# Circle compactifications:

## index, Chern-Simons terms, and chiral dynamics

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arxiv:0812.2085[hep-th](JHEP0903:027, 2009) +
work in progress

## outline

motivation & brief review of older and recent work on $R^3 x S^1$

our work on the index theorem and a few remarks on Chern-Simons terms (not too technical, I hope)

a study of a chiral gauge theory example and an exotic mechanism of confinement (showing chiral theories are weird)

finally, to earn the right to speak at this
**wonderful "SUSY Breaking '09" workshop** -
an (incomplete) argument about the I=3/2 SU(2) (old) ISS proposal of SUSY breaking

conclusions

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rigour

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"circle compactification" = $R^3 \times S^1$

L - circumference of *nonthermal* circle
R-radius, whenever difference matters
so more precisely $R^{2,1} \times S^1$

**why bother?**

various "deformations" of 4d field theories have been useful to study aspects of nonperturbative dynamics

**especially true in supersymmetry, where consistency with all calculable deformations play an important role, e.g.:**

- circle compactification of N=2 4d SYM
  (Seiberg, Witten)
- circle compactification of N=1 4d SYM
  (Aharony, Intriligator, Hanany, Seiberg, Strassler; Dorey, Hollowood, Khoze, Mattis)

in the supersymmetric case, using holomorphy, one argues that with supersymmetric b.c. there is a smooth 4d limit

- many cases studied about 10 years ago

for nonsupersymmetric theories, interest in "circle compactification" deformations has been rekindled more recently
(Unsal w/ Shifman & Yaffe, in various combinations since about 2007)

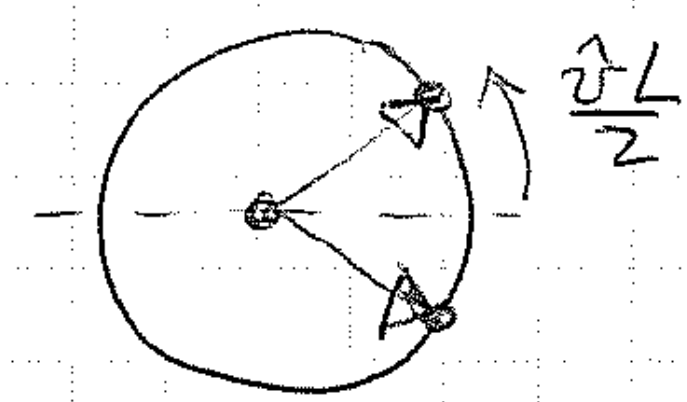
for pure YM theory, $R^3 \times S^1$ is equivalent to a thermal setup - as temperature increases, thermal fluctuations cause a deconfinement phase transition - center symmetry breaks and the trace of the Polyakov loop obtains a nonzero expectation value:

e.g., SU(2):

eigenvalues of Wilson line ("Polyakov loop")

$$A_4|_\infty = \hat{v}\, t_3 \,, \text{ say } t_3 = \frac{\sigma_3}{2}$$

$$e^{\pm i \frac{\hat{v} L}{2}}$$




$$\Omega|_\infty = P \exp \oint_{S_1} A_4(\infty, x_4)\, dx^4$$

if $\frac{\hat{v}L}{2} \neq 0, \pi, \ldots$ SU(2) broken to U(1) at high scale, e.g. $\hat{v} = \frac{\pi}{L}$

for centrally symmetric vacuum, $\mathrm{Tr}\Omega = e^{i\frac{\hat{v}L}{2}} + e^{-i\frac{\hat{v}L}{2}} = 0$

and theory weakly coupled (if no electrically charged light states)

- however, "Casimir energy" in pure YM makes eigenvalues attract forcing and breaking center symmetry (Gross, Pisarski & Yaffe...):

$$\mathrm{Tr}\Omega = e^{i\frac{\hat{v}L}{2}} + e^{-i\frac{\hat{v}L}{2}} \neq 0 \quad \hat{v} = 0$$

- on thermal circle in theories with fermions this is generic - thermal fluctuations always cause deconfinement, assuming 4d theory confines (see, e.g., various Casimir calculations in Unsal & Yaffe)

- **bad news - as far as learning about 4d theory:**

1.) means phase transition with L in theories with (approximate) center symmetry, so no smooth
2.) loss of calculability - abelianization - at small L - since the idea is to have a calculable small-L limit which is smoothly connected to 4d

however, with periodic (non-thermal) b.c. this is not always so:

- if fermion rep judiciously chosen, at small circle, Casimir energy may cause eigenvalues to repel and thus pick center-symmetric vacuum (Unsal & Yaffe - e.g., many adjoints + possibly a few other complex representations) a particular case of the above is supersymmetry, where Casimir energy = 0, so, can simply pick center-symmetric vacuum as a point on moduli space

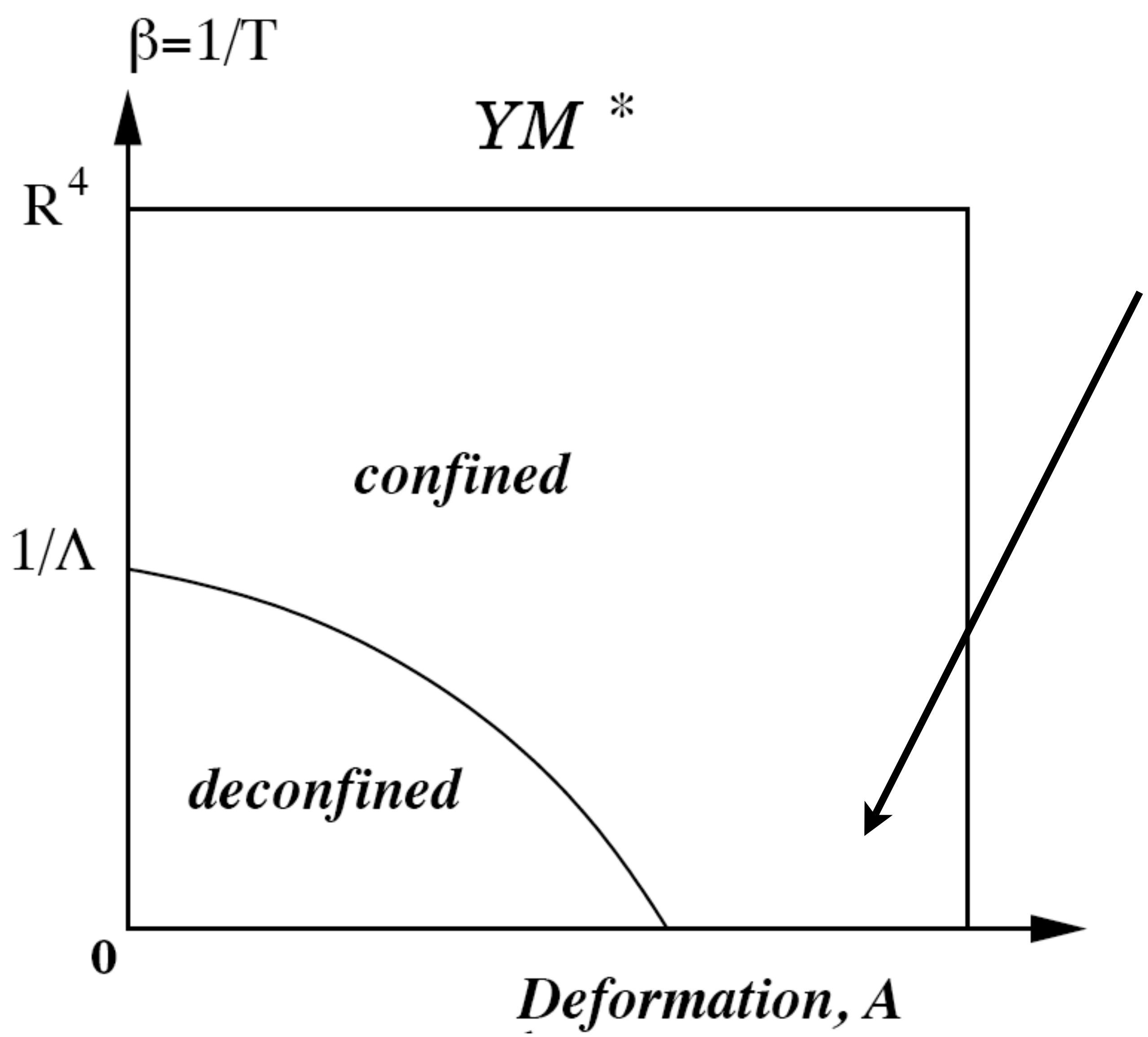
- else, one can apply a "double-trace deformation" on the circle, forcing a center-symmetric expectation value for general representations (Shifman &Unsal)

deformation term ~ $$\frac{1}{L^3} \int d^3x \sum_n a_n | \, \mathrm{Tr}\Omega(x)^n |^2$$

the role of the deformation is two-fold:

1.) at infinite L the deformation is turned off - if theory has no anomaly free continuous global symmetries which could break as L changes, there is no other obvious phase transition that can occur with L (it appears that discrete chiral symmetries broken at any L); center symmetry unbroken at small L as well as large L =**“smoothness conjecture”**

2.) ensures center symmetric vacuum, and thus calculability, at small L - **both perturbative and nonperturbative dynamics is then under control**

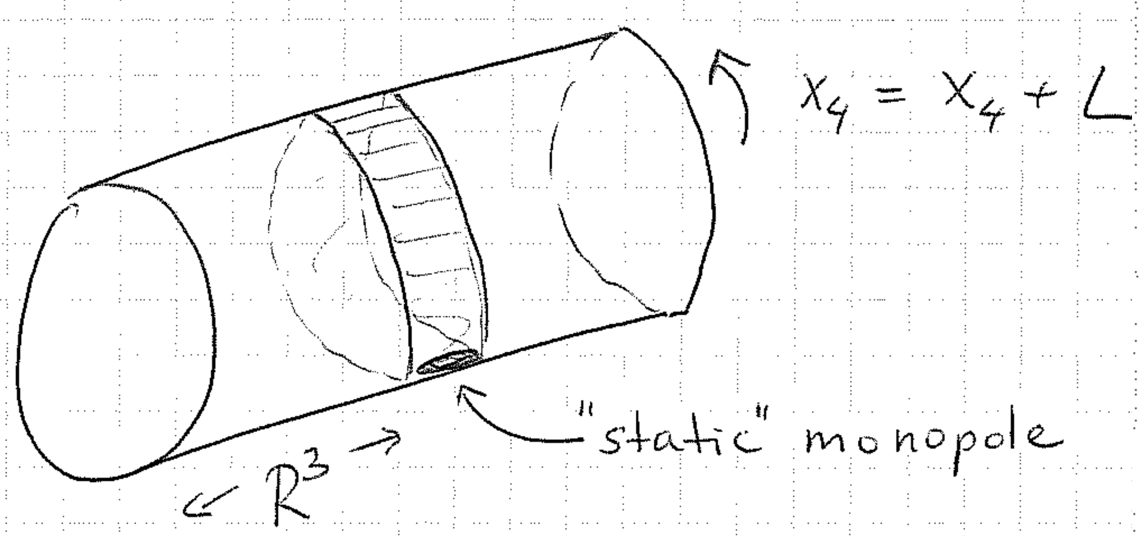


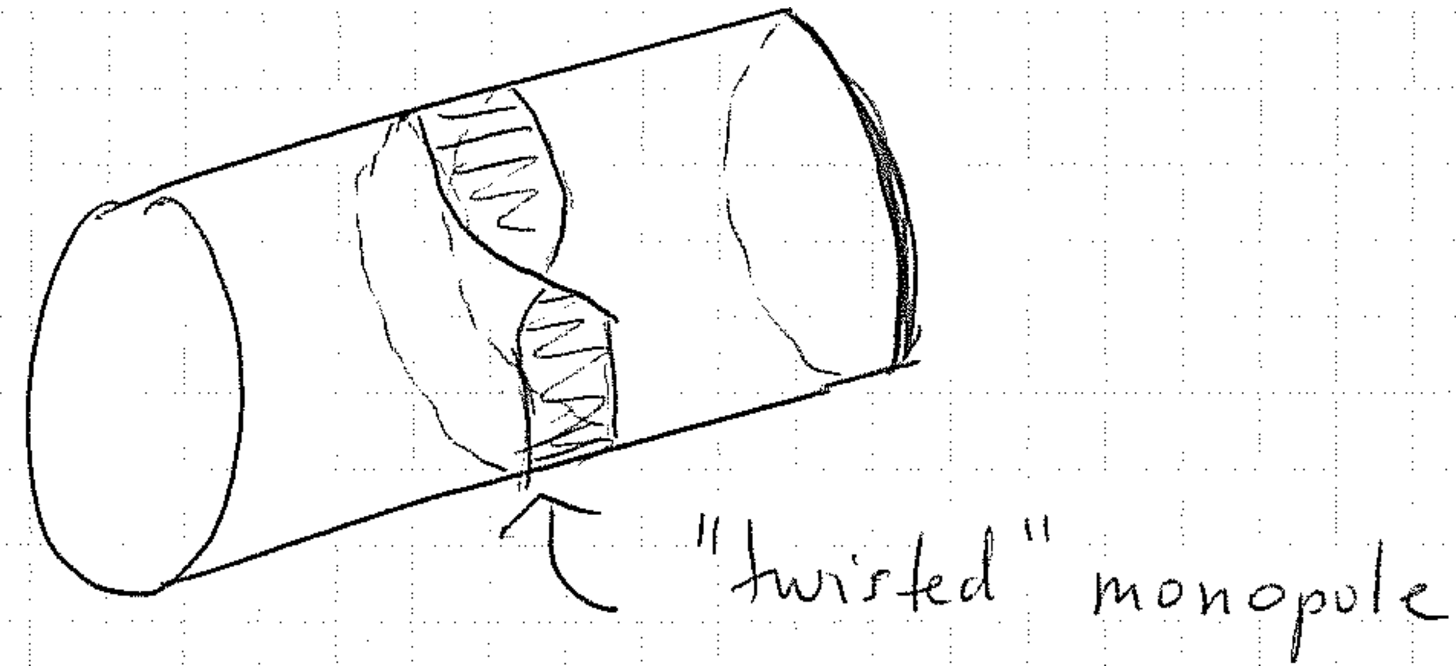

- here the theory is solvable by abelian duality: abelian confinement and mass gap can be show analytically (cf Seiberg-Witten theory)

- continuous connection to large radius as no gauge invariant order parameter can distinguish

- in some cases there already exist lattice studies of this story at various L - seem consistent with smoothness conjecture modulo usual (here: technical) issues of chiral limit on the lattice ... but stay tuned.

SU(2) broken to U(1): "static" Prasad-Sommerfield monopole is the main topological background - an instanton in the $R^3 \times S^1$ theory - all other instantons can be obtained from it by judicious combinations of "gauge transformations" and holonomy shifts




"static" (BPS) monopoles

"KK monopoles"
(P. Yi & K. Lee; P. van Baal ~ 1997)
- clearly, specific to locally 4d case
- have opposite magnetic charge to that of self-dual (BPS) monopole

monopoles can destabilize perturbative vacuum and generate mass of the dual photon (Polyakov, 1977) [ + KK monopoles in the locally 4d case of interest to us] *"monopoles can destabilize perturbative vacuum and generate mass of the dual photon"* **- a quick reminder:**

abelian duality in 3d
photon - scalar

$$F = *d\sigma \qquad L = \frac{1}{2}(\partial\sigma)^2$$

masslessness of scalar

$$U(1)_{\text{flux}} : \sigma \to \sigma + c$$

topological current in electric theory

$$\mathcal{J}_\mu = \partial_\mu \sigma = \frac{1}{2}\epsilon_{\mu\nu\rho}\bar{F}_{\nu\rho} = F_\mu$$

conservation - absence of magnetic charge

$$\partial_\mu \mathcal{J}_\mu = \partial_\mu F_\mu = 0$$

presence of monopoles means continuous symmetry reduced

$$\partial_\mu \mathcal{J}_\mu = \partial_\mu F_\mu = \rho_m(x)$$

monopole-induced mass of dual photon - physics of Debye mechanism (Polyakov, 1977)

$$L = \frac{1}{2}(\partial\sigma)^2 - e^{-S_0}(e^{i\sigma} + e^{-i\sigma})$$

with monopoles included, only discrete shift of dual photon remains:

$$\sigma \to \sigma + 2\pi$$

$e^{-S_0}\cos\sigma$ - example of a **“topological flux operator”**, i.e., induced by topological objects with nonzero magnetic charge; here given in pure SU(2) broken to U(1) YM 3d; similar in locally 4d

$U(1)_{\text{flux}}$ if present, forbids (magnetic) flux carrying operators.

- in theories with fermions **“topological flux operators”** due to monopoles and KK monopoles will carry fermion zero modes

- what are the relevant index theorems?

$R^3$ Callias, 1978 (E. Weinberg, 1980)

$R^3 \times S^1$ Nye & M. Singer, 2000 (Unsal & EP, 2008)

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# An $L^2$-Index Theorem for Dirac Operators on $S^1 \times \mathbb{R}^3$

Tom M. W. Nye and Michael A. Singer

where, in APPENDIX A. ADIABATIC LIMITS OF $\eta$-INVARIANTS

we found:

$$\text{ind}\,(D^+_{\mathbb{A}}) = \int_X \text{ch}(\mathbb{E}) + \frac{1}{\mu_0} \sum_\mu \epsilon_\mu c_1(E_\mu)[S^2_\infty]$$
$$= \int_X \text{ch}(\mathbb{E}) - \frac{1}{2}\overline{\eta}_{\text{lim}} \quad (22$$

two obvious questions:

1.) where does this come from?

2.) what number is it equal to in a given topological background?

Unsal, EP; arxiv:0812.2085[hep-th](JHEP0903:027, 2009)

we gave a derivation along physicist's lines (i.e. one we can understand) generalizing E. Weinberg's work on Callias index in monopole background on R3 to R3 x S1

calculated index for various representations/backgrounds

showed & explained jumps of index as ratio holonomy/radius varied

finally, techniques used to calculate index also good to study generation of CS terms and argue that some QCD-like theories should possess a CS phase on R3xS1 operator trace identities (as in E.Weinberg) + anomaly equation (new element, as theory is locally 4d)

. . . . . . . . . .

as per Nye-Singer formula, the index has two contributions:

$$I_{\mathcal{R}} \equiv I^1_{\mathcal{R}} + I^2_{\mathcal{R}}$$

**topological charge contribution:**

$$I^2_{\mathcal{R}}(0) = -2T(\mathcal{R})Q = -\frac{T(\mathcal{R})}{16\pi^2}\int d^3x \int_0^L dy\; G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}$$

## “eta-invariant” contribution:

$$I^1_{\mathcal{R}}(0) = -\frac{1}{2} \sum_{j=1}^{N} (n_j - n_{j-1}) \sum_{p=-\infty}^{\infty} \frac{\hat{v}_j + \frac{2\pi p}{L}}{|\hat{v}_j + \frac{2\pi p}{L}|} \quad = -\frac{1}{2} \sum_{j=1}^{N} (n_j - n_{j-1})\, \eta_j[0]$$

$$A_4\big|_{\infty} = \mathrm{diag}(\hat{v}_1, \hat{v}_2, \ldots \hat{v}_N) \qquad \hat{v}_1 < \hat{v}_2 < \ldots < \hat{v}_N, \qquad \sum_{i=1}^{N} \hat{v}_i = 0$$

KK sum = eta invariant of operator $i\frac{d}{dy} + \hat{v}_j$

with eigenvalues $\hat{v}_j + \frac{2\pi p}{L}$ def. by analytic continuation of $\eta[v_j, s] \equiv \eta_j[s] \equiv \sum_{\lambda \neq 0} \frac{\mathrm{sign}\lambda}{|\lambda|^s}$

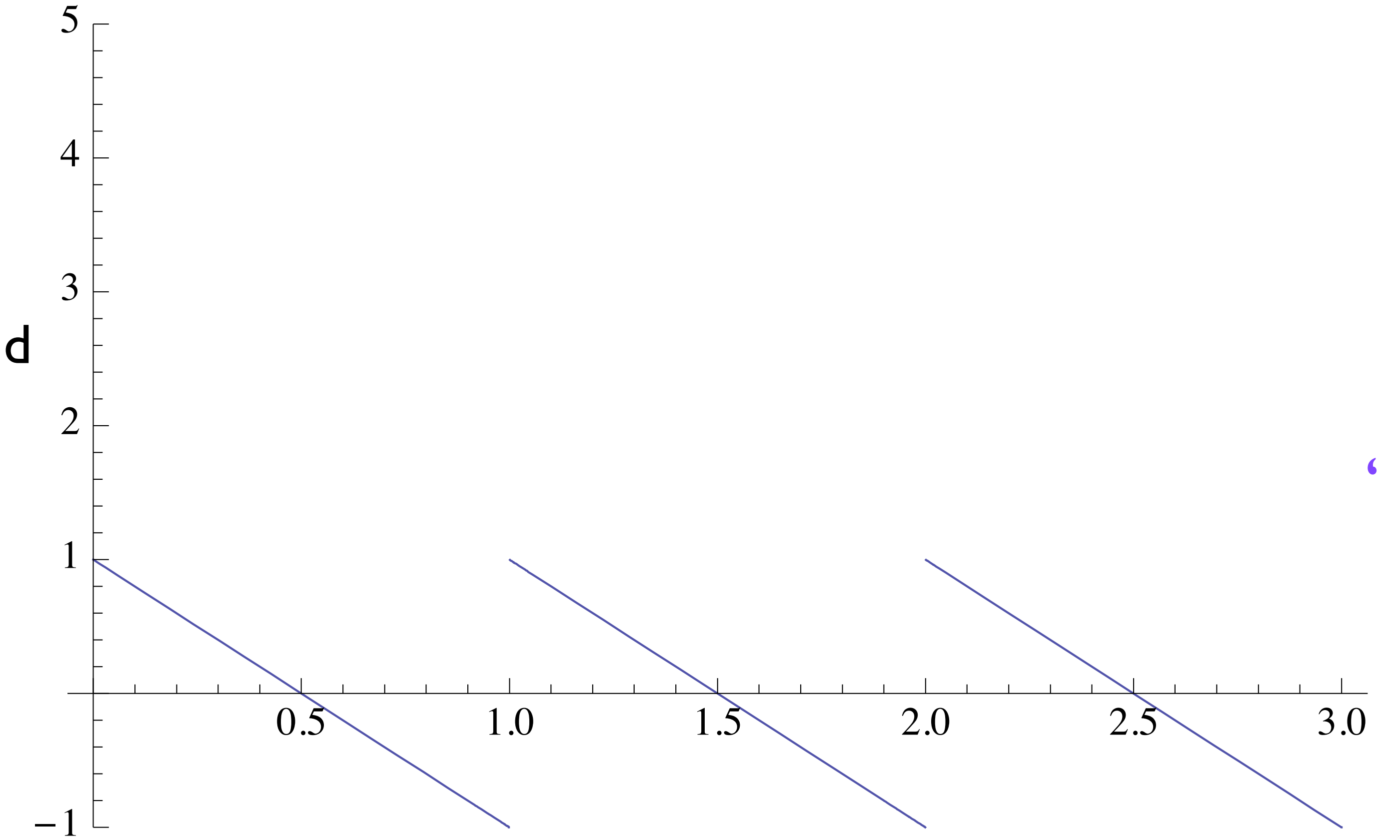
**both terms are not integers, but their sum is**

- best to look at plot, e.g., for SU(2) fundamental:

## “eta-invariant” contribution:

for SU(2): one kind of monopole, one value of holonomy - $\frac{\hat{v}L}{4\pi}$

index in charge-1 static monopole background




“sawtooth” function

$\frac{\hat{v}L}{4\pi}$

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nonperiodic (linear function of v)

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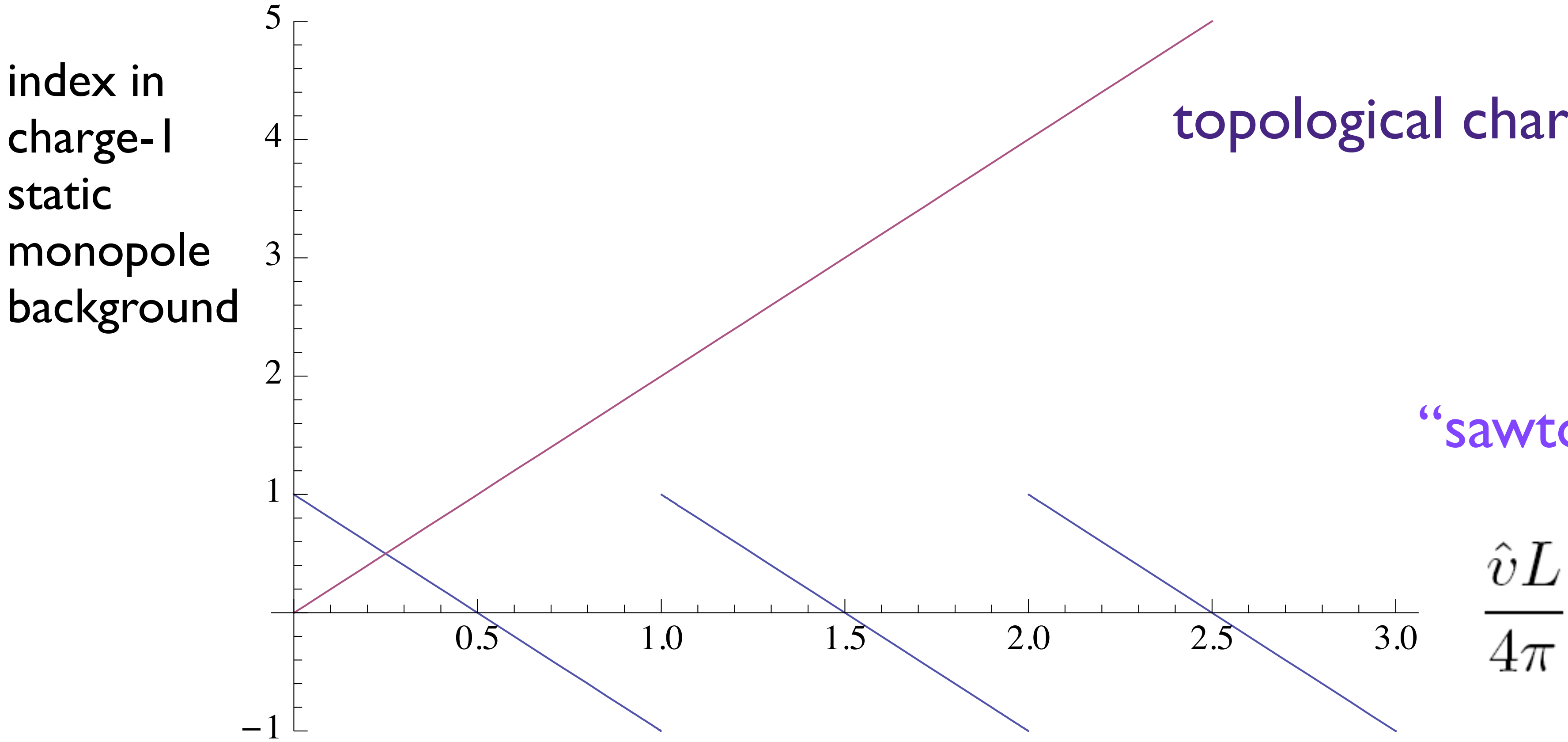

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"sawtooth" function, periodic

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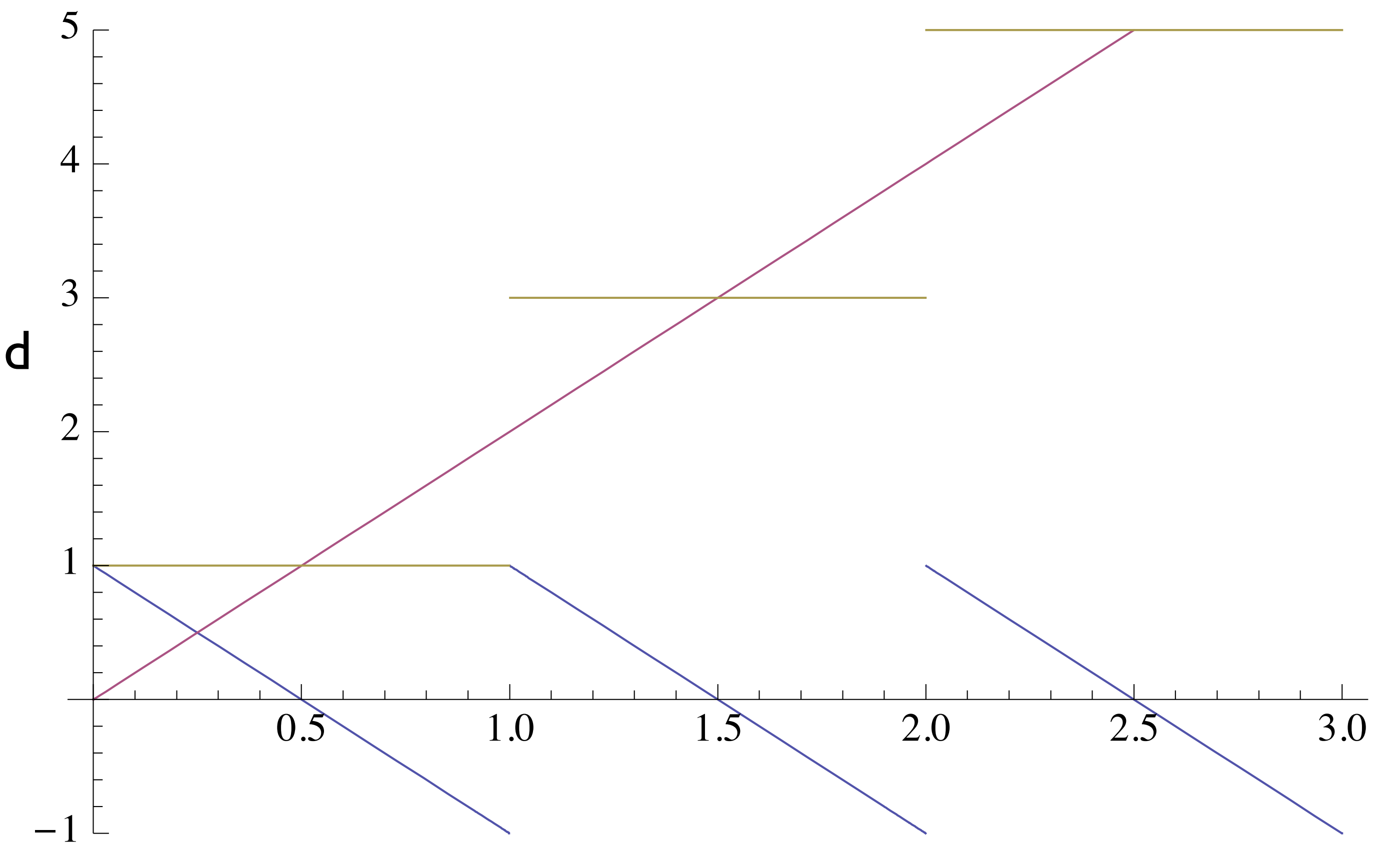


$\frac{\hat{v}L}{4\pi}$

sum = index

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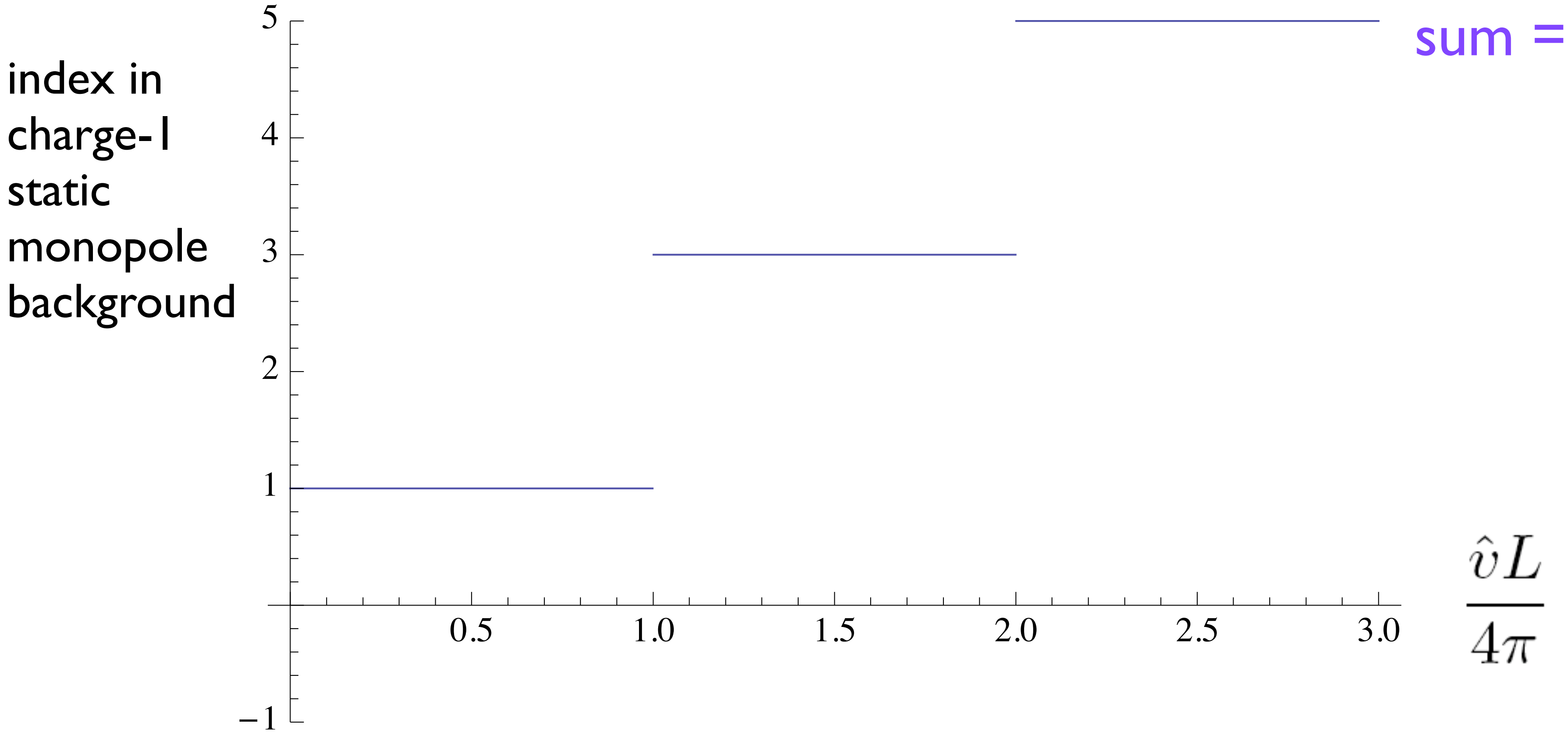


sum = index

# comments on the index formula:

**v < 1/(2R), R=L/(2 pi) -**
should get 3d result, KK scale
and monopole scale
separated

index in
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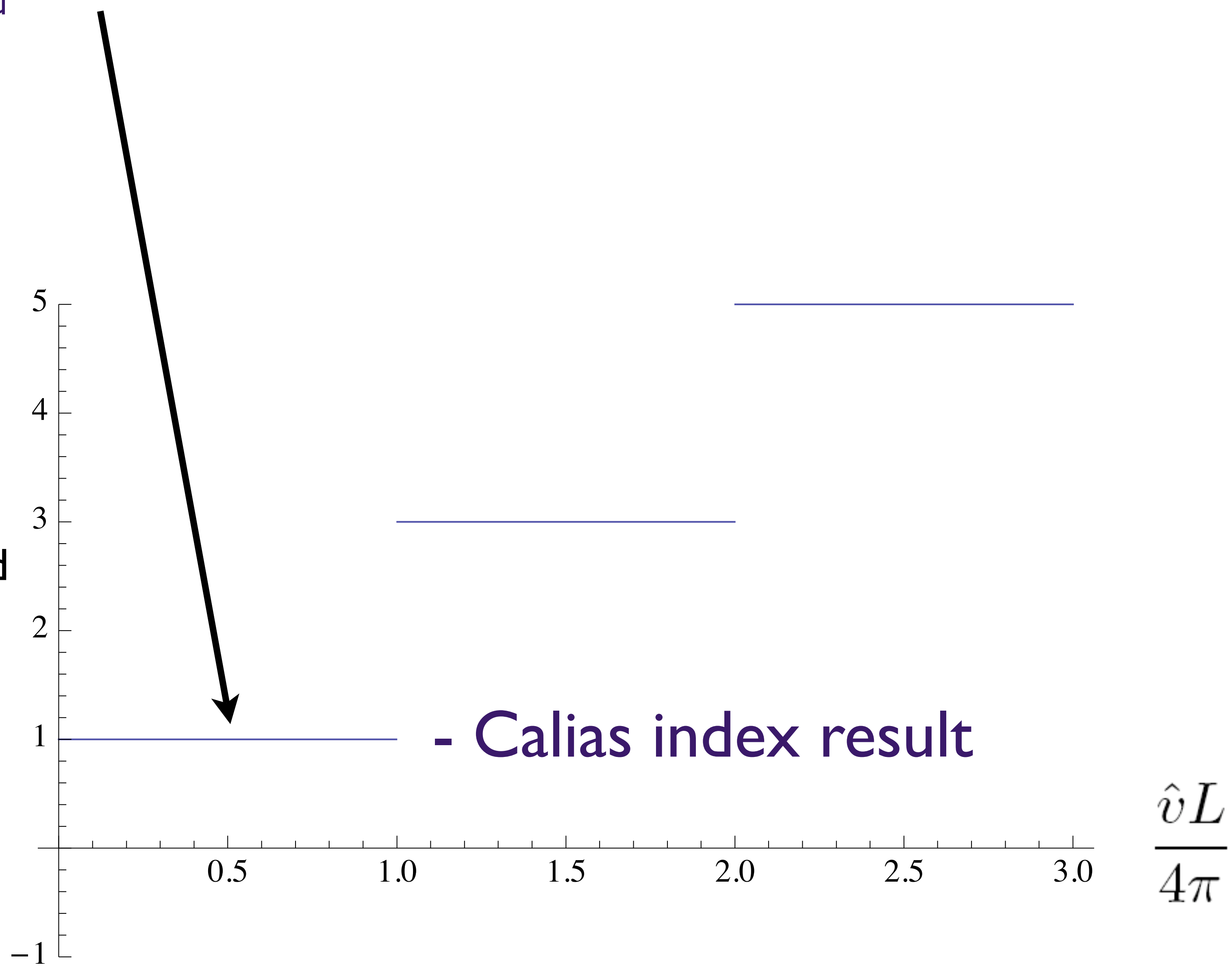


- Calias index result

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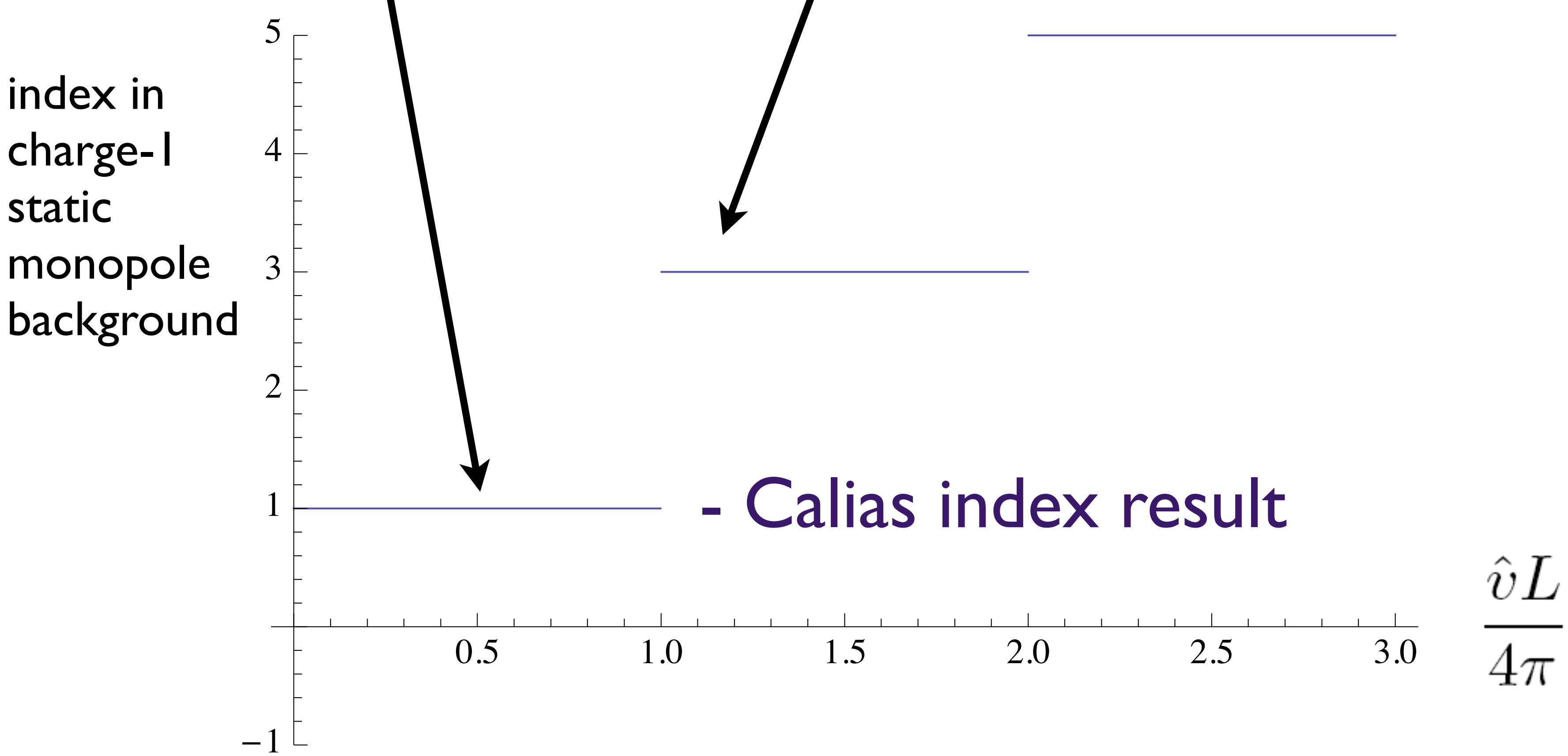
**index jumps as v crosses each 1/(2R) KK threshold**
**-** non-normalizable zero modes of KK fermions become normalizable two per 3d (static) zero mode, so jump by 2



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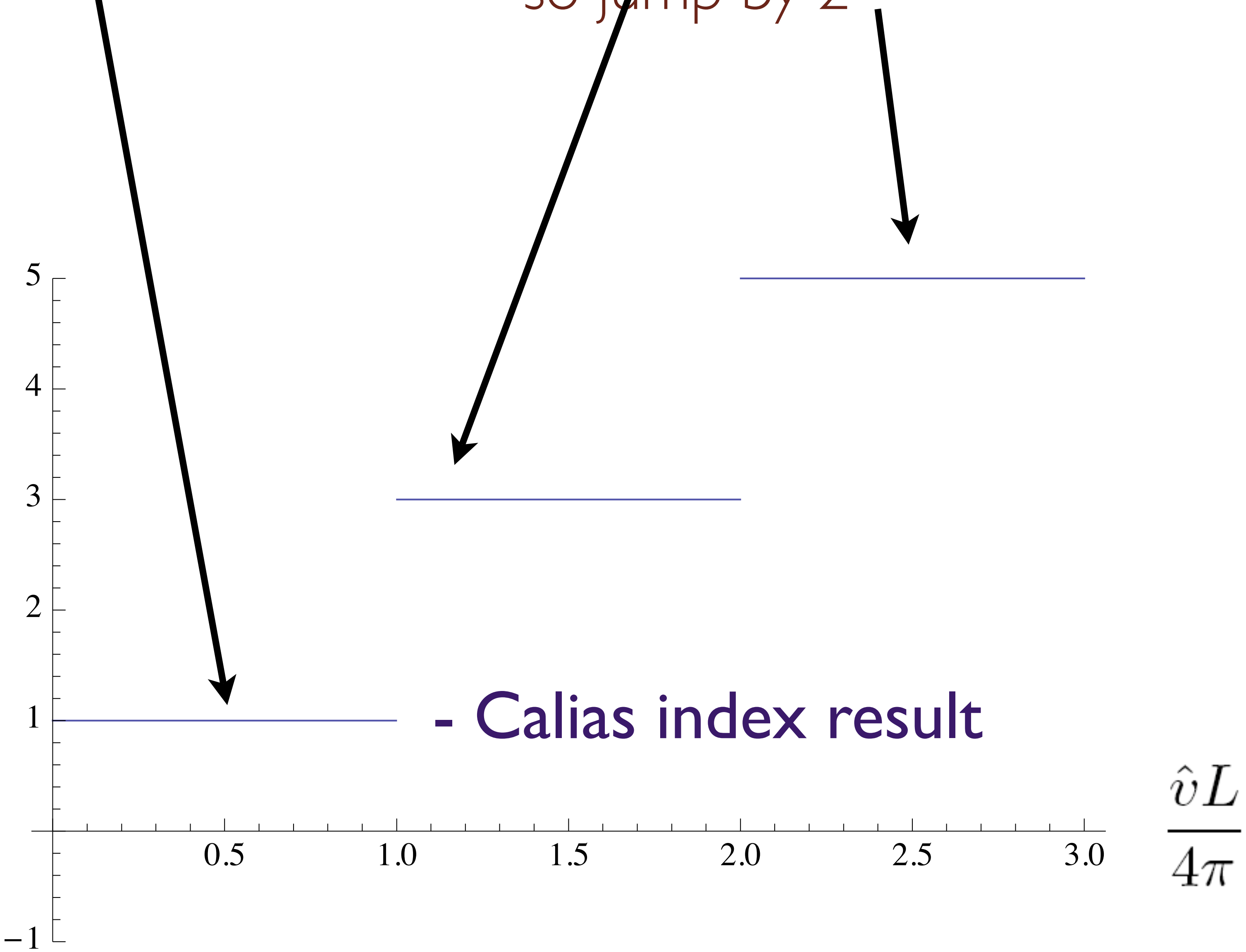


**- Calias index result**

satisfying, nice math, etc., but should we ever care about v>1/(2R)...?

should we ever care about $v > 1/(2R)$...?

- in a non-SUSY theory, probably not (at least in center-stabilized setup)

- in SUSY theory, with SUSY b.c., perturbative potential for v vanishes (Casimir energy=0)

hence, nonperturbative (super)potential can be generated by monopoles&KK monopoles

semiclassically calculable: Davies, Hollowood, Khoze, Mattis, 1999, schematically:

$$W = \exp(Z) + c\,\exp(-Z), \quad \mathrm{Re}\, Z \sim v$$

used along with holomorphy to obtain 4d value of gaugino condensate agreement with weakly coupled 4d instanton calculation (remember weak vs. strong instanton calculation issue in SYM) however, W must be periodic function of 2Rv just as Casimir energy is (after KK sum) thus need to sum over KK partners of monoples and KK monopoles,

these are obtained by starting with static solutions in vacua with 2Rv>1

**this was done in a $R^4 \times S^1$ study of compactified 5d Seiberg-Witten curves, but not in the 4d SYM setup** Csaki, Erlich, Khoze, EP, Shadmi, Shirman, 2001

**hence these jumps of index at 2Rv>1 would be relevant for a calculation of W in pure SYM that would give periodic answer**

- such as found by Dorey in N=1* theory, alas not by an explicit calculation answers I told you so far:

we gave a derivation along physicist's lines (i.e. one we can understand) generalizing E. Weinberg's work on Callias index in monopole background on R3 to R3 x S1

calculated index for various representations/backgrounds

showed & explained jumps of index as ratio holonomy/radius varied

answers about index left to talk about:

finally, techniques used to calculate index also good to study generation of CS terms and argue that some QCD-like theories should possess a CS phase on R3xS1 same tools (eta-invariant) give a general formula for the CS term in this geometry, as a function of matter representation and Wilson line that is turned on

for example, if $A_4$ is the Wilson line on the circle:

$$k_{ab} = -\mathrm{tr}(\{T^a, T^b\} A_4)\frac{L}{2\pi} + \mathrm{tr}(T^a T^b \mathrm{sign} A_4)$$

↑ "4d" contribution (~chiral anomaly)

↑ "3d" contribution (~"parity anomaly")

- sometimes making sure 3d contribution vanishes requires choosing background with care

- also, one can turn on Wilson lines for anomalous U(1)

anomaly-free U(1) bckgd Wilson lines do not generate CS, except may be by 3d term these must be discrete Wilson lines, since Wilson lines = b.c. on circle , must be in anomaly-free subgroup of U(1) in order to make sense; equivalently - above CS is then properly quantized

discrete Wilson lines give rise to gauge invariant CS terms which dominate at long distances (monopoles & friends are "excised") - topological phase in the IR

these are "chirally twisted" vectorlike theories: e.g. YM with a number of adjoint Weyl fermions - in SYM, i.e. one adjoint (say, SUSY inessential here), twist by an element of the $Z_{2N}$ anomaly-free subgroup of $U(1)_R$ - CS term generated for all b.c. but the periodic and antiperiodic one

**generally, one finds a rich phase structure as a function of various allowed deformations - most of it specific to circle compactification, so perhaps of interest to cond.-mat. quantum critical points etc.?**

**as an application**, consider an example of a chiral gauge theory with only a discrete global symmetry - where smoothness conjecture is expected to hold:

how does SU(2) theory with a single I=3/2 Weyl fermion behave?
chiral gauge theory, asymptotically free, (Witten) anomaly free

do calculable $R^3 x S^1$ deformations have to say anything useful about SU(2) I=3/2 theories?

consider theory with center-stabilizing deformation

index theorem says that monopoles (1) and KK-monopoles (2) have:

$$\mathcal{I}_1 = 4, \qquad \mathcal{I}_2 = 6, \qquad \mathcal{I}_{\rm inst} = \mathcal{I}_1 + \mathcal{I}_2 = 10$$

- so, the corresponding topological flux operators are

M $$\mathcal{M}_1 = e^{-S_0} e^{i\sigma} \psi^4, \qquad \overline{\mathcal{M}}_1 = e^{-S_0} e^{-i\sigma} \bar{\psi}^4,$$ anti-M

KK $$\mathcal{M}_2 = e^{-S_0} e^{-i\sigma} \psi^6, \qquad \overline{\mathcal{M}}_2 = e^{-S_0} e^{i\sigma} \bar{\psi}^6,$$ anti-KK

$S_0 = \frac{8\pi^2}{Ng^2} = \frac{8\pi^2}{2g^2}$ - monopole action,

$d\sigma = *F$ is the dual photon

- anomaly-free discrete chiral symmetry is an exact symmetry of microscopic theory, hence topological shift symmetry intertwines with it to maintain invariance of monopole/KK operators:

$$\mathbb{Z}_{10}: \qquad \psi^4 \to e^{i\frac{8\pi k}{10}} \psi^4, \qquad \sigma \to \sigma - \frac{4\pi k}{5}$$

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**implies that the leading purely bosonic term -** the only one that can generate a mass gap in the gauge sector **- allowed in the dual photon action is**

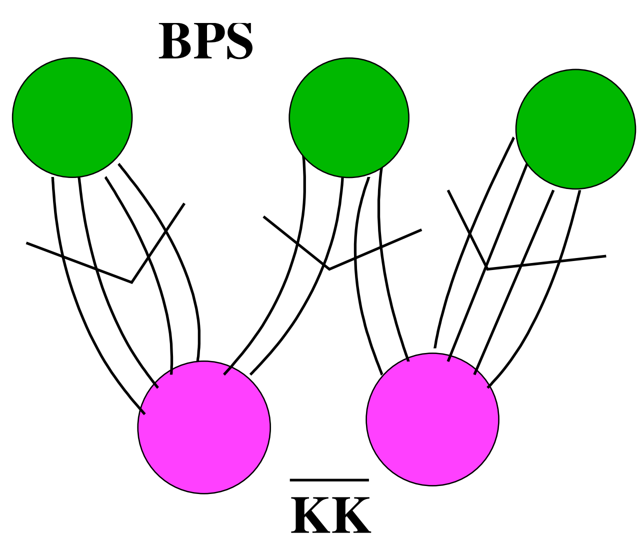
$$e^{-5S_0}(e^{i5\sigma} + e^{-i5\sigma}) \sim e^{-5S_0}\cos 5\sigma$$

but what is the topological object that gives rise to this dual-photon mass?

- must have magnetic charge 5
- must have no fermion zero modes to generate Debye mass for dual photon

inspect charges of M (+1) and anti-KK (+1) as well as # of fermion zero modes M (4) and KKbar (6)

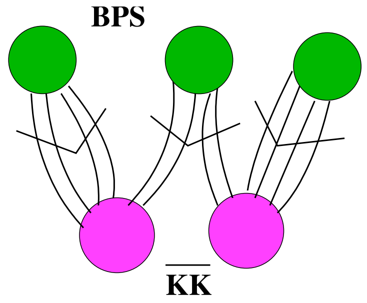
so, the leading cause of mass gap for the dual photon in chiral SU(2) with I=3/2 must be a "magnetic quintet", a bound state of 3 BPS and 2 anti-KK monopoles with magnetic charge 5




cartoon of "magnetic quintet"

that the theory is locally 4d is crucial for having nonzero mass gap
(else exact U(1) flux symmetry forbids it):

$$m_\sigma \approx \Lambda(\Lambda L)^4 \text{ for } \Lambda L \ll 1 \text{ with 4d scale } \Lambda$$




cartoon of "magnetic quintet"

- monopoles and anti-KK monopoles repel each other electromagnetically (same magnetic charge objects)
- however, fermions are known to generate attractive interactions between instantons and should be responsible for "gluing" 2M + 3 anti-KK - if indeed the theory confines
- this is like the "magnetic bion" of Unsal's that generates mass gap in QCD with adjoints, including SYM - a bound state of M and anti-KK (and we know it should exist because of SUSY)
- except, unlike Mithat's "bion", here the attraction is short range as fermions are massive and it is hard to analytically establish existence of object - and hence *show* that the theory has confinement at small L - the dynamics is likely to involve the nonabelian sector and fermion back-reaction

*- despite chiral nature of theory, rep. is pseudoreal, has real determinant and so it can be studied on the lattice (phase of chiral det is the main difficulty for lattice chiral theories); "cooling" (i.e. smoothing) of lattice field configurations one could look for charge-5 objects (as usual, issues with taking chiral limit will slow progress, but situation is bound to improve in future)*

*- so, this is an, in principle, testable story...*

## **another application:** ISS(henker) SU(2) SUSY - breaking proposal

some ancient history, 1995:

$$I = 3/2 \qquad Q_{abc} \qquad u = Q^4$$

$$U(1)_R \qquad [\lambda] = +1,$$

$$[Q] = \frac{3}{5}, \qquad [\psi] = -\frac{2}{5},$$

$$[u] = \frac{12}{5}, \qquad [\psi_u] = [q^3\psi] = \frac{7}{5}$$

$W = cu^{5/6}\Lambda^{-1/3}$ allowed by symmetries but bad weak-coupling limit, so c=0

**if theory confines**, with u - the single massless composite saturating ‘t Hooft (as is easily checked), adding W = u gives “simplest” susy breaking theory in IR

**does it?** - probably not, most likely CFT: $b_0$ = 1; also Intriligator, 2005 (“a-maximization”)

hard to be sure, ‘cause difficult to study: strong coupling, none of the usual SUSY deformations

does circle compactification deformation - the only one available - say anything?

## start in 3d, work our way "up" to 4d:

| | $U(1)_{R'}$ | $U(1)_A$ |
|---|---|---|
| $\lambda$ | $1$ | $0$ |
| $\psi$ | $-1$ | $1$ |
| $Q$ | $0$ | $1$ |
| $Y$ | $2$ | $-4$ |

$Y \sim e^{-\phi + i\sigma + \dots}$ $\qquad u = Q^4$

$W[Y, u] = Yu$ $\qquad$ is it there?

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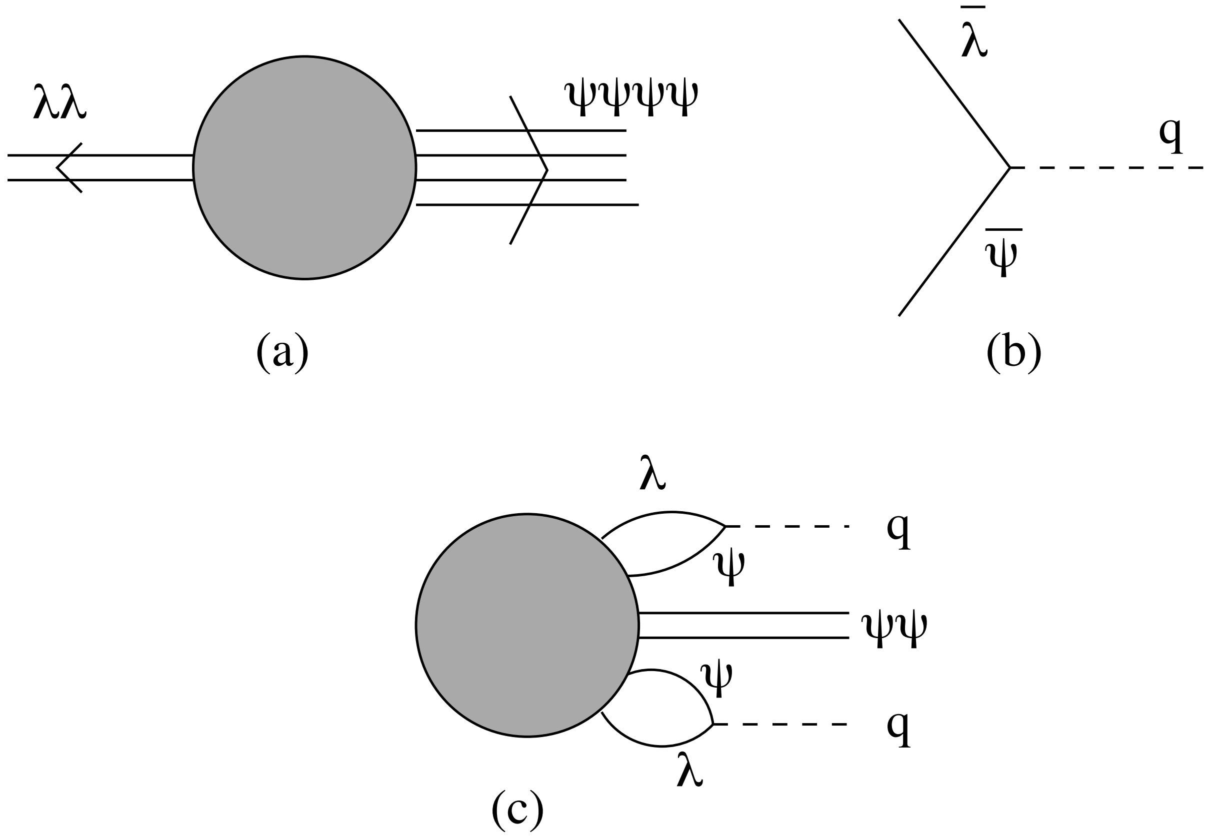
$W[Y,u] = Yu$ **is it there?**



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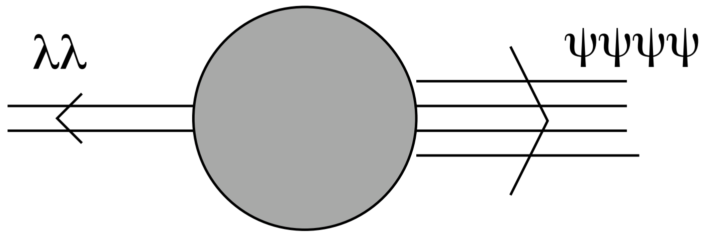
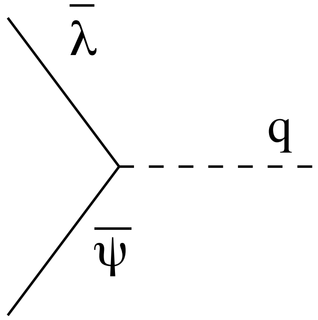
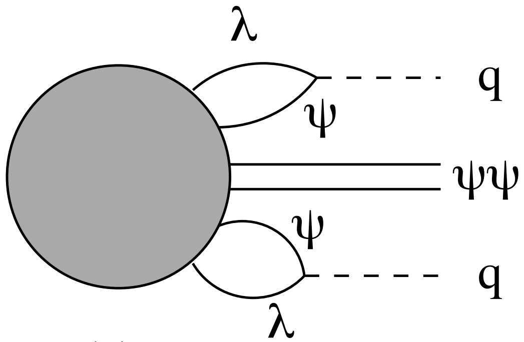
| $\psi$ | $-1$ | $1$ |
| $Q$ | $0$ | $1$ |
| $Y$ | $2$ | $-4$ |



(a)



(b)



(c)

$Y \sim e^{-\phi+i\sigma+\ldots}$ $\qquad u = Q^4$

$W[Y,u] = Yu$ $\qquad$ is it there?

$$e^{-S_0}e^{-\phi+i\sigma}\psi^4\lambda^2(x)\left(\int d^3y\; q\bar{\lambda}\bar{\psi}(y)\right)^2 \longrightarrow \widetilde{\mathcal{M}}_1 \equiv e^{-S_0}e^{-\phi+i\sigma}q^2\psi^2 \ .$$

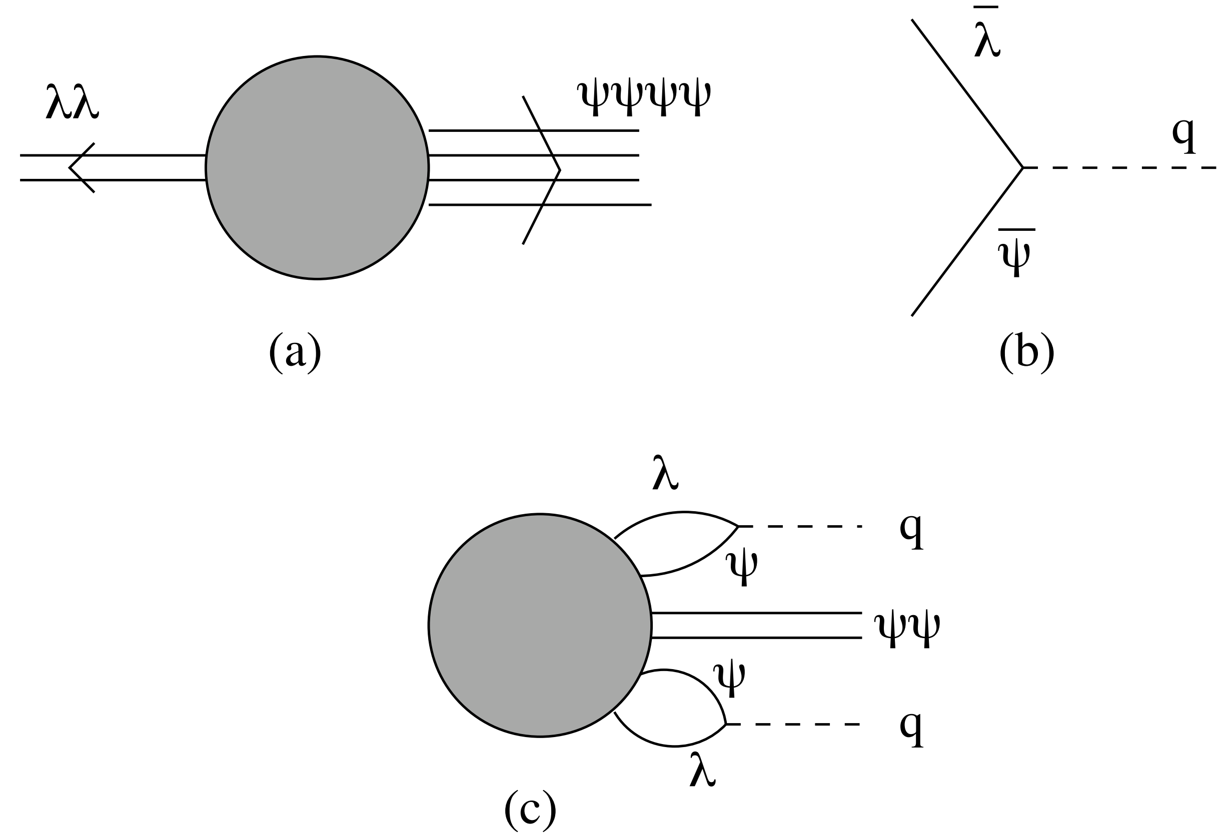
$$W[Y,u] = Yu \ = YQ^4, \qquad \widetilde{\mathcal{M}}_1 = \frac{\partial^2 W}{\partial q^2}\psi\psi$$

$$V(\phi, q) = e^{-2S_0}e^{-2\phi}q^6(1+\mathcal{O}(q^2))$$

, so Coulomb branch not lifted (not no region where Y and U both light)

# start in 3d, work our way "up" to 4d:

| | $U(1)_{R'}$ | $U(1)_A$ |
|---|---|---|
| $\lambda$ | $1$ | $0$ |
| $\psi$ | $-1$ | $1$ |
| $Q$ | $0$ | $1$ |
| $Y$ | $2$ | $-4$ |




$Y \sim e^{-\phi+i\sigma+\dots}$ $\qquad u = Q^4$

$W[Y,u] = Yu$ $\qquad$ is it there?

$$e^{-S_0}e^{-\phi+i\sigma}\psi^4\lambda^2(x)\left(\int d^3y\, q\bar{\lambda}\bar{\psi}(y)\right)^2 \longrightarrow \widetilde{\mathcal{M}}_1 \equiv e^{-S_0}e^{-\phi+i\sigma}q^2\psi^2 \ .$$

$$W[Y,u] = Yu \ = YQ^4, \qquad \widetilde{\mathcal{M}}_1 = \frac{\partial^2 W}{\partial q^2}\psi\psi$$

$$V(\phi,q) = e^{-2S_0}e^{-2\phi}q^6(1+\mathcal{O}(q^2))$$

, so Coulomb branch not lifted (not no region where Y and U both light)

- Y and u do not obey 't Hooft for R', A parity anomalies
- at origin need new degrees of freedom
- most likely 3d CFT of strongly coupled "quarks", gluons, gluinos

compare with "similar" vectorlike theory- Aharony, Hanany, Intriligator, Seiberg, Strassler 1997

SU(2) with 4 doublets, also start in 3d work towards 4d:

$$W = -Y\ \mathrm{Pf}(M) \sim -Y M_{12} M_{34}$$

$$\mathcal{M}_1 = e^{-S_0} e^{-\phi + i\sigma} \psi_1 \psi_2 \psi_3 \psi_4 \lambda^2$$

↓ Yukawa "lifting"

$$\widetilde{\mathcal{M}}_1 \equiv e^{-S_0} e^{-\phi + i\sigma} (q_1 q_2 \psi_3 \psi_4 + \ldots) = \sum_{a,b} \frac{\partial^2 W}{\partial q_a \partial q_b} \psi_a \psi_b\ .$$

as far as I can tell, ours is the first explanation of origin of this W

- here, in contrast, Y and M obey 't Hooft for parity anomalies
- at origin no need for new degrees of freedom 3d CFT of Y, M composites - recall cubic superpotential relevant in 3d (Wilson-Fisher fixed point)
- "turning on" finite radius - new finite action topological objects - the KK monopoles - contribute to W - two zero modes (fund. only)

$$W = -Y\ \mathrm{Pf}(M) + \eta Y \longrightarrow \mathrm{Pf}(M) = \eta, \quad Y = 0.$$

$$V(\phi, q) = e^{-2S_0} e^{-2\phi} q^6 (1 + \mathcal{O}(q^2)) + e^{-2S_0} e^{-2\phi}$$

looks "runaway" but recall periodic....

- Coulomb branch lifted, hence vacuum at strong coupling (Y=0)
- Y gets mass, M's become free - integrating out Y gives 4d quantum constraint - nice match to known 4d results, consistent with various flows

back to SU(2) with I=3/2 - "turn on" nonzero L:

as already shown in 3d, monopoles contribute to superpotential on C-branch:

$$\mathcal{M}_1 = e^{-S_0} e^{-\phi + i\sigma} \psi^4 \lambda^2 \longrightarrow \widetilde{\mathcal{M}}_1 = e^{-S_0} e^{-\phi + i\sigma} q^2 \psi^2 \qquad W[Y, Q] = Y Q^4$$

however, as opposed to vectorlike theory, index of KK monopoles too big

$$\mathcal{M}_2 = e^{-S_0} e^{+\phi - i\sigma} \psi^6 \lambda^2 \longrightarrow \widetilde{\mathcal{M}}_2 = e^{-S_0} e^{+\phi - i\sigma} \psi^4 q^2$$

so, Coulomb branch, unlike vectorlike example, is not lifted by KK monopoles

symmetry-wise: R-symmetry intertwined with topological shift symmetry

$$\psi^4 \lambda^2 \to e^{i \frac{2\alpha}{5}} \psi^4 \lambda^2, \qquad \psi^6 \lambda^2 \to e^{-i \frac{2\alpha}{5}} \psi^6 \lambda^2 \quad \sigma \to \sigma - \frac{2}{5}\alpha, \qquad [Y] = -\frac{2}{5}$$

hence no mass gap in the gauge sector at small L, origin-CFT (not one of Y,u)

as L increases, can imagine generating mass gap due to R symmetry breaking from fermion condensates (strong multifermion interactions - NJL) but not consistent with SUSY

thus our story seems consistent with 4d arguments that theory is a CFT, and not confining - hence no SUSY breaking upon addition of W = u
(u is quite irrelevant at f.p.)

## conclusions

generally, the moral was that, along with center-stabilized deformations (in non-SUSY case), these $R^3 \times S^1$ compactifications give a calculable regime where the IR physics, including nonperturbative effects is under quantitative control

**in some cases, one argues that the dynamics is smooth as the size of the circle varies - some (preliminary) lattice studies seem to support this**

confinement, when it occurs, is due to condensation of objects of nonzero magnetic charge - similar to Polyakov's 3d mechanism - but often of quite exotic objects, with constituents that only exist on locally-4d manifolds, for example:

- QCD with adjoints "bions" (Unsal 2007)
- chiral I=3/2 SU(2) "quintets" (Unsal, EP 2009)
- other weird ones exist as well

in many cases, lattice can be used to verify the qualitative picture that has emerged and study how it evolves in "infinite"-4d limit

finally, perhaps other SUSY theories, left out 10 yrs ago, can be (benefitially?) studied with this deformation, and some lose ends tied...