Gauge/Gravity Duality in Supersymmetric and Non-Supersymmetric Vacua

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Plan of the talk:

- (Biased) Introduction to the Gauge/Gravity duality
- The orbifold of the conifold as a specific example
- Supersymmetric vacua: confining or Coulomb branch
- Supergravity solutions
- Non-supersymmetric (metastable) vacua: anti-D3 branes

Based on:

- RA, Bertolini, Franco, Kachru
  `arXiv:hep-th/0610212,0703236`
- RA, Benini, Bertolini, Closset, Cremonesi
  `arXiv:0804.4470 [hep-th]`

Gauge/Gravity Duality is a generalization of AdS/CFT correspondence.

AdS/CFT

(IIB) String theory on $AdS_5 \times X^5$ is equivalent to a gauge theory with (super)conformal symmetry at the quantum level.

Gauge/Gravity

String theory on some deformation (more or less radical) of $AdS_5 \times X^5$ is equivalent to a gauge theory with non-conformal (and thus possibly richer) quantum dynamics.

Several examples!

Gauge/Gravity Duality is a very important tool to address issues relating to the strong coupling behavior of gauge theories and of string theory.

We will concentrate here on using the duality to understand strongly coupled supersymmetric gauge theories from the weakly coupled, supergravity limit of string theory.

The potential is enormous: in principle, one should be able to

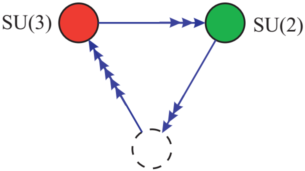
- encode the holomorphic data of the gauge theory in some asymptotic properties of the supergravity solution.
- find the full SUGRA solution corresponding to a specific vacuum.
- derive all the non-holomorphic properties of the theory, down to low-energies, in that vacuum.

This is a top-down, unified approach to studying low-energy strongly coupled dynamics of gauge theories.

In this spirit, we aim at having no external parameters but only moduli in the supergravity solution. In particular, we will avoid introducing “flavor” branes with non-dynamical parameters.

It is however difficult to engineer in this way simple theories such as SQCD. Because of the (super)conformal origin of the duality, it turns out that it is more natural to consider quiver gauge theories, with several nodes (gauge groups) but very few independent superpotential couplings.

MSSM is a quiver after all



Our favourite set up

$\mathcal{N} = 1$ quiver gauge theories dual to regular and fractional $D3$-branes at Calabi-Yau singularities.

The presence of fractional branes induces a non-trivial RG flow which breaks conformality.

At low-energies, there is a wealth of expected behaviors:

- confinement (and chiral symmetry breaking)
- abelian Coulomb phase ($\mathcal{N} = 2$ or $\mathcal{N} = 4$)
- Supersymmetry breaking by runaway or in a metastable vacuum (DSB)

These different behaviors can appear in the same quiver gauge theory in different regions of field space (branches of moduli space) or for different ranks of the gauge groups.

Field theory point of view

fix the theory in the UV and then see that for different values of the ranks, there are different moduli spaces which can have several branches (and possibly runaway potentials)

String theory point of view

fix the asymptotic geometry and the number of fractional branes. Different kinds of fractional branes lead to different effects (transitions, smoothings) on the singular geometry

There is a dictionary between geometric and gauge theoretic effects.

The UV completion of the low-energy theories usually involves an infinite number of degrees of freedom.

This is of course related to the AdS/CFT origin of the non-conformal theories that we are dealing with, the ultimate UV completion being string theory.

Rough dictionary

- Confinement: deformation of the singularity in the IR, cascade of Seiberg dualities as UV completion. [Klebanov Strassler 00, Maldacena Nunez 00, Vafa 00]
- $\mathcal{N} = 2$ Coulomb branch: enhançon radius in the IR, UV completion in terms of a cascade of higgsings.

  [Polchinski 00, Bertolini et al. 00, Aharony 01]
- Runaway: obstructed deformation in IR, cascade of Seiberg dualities in UV in some cases. [Berenstein et al. 05, Bertolini et al. 05, Franco et al. 05]
- Metastable vacua: $\overline{D3}$-branes at the tip of a (smooth) geometry with fluxes instead of (regular) $D3$-branes. [Kachru Pearson Verlinde 01]

Conifold$/\mathbb{Z}_2$

In the rest of this talk, we will consider this very simple geometry where most of the behaviors above can be reproduced.

It is obtained starting from the Conifold

$$xy = zt$$

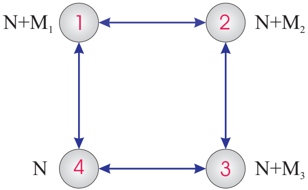
and modding by the $\mathbb{Z}_2$ identification

$$(x, y, z, t) \to (x, y, -z, -t)$$

so that it is defined by the simple equation in $\mathbb{C}^4$

$$\boxed{x^2y^2 = uv}$$

The quiver corresponding to the Conifold$/\mathbb{Z}_2$ is



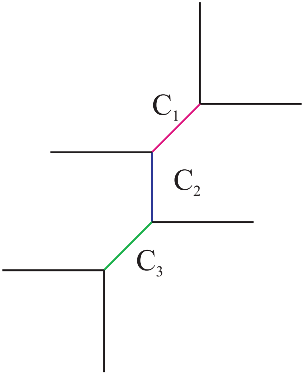
with superpotential

$$W = h \sum_{i=1}^{4} (-1)^{i+1} X_{i,i+1} X_{i+1,i+2} X_{i+2,i+1} X_{i+1,i}$$

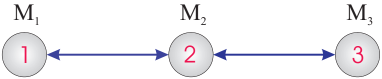
Also seen as a $\mathbb{Z}_2$ "orbifold" of the 2-node quiver for the Conifold.

It is non-chiral, and the ranks can be chosen freely.

Geometrically, there are 3 independent fractional branes, corresponding to the 3 independent resolutions of the singularity.

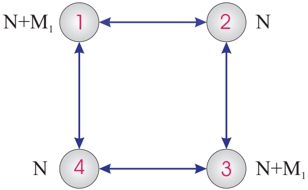


When there are only fractional branes ($N = 0$), as should be for low enough energies (bottom of some cascading RG flow):



$$W = hX_{12}X_{23}X_{32}X_{21}$$

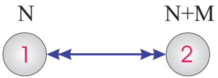
For different values of the ranks M_1, M_2, M_3 we find a variety of behaviors in the IR: confinement, Coulomb branch ($\simeq \mathbb{C}$), runaway and even metastable DSB.



Confinement:

$M_1 = M_3, M_2 = 0$

This is exactly the $\mathbb{Z}_2$ orbifold of the quiver for the Conifold with a fractional brane:



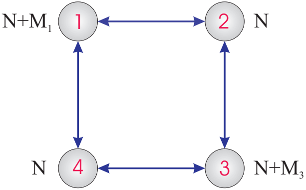
The dual geometry undergoes two geometric transitions,

with an equal deformation parameter: $(xy - \epsilon)^2 = uv$

The full SUGRA solution of the dual description is given by the solution of Klebanov and Strassler modded by $\mathbb{Z}_2$.

More confinement:

$M_1 \neq M_3$, $M_2 = 0$



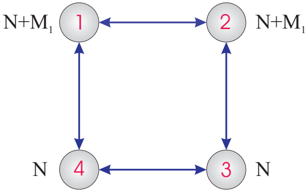
The geometry has now two different deformation parameters and is completely smooth:

$$(xy - \epsilon_1)(xy - \epsilon_3) = uv$$

The case $M_3 = 0$, $\epsilon_3 = 0$ is just a special case.

Abelian Coulomb phase ($\mathcal{N}=2$):

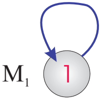
$M_1 = M_2, M_3 = 0$



The RG flow is a cascade of higgsings (more similar to $\mathcal{N}=2$ baryonic roots rather than $\mathcal{N}=1$ Seiberg dualities). [See Benini Bertolini Closset Cremonesi 08]

At the bottom it reduces to a two node quiver with $W = 0$

which has a single node $\mathcal{N}=2$ Coulomb branch



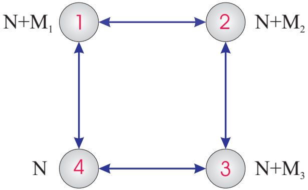
The abelian Coulomb vacua are really deriving from an $\mathcal{N} = 2$ system of fractional branes at a $\mathbb{C}^2/\mathbb{Z}_2$ singularity.

The identification

$$(x, y, z, t) \to (x, y, -z, -t)$$

has its fixed points at $z = 0 = t$. This is a one-complex dimensional locus composed of two branches $\simeq \mathbb{C}^*$:

$$x = 0 \text{ and } y = 0.$$



The more general case

shows an interplay of the previous RG flows

This interplay can lead to

- several branches displaying different behaviors
- or to a more destructive runaway behavior

We will even see an instance of a metastable vacuum “between” a SUSY confining vacuum and a Coulomb branch.

Description in term of a supergravity solution

[ABBCC 08]

The ansatz is the usual $D3$-brane one (we fix $\tau = i$)

$$\begin{aligned} ds^2_{10} &= h^{-1/2} dx^2_{1,3} + h^{1/2} ds^2_6 \\ F_5 &= (1 + *_{10})\, dh^{-1} \wedge d\mathrm{Vol}_{3,1} \end{aligned}$$

- In the UV, the 6-dimensional space is simply the singular Calabi-Yau cone $x^2 y^2 = uv$.
- In the presence of fractional branes, a 3-form flux $G_3 = F_3 + iH_3$ is turned on.
- $\mathcal{N} = 1$ SUSY is preserved when the flux is $(2,1)$ primitive, in particular $*_6\, G_3 = i\, G_3$ [Grana Polchinski 00]

F_3 is fixed by matching the brane charges at large radii, H_3 follows by the above condition.

F_3 is fixed in terms of the ranks of the quiver.

By the correspondence between ranks and branes wrapping specific cycles, one can determine

$$
\begin{aligned}
F_3 \quad \sim \quad & (M_1 - M_2 + M_3)\, \omega_3^{CF} \\
& + (M_1 - M_2 - M_3)\, d\psi' \wedge \omega_2^{x=0} \\
& + (-M_1 - M_2 + M_3)\, d\psi'' \wedge \omega_2^{y=0}
\end{aligned}
$$

- ω_3^{CF} is the same "untwisted" flux term appearing in the KT/KS solutions. [Klebanov Tseytlin 00, Klebanov Strassler 00]
- In the "twisted" flux terms, $\omega_2^{x,y=0}$ is the 2-form on the exceptional 2-cycle of the A_1-singularity on the $x = 0$ or $y = 0$ lines. The angular variables on the two lines are related by $\psi' \sim 2\pi - \psi''$.

Warp factor:

$$\Delta h = *_6(F_3 \wedge H_3)$$

Twisted and Untwisted pieces in the flux source term do not mix.

Solution is thus

$$h = h^{KT} + h^{\mathrm{twisted}}$$

with the twisted part similar in spirit to the warp factor for branes on $\mathbb{C}^2/\mathbb{Z}_2$. [Bertolini et al. '00]

There will be generically an enhançon radius.

What about the IR dynamics of the theory?

We still expect a warped geometry like

$$ds_{10}^2 = h^{-1/2} dx_{1,3}^2 + h^{1/2}\, ds_6^2$$

but now the 6-dimensional space ds_6^2 is no longer necessarily a cone.

It should be related now to the deformations of the orbifolded conifold.

- There is an additional dimensionful quantity R_ϵ related to the complex deformation by $\epsilon \equiv R_\epsilon^3$.

If there are $\mathcal{N} = 2$ branes around, they introduce a twisted sector

- It implies the presence of an enhançon radius ρ_c.

It all depends on whether $\rho_c \gg R_\epsilon$ or $\rho_c \ll R_\epsilon$.

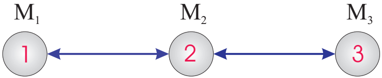
If $\rho_c \gg R_\epsilon$

- The deformation is cloaked by the enhançon.
- The UV solution is a very good approximation in the IR.
- The second node is the one with the dominant dynamics $\Lambda_2 \gg \Lambda_1, \Lambda_3$.

If $\rho_c \ll R_\epsilon$

- We expect a solution that differs in the IR with respect to the UV one.
- It should correspond to an orbifold of KS with twisted fluxes.
- It is related to the situation where $\Lambda_1, \Lambda_3 \gg \Lambda_2$.

In the latter case, some checks can be made on the geometry. In particular, the Gukov-Vafa-Witten W reproduces the Taylor-Veneziano-Yankielowicz W computed in the gauge theory.

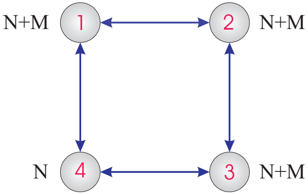


In summary:

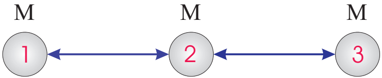
- If $M_2 = 0$, geometric transition to a smooth background: confinement
- If $M_2 \neq 0$, there are still some wrapped branes around: Coulomb branch

A special case:

$M_1 = M_2 = M_3 \equiv M$ [ABFK 06]



After a cascade of Seiberg dualities one ends up in a 3-node quiver:



$$W = hX_{12}X_{23}X_{32}X_{21}$$

Both node 1 and 3 have $N_f = N_c$. At scales of the order of Λ_1, Λ_3, they confine and the dynamics is given in terms of mesons and baryons.

[Seiberg 94]

$$W = h\mathcal{M}_1\mathcal{M}_3 + \lambda_1(\det \mathcal{M}_1 - \mathcal{B}\tilde{\mathcal{B}} - \Lambda_1^{2M}) + \lambda_3(\det \mathcal{M}_3 - \mathcal{C}\tilde{\mathcal{C}} - \Lambda_3^{2M})$$

The deformed moduli space has actually two distinct branches for each node because of the quartic coupling h: Baryonic and Mesonic.

On Baryonic$_1$-Baryonic$_3$ branch

- Left with a pure $SU(M)$ SYM at node 2, with confining IR dynamics.
- Gravity dual based on the deformed geometry $(xy - \epsilon)xy = uv$, flux but no physical branes in the background.

On Mesonic$_1$-Mesonic$_3$ branch

- Left with the $SU(M)$ gauge group at node 2 with a massless adjoint given by $\mathcal{M}_1 \propto \mathcal{M}_3^{-1}$. It has $\mathcal{N} = 2$ SUSY at low energies, and an abelian Coulomb branch.
- Gravity dual is based on $(xy - \epsilon)^2 = uv$ with flux and M fractional $D3$-branes wrapped on the exceptional cycle of the A_1-singularity.

What about Mesonic$_1$-Baryonic$_3$?

There is a tension among the different F-terms, which cannot be solved simultaneously.

- $\langle \mathcal{M}_1 \rangle = \Lambda_1^2 \neq 0$ acts as an effective mass term for the flavors of node 3.
- This is actually SUSY breaking by the rank condition of ISS (in the limiting case of $N_f = N_c$), with dynamically generated masses.

  [Intriligator Seiberg Shih 06]
- We have thus a (candidate) metastable vacuum on the field theory side.

[Disclaimer: incalculable instability towards turning on the baryons]

Gravity dual of this metastable state?

Note first that

$$Q_3 \equiv \int H_3 \wedge F_3 + N_3 - \bar{N}_3$$

A shift in $\int_B H_3$ can be compensated by the presence of $D3$s or $\overline{D3}$s.

For instance, for $\int_A F_3 = M$ and $\bar{N}_3 = 0$, a situation with

$$\int_B H_3 = k \text{ and } N_3 = 0$$

has the same charge at infinity (and thus describes the same theory) as a situation with

$$\int_B H_3 = k - 1 \text{ and } N_3 = M.$$

This is interpreted as going over the ($\mathcal{N} = 4$) mesonic branch one step up in the cascade.

But we can also shift in the other direction: [Kachru, Pearson, Verlinde '01]

$$\int_B H_3 = k + 1 \text{ and } \bar{N}_3 = M$$

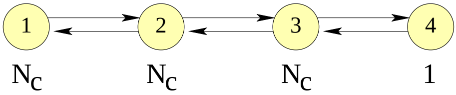
This configuration has the same value of Q_3, and the same asymptotics, but it breaks SUSY!

We interpret it as the $(xy - \epsilon)^2 = uv$ geometry with flux, M fractional $D3$-branes and M $\overline{D3}$-branes.

The $\overline{D3}$-branes cannot annihilate with the fractional $D3$-branes, but are rather dissolved in them as (SUSY breaking) gauge flux.

A more refined 4-node model

Embeddable in Conifold$/\mathbb{Z}_N$ [ABFK 07]



$$W = h(X_{12}X_{23}X_{32}X_{21} - X_{23}X_{34}X_{43}X_{32}) + mX_{43}X_{34}$$

Main improvements:

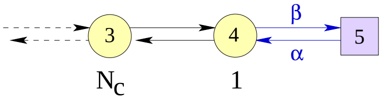
- IR free window: $N_f = N_c + 1$ SQCD (with quartic coupling)
- R-symmetry is broken [Kitano Ooguri Ookouchi 06]

Retains nice features of previous model:

- All masses generated dynamically, even m
- Geometrical picture unchanged (up to $1/N_c$ corrections)

A very useful stringy instanton

The mass coupling m is generated by a stringy instanton related to the (unoccupied) 5th node



$$W_{\text{inst}} \propto \int d\alpha d\beta \, e^{-\alpha X_{43} X_{34} \beta} \propto c \, X_{43} X_{34} \, e^{-\text{Area}} \equiv m X_{43} X_{34}$$

An orientifold projection is actually needed to avoid unwanted extra fermionic zero modes [see e.g. RA, Bertolini, Ferretti, Lerda, Petersson 07]

Conclusions and Outlook

- SUSY breaking on the gauge theory side is achieved through $\langle F \rangle \neq 0$ (ISS), while on the gravity side $\overline{D3}$-branes do the job (KPV).
- The scale of SUSY breaking is hierarchically small on both sides, and is directly related to one of the dynamical scales of the set up (no external parameters as in ISS).
- There is still some work to be done to flesh out this proposal. Beyond the backreaction of the fractional $D3$-branes in the SUSY backgrounds, one needs to compute the backreaction of the $\overline{D3}$-branes in the metastable background. [De Wolfe, Kachru, Mulligan '08]