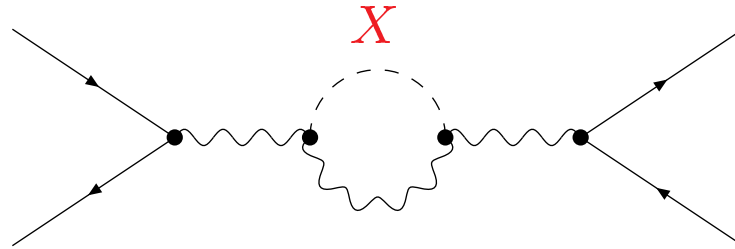
Benchmark scenarios

“Snowmass points and slopes” (SPS),
hep-ph/0202233

examples (mSUGRA):

- SPS1a: $m_0 = 100 \text{ GeV}$, $m_{1/2} = 250 \text{ GeV}$, $A_0 = -100$,
 $\tan \beta = 10$, $\mu > 0$.
- SPS1b: $m_0 = 200 \text{ GeV}$, $m_{1/2} = 400 \text{ GeV}$, $A_0 = 0$,
 $\tan \beta = 30$, $\mu > 0$.

indirect access to SUSY particles through quantum loops



$X =$ Higgs bosons, SUSY particles

- μ lifetime: $M_W \leftrightarrow M_Z, G_F$
- Z observables: $g_V, g_A, \sin^2 \theta_{\text{eff}}, \Gamma_Z, M_Z, \dots$

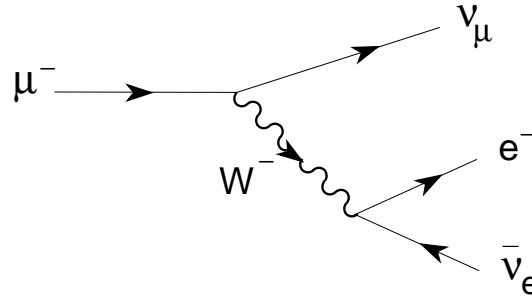
[Heinemeyer, WH, Weiglein, Phys. Rep. 425 (2006) 265]

2-loop improvements $\mathcal{O}(\alpha\alpha_s, \alpha_t^2, \alpha_b^2, \alpha_t\alpha_b)$
and complex parameters

[Heinemeyer, WH, Stöckinger, A. Weber, Weiglein 06]

[Heinemeyer, WH, A. Weber, Weiglein 07]

$M_W - M_Z$ correlation



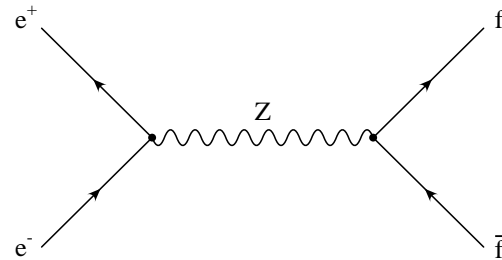
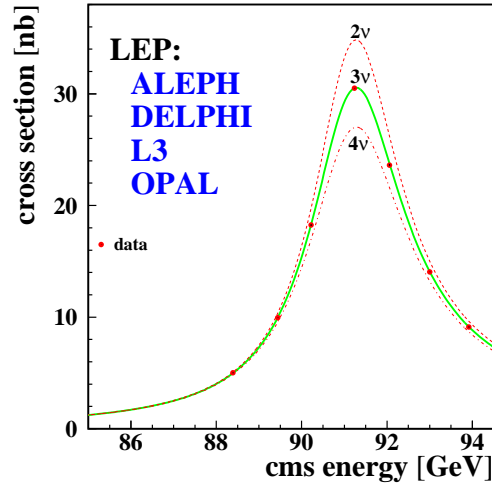
$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{M_W^2 (1 - M_W^2/M_Z^2)} (1 + \Delta r)$$

Δr : quantum correction, $\Delta r = \Delta r(m_t, X_{\text{SUSY}})$

$\rightarrow M_W = M_W(\alpha, G_F, M_Z, m_t, X_{\text{SUSY}})$

X_{SUSY} = set of non-standard model parameters

Z resonance



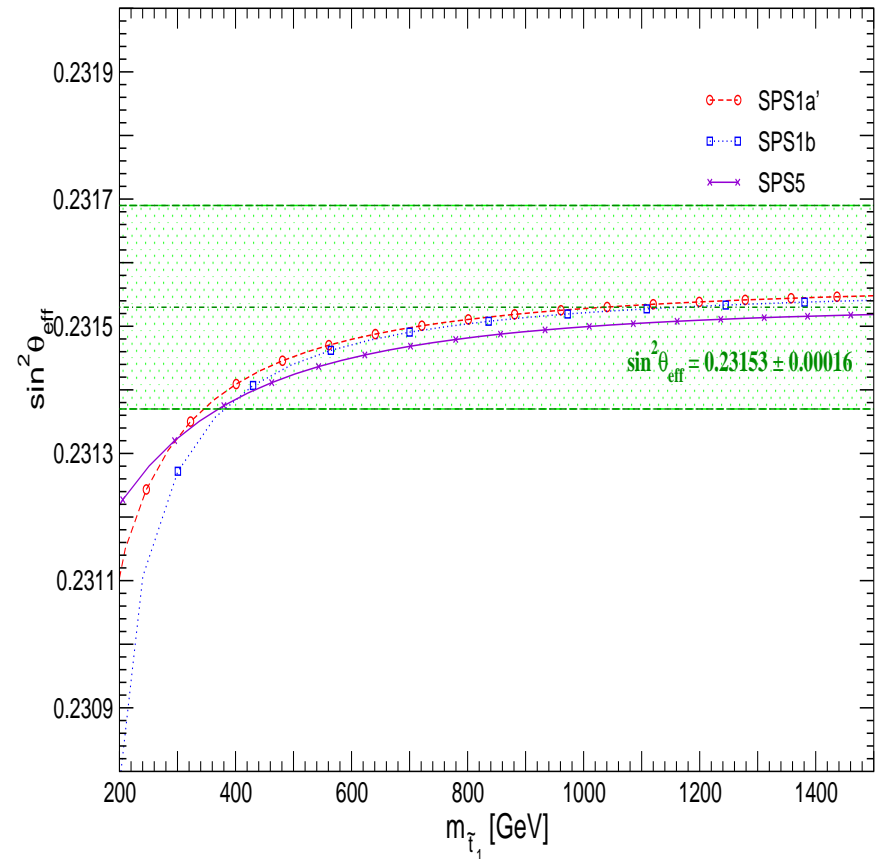
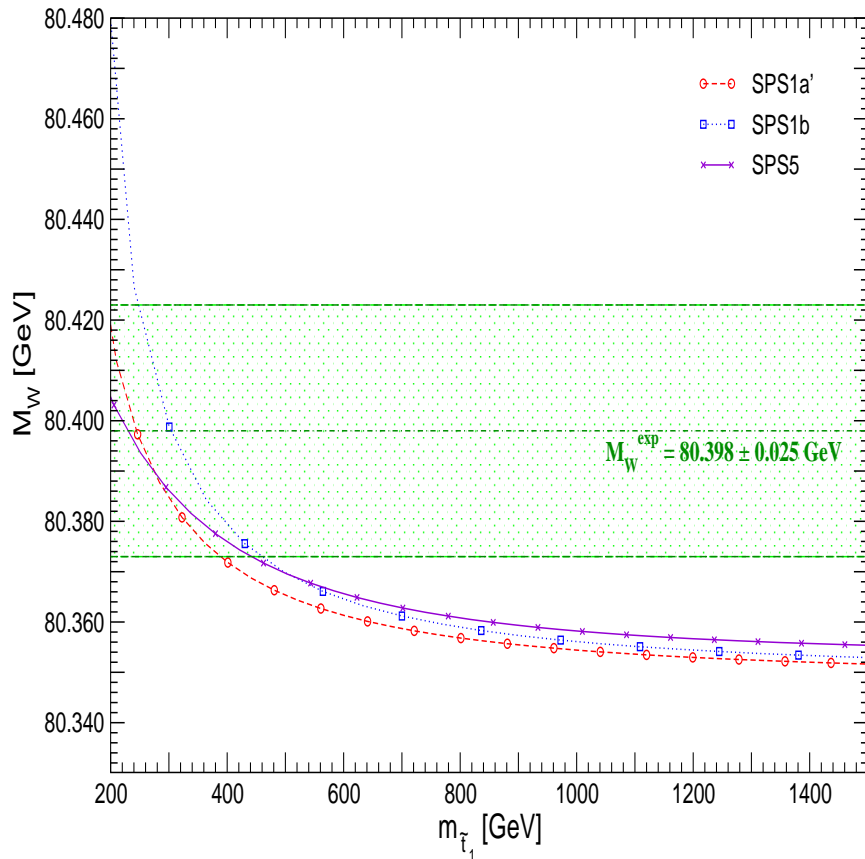
effective Z boson couplings

$$g_V^f \rightarrow g_V^f + \Delta g_V^f, \quad g_A^f \rightarrow g_A^f + \Delta g_A^f$$

with higher order contributions $\Delta g_{V,A}^f(m_t, X_{\text{SUSY}})$

$$\sin^2 \theta_{\text{eff}} = \frac{1}{4} \left(1 - \text{Re} \frac{g_V^e}{g_A^e} \right) = \kappa \cdot \left(1 - \frac{M_W^2}{M_Z^2} \right)$$

M_W and $\sin^2 \theta_{\text{eff}}$ for varied SUSY-scale

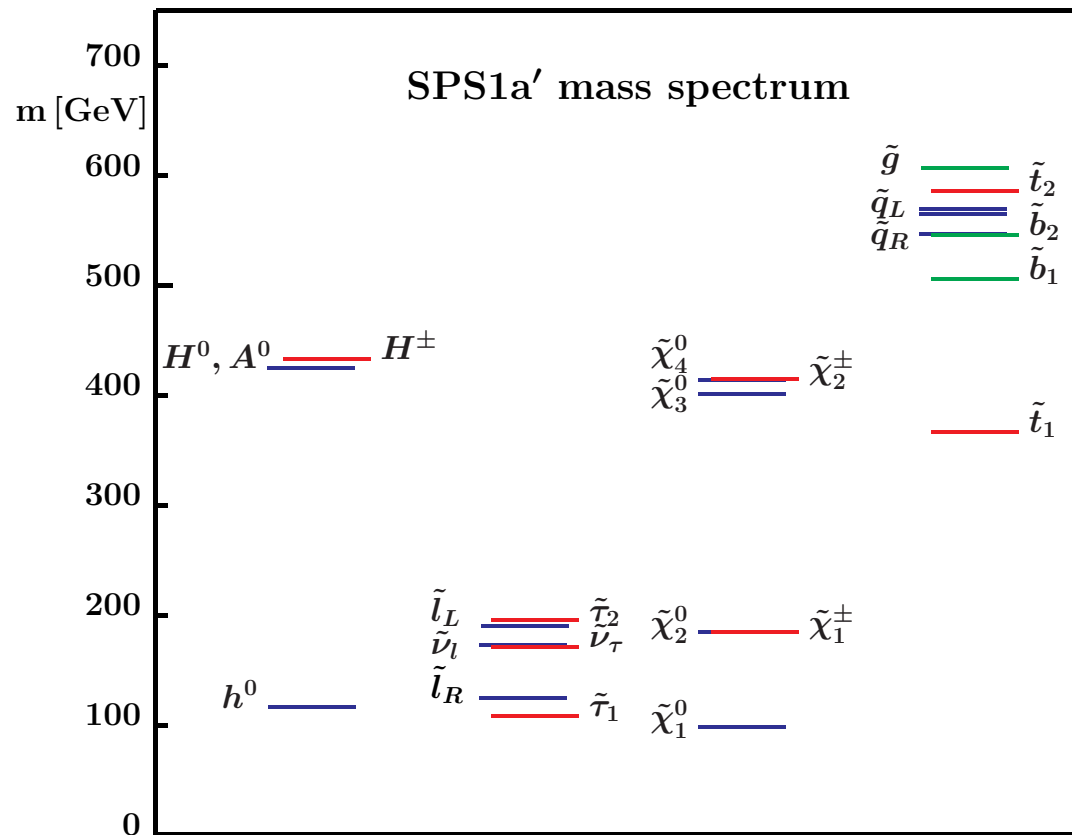


Fortran Code SUSYPOPE [A. Weber, PhD thesis, Munich 2008]

also used in recent fits by *AbdusSalam, Allanach, Quevedo, Feroz, Hobson,*
arxiv:0904.2548

SPS1a' scenario

$$\begin{array}{llll}
 M_{1/2} & = & 250 \text{ GeV} & \text{sign}(\mu) = +1 \\
 M_0 & = & 70 \text{ GeV} & \tan \beta(\tilde{M}) = 10 \\
 A_0 & = & -300 \text{ GeV} &
 \end{array}$$



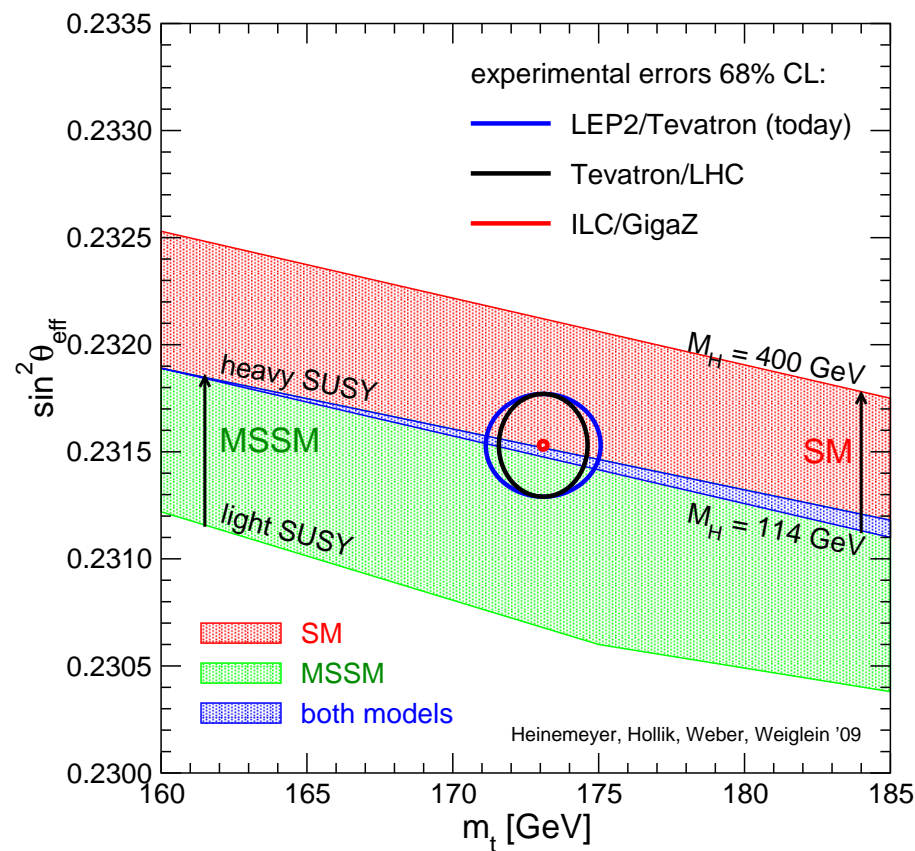
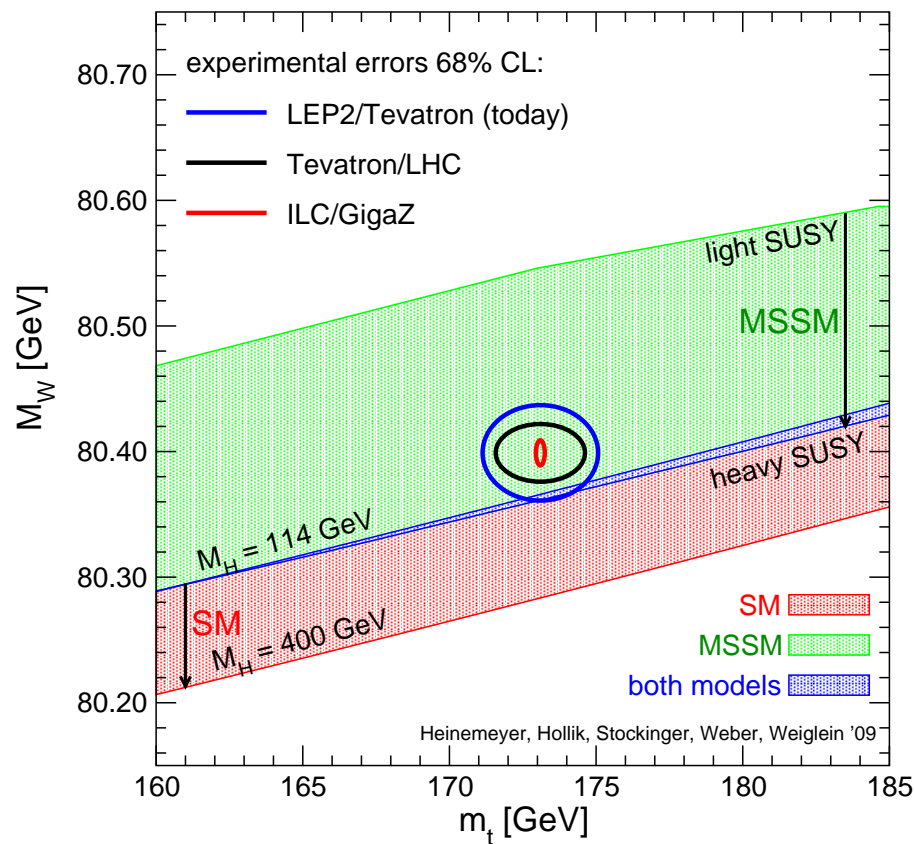
Scatter plots for M_W & $\sin^2 \theta_{\text{eff}}$

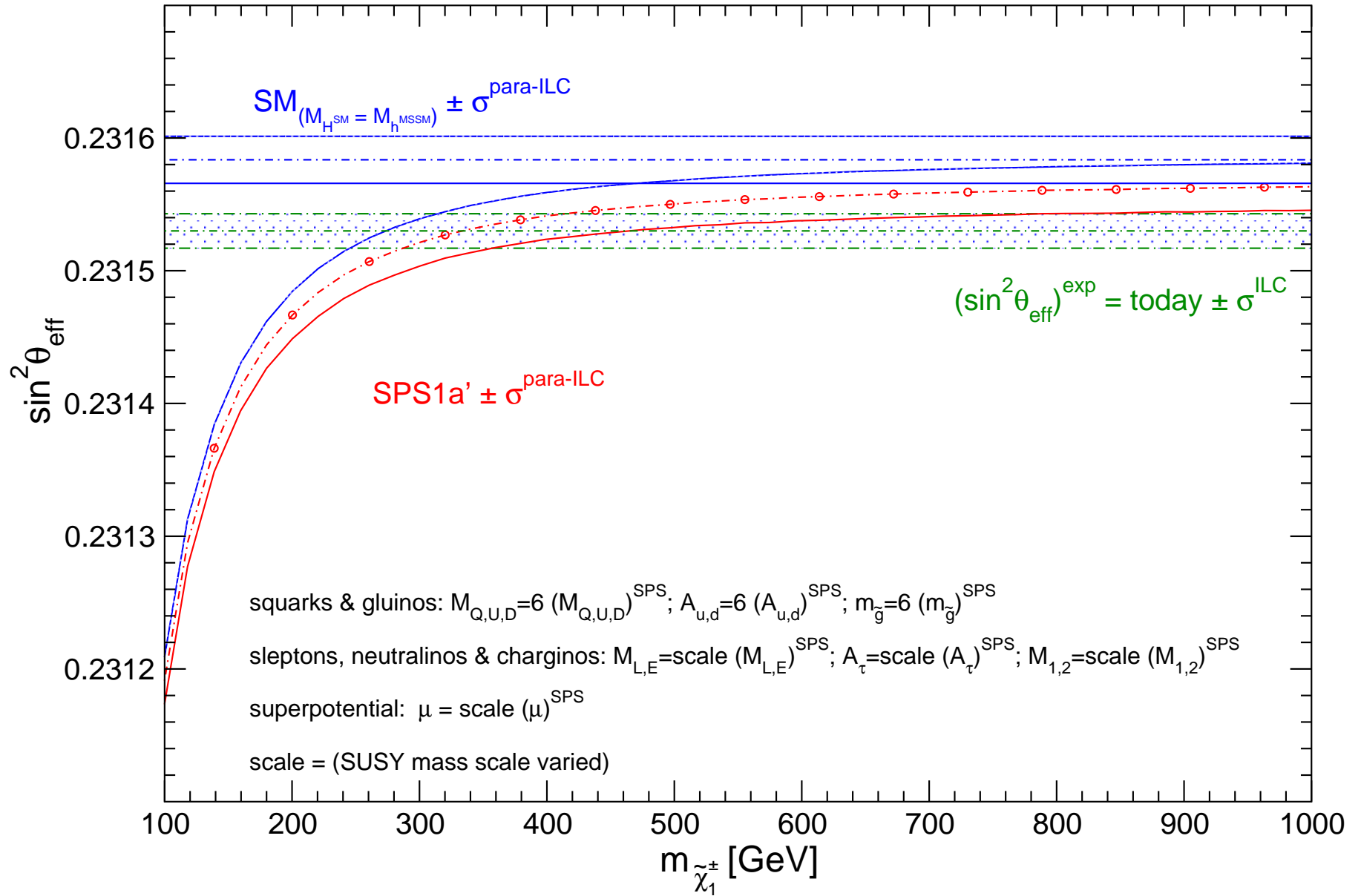
■ SUSY parameters:

$$\begin{aligned} \text{sleptons} & : M_{\tilde{F}, \tilde{F}'} = 100 \dots 2000 \text{ GeV} \\ \text{light squarks} & : M_{\tilde{F}, \tilde{F}'_{\text{up/down}}} = 100 \dots 2000 \text{ GeV} \\ \tilde{t}/\tilde{b} \text{ doublet} & : M_{\tilde{F}, \tilde{F}'_{\text{up/down}}} = 100 \dots 2000 \text{ GeV} \\ & A_{t,b} = -2000 \dots 2000 \text{ GeV} \\ \text{gauginos} & : M_{1,2} = 100 \dots 2000 \text{ GeV} \\ & m_{\tilde{g}} = 195 \dots 1500 \text{ GeV} \\ & \mu = -2000 \dots 2000 \text{ GeV} \\ \text{Higgs} & : M_A = 90 - 1000 \text{ GeV} \\ & \tan \beta = 1.1 \dots 60 \end{aligned}$$

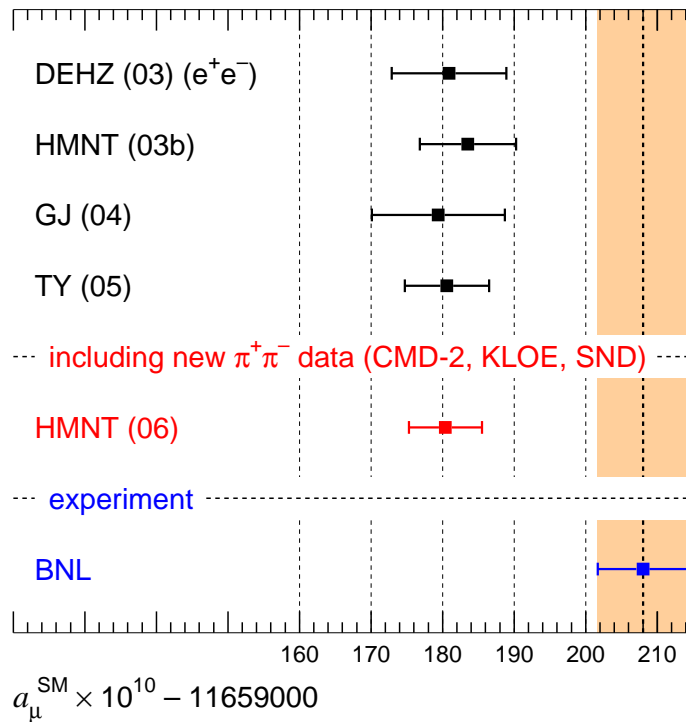
- Unconstrained scan, only Higgs mass required to be in agreement with LEP data.

[Heinemeyer, Hollik, Stöckinger, Weber, Weiglein]





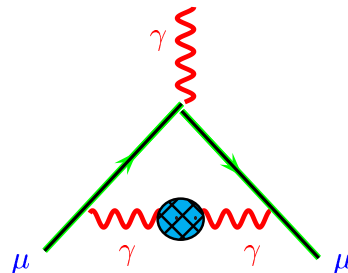
Anomalous g-factor of the muon



Hagiwara, Martin, Nomura, Teubner

e^+e^- data based SM prediction: 3.4σ below exp. value

theory uncertainty from hadronic vacuum polarization



$g - 2$ with supersymmetry

new contributions from virtual SUSY partners of μ , ν_μ and of W^\pm , Z



extra terms

$$+ \frac{\alpha}{\pi} \frac{m_\mu^2}{M_{\text{SUSY}}^2} \cdot \frac{v_2}{v_1}$$

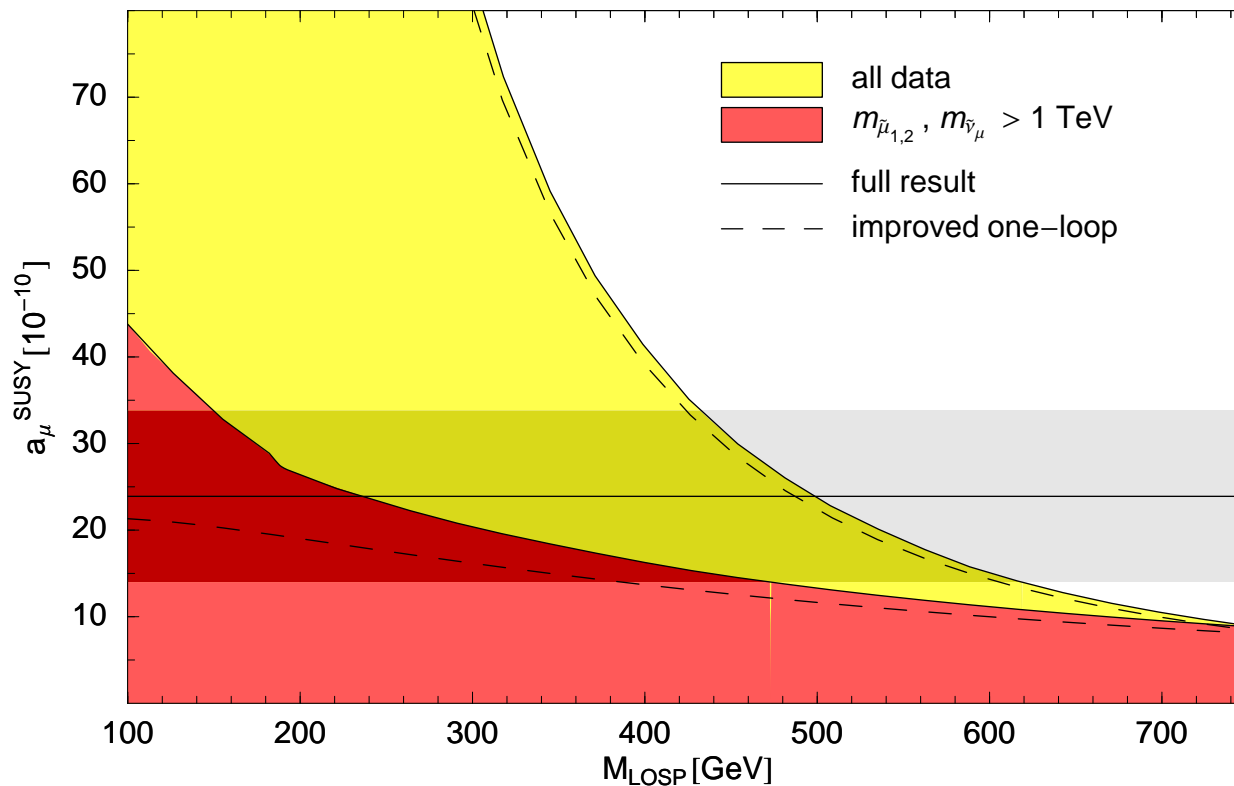
can provide missing contribution for

$$M_{\text{SUSY}} = 200 - 600 \text{ GeV}$$

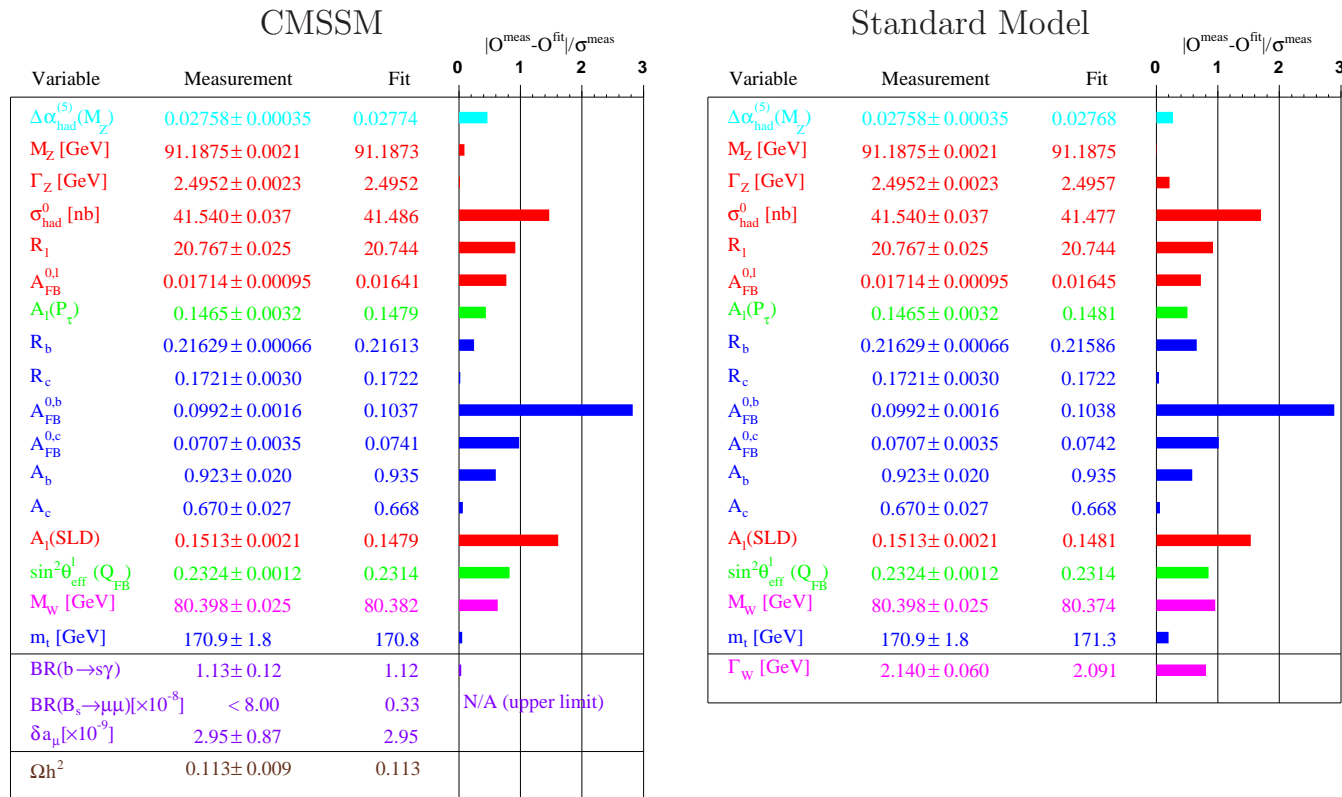
2-loop calculation [*Heinemeyer, Stöckinger, ...*]

scan over SUSY parameters compatible with EW and $b \rightarrow s\gamma$ constraints $(\tan \beta = 50)$

[Stöckinger]

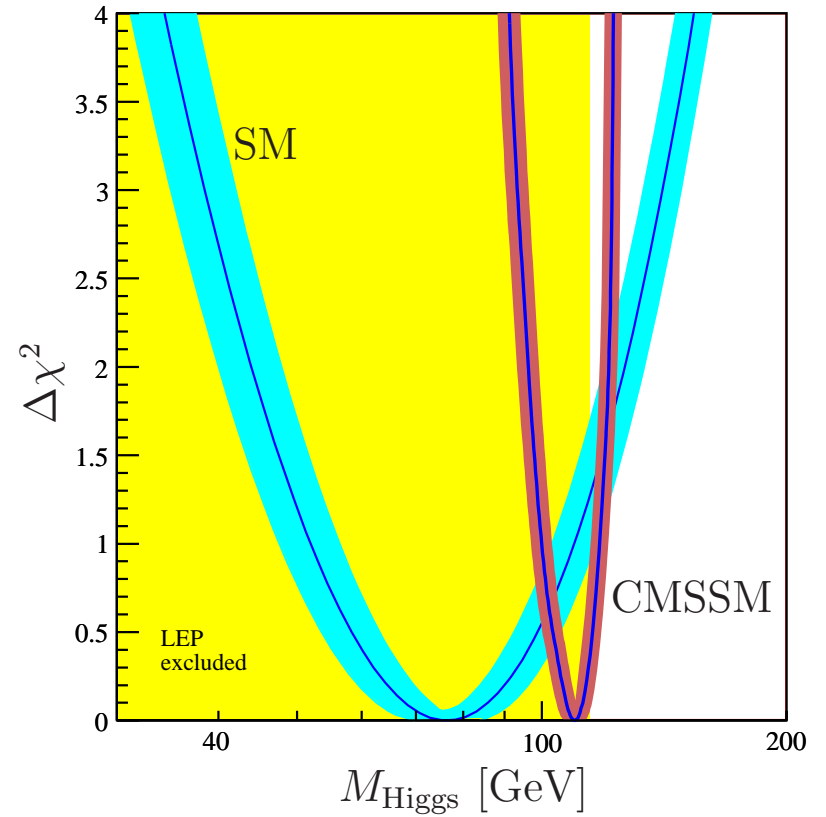
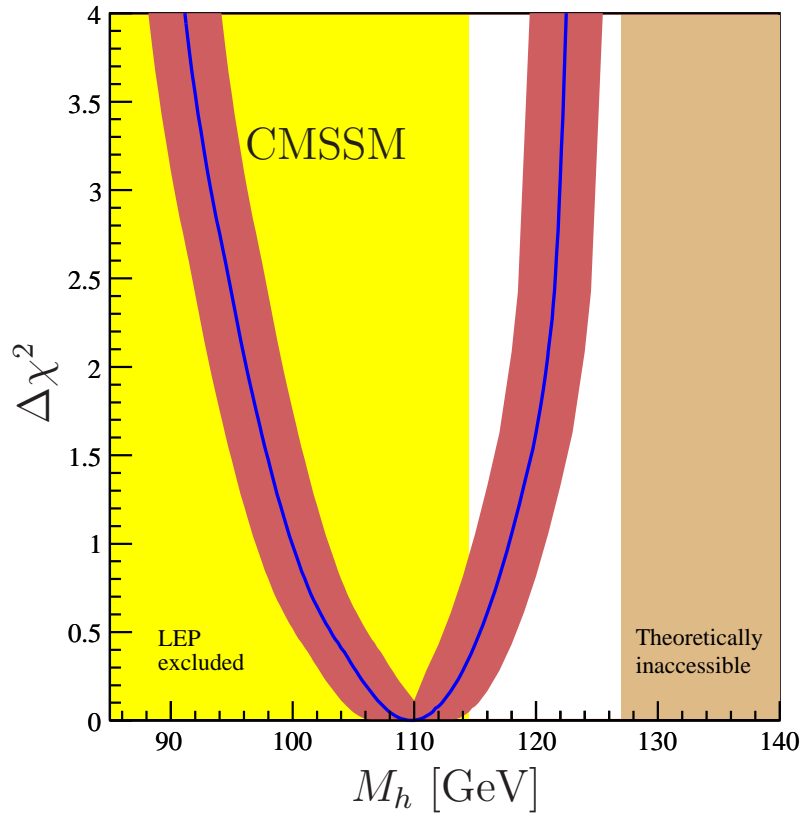


LOSP = lightest observable SUSY particle ($\chi_1^\pm, \chi_2^0, \dots$)

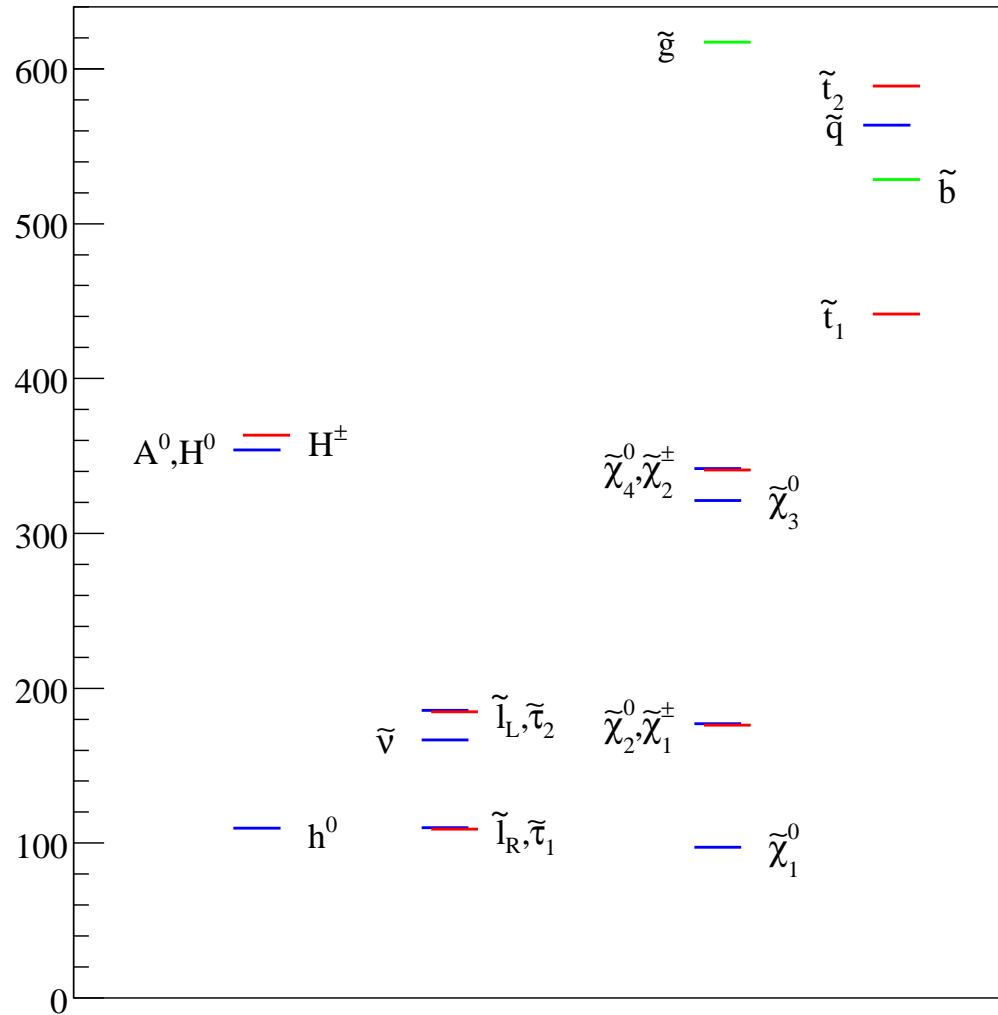


global fit in the constrained MSSM including data from $g - 2$, B physics, and cosmic relic density

[O. Buchmueller, ..., Weber, Weiglein]



$$M_h = 110^{+8}_{-10} \text{ GeV}$$



best fit particle spectrum

Outlook –Possible scenarios

- a single light Higgs boson
 - SM Higgs boson?
 - SUSY light Higgs boson?
 H, A, H^\pm heavy (decoupling scenario) $h \sim H_{\text{SM}}$
- a light Higgs boson + more (H, A, H^\pm)
 - SUSY Higgs?
 - non-SUSY 2-Higgs-Doublet model?
- a single heavy Higgs boson ($\gg 200$ GeV)
 - SUSY ruled out
 - SM + (?) strong interaction?
- no Higgs boson
 - strongly interacting weak interaction
new strong force \sim TeV scale

Conclusions

- **Electroweak precision physics**
 - sensitive to quantum structure
 - constraints on unknown parameters
- **precision tests of the Standard Model have established the SM as a quantum field theory**
- **MSSM is competitive to the SM**
 - global fits of similar quality (even better)
 - natural: light Higgs boson h^0
- **future experiments at colliders**
 - discovery of Higgs and SUSY at LHC (?)
 - precision studies at e^+e^- Linear Collider