$B \to K^*$ Decays: SM and Beyond

Aoife Bharucha with W. Altmannshofer, Patricia Ball,
A.J. Buras, D. Straub and M. Wick (arXiv:0811.1214 [hep-ph])
also with William Reece and Thorsten Feldmann

IPPP

Internal Seminar, 26th June 2009

Soon Launching Expedition to 14TeV

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Some Structure

- Tools for calculating $B \to K^*$ Decays
- Theoretical Predictions
- Prospects at LHCb

A B Physicists ToolBox

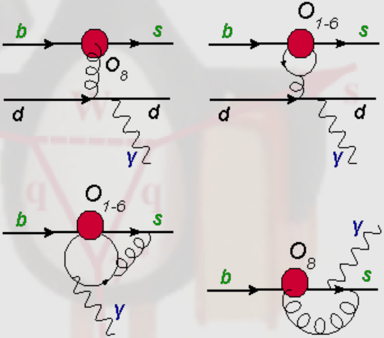
WILSON COEFFICIENTS

Contain short distance effects, and possibly NP

HADRONIC MATRIX ELEMENTS

$\langle B|J|K^*\rangle$ described by Form Factors

HARD SPECTACTOR EFFECTS- For $B \to K^* \mu^+ \mu^-$ QCD factorization/ SCET/ HQET...



Not so brief Interlude-Form Factors

Matrix elements responsible for $B \to K^* l^+ l^- / \nu\bar{\nu}$ can be expressed as:

8 Full form factors (FF's)

$$\langle K^*(p)|\bar{s}\gamma_\mu\gamma_L b|\bar{B}(p_B)\rangle = -ie^*_\mu(m_B + m^*_K)\mathbf{A_1}(q^2) + i(p_B + p)_\mu e^*.q\frac{\mathbf{A_2}(q^2)}{m_B + m^*_K}$$

$$+ iq_\mu(e^*.q)\frac{2m_{K^*}}{q^2}\left(\mathbf{A_3}(q^2) - \mathbf{A_0}(q^2)\right) + \epsilon_{\mu\nu\rho\sigma}e^{*\nu}p^\rho_B\, p^\sigma\,\frac{2\mathbf{V}(q^2)}{m_B + m^*_K}$$

$$\langle K^*(p)|\bar{s}\sigma_{\mu\nu}q^\nu\gamma_L b|\bar{B}(p_B)\rangle = i\epsilon_{\mu\nu\rho\sigma}e^{*\nu}p^\rho_B p^\sigma 2\mathbf{T_1}(q^2) + \mathbf{T_2}(q^2)e^*_\mu(m^2_B - m^2_{K^*})$$

$$-\mathbf{T_2}(q^2)(e^*.q)\,(p_B + p)_\mu + \mathbf{T_3}(q^2)(e^*.q)\left\{q_\mu - \frac{q^2}{m^2_B - m^2_{K^*}}(p_B + p)_\mu\right\}$$

Form Factor Predictions

Theoretical Prections:

- **Lattice**: High q^2, Unstable particles eg. K^* difficult
- **Light Cone Sum Rules**: Low q^2

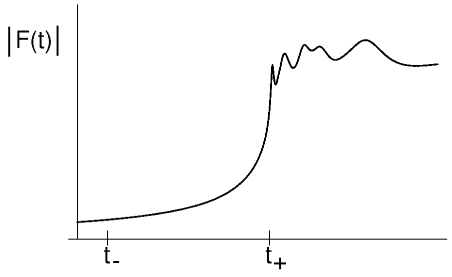
Range of Form Factors:

- Kinematic Range: $0 \leq q^2 \leq 20\mathrm{GeV}^2$
- QCDF Range for $B \to K^* \mu^+ \mu^-$: $1 \leq q^2 \leq 6\mathrm{GeV}^2$

Extrapolation to high q^2?

Form Factor Generalities

- Cut at t_+, $t_\pm = (m_H \pm m_L)^{1/2}$
- Series of poles at $q^2 > t_+$
- May be poles at m_R in $t_- < q^2 < t_+$[a]



[a] W. A. Bardeen, E. J. Eichten and C. T. Hill, arXiv:hep-ph/0305049

- $$F(q^2) = \frac{F(0)/(1-\alpha)}{1 - \frac{q^2}{m_R^2}} + \frac{1}{\pi}\int_{t_+}^{\infty} dt\, \frac{\mathrm{Im} F_+(t)}{t - q^2 - i\epsilon}$$
- $$F(q^2) = \frac{F(0)/(1-\alpha)}{1 - \frac{q^2}{m_R^2}} + \sum_{k=1}^{N} \frac{\rho_k}{1 - \frac{1}{\gamma_k}\frac{q^2}{m_R^2}}$$

Parameterizations differ in approach:

- Make a heuristic, pragmatic ansatz
- Based on first-principles eg. unitarity, analyticity

Pole Type Parameterisations

BK (Becirevic-Kaidalov[1])

$$f(q^2) = \frac{r_1}{1 - q^2/m_{\rm R}^2} + \frac{r_2}{1 - \alpha q^2/m_{\rm R}^2}$$

BZ(Ball-Zwicky[2])

$$f(q^2) = \frac{r_1}{1 - \alpha\, q^2/m_{\rm R}^2} + \frac{r_2}{(1 - \alpha\, q^2/m_{\rm R}^2)^2}$$

[1] D. Becirevic and A. B. Kaidalov, arXiv:hep-ph/9904490

[2] P. Ball and R. Zwicky,arXiv:hep-ph/0406232, arXiv:hep-ph/0406261

Series Type Parameterisations

SE (Series Expansion[3])

$$f(t) = \frac{1}{B(t)\phi_f(t)} \sum_{k=0}^{\infty} a_k z^k(t)$$

with

$$z(t) = z(t, t_0) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}} \qquad \sum_{k=0}^{\infty} a_k^2 \leq 1$$

t_0 is a free parameter, optimised to reduce $|z(t)|_{\max}$, and
$B(t) = z(m_R^2, t_0)$.
SSE (Simplified Series Expansion[4])

$$f(t) = \frac{f_0}{1 - q^2/m_{\mathrm{R}}^2} \frac{\sum_{k=0}^{\infty} c_k z(t, t_0)}{\sum_{k=0}^{\infty} c_k z(0, t_0)}$$

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[3]C. G. Boyd, B. Grinstein and R. F. Lebed, arXiv:hep-ph/9412324.

[4]C. Bourrely, I. Caprini and L. Lellouch,arXiv:0807.2722

Dispersive Bounds

$$\Pi_X(q^2) = i \int d^4x e^{iqx} \langle 0 | \mathrm{T} J_X(x) J_X^\dagger(0) | 0 \rangle$$

$\Pi_x(q^2)$

OPE

Unitarity Considerations

$$\Pi_x(q^2) = \sum_n C_n < O_n >$$

$$\mathrm{Im}\, \Pi_x(q^2) > \mathrm{Im}\Pi_X{}^{H_1 H_2}(q^2)$$

A few details..

$$\Pi_X(q^2) = i \int d^4x e^{iqx} \langle 0 | \mathrm{T} J_X(x) J_X^\dagger(0) | 0 \rangle$$

Unitarity approach: Quantity analytic so use unitarity relations

$$\mathrm{Im}\Pi_X = \frac{1}{8\pi^2} \sum_\Gamma \int \frac{d^3p_1 d^3p_2}{(2E_1)(2E_2)} \delta^4(q - p_1 - p_2) \langle 0 | J_X^\dagger | \Gamma \rangle \langle \Gamma | J_X | 0 \rangle$$

Unitarity/Positivity

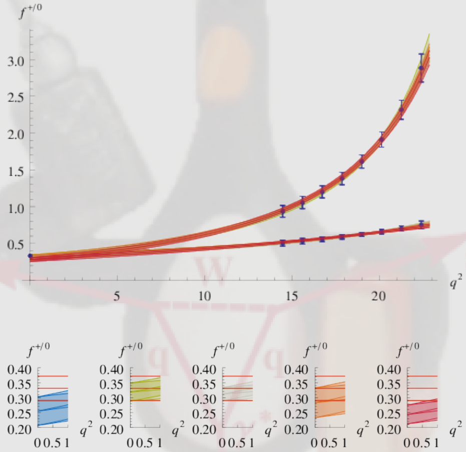
- Only Positive terms in sum $|\Gamma\rangle = |BK^*\rangle$,
- $\mathrm{Im}\Pi_X > \mathrm{Im}\Pi_X^{BK^*}$
- Use crossing symetry to express $\Pi_X^{BK^*}$ in terms of form factors

OPE

- $J_X(x) J_X^\dagger(0) = \sum_{n=1}^{\infty} C_n O_n$
- $\Pi_X(q^2) = \sum_{n=1}^{\infty} C_n \langle O_n \rangle$
- Operators are condensates: $\mathbb{I}$, $\langle m q \bar{q} \rangle$ and $\langle \frac{\alpha_s}{\pi} G^2 \rangle$

Preliminary Results for B to K

Combining Lattice[5] and LCSR[6] results:



[5] A. Al-Haydari *et al.* [QCDSF Collaboration], arXiv:0903.1664 [hep-lat]

[6] P. Ball and R. Zwicky, arXiv:hep-ph/0406261

Application to $B \to K^*$

- $B \to K^* l^+ l^-$ QCDF predictions valid in $1 - 6\,\mathrm{GeV}^2$
- $B \to K^* \nu\bar{\nu}$ higher q^2 form factors required
- Very limited lattice results for tensors only LCSR's valid at lower q^2

Can Dispersive Bounds provide the answer?

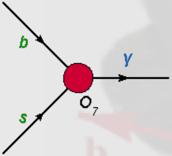
- Can provide theoretical input to constrain the shape
- Calculated bounds, from Wilson coefficients of OPE
- Also include constraint from relations at large recoil
- Results to appear soon

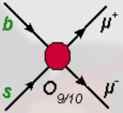
Relating Observables to NP: EFTs

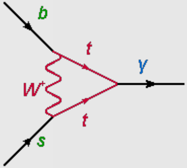
$$\mathcal{L} = \sum_i C_i O_i$$

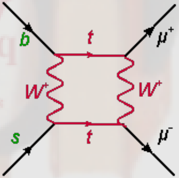
For $B \to K^*$ decays, important Operators are..

Electromagnetic Dipole O_7 Vector/Axial Current $O_{9(10)}$









Relating Observables to NP: EFTs

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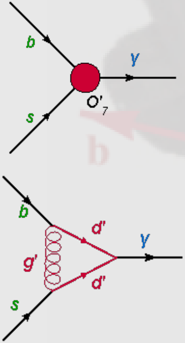
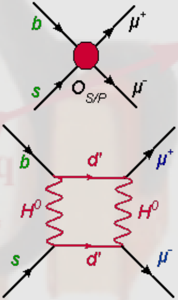
For $B \to K^*(\to K^-\pi^+)\mu^+\mu^-$, important NP O's are..

Spin-Flipped EM Dipole O_7' Scalar/Pseudoscalar $O_{S(P)}$





What will the Flavour Telescope see?

- SM CP violation is doubly Cabibbo suppressed.
- 4 body decays, many angular observables, sensitive to different Wilson Coefficients.

So we Focus on Additional..

- **CP Violation**
- **Operators**

Keeping in Mind Bounds from..

- EDM's, CP Asymmetries....
- $B_s \to \mu^+\mu^-$, $B \to X_s\gamma$, $B \to X_s\mu^+\mu^-$

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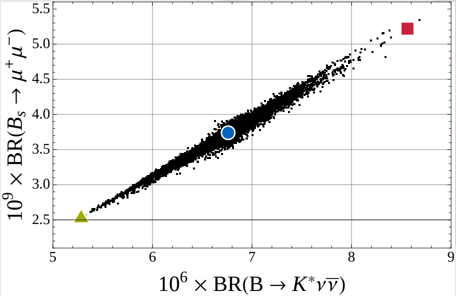
Prospects for $B \to K^* \nu\bar{\nu}$



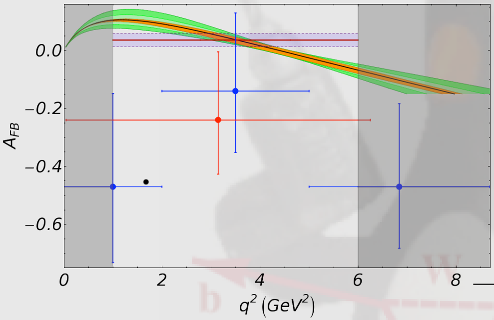
Correlation between $\text{BR}(B \to K^* \nu\bar{\nu})$ and $\text{BR}(B_s \to \mu^+\mu^-)$ in the considered MSSM scenario. The blue circle represents the SM point, while the red square (green diamond) corresponds to the MSSM parameter set I (II).[a]

[a] W. Altmannshofer, A. J. Buras, D. M. Straub and M. Wick, arXiv:0902.0160

Parameter Set	$\tan\beta$	μ	M_2	$m_{\tilde{Q}}$	$m_{\tilde{U}}$	A_t	$(\delta_u^{RL})_{32}$
I	5	500	800	500	400	-800	0.75
II	5	120	700	400	800	-700	-0.5

Table: Two example MSSM parameter sets giving large effects in $b \to s\nu\bar{\nu}$ transitions. Dimensionful quantities are expressed in GeV.

Current Status/Prospects at LHCb



- Belle (2008) - ICHEP
  200 events
- BaBar (2008) 0804.4412
  100 events
- LHCb $< 0.1\,\text{fb}^{-1}$ 100 events
  $2\,\text{fb}^{-1}$ 7200 events
  $q^2 < m^2_{J/\psi}$, 3900 events[a]

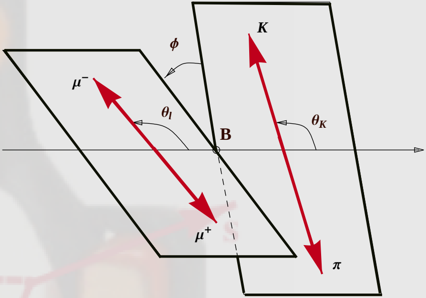
[a] J. Dickens, V. Gibson, C. Lazzeroni and M. Patel, CERN-LHCB-2007-038

- EvtGen Model for $B \to K^* \mu^+ \mu^-$ at NLO
- Includes trigger studies, acceptance effects
- Finds sets of allowed WCs using current experimental constraints

Angular Observables for $B \to K^*\mu^+\mu^-$

Choosing a good place to look..

$$\frac{d^4\Gamma}{dq^2 d\Omega} = \frac{9}{32\pi} I(q^2, \theta_l, \theta_K, \phi)$$



..where $I(q^2, \theta_l, \theta_K, \phi) = \sum_{i=1}^{9} I_i^{(s/c)}(q^2)\, \omega_i(\theta_l, \theta_K, \phi)$

Emphasize CP Conserving and CP Violating[7] Effects

$$S_i^{(a)} = \frac{I_i^{(a)} + \bar{I}_i^{(a)}}{d(\Gamma+\bar{\Gamma})/dq^2} \qquad A_i^{(a)} = \frac{I_i^{(a)} - \bar{I}_i^{(a)}}{d(\Gamma+\bar{\Gamma})/dq^2}$$

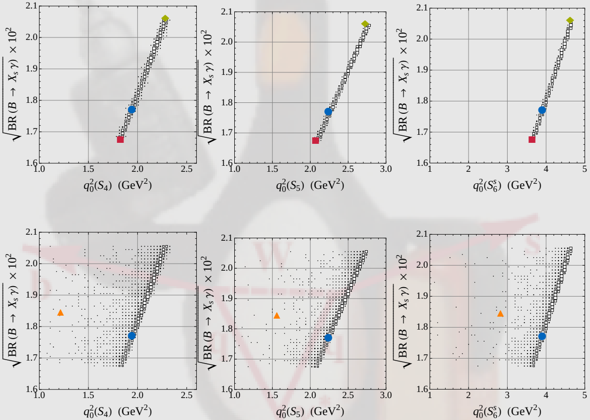
[7] Also considered in C. Bobeth, G. Hiller and G. Piranishvili arXiv:0805.2525

Model Independent Analysis

Observable	Most affected by
$S_1^s,\ S_1^c,\ S_2^s,\ S_2^c$	$C_7,\ C_7',\ C_9,\ C_9',\ C_{10},\ C_{10}'$
S_3	$C_7',\ C_9',\ C_{10}'$
S_4	$C_7,\ C_7',\ C_{10},\ C_{10}'$
S_5	$C_7,\ C_7',\ C_9,\ C_{10}'$
S_6^s	$C_7,\ C_9$
A_7	$C_7,\ C_7',\ C_{10},\ C_{10}'$
A_8	$C_7,\ C_7',\ C_9,\ C_9',\ C_{10}'$
A_9	$C_7',\ C_9',\ C_{10}'$
S_6^c	$C_S - C_S'$

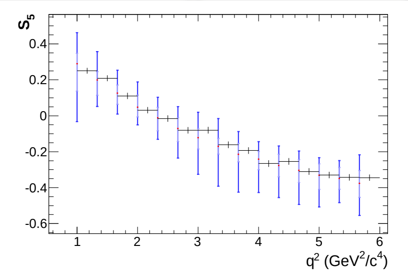
Theoretical Predictions: Specific Scenarios

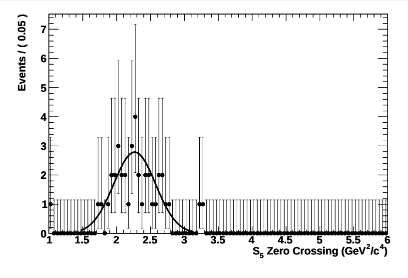
Correlate zeros of S_4, S_5, S_6^s with $B(b \to s\gamma)$

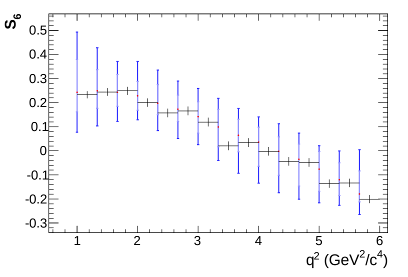


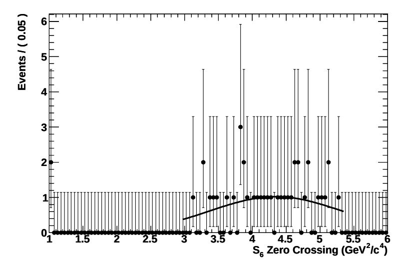
Bound on C_7 from $b \to s\gamma$ weakened if complex C_7 due to additional CP violating phases..

S_5 and S_6 at LHCb

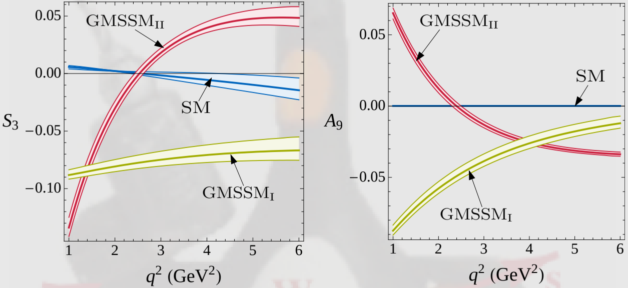


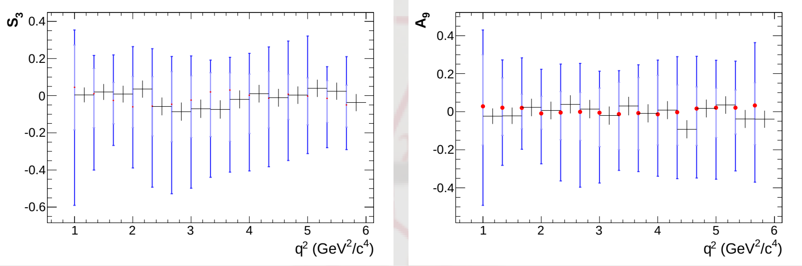






Prospects at LHCb: Additional Operators (O_7')





Summary

- Form Factors for B decays are critical to the success of LHCb
- $\mathbf{B} \rightarrow \bar{\mathbf{K}}^{*} \mu^{+} \mu^{-}$ observables provide insight into NP
- New **NLO EvtGen model**, promising preliminary results for zero's of S_5, S_6
- Early observation of S_3 and A_9 possible, but more data required to prove BSM effects.

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Back-up slides

- A natural expansion parameter for a function in an interval are orthonormal polynomials in the interval
- Domain of convergence is the ellipse passing through 1st singularity of function[8]
- Accelerate rate of convergence by mapping interior of cuntions analyticity domain to inside of ellipse, such that the region of interest is mapped to the segment between focal points[9]
- When the physical interval is far from the 1st singulartiy, ellipse becomes close to a circle, so a simple Taylor expansion about the centre has roughly the same radius of convergence as expansion in above polynomials.

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[8] Walsh 1956

[9] Cutkosky and Deo 1968