On Goldstino Interactions

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Motivations

Supersymmetry (and especially supersymmetry breaking) is important for particle physics, cosmology and string theory.

Global $\mathcal{N} = 1$ supersymmetry predicts the existence of a massless Weyl fermion G_α whose dynamics could shed light on physics of higher energies. Vice versa, by understanding better the constraints on the dynamics of G_α we can hope to learn something about general properties of SUSY breaking.

Open Questions

- Is there any difference between D-term and F-term breaking ?

Nati has given some aspects of the answer, but there is more to say. Does the Goldstino know anything about D-terms or F-terms?

Open Questions

- What are the interactions of the Goldstino particle with light matter fields and with itself?

The self-interactions of the Goldstino are often claimed to be captured by the Akulov-Volkov Lagrangian:

$$\mathcal{L}_{A-V} \sim F^2 \det\left[\delta^\mu_\nu - \frac{i}{F^2}\partial_\nu G\sigma^\mu\bar{G} + \frac{i}{F^2}G\sigma^\mu\partial_\nu\bar{G}\right]$$

$$\sim F^2 + G\sigma^\mu\partial_\mu\bar{G} + \cdots$$

Are there corrections? What are we expanding in? Is there a simple description of this Lagrangian? Why is there a symmetry $G_\alpha \rightarrow e^{i\theta}G_\alpha$?

What are the couplings of G_α to matter fermions?

Open Questions

- What is the connection between the UV physics and the theory of Goldstino?

Models of SUSY breaking are sometimes uncalculable and even worse, there is no superpotential. Can we say which UV operators are associated to the low energy Goldstino? Is it well a defined question? Is there a useful superspace description at low energies?

Outline

Our goal here is to give a flavor on how these questions can be addressed and provide some answers

- Symmetry breaking
- The supercurrent multiplet
- Relation to Goldstino
- Connecting the UV and the IR
- Comments

Symmetry Breaking

If a QFT has symmetry generated by some charge Q then we can identify a conserved current

$$\partial^\mu j_\mu = 0 \ .$$

In case the symmetry is spontaneously broken, the operator equation $\partial^\mu j_\mu = 0$ still holds but $Q = \int d^3x j_0$ does not exist. It has a divergence in the IR due to the massless particle.

In spite of this,

$$[Q, \mathcal{O}] \ ,$$

where $\mathcal{O}$ is any local operator is a well defined local operator. The divergent piece cancels when one computes commutators.

Symmetry Breaking

We conclude that even if a symmetry is spontaneously broken,

All the operators sit in representations of the symmetry.

The Supercurrent Multiplet

$\mathcal{N}=1$ supersymmetric theories have a conserved supercurrent $S_{\mu\alpha}$,

$$\partial^\mu S_{\mu\alpha} = 0\ .$$

We can study the multiplet in which this object sits, i.e. calculate $\{Q, S_{\mu\alpha}\}$, $\{Q^\dagger, S_{\mu\alpha}\}$ and so on. The answer was given by Ferrara and Zumino. It can be embedded in a real multiplet with a vector index, $\mathcal{J}_\mu$ (or $\mathcal{J}_{\alpha\dot\alpha}$), such that the expansion in components is

$$\mathcal{J}_\mu = j_\mu + \theta^\alpha\left(S_{\mu\alpha} + \frac{1}{3}(\sigma_\mu\bar\sigma^\rho S_\rho)_\alpha\right) + c.c.$$

$$+(\theta\sigma^\nu\bar\theta)\left(2T_{\nu\mu} - \frac{2}{3}\eta_{\nu\mu}T - \frac{1}{2}\epsilon_{\nu\mu\rho\sigma}\partial^\rho j^\sigma\right) + \frac{i}{2}\theta^2\partial_\mu x^\dagger - \frac{i}{2}\bar\theta^2\partial_\mu x\cdots\ .$$

where $\cdots$ stand for unimportant terms.

The Supercurrent Multiplet

We see that the supercurrent is a friend of:

- The energy momentum tensor (which is also conserved, $\partial^\mu T_{\mu\nu} = 0$).
- An R-current j_μ. It does not have to be conserved. (The R-charge of all the scalars is $2/3$ and all the chiral multiplet fermions have charge $-1/3$.)
- A mysterious complex scalar field x. (This will be our hero.)

The Supercurrent Multiplet

The information encoded in the superfield is equivalent to the current algebra

$$\{Q^{\dagger}_{\dot\beta}, S_{\mu\alpha}\} = \sigma^{\nu}_{\alpha\dot\beta}\left(2T_{\mu\nu} + i\eta_{\nu\mu}\partial j - i\partial_\nu j_\mu - \frac{1}{4}\epsilon_{\nu\mu\rho\sigma}\partial^{[\rho}j^{\sigma]}\right)$$

$$\{Q_\beta, S_{\mu\alpha}\} = 2i(\sigma_{\mu\nu})_{\alpha\beta}\partial^\nu x^\dagger$$

The current algebra holds even if SUSY is broken. We see that the mysterious x is a well defined operator in the theory that can be obtained by varying the supercurrent.

The Supercurrent Multiplet

The best way to package this information is to note that the superfield $\mathcal{J}_\mu$ can be obtained as a solution of

$$\bar{D}^{\dot{\alpha}}\mathcal{J}_{\alpha\dot{\alpha}} = D_\alpha X$$

where X is some chiral field. Solving this in components we discover the superfield $\mathcal{J}$ and the conservation equations. X is given by

$$X = x + \theta\psi + \theta^2 F \ ,$$

with

$$\psi = \frac{2}{3}\sigma^\mu_{\alpha\dot{\alpha}}\bar{S}^{\dot{\alpha}}_\mu \ , \qquad F = \frac{2}{3}T^\mu_\mu + i\partial_\mu j^\mu \ .$$

The operator x is therefore the lowest component of this chiral superfield.

Relation to Goldstino

The supercurrent $S_{\mu\alpha}$ has two different Lorentz representations $(1, 1/2)$, $(0, 1/2)$. If supersymmetry is broken then at the deep IR the $(1, 1/2)$ component is unimportant while the $(0, 1/2)$ component is the Goldstino.

Recall that in the chiral superfield X the fermion $\psi = \frac{2}{3}\sigma^{\mu}_{\alpha\dot{\alpha}}\bar{S}^{\dot{\alpha}}_{\mu}$ is just the spin $1/2$ projection. Therefore, at very long distances,

$$\psi_\alpha \sim G_\alpha \ .$$

More generally, supersymmetry is realized in some way at low energies and therefore the chiral superfield X must flow at low energies to a chiral superfield which contains the Goldstino!

Relation to Goldstino

This superfield X is expected to be nonlinear at low energies, and we see that the operator x flows to be the friend of the Goldstino at low energies

$$\{Q^\dagger, G\} \sim \sigma^\mu \partial_\mu x \ .$$

In addition, the F component of X is just the vacuum energy T^μ_μ.

Relation to Goldstino

So far we have learned that

- The Goldstino can always be embedded in a chiral superfield which can be defined in the UV. This makes perfect sense even in uncalculable examples. So, we know something about certain local operators even in strongly coupled cases.
- X in some sense generalizes the SUSY-breaking “spurion." Therefore, a chiral spurion superfield exists not only in weakly coupled examples!
- The Goldstino can be described in a chiral superfield regardless of whether there are non-zero D-terms for some vector superfields.

Connecting the IR and the UV

Consider the simplest model of SUSY breaking

$$\mathcal{L} = \int d^4\theta \bar{\Phi}\Phi + \int d^2\theta f\Phi + c.c. \ ,$$

with

$$\Phi = \phi + \sqrt{2}\theta\psi_\phi + \theta^2 F_\phi \ .$$

The vacuum energy is $|f|^2$, the Goldstino is ψ_ϕ and we can calculate

$$\mathcal{J}_{\alpha\dot{\alpha}} = 2D_\alpha\Phi\bar{D}_{\dot{\alpha}}\bar{\Phi} - \frac{2}{3}[D_\alpha, \bar{D}_{\dot{\alpha}}]\bar{\Phi}\Phi \ .$$

$$X = \frac{8}{3}f\Phi \ .$$

Connecting the IR and the UV

In this case ϕ is massless and we should keep it at low energies. We see that the Goldstino resides in a chiral multiplet Φ and X naturally coincides with it.

Let us consider a more elaborate “generic" example

$$\mathcal{L} = \int d^4\theta \left(\bar{\Phi}\Phi - \frac{1}{M^2}(\bar{\Phi}\Phi)^2 \right) + \int d^2\theta f \Phi + c.c. \; .$$

$\phi = 0$ is a good vacuum. The vacuum energy is $|f|^2$ and the spectrum is

$$m_\phi^2 = |f|^2/M^2 \; , \qquad m_{\psi_\phi} = 0 \; .$$

ψ_ϕ is the Goldstino. Our description at low energies should include only ψ_ϕ as a physical field. We should integrate out ϕ.

Connecting the IR and the UV

Let us go to zero-momentum, the Lagrangian is

$$\mathcal{L}_{zm} = -\frac{1}{M^2}(2\phi F_\phi - \psi_\phi^2)(2\phi^\dagger F_\phi^\dagger - \bar{\psi}_\phi^2) \ .$$

We integrate out ϕ and get

$$\phi = \frac{\psi_\phi^2}{2F_\phi} \ .$$

This is independent of the high energy parameter M. We describe the low energy theory with ψ_ϕ and F_ϕ. We see that we need to use the non-linear superfield

$$\Phi_{NL} = \frac{\psi_\phi^2}{2F_\phi} + \sqrt{2}\theta\psi_\phi + \theta^2 F_\phi \ .$$

Connecting the IR and the UV

$$\Phi_{NL} = \frac{\psi_\phi^2}{2F_\phi} + \sqrt{2}\theta\psi_\phi + \theta^2 F_\phi$$

satisfies the interesting constraint

$$\Phi_{NL}^2 = 0 \ .$$

We should use this field to write the low energy action. The term $(\bar{\Phi}\Phi)^2$ vanishes of course, so we remain with

$$\mathcal{L} = \int d^4\theta \bar{\Phi}_{NL}\Phi_{NL} + \int d^2\theta f \Phi_{NL} + c.c. \ .$$

This gives the Akulov-Volkov action. We see that it can be described very simply with a constrained superfield $\Phi_{NL}^2 = 0$. (Rocek reached a similar conclusion by studying nonlinear representations of the SUSY algebra.)

Connecting the IR and the UV

After solving for the auxiliary field we get

$$\mathcal{L} = f^2 - i\psi_\phi \sigma^\mu \partial_\mu \bar{\psi}_\phi + \frac{1}{f^2}\psi_\phi^2 \partial^2 \bar{\psi}_\phi^2 + \cdots$$

Note that the last term is just substituting $\phi = \frac{\psi_\phi^2}{2F_\phi}$ in $\phi\partial^2\phi^\dagger$. This is just the Akulov Volkov Lagrangian.

We see that

- The derivation and result are independent of M.
- All the terms in the A-V action have twice more fermions than derivatives.

Connecting the IR and the UV

This integration out procedure can be continued and the result is that the expansion is in a “scaling" number

$$\partial^k \psi^l$$

is assigned scaling $S = 2k - l$. The A-V terms have $S = 0$ and are universal. Corrections have higher S and are dependent on high energy physics in general.

The A-V Lagrangian is invariant under $\psi_\phi \to e^{i\theta}\psi_\phi$. This must be the case since it is independent of the UV physics! Higher scaling corrections break this symmetry.

Connecting the IR and the UV

Having understood the IR, let us connect with $\mathcal{J}$ and X. We get that the exact expression for X is

$$X = \frac{8}{3} f \Phi - \frac{1}{3M^2} \Phi^2 \bar{D}^2 (\bar{\Phi}^2) \ .$$

However, at low energies $\Phi^2 = 0$ and therefore at low energies

$$X = \frac{8}{3} f \Phi_{NL} \ .$$

Thus the operator X is itself nilpotent at low energies:

$$X^2 = 0 \ .$$

This operator equation holds at long distances in every SUSY breaking theory!

Comments

We can add matter field. Repeating the same procedure and get the two operator equations

$$\Phi_{NL}^2 = 0 \ ,$$

$$\Phi_{NL} Q_{NL} = 0 \ ,$$

where Q_{NL} is a nonlinear version of some matter chiral field. The universal part of the Lagrangian is given by

$$\mathcal{L} = \int d^4\theta \, (\bar{\Phi}_{NL}\Phi_{NL} + \bar{Q}_{NL}Q_{NL}) + \int d^2\theta f \Phi_{NL} + c.c.$$

Solving the constraints we find the leading interaction

$$\frac{1}{f^2}\bar{\psi}_q\bar{\psi}_\phi\partial^2\psi_q\psi_\phi \ .$$

Comments

Many theories of SUSY breaking have a spontaneously broken R-symmetry, one could ask what is the low energy description. By similar methods we can see that the constraint becomes

$$\left(\bar{\Phi}_{NL}\Phi_{NL} - v^2\right)^3 = 0 \ .$$

Solving this in components shows that the radial mode disappears and we remain with the R-axion and the Goldstino.

Conclusions

- Supersymmetry and superspace are useful even when SUSY is broken.
- We can follow the supercurrent multiplet $\mathcal{J}_{\alpha\dot{\alpha}}$ and the associated X along the flow.
- X flows to the Goldstino multiplet and satisfies $X^2 = 0$ at long distances.
- We can efficiently find the interesting interactions of the Goldstino and other particles, as well as additional axions by using constrained superfields and writing supersymmetric actions with them.
- The deep low-energy theory is completely universal and cannot tell apart pure F-term breaking from cases in which there are also D-terms.

More Applications

- Coupling of two Goldstino particles to one gauge field:

$$\frac{< D >}{F^2} F_{\mu\nu} \partial^\mu \bar{G} \bar{\sigma}^\nu G \ .$$

This operator can arise with some coefficient which is model dependent. The operator is nonzero only if the gauge field is massive.

- Integrating out this gauge field we get the operator

$$\partial_\mu (G) \partial^\mu \bar{(}G) \psi \bar{\psi} \ .$$

This is a different linear combination from the operator we have seen before but with the same scaling. There are also other sources for this operator.