Generalised fluxes and de Sitter vacua

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w/ A. Guarino, J.M. Moreno, in progress

Contents

Moduli stabilisation

- Moduli are present in any string model
- Many of them parametrise physical quantities, i.e. must acquire a VEV if we are to obtain the Standard Model at low energies
- Their stabilisation is likely to be linked to the breakdown of SUSY
- They have potential cosmological interest

Past history

Nilles'84

- Partial success in stabilising moduli through non-perturbative effects: multiple gaugino condensation in the heterotic

Krasnikov'87

Casas, Lalak, Munoz, Ross'90

- Minima that broke SUSY were AdS

BdC, Casas, Munoz '92

- Very steep potentials: runaway dilaton

Brustein, Steinhardt'93

N=1, D=4 SUGRA

Scalar potential:

$$V_F = \mathrm{e}^K \{ K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2 \}$$

K, Kähler potential, W superpotential

$$D_I W = K_I W + W_I$$ Kähler derivative

In general this is a multivariable potential

SUSY preserved: $D_I W = 0\,, \forall I$

AdS -> $V = -3\mathrm{e}^K |W|^2$

Minkowski -> $V = W = 0$

SUSY broken: $D_I W = 0\,, \text{for some } I$

$$\left.\begin{array}{c}\text{AdS}\\ \text{Minkowski}\\ \text{dS}\end{array}\right\}\quad \text{all possible}$$

but mostly AdS solutions found!

Recent progress

Flux compactification in type IIB opens up a new path in model building

Giddings, Kachru, Polchinski'02

KKLT proposal: combine fluxes and np effects

Kachru, Kallosh, Linde, Trivedi'03

$$K = -3\ln(T + \bar{T}), \quad W = W_0 + A\mathrm{e}^{-aT}$$

T overall modulus, W_0 effective flux parameter

Np piece generated by D3 branes or gaugino condensation

This potential has a SUSY preserving minimum

$$D_T W = 0\,, W \neq 0$$ -> AdS

KKLT add anti-D3 branes, parametrised as a D-term

$$V = V_F + \frac{k}{T_R^2}, \;\; T_R \equiv \mathrm{Re}T$$

The new piece uplifts the minimum to dS and breaks SUSY explicitly

In a SUGRA context, the D-term can come from an anomalous U(1)

Burgess, Kallosh, Quevedo'03

Achucarro, BdC, Casas, Doplicher'06

In type IIA it is possible to generate a superpotential for all closed string moduli just from fluxes

Grimm, Louis'05

Derendinger, Kounnas, Petropoulos, Zwirner'05

Villadoro, Zwirner'05

DeWolfe, Giryavets, Kachru, Taylor'05

Cámara, Font, Ibáñez'05

These can be NS-NS (H), R-R (F) and also geometric (f) fluxes

Graña, Minasian, Petrini, Tomasiello'06

Andriot'08

Caviezel, Koerber, Kors, Lüst, Tsimpis, Zagermann '08

Aldazábal, Font'08

To recover T-duality between IIA and IIB we have to introduce non-geometric (Q,R) fluxes in the latter

Shelton, Taylor, Wecht'05,'06

For a symmetric orbifold, $T^6/Z_2 x Z_2$

$$K = -\ln(-i(S-Sb))-3\ln(-i(\tau-\tau b))-3\ln(-i(U-Ub))$$

$$W = P_1(\tau) + S\, P_2(\tau) + U\, P_3(\tau)$$

- S is the axiodilaton
- U is the Kähler (IIB), complex structure (IIA) modulus
- τ is the complex structure (IIB), Kähler (IIA) modulus

W is linear in S and U, whereas

$$P_1(\tau) = a_0 - 3a_1\tau + 3a_2\tau^2 - a_3\tau^3$$

$$P_2(\tau) = -b_0 + 3b_1\tau - 3b_2\tau^2 + b_3\tau^3$$

$$P_3(\tau) = 3(c_0 + (\hat{c}_1 + \check{c}_1 - \c{c}_1)\tau - (\hat{c}_2 + \check{c}_2 - \c{c}_2)\tau^2 - c_3\tau^3)$$

In type IIB language

$a_0,\ldots,a_3$ given by R-R fluxes

$b_0,\ldots,b_3$ given by NS-NS fluxes

$c_0,\ldots c_3$ given by Q fluxes ← non geometric

They all must be integers

Comments on method

- The scalar potential is a function of 3 complex fields (S, U, T), or 6 real variables
- It contains polynomials of high degree
- The stationary conditions, $\partial V=0$, are difficult to solve in general
- Most results in the literature look for SUSY solutions, solving the F-equations

Shelton, Taylor, Wecht'06
Font, Guarino, Moreno'08

- Powerful techniques based on computational algebra are available

Gray, He, Lukas, Ilderton'09

Further analytic insight

Gomez-Reino, Scrucca'06

A calculation of the Hessian matrix, imposing V=0 and the existence of a minimum, returned a powerful constraint on the curvature only

$$\sum_k R_k \left(\frac{G_k G_{\bar{k}}}{G_{k\bar{k}}} \right)^2 < 6$$

where

$$G_k = K_k + \frac{W_k}{W}$$

$$R_k = \frac{G_{kk\bar{k}\bar{k}}}{G_{k\bar{k}}^2} - \frac{G_{kk\bar{k}} G_{kk\bar{k}}}{G_{k\bar{k}}^3}$$

Curvature bound

In particular, for moduli fields in string models

$$K = -\sum_{i=1}^{n} n_i \ln(\phi_i + \bar{\phi}_i), \; W = W(\phi_1, \ldots, \phi_n)$$

The necessary condition for having a minimum with V=0 becomes

$$\sum_{i=1}^{n} n_i > 3$$

independent of W!

Summarising so far

enormous efforts have been devoted to understanding flux compactification in string theory

non geometric fluxes provide a T-dual description between IIA and IIB

the resulting potential is a polynomial in the different fields and difficult to minimise

so far only SUSY and/or AdS solutions have been found

Goal: can we find SUSY breaking, dS solutions?

To achieve this we combine two different pieces of research

The classification of all subalgebras satisfied by Q fluxes in IIB (on $T^6/Z_2 x Z_2$)

Font, Guarino, Moreno'08

A no-go theorem on the existence of de Sitter vacua and inflation in IIA

Hertzberg, Kachru, Taylor, Tegmark'07

Generalised fluxes and algebras

Font, Guarino, Moreno'08

Consider IIB compactified on $(T^2 x T^2 x T^2)/(Z_2 x Z_2)$

NS-NS (H) and Q fluxes can be regarded as structure constants of an extended (12d) symmetry algebra of the compactification

Shelton, Taylor, Wecht'06

Dabholkar, Hull'06

The algebra has isometry generators (Z) and gauge symmetry generators (X)

$$[X,X] = Q\,X, \quad [Z,X] = Q\,Z, \quad [Z,Z] = H\,X$$

Jacobi identities (H Q=0, Q Q =0) and tadpole cancellation conditions restrict the possible values of the flux constants (a,b,c)

Even more, Q can only be one of 5 possible 6d subalgebras: SO(4), SU(2)+U(1)^3, SU(2)$\oplus$U(1)^3, nil, SO(3,1)

A suitable reparametrisation of the U modulus makes it modular invariant

W is now simpler and the F-equations can be solved analytically

No-go theorem and inflation

Hertzberg, Kachru, Taylor, Tegmark'07

Instead of looking at V in terms of W and K, let's write it in terms of the contributions from the different fluxes in IIA

$$V = V_H + V_{Fp} + V_{D6} + V_{O6}$$

$V_H \sim 1/y^3\sigma^2$

$V_{Fp} \sim 1/y^{3-p}\sigma^4$

$V_{D6} \sim 1/\sigma^3$

$V_{O6} \sim -1/\sigma^3$

$\sigma = \mathrm{Im}(S)$, $y = \mathrm{Im}(\tau)$

V_H and V_p are positive definite

This potential satisfies

$$-y\, \partial V/\partial y - 3\sigma\, \partial V/\partial \sigma = 9V + \sum pV_p$$

Then, at an extremum, V<0!

Way out: consider geometric (V_f) and non geometric (V_Q, V_R) fluxes.The previous condition reads

$$-y\, \partial V/\partial y - 3\sigma\, \partial V/\partial \sigma = -2\, V_f - 4V_Q - 6V_R + 9V + \sum pV_p$$

V_f used to construct de Sitter vacua

Silverstein'08

Haque, Shiu, Underwood, Van Riet'08

Our work

BdC, Guarino, Moreno'09

We consider N=1 orientifolds of the $Z_2 x Z_2$ orbifold

- it is its own mirror under T-duality (U and τ swap roles)

- IIA and IIB compactified on these structures are equivalent under T-duality

IIB with O3/O7 $\leftrightarrow$ IIA with O6 $\leftrightarrow$ IIB with O5/O9

Strategy:

i) we have a complete classification of allowed fluxes in IIB (based on the Q subalgebra)

ii) we can map this IIB potential to a IIA one and use no-go theorems to look for de Sitter vacua

Moreover: it seems that only compactifications on $Z_2 x Z_2$ may allow for inflation

Flauger, Paban, Robbins, Wrase'08

The generalised scalar potential

We work with real fields,

$$S = s + i\,\sigma,\ U = t + i\,\mu,\ \tau = x + i\,y$$

in terms of which the potential reads

$$V = a(y,\mu,\phi)/\sigma^2 + b(\mu)/\sigma^3 + c(y,\mu,\phi)/\sigma^4$$

where $\phi=(s,t,x)$ are the axions and

a accounts for generalised NS-NS fluxes

b accounts for localised sources

c accounts for R-R fluxes

We can now study moduli stabilisation in a systematic way, relating a, b, c to the original flux parameters in IIB

This already tells us the signs of the different contributions, facilitating the search for de Sitter vacua

Most of the search can be done analytically because S and U enter W linearly.

$$\partial V/\partial s = 0 \Rightarrow s_0 = s_0(x_0, y_0)$$

$$\partial V/\partial t = 0 \Rightarrow t_0 = t_0(x_0, y_0)$$

The physical parts of S,U (σ,μ) can be stabilised analytically by imposing V=0

$$\sigma_0 = \sigma_0(x_0,y_0)$$

$$\mu_0 = \mu_0(x_0,y_0)$$

We are left with $\partial V/\partial x=\partial V/\partial y = 0$

After replacing all other fields these are two nasty equations that require numerical analysis

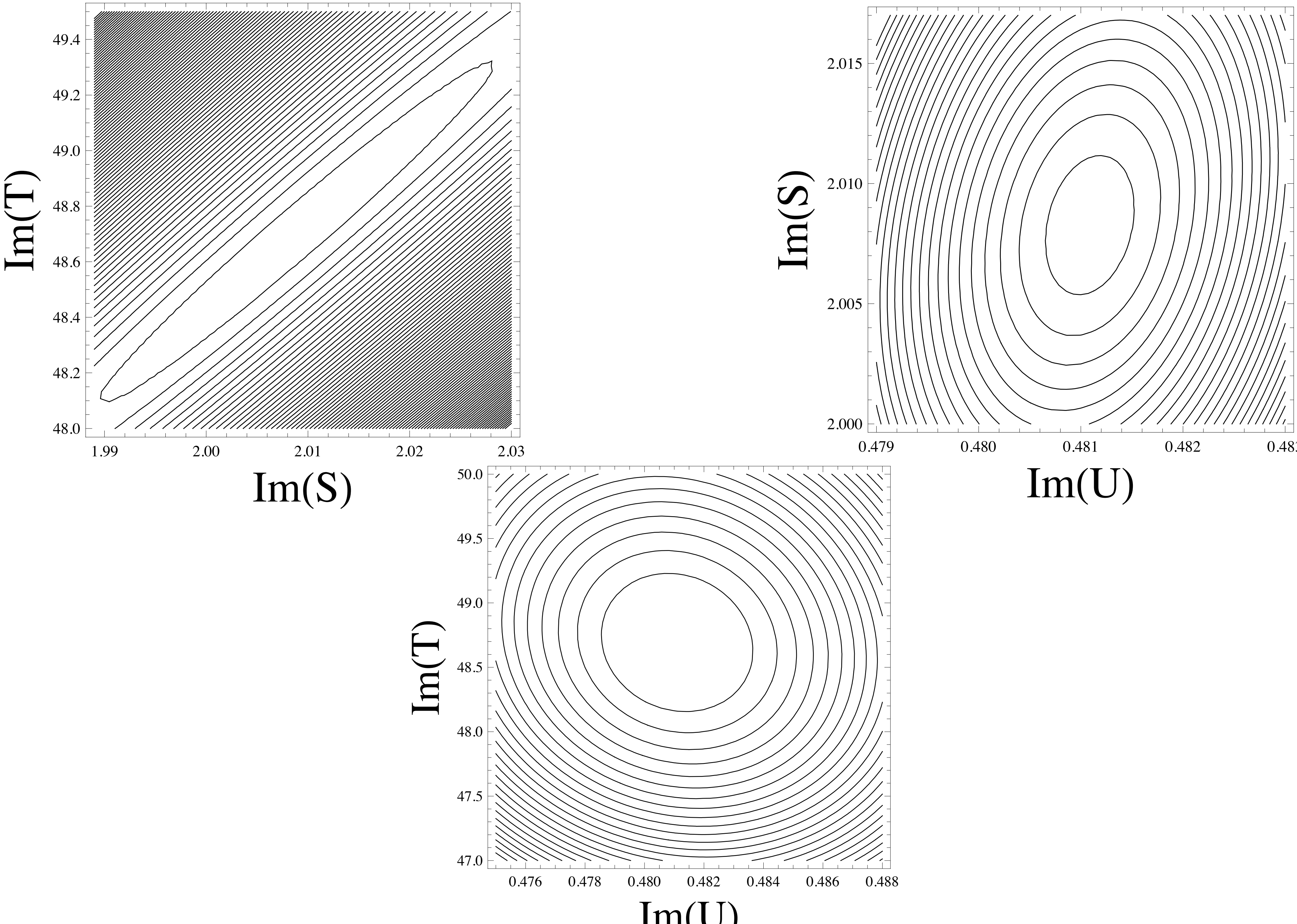
Results

SO(4), SU(2)+U(1)^3, SU(2)$\oplus$U(1)^3, nil, SO(3,1)

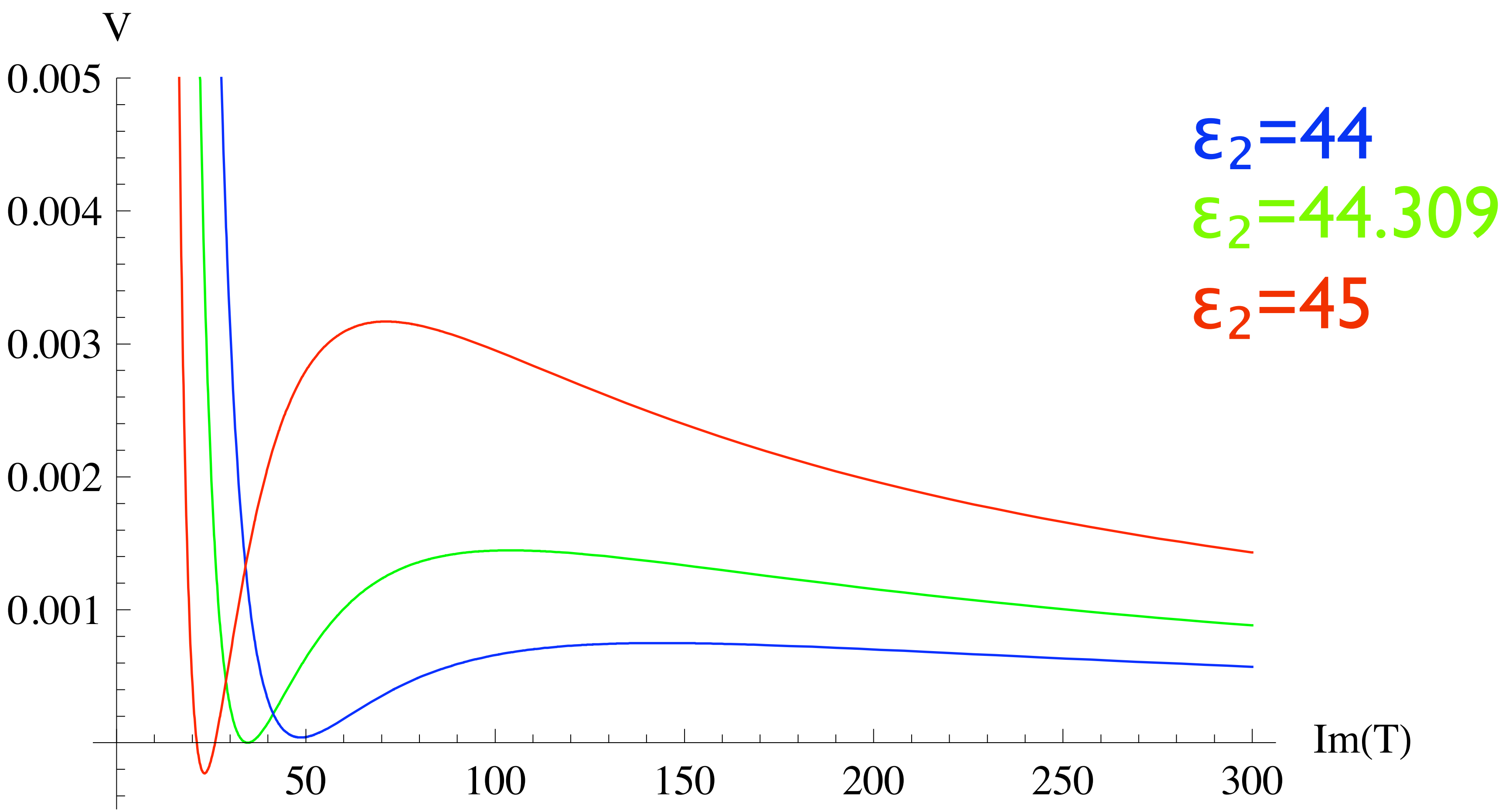
Of the 5 possible algebras, three of them cannot give de Sitter vacua (no-go theorem applies)

At most one can have Minkowski minima with one tachyonic direction

However SO(3,1) contains plenty of de Sitter vacua with all moduli stabilised


Im(T)
49.4
49.2
49.0
48.8
48.6
48.4
48.2
48.0
1.99
2.00
2.01
2.02
2.03
Im(S)
Im(S)
2.015
2.010
2.005
2.000
0.479
0.480
0.481
0.482
0.483
Im(U)
Im(T)
50.0
49.5
49.0
48.5
48.0
47.5
47.0
0.476
0.478
0.480
0.482
0.484
0.486
0.488
Im(U)

The vacua oscillate between AdS and dS according to the value of a certain flux parameter (ε_2)



Conclusions (I)

- The last few years have seen exciting progress in the field of moduli stabilisation in string theory, due to fluxes
- In particular we have discussed T-dualities between IIA and IIB using non geometric fluxes
- The resulting W can stabilise all moduli, but the potential is quite involved
- Treating it requires new systematic, analytic and numerical methods

Conclusions (II)

- Strategy: use algebraic results in IIB, which simplifies W and the number of fluxes and no-go theorems on the existence of de Sitter vacua in IIA
- All combined results in a systematic and feasible search with plenty of de Sitter (SUSY breaking) minima
- Still lots to do...