Multiparton amplitudes at NLO with BlackHat+Sherpa

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in collaboration with

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H. Ita, D. Kosower
T. Gleisberg

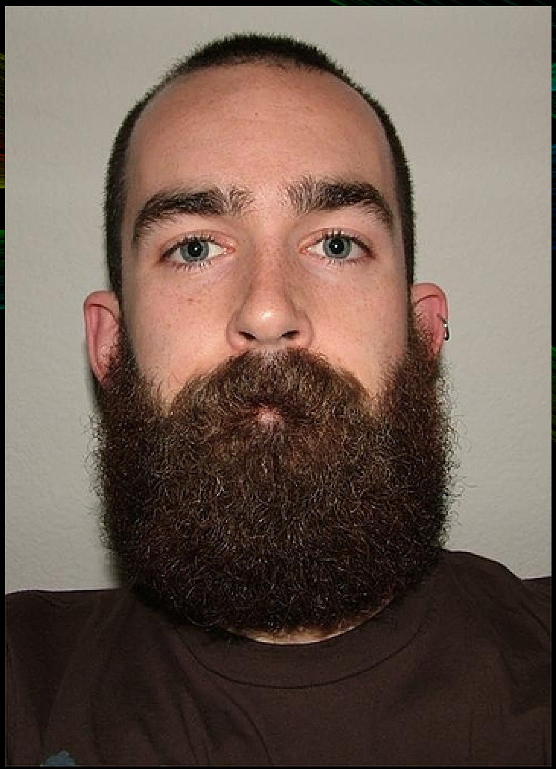
Durham, 01 April 2009

Based on: arXiv:0803.4180, arXiv:0808.0941, arXiv:0902.2760

Outline


New!
5 Blade Shaving Surface™
+1 Precision Trimmer
BlackHat
Fusion™
POWER
4

Outline



W+1 jet

W+2 jet

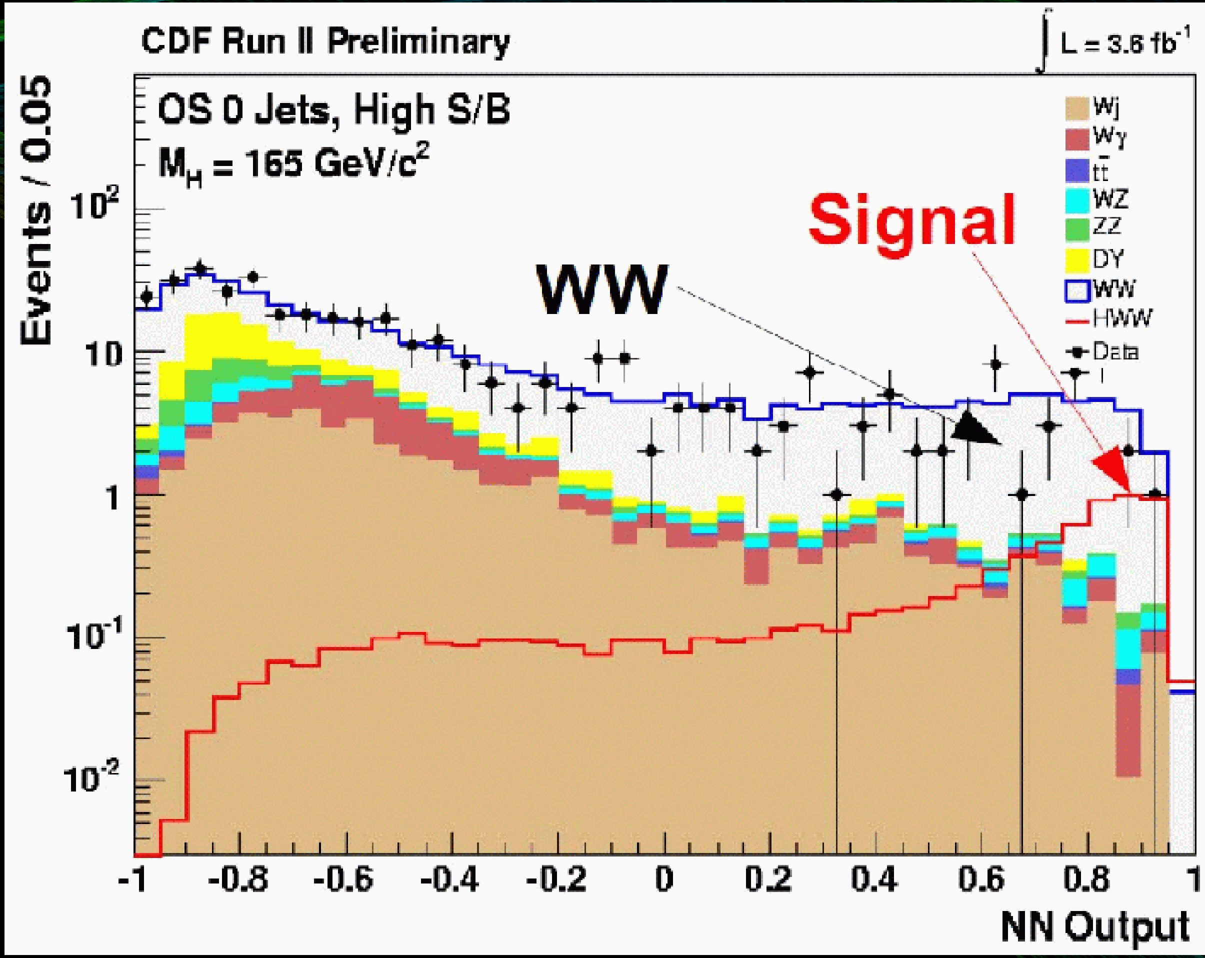
W+3 jet

Motivation

- Theoretical predictions for QCD processes are crucial for physics program at a hadron collider
  - Signal
  - Background
- Many measurements are limited by theory
- Some (most) theory predictions need to be improved!

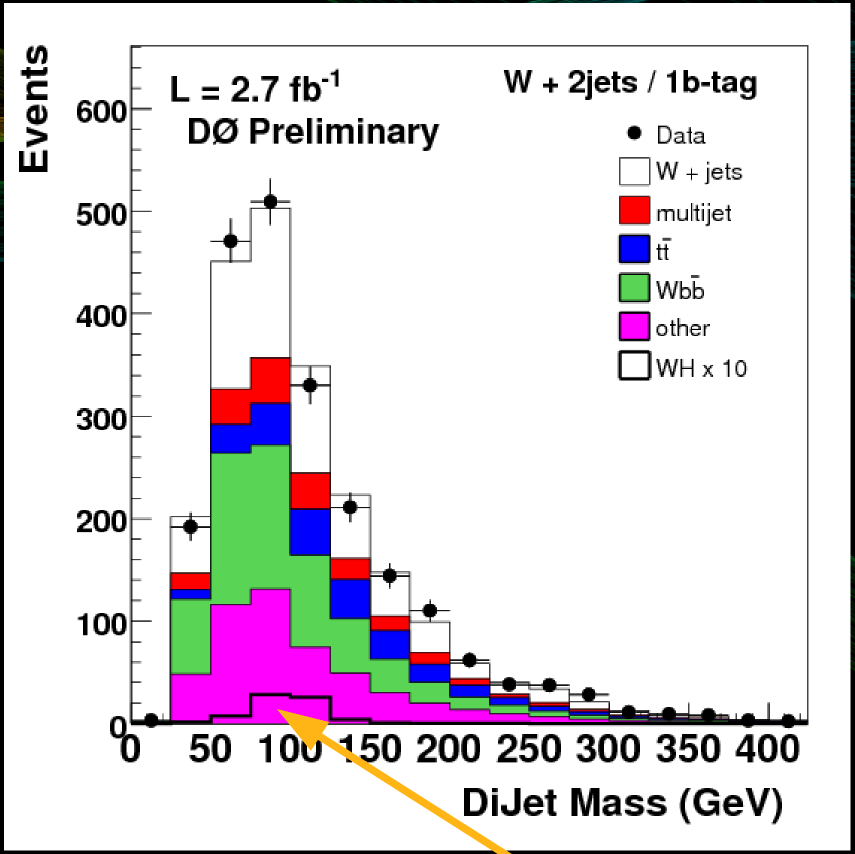
Motivation

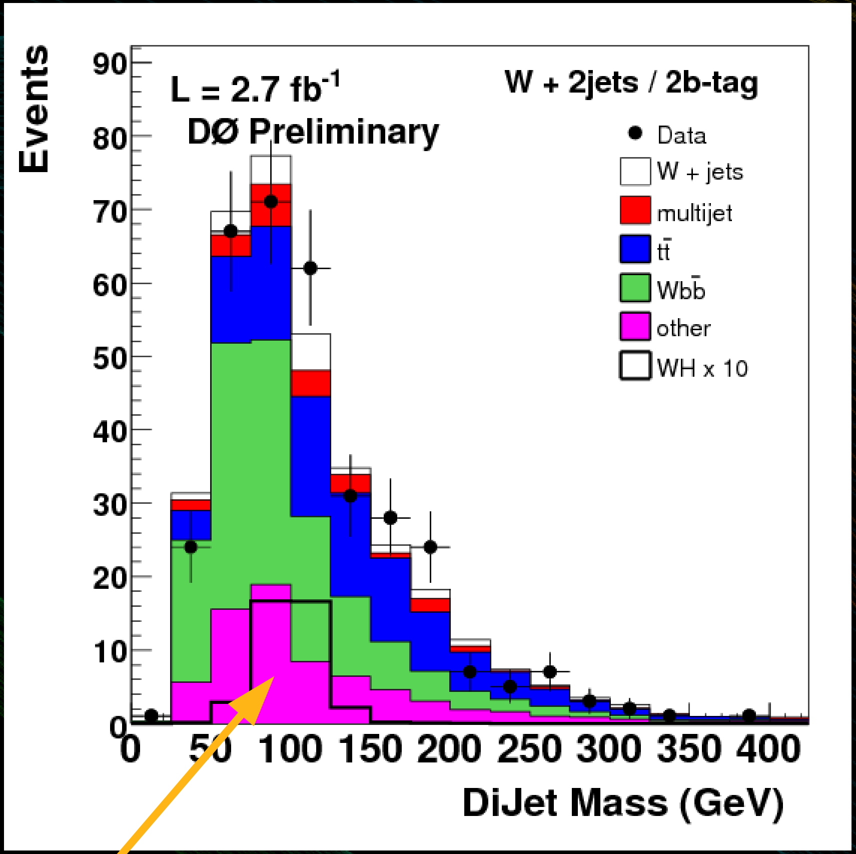
- Higgs →WW search @ CDF



Motivation

- Higgs associated production WH ($H \rightarrow b\bar{b}$)





Signal x 10

NLO Corrections

- NLO corrections are needed for a good theoretical understanding of QCD processes
- Improve theory prediction for
  - Absolute normalization
    - Corrections can be very large
    - Reduce renormalization scale dependency

Number of jets	LO	NLO
1	16%	7%
2	30%	10%
3	42%	8%

  - Shape of distributions

NLO Corrections

- Two parts
  - Real radiation
  - Virtual corrections
- Real radiation is automated
  [Gleisberg,Krauss;
  Seymour,Tevlin;
  Hasegawa,Moch,Uwer;
  Frederix,Gehrmann,Greiner]
- Virtual part is the bottleneck for processes with 5 or more external particles

NLO Corrections

- NLO Cross section:

$$\sigma_n^{NLO} = \int_n \sigma_n^{tree} + \int_n \sigma_n^{virt} + \int_{n+1} \sigma_{n+1}^{real}$$

- Real & virtual corrections have infrared divergences
  - Combination is free of infrared divergences
  - The cancellation is between objects living in two different phase spaces

NLO Subtraction

- Introduce subtraction term σ_{n+1}^{sub}
  - Same soft/collinear singularity structure as n+1 MEs

$$\int_{n+1} \left(\sigma_{n+1}^{real} - \sigma_{n+1}^{sub}\right) \quad \text{is finite}$$

  - Easy enough to be integrated over the singular PS

$$\int_{n+1} \sigma_{n+1}^{sub} = \int_n \int_1 \sigma_{n+1}^{sub} = \int_n \Sigma_n^{sub}$$

NLO with Blackhat+Sherpa

NLO cross section

$$\sigma_n^{NLO} = \int_n \sigma_n^{tree} + \int_n (\sigma_n^{virt} + \Sigma_n^{sub}) + \int_{n+1} (\sigma_{n+1}^{real} - \sigma_{n+1}^{sub})$$

BlackHat

Sherpa

Sherpa

[Gleisberg,Hoeche,Krauss,Schoenherr,Schumann,Siegert,Winter]

Provides

- Efficient phase space integration
- Event generation
- Analysis framework
- Automated dipole subtraction for the real part [Catani,Seymour] [Gleisberg,Krauss]
- (and much more)
- Is written in C++

BlackHat

[Berger, Bern, Dixon, Febres Cordero, Forde, Ita, Kosower, DM]

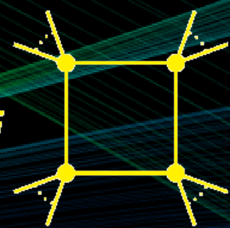
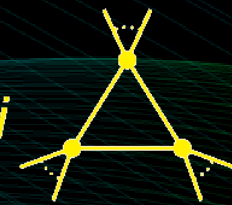
- Goal : automate computation of virtual 1-loop amplitudes for QCD processes
- C++ framework
- Uses new progress in the use of unitarity techniques, spinor formalism, complex momenta
  [Ossola,Papadopoulos,Pittau;Forde]
- Cut containing part: 4 Dim, using Forde's method
- Rational part: 1- loop recursion (reuse of lower point results) [Berger,Bern,Dixon,Forde,Kosower]

Advantages of unitarity

- Advantages of unitarity vs Feynman diagrams
  - Work with simpler on-shell objects
    - No gauge dependence
    - No off-shell information
    - More compact results
    - Numerically more stable
  - Unitarity method scales better with increasing number external legs

One-loop decomposition

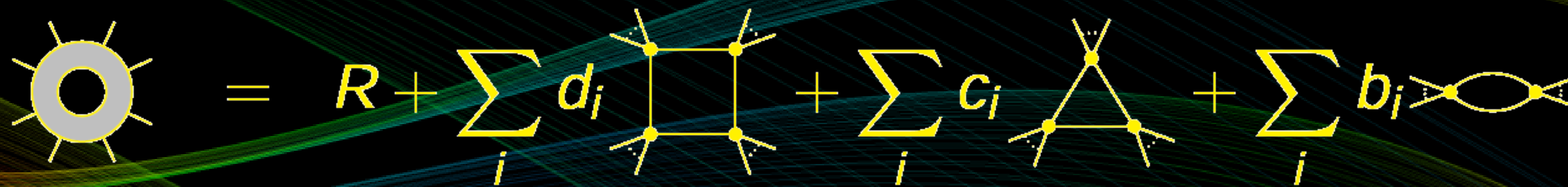
$$A = R + C$$

$C = \sum_i b_i$  $+ \sum_i c_i$  $+ \sum_i d_i$ 

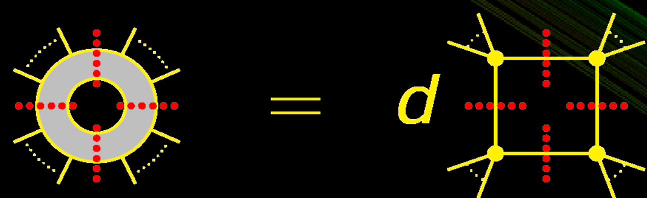
- The task is reduced to determining the coefficients
- The coefficients b_i, c_i, d_i are computed using generalized unitarity techniques
- Coefficient computed through unitarity techiques are simpler than those obtained with Feynman diagram techniques

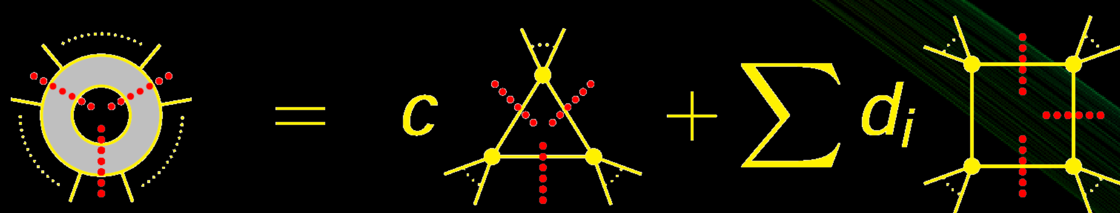
Scalar integral coefficients

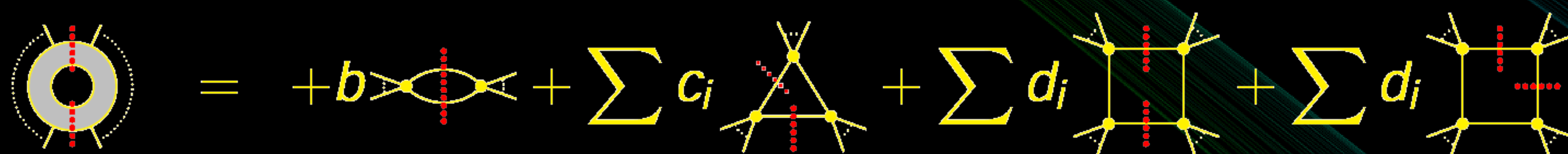
- Cuts can be used to isolate the coefficients of scalar integrals



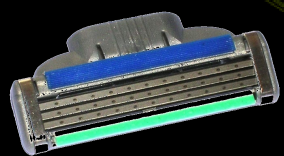
- Apply generalized cuts on both sides of the equation



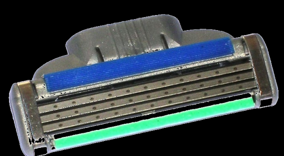




D vs 4 Dim Unitarity



D = 5



D = 6

4 Dim cut

Recursive Rational

Cuts in practice

- Given external momenta configuration:
  - Generate loop momenta configurations that satisfy the cut conditions (complex momenta)
  - For each configuration, compute and multiply the trees at the corner of the cut diagram
  - Combine the results appropriately

All the integral coefficients

Recursion relations

- Recursion relations allow to compute amplitudes from lower multiplicity amplitudes.
- Based on the analytic properties of the amplitudes and on the factorization properties on multi-particle poles
- Complex shift:

$$p_1 \to p_1(z), \qquad p_2 \to p_2(z)$$

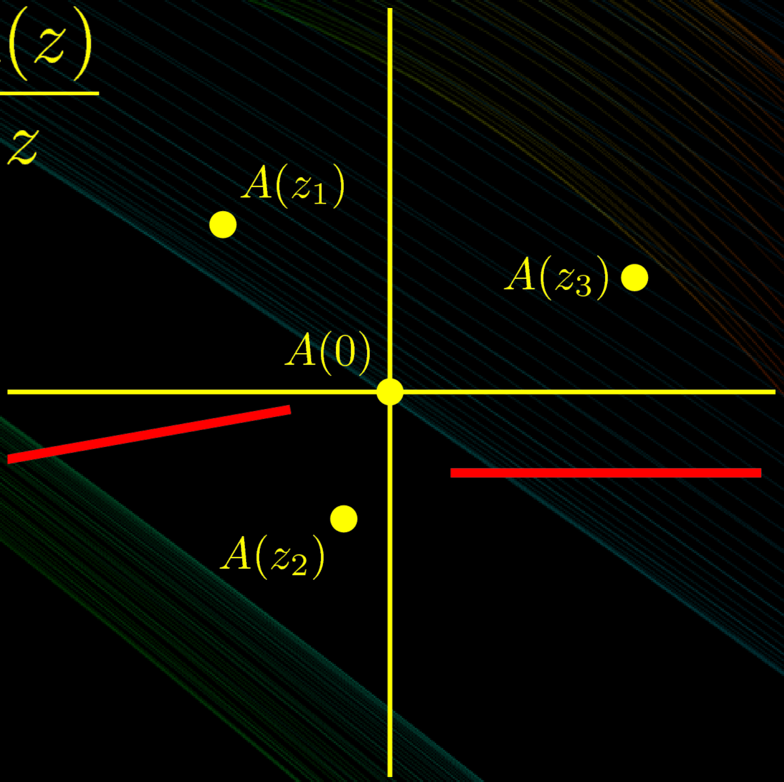
- Linear transformation that preserves
  - Onshell properties: $p_1(z)^2 = 0, \qquad p_2(z)^2 = 0$
  - Momentum conservation: $p_1 + p_2 = p_1(z) + p_2(z)$
- A → A(z), physical amplitude is *A(0)*
- Use the analytic properties of *A(z)* to construct *A(0)*

Analytic structure of the amplitude

- Use the analytic properties of the one-loop amplitude to construct the rational term
- Use a complex shift on the full amplitude

$$p_1 \to p_1(z), p_2 \to p_2(z), \quad A \to A(z)$$

- Consider the complex function $\frac{A(z)}{z}$
  - Poles, $s_{i...j}(z) \to 0$
  - Branch cuts: $log(s_{i...j}(z))$

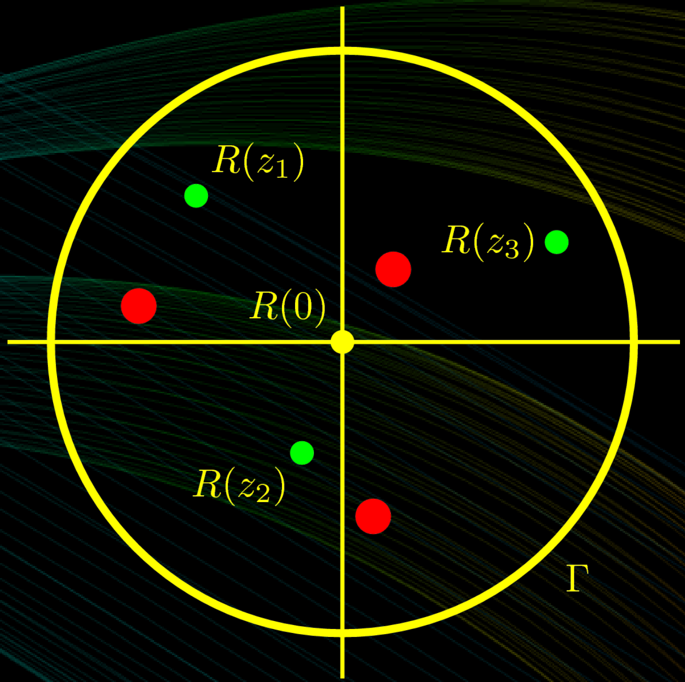


Rational term

- Consider *R(z)*

$$\frac{1}{2\pi i}\int_\Gamma \frac{R(z)}{z} = R_\infty$$

- The value R_∞ of the contour integral at ∞ can be constructed using an auxiliary recursion.



$$R(0) = R_\infty - \sum_{\text{poles}\alpha} \text{Res}_{z=z_\alpha} \frac{R(z)}{z}$$

- Two types of poles: Physical and Spurious

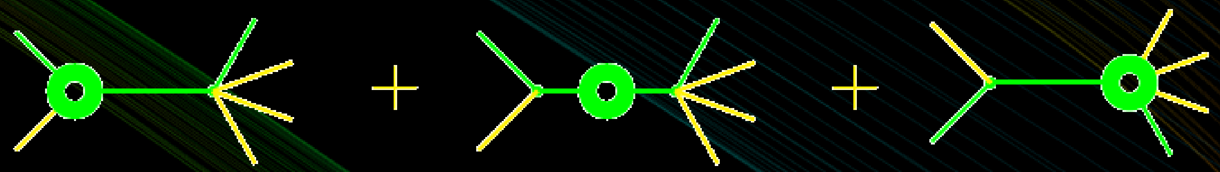
$$R(0) = R_\infty - \sum_{\text{phys}} \text{Res}_{z_p} \frac{R(z)}{z} - \sum_{\text{spur}} \text{Res}_{z_s} \frac{R(z)}{z}$$

Rational Term: Recursive Part

$$R(0) = R_\infty - \sum_{\text{phys}} \text{Res}_{z_p} \frac{R(z)}{z} - \sum_{\text{spur}} \text{Res}_{z_s} \frac{R(z)}{z}$$

R(z) factorizes at the physical pole locations, so that we can use recursion relations. [Bern,Dixon,Kosower]

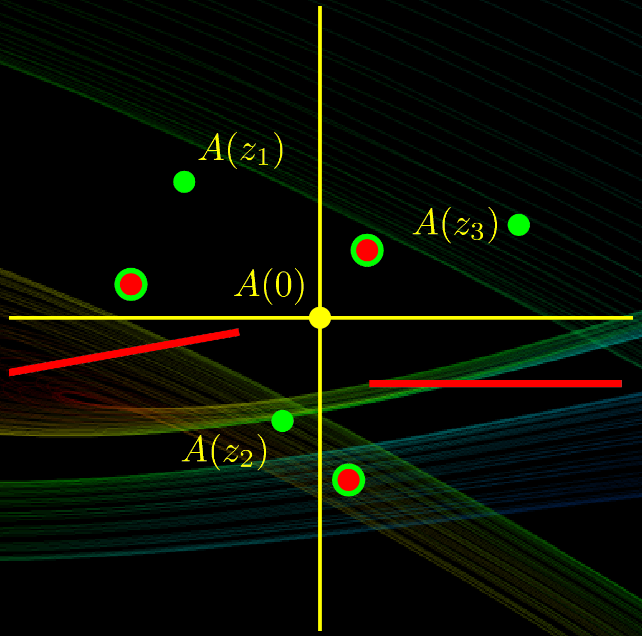
$$\text{Res}_{z_p} \frac{R(z_p)}{z_p} =$$



- This part can be constructed from lower point results

$$R_D = -\sum_{z_p} \text{Res}_{z_p} \frac{R(z_p)}{z_p}$$

Recursion for Rational Terms: Spurious Part



$$\frac{A(z)}{z} = \frac{R(z)}{z} + \frac{C(z)}{z}$$

- Spurious poles appear in *C(z)* and *R(z)* due to Gram determinants
- The residues of *R(z)/z* and *C(z)/z* at the unphysical poles have to cancel since *A(z)* has no spurious poles.

$$\mathrm{Res}_{z_s} \frac{R_S(z_s)}{z_s} = -\mathrm{Res}_{z_s} \frac{C(z_s)}{z_s}$$

Numerical extraction

- We compute numerically

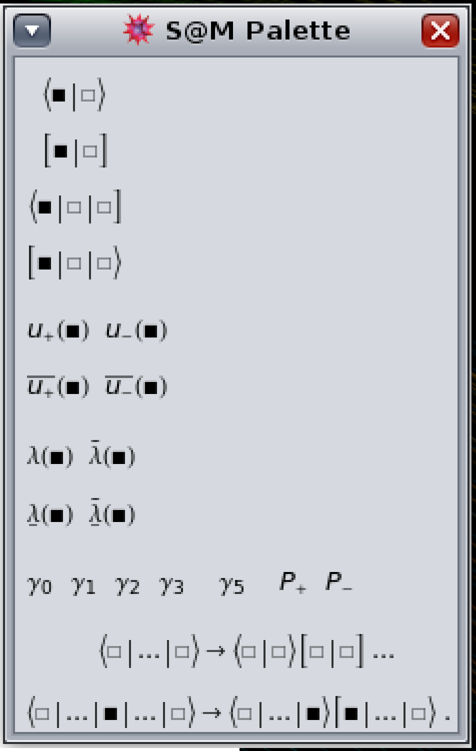
$$\sum_{\text{spur}} \text{Res}_{z_s} \frac{C(z_s)}{z_s}$$

- Numerical spurious extraction is tricky, but possible because
  - Precise cut part input
  - Location of the spurious poles is known a priori
  - Only need to evaluate a small part of *C(z)* around the pole.
  - Only need rational part of the expansion of the integral functions around vanishing Gram determinant

W + jets...

S@M [DM,P. Mastrolia arXiv:0710.5559]

- Mathematica implementation of the spinor-helicity formalism
- Numerical evaluation
- Complex shifts
- 2-dim 4-dim spinors
- < > and [ ] notation
- …



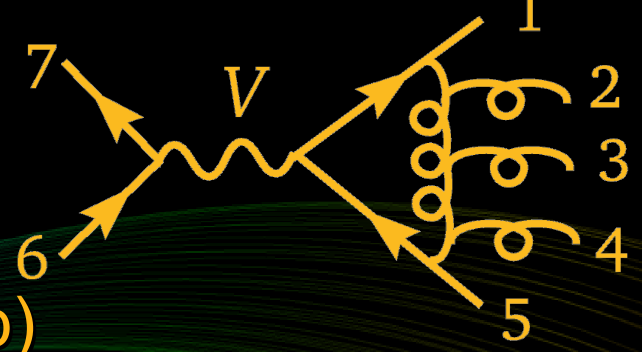
In[2]:=

```
Spaa[1, 2]^4 /
  (Spaa[1, 2] Spaa[2, 3] Spaa[3, 4] Spaa[4, 5]
    Spaa[5, 1]) - Spab[1, 2, 4] / Spbb[1, 3, 5, 2]
```

Out[2]=

$$-\frac{\langle 1 | 2\rangle^3}{\langle 1 | 5\rangle \langle 2 | 3\rangle \langle 3 | 4\rangle \langle 4 | 5\rangle} + \frac{\langle 1 | 2 | 4]}{[2 | 5 | 3 | 1]}$$

W+jets



- W/Z+jets processes are important
  - For SM physics (Higgs, $t\bar{t}$, single top)
  - Background to new physics
  - Luminosity determination
- So far
  - MCFM [John Campbell, Keith Ellis]
    - NLO W+1 jet (Feynman diagrams)
    - NLO W+2 jets (amplitudes from (early) unitarity methods)
  - Leading color primitive amplitudes (2q3gW) [BlackHat]
  - All primitive amplitudes [Ellis,Giele,Kunszt,Melnikov,Zanderighi]
  - Leading color W+3 jets (2q3gW) [Ellis,Melnikov,Zanderighi]
  - Leading color W+3 jets (all subprocesses) [BlackHat]

W+jets @ Tevatron

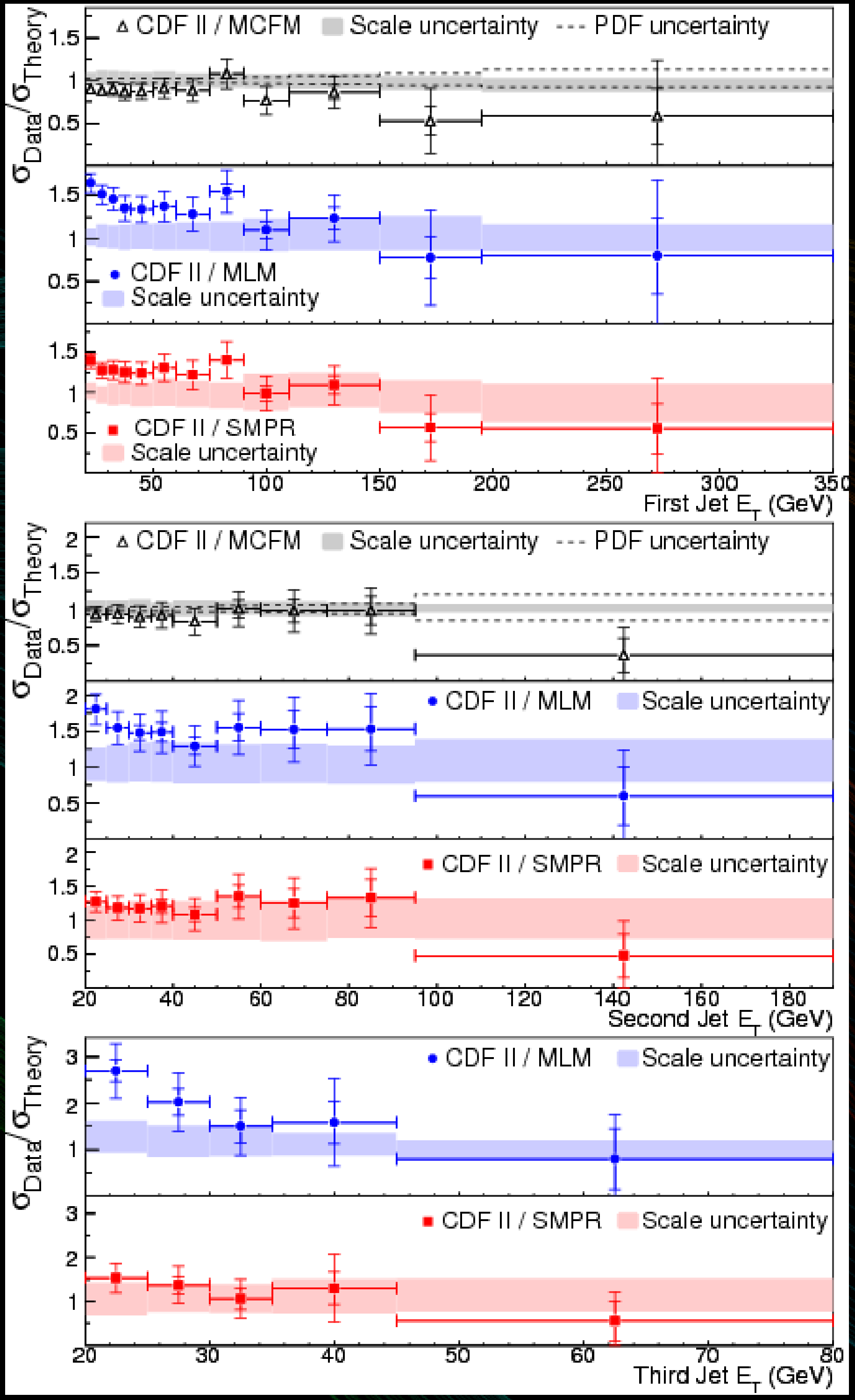
- CDF Collaboration
  - $320pb^{-1}$
  - Corrected for comparison with particle level
  - Comparison with
    - NLO: MCFM
    - MLM = Alpgen+Herwig
    - SMPR = Madgraph+Pythia

$E_T^e > 20$ GeV $\quad E_T^{\text{jets}} > 20$ GeV

$|\eta^e| < 1.1 \quad \not{E}_T > 30$ GeV

$|\eta^{\text{jets}}| < 2 \quad E_T^{\text{jets}} > 20$ GeV

$M_T^W > 20$ GeV



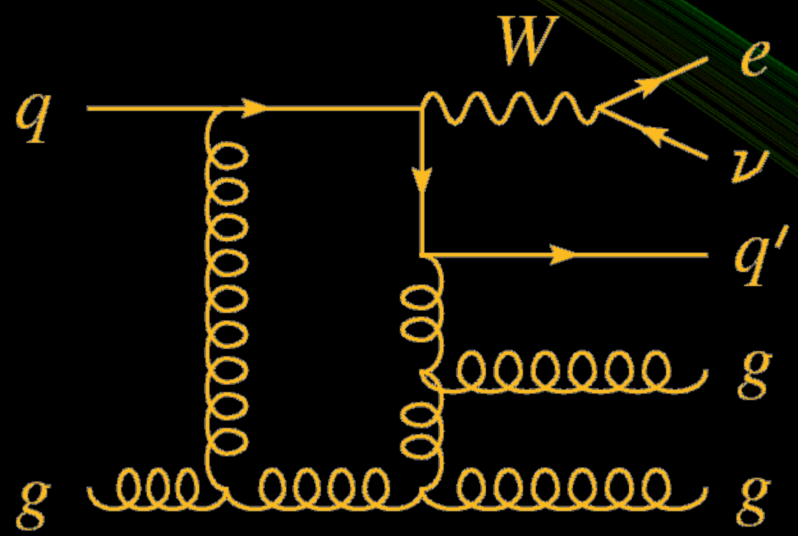
[CDF Collaboration PRD 77 011108, Arxiv:0711.4044]

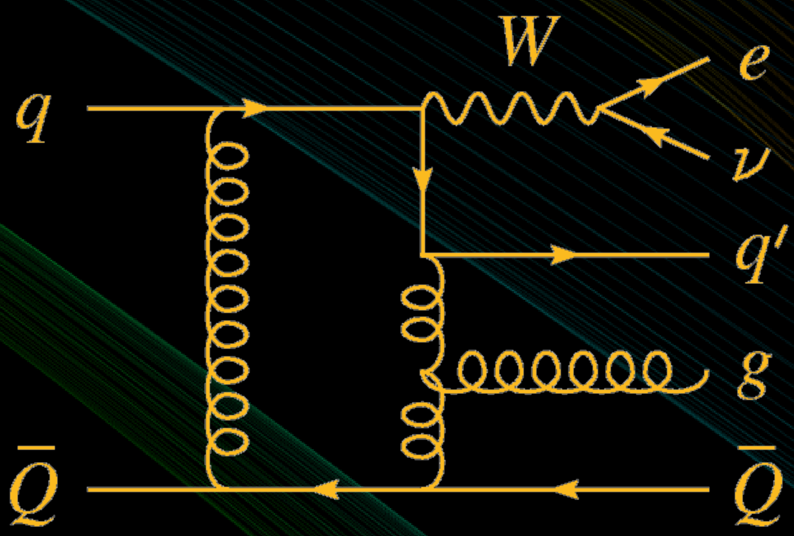
Leading color approximation

- Neglect terms of order

  $\frac{1}{N_c^2}$ (subleading color), $\frac{N_f}{N_c}$ (closed fermion loop)

  in finite part of the virtual amplitude
- Works for W+1,2 jets within 3 %



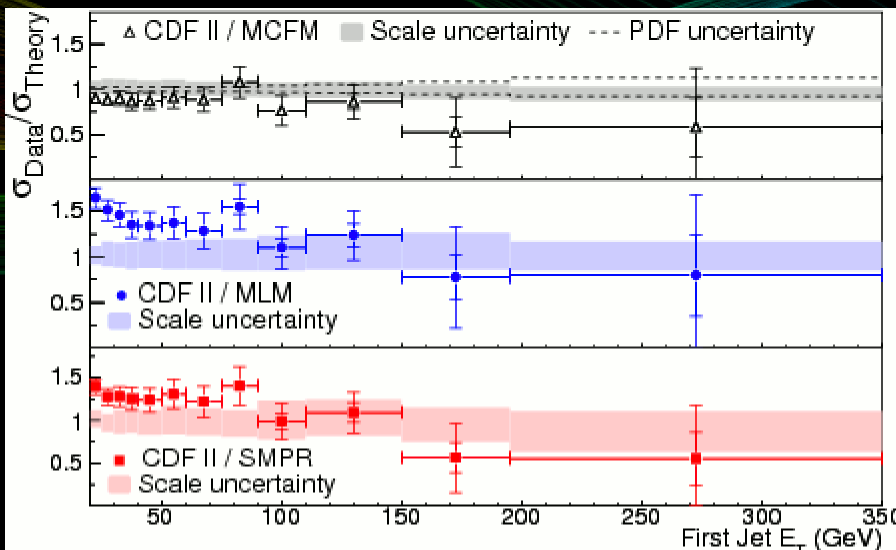


LC Approximation

- Validated at 1,2 jets
- Total cross section ($E_T^{nth-\mathrm{jet}} > 25\ \mathrm{GeV}$)

number of jets	CDF	LC NLO	NLO
1	53.5 ± 5.6	$58.3^{+4.6}_{-4.6}$	$57.8^{+4.4}_{-4.0}$
2	6.8 ± 1.1	$7.81^{+0.54}_{-0.91}$	$7.62^{+0.62}_{-0.86}$
3	0.84 ± 0.24	$0.826^{+0.049}_{-0.084}$	—

W+1 jet @ Tevatron



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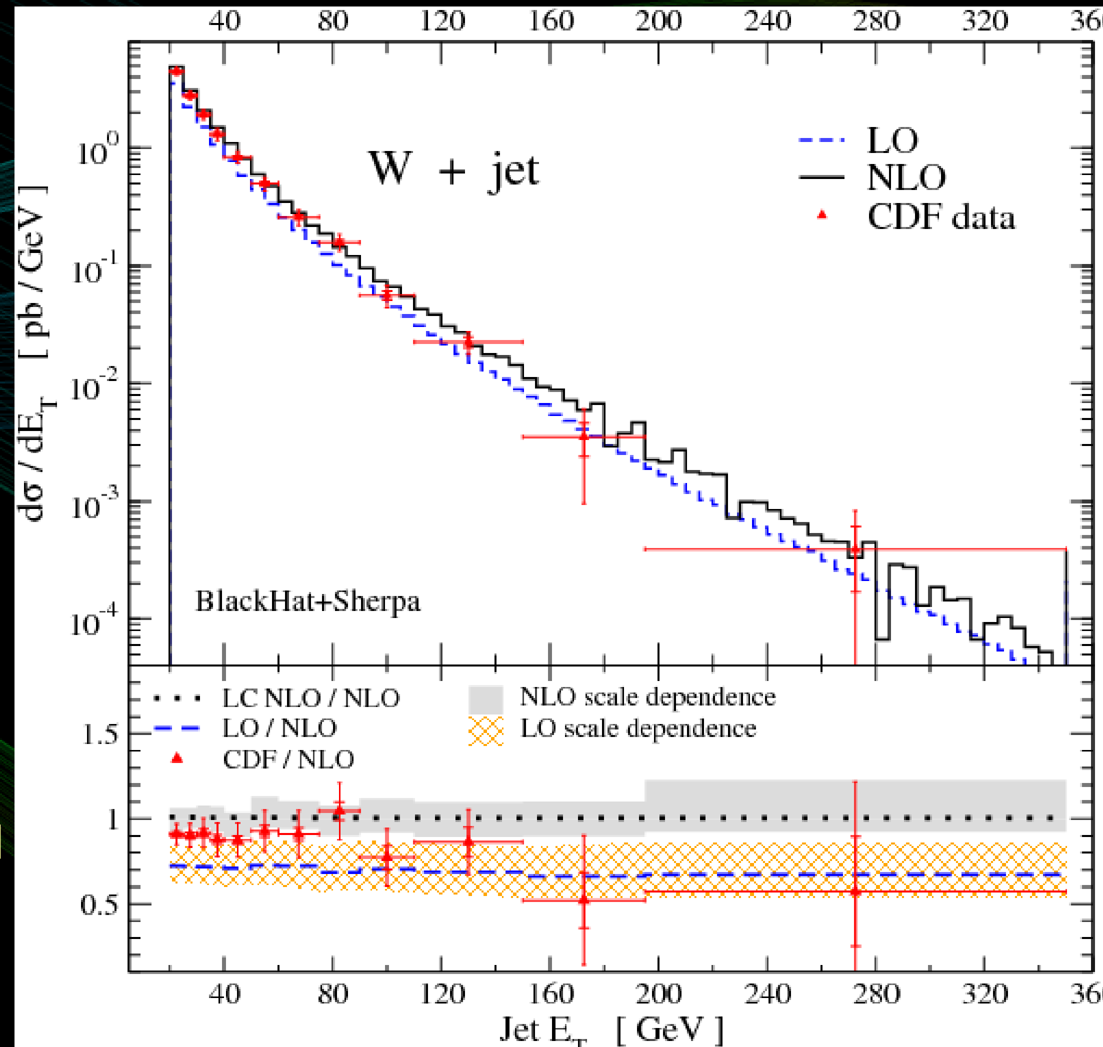
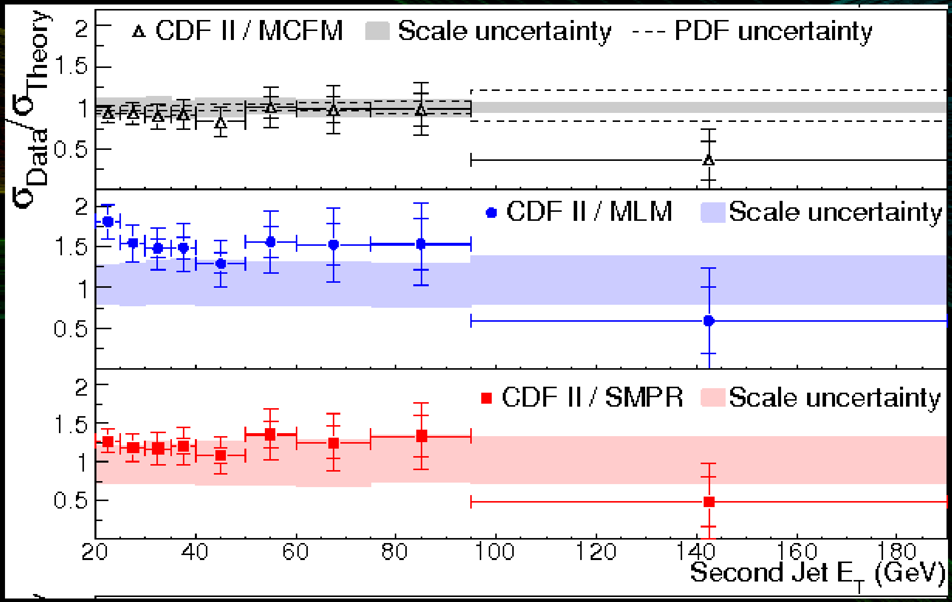


$$\mu = \sqrt{m_W^2 + p_T^2(W)}$$

PDF: CTEQ6M

Jet algorithm: SISCone [Salam,Soyez]

W+2 jets @ Tevatron



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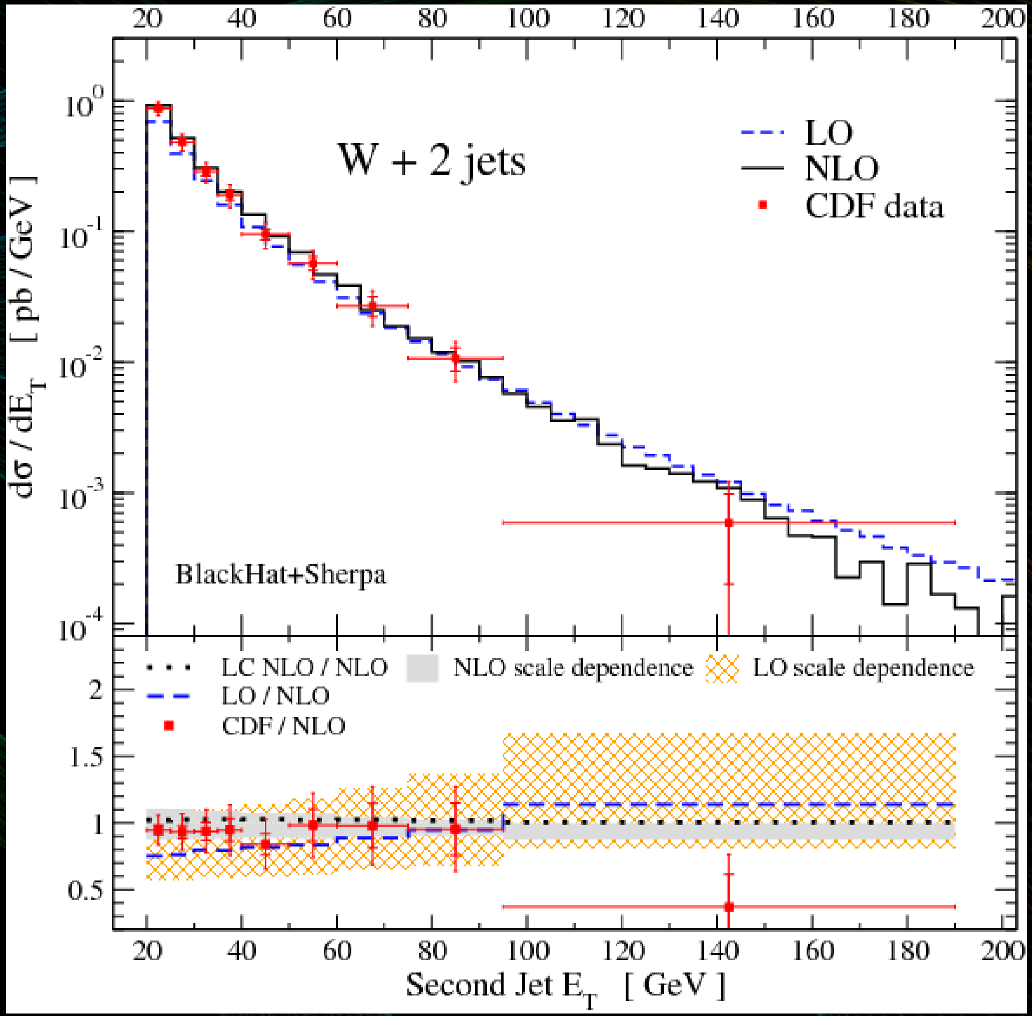

$$\mu = \sqrt{m_W^2 + p_T^2(W)}$$

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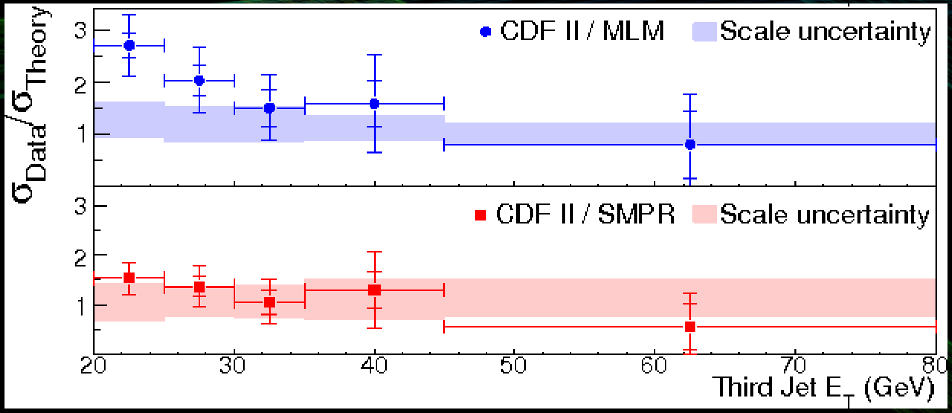
W+3 jet @ Tevatron



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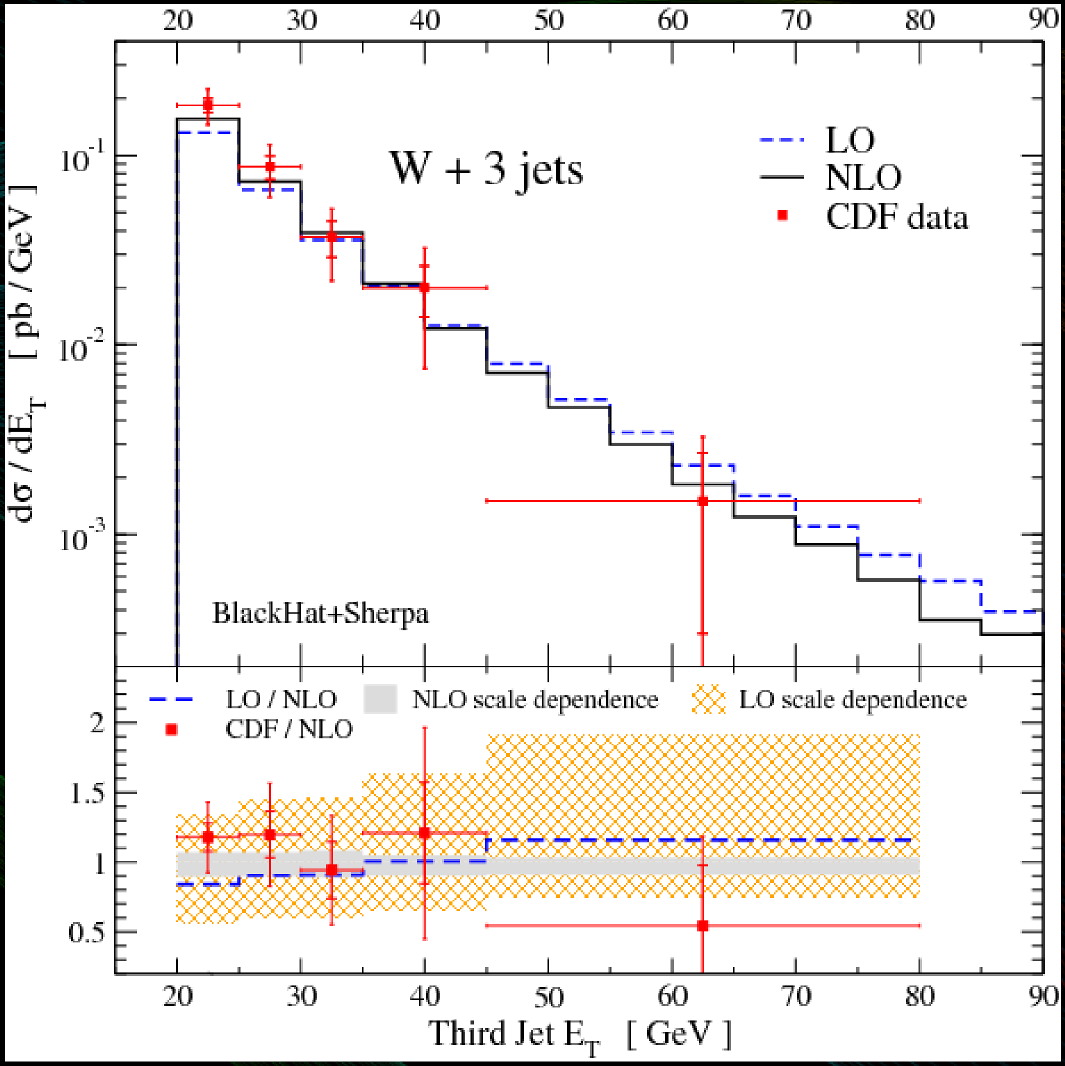

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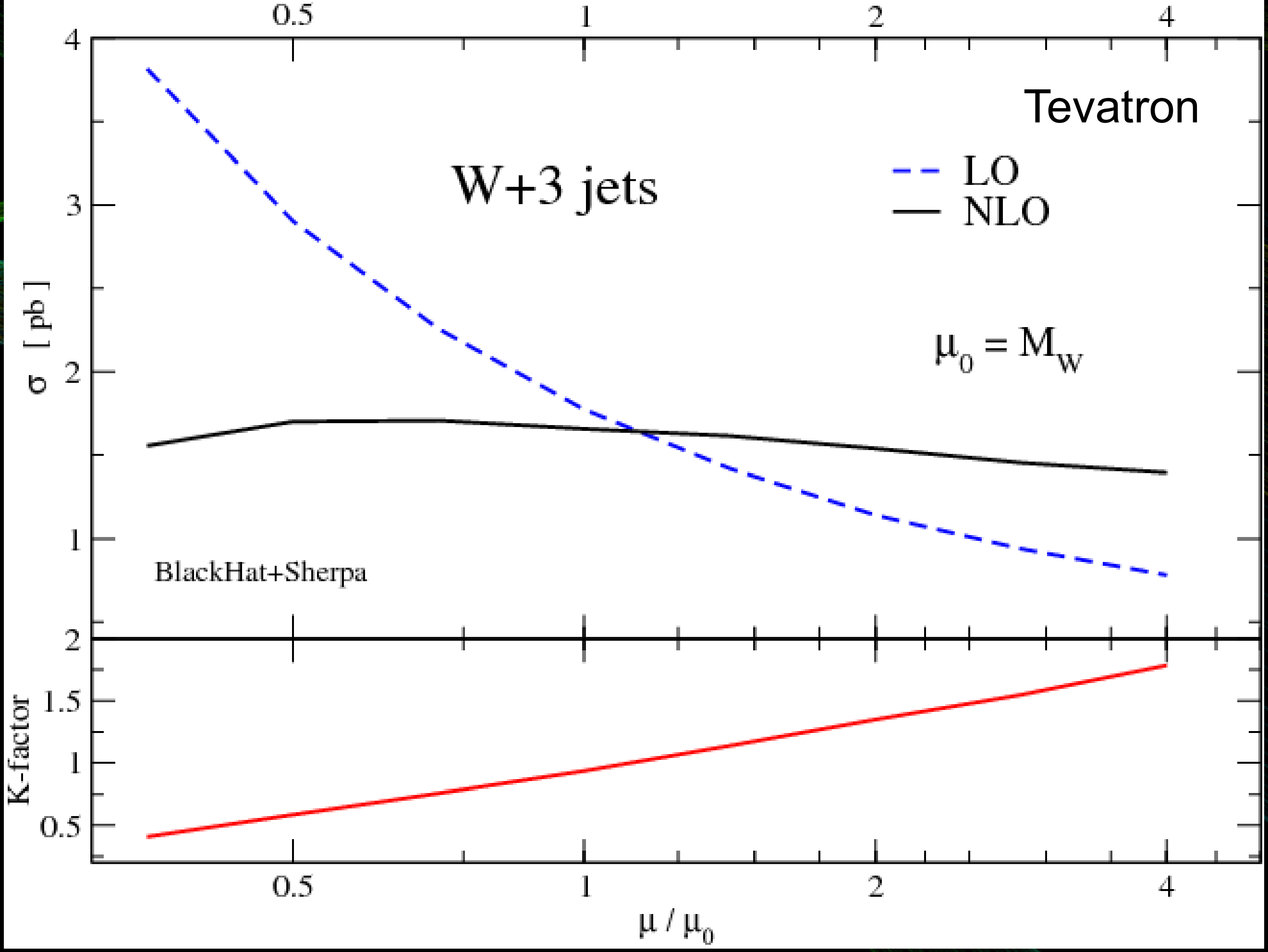
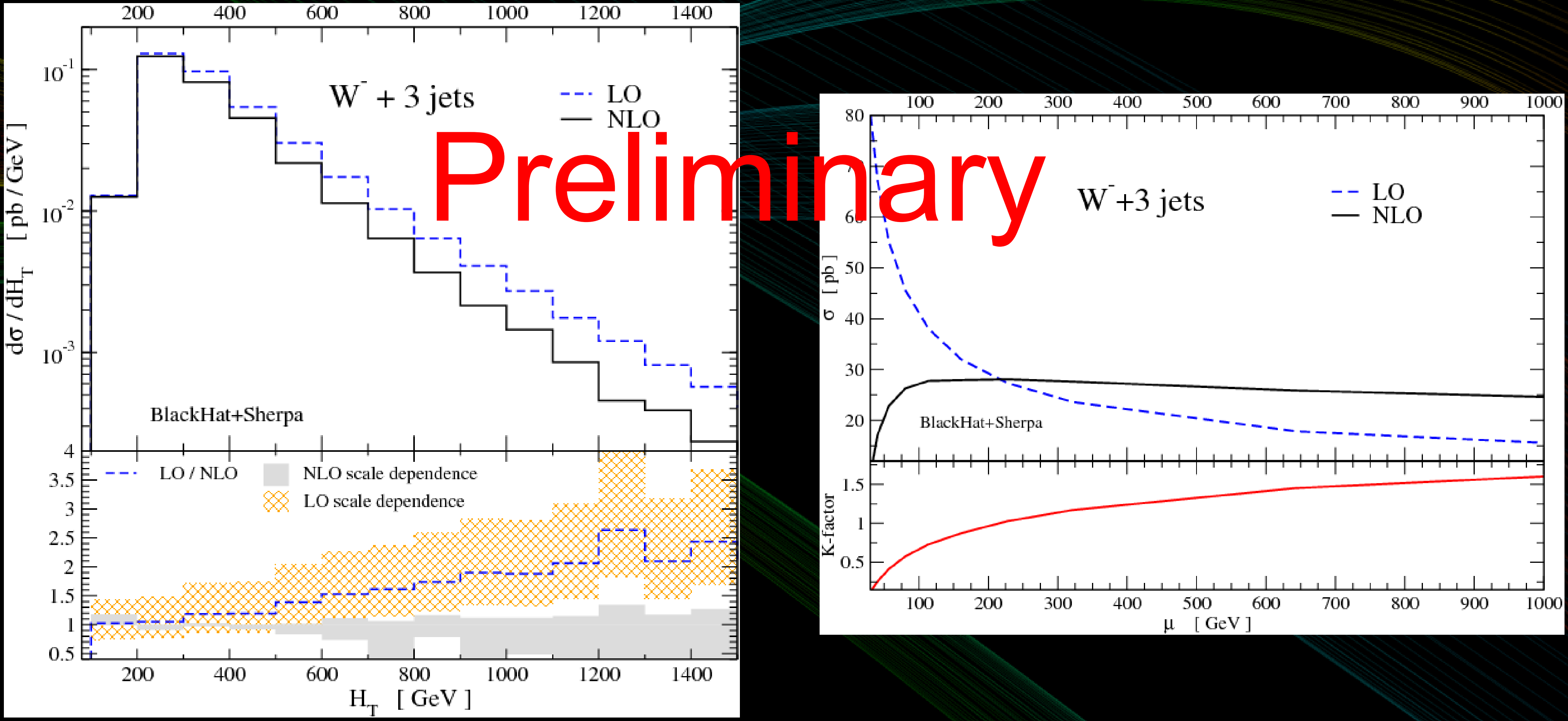
Scale dependence

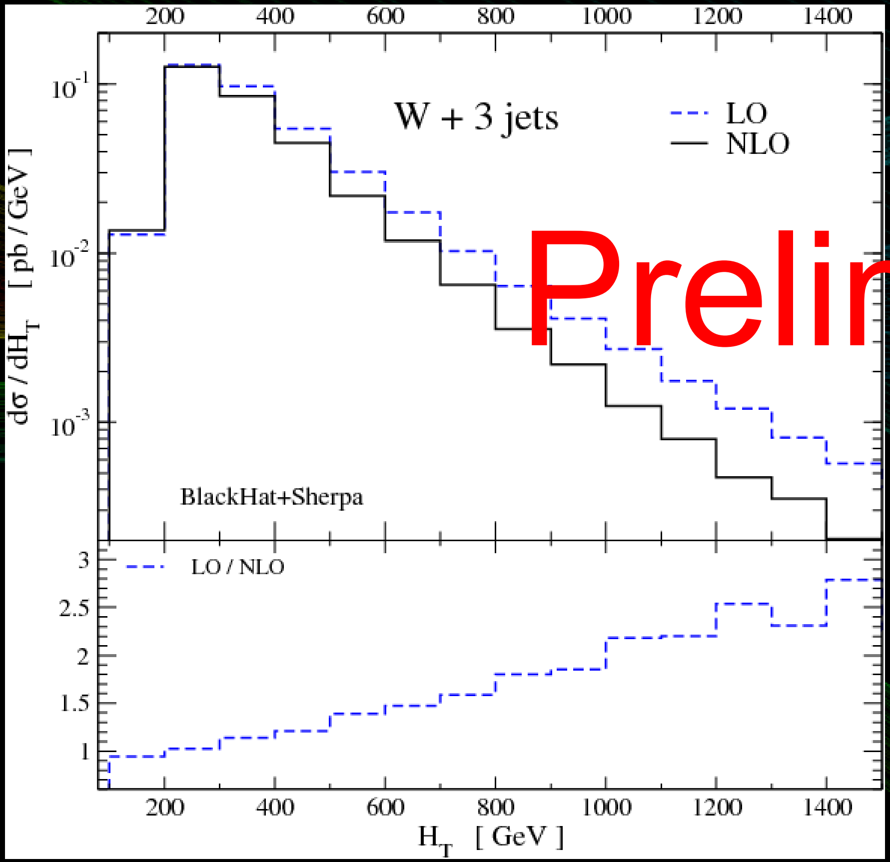
- NLO scale dependency much smaller than at LO

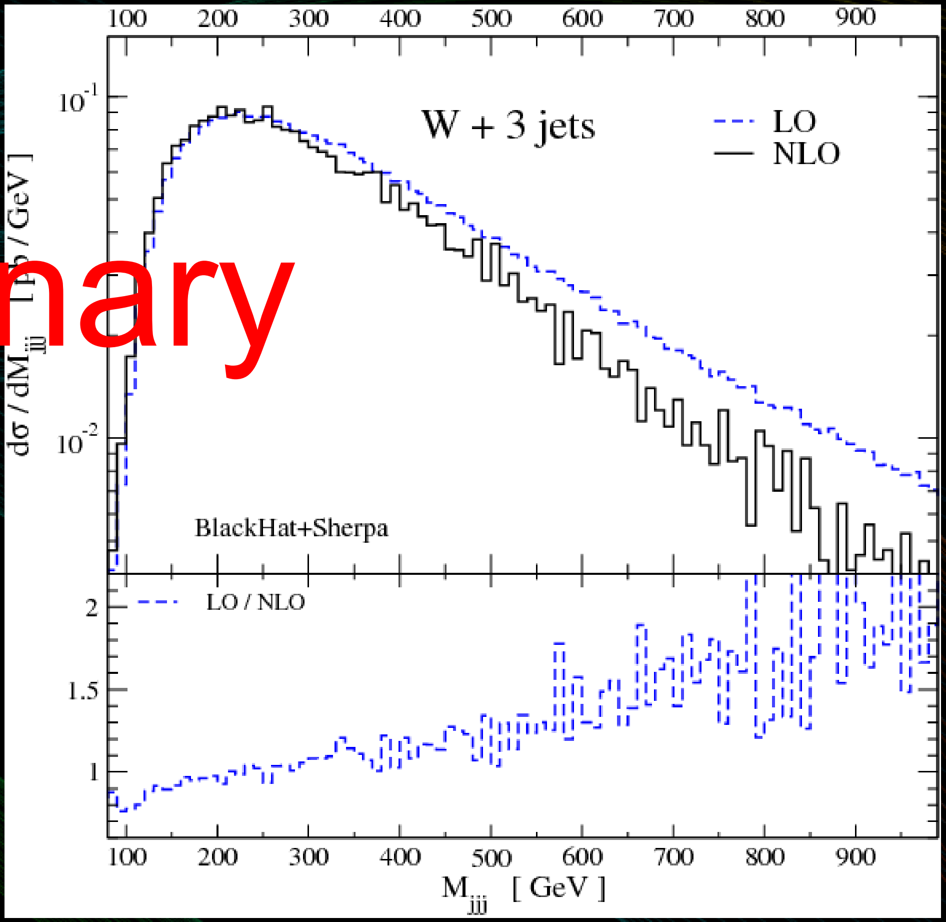


W+3 jets @ LHC


Preliminary
W- + 3 jets
LO
NLO
BlackHat+Sherpa
dσ / dH_T [pb / GeV]
LO / NLO
NLO scale dependence
LO scale dependence
H_T [GeV]
W-+3 jets
LO
NLO
BlackHat+Sherpa
σ [pb]
K-factor
μ [GeV]

W+3 jets @ LHC





Preliminary

$$H_T = \sum_{j=1,2,3} E_{T,j}^{\text{jet}} + E_T^e + \not{E}_T$$

$$M_{3jet} = \sqrt{(k_{j1} + k_{j3} + k_{j2})^2}$$

Conclusion

- Numerical implementations of unitarity+on-shell recursion can produce phenomenologically useful results
- First comparison of NLO W+3 jets and experimental data from the Tevatron
- Presented first prediction for NLO W+3 jets at LHC
- Show potential of unitarity techniques

Conclusion

- Numerical implementations of unitarity+on-shell recursion can produce phenomenologically useful results
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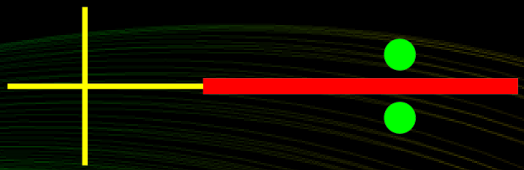
More stuff

Illustration

$$Disc_z f = \lim_{\epsilon \to 0} f(z+i\epsilon) - f(z-i\epsilon)$$



Use discontinuity to solve "complicated" integral

$$I \equiv \int_0^1 dx \log(1-xy) = \frac{y-1}{y}\log(1-y) - 1$$

- Make an ansatz : $I = a\log(1-y) + b$

for y>1:

$$\int_0^1 dx\ (-2\pi i)\Theta(xy-1) \quad = \quad a\ (-2\pi i)$$

$$\Rightarrow a = \int_0^1 dx\Theta(xy-1) = \int_{1/y}^1 dx = 1 - \frac{1}{y} = \frac{y-1}{y}$$

Illustration

- Observations
  - "cut" integral $\int_0^1 dx\Theta(1-xy)$

    is easier than original integral $\int_0^1 dx \log(1-xy)$

  - Need to have an ansatz $I = a\log(1-y) + b$
  - We have to get the part that has no cut by another means

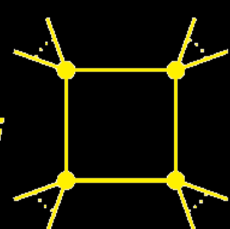
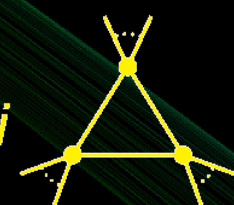
$$I \equiv \int_0^1 dx \log(1-xy) = \frac{y-1}{y}\log(1-y) - 1$$

?

Illustration

- Integration variable $x \rightarrow$ loop momentum
  External variable $y \rightarrow$ invariants
  Coefficient of $\log \rightarrow$ scalar integrals
  non log part $1 \rightarrow$ rational terms
- A one-loop amplitude can be decomposed into a sum of coefficients multiplying scalar integrals and rational terms.

$$A = R + C$$

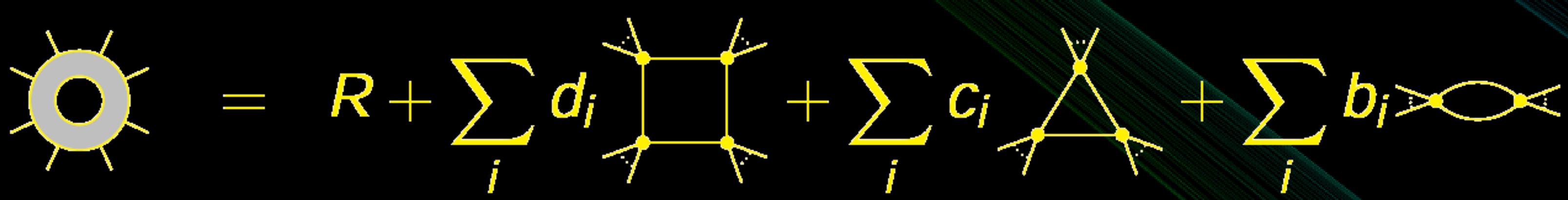
$$C = \sum_i b_i$$  $$+ \sum_i c_i$$  $$+ \sum_i d_i$$ 

Unitarity

- We want to isolate the “bubble” information in the amplitude
- The massless scalar bubble function, viewed as an analytic function of *s* has a discontinuity

$$B_0(s) \sim \int d^{4-2\epsilon} l \frac{1}{l^2} \frac{1}{(l-p)^2} \sim \frac{1}{\epsilon} + \log(-s) + 2 + \mathcal{O}(\epsilon)$$

- The one-loop amplitude also has a discontinuity
- Compute the discontinuity on both sides of the equation



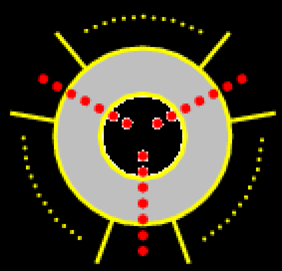
Generalized unitarity

- A unitarity cut is the replacement

$$\frac{i}{p^2 - m^2 + i\epsilon} \to 2\pi\delta(p^2 - m^2)$$

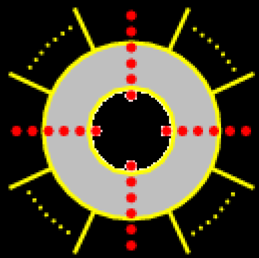
Can exchange more than two propagators for delta functions.

- triple cut



$$= \int d^4l\delta(l^2)\delta((l-K_1)^2)\delta((l-K_1-K_2)^2)A_1A_2A_3$$

- quadruple cut

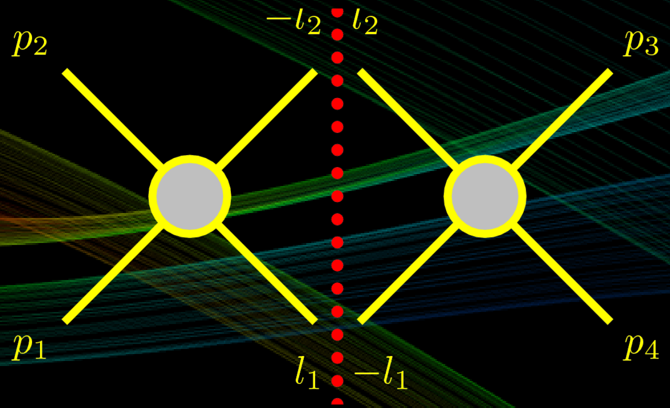


$$= \int d^4l\delta(l^2)\delta((l-K_1)^2)\delta((l-K_1-K_2)^2)\delta((l+K_4)^2)A_1A_2A_3A_4$$

Unitarity cut

- The discontinuity can be computed by replacing the propagators by delta functions under the loop integral

[Cutkosky]



$$\frac{i}{l_1^2} \to 2\pi\delta^+(l_1^2) \quad \frac{i}{l_2^2} \to 2\pi\delta^+(l_2^2)$$

$$-i\mathrm{Disc}A_4^{(1L)}(1,2,3,4)\Big|_{12\mathrm{cut}} = \int \frac{dl^4}{(2\pi)^4} 2\pi\delta^+(l_1^2) 2\pi\delta^+(l_2^2)$$

$$\times A_4^{\mathrm{tree}}(l_1,1,2,-l_2) A_4^{\mathrm{tree}}(l_2,3,4,-l_1)$$

- Relates one loop amplitudes to products of tree amplitudes
- Exchange loop integral for a phase-space integral (and don't do the integral)

Double cut

- Momentum parametrization with two massless four vectors

$$l^\mu = \alpha_1 k_1^\mu + \alpha_2 k_2^\mu + \frac{\alpha_3}{2}\langle k_1|\gamma^\mu|k_2] + \frac{\alpha_4}{2}\langle k_2|\gamma^\mu|k_1]$$

- The two delta functions leave two free parameters

$$l^\mu = y\tilde{K}_1^\mu + (1-y)\tilde{\chi}^\mu + \frac{t}{2}\left\langle \tilde{K}_1|\gamma^\mu|\chi\right] + \frac{y(1-y)}{2t}\left\langle \chi|\gamma^\mu|\tilde{K}_1\right]$$

- Double cut integrand is a function of *t* and *y*

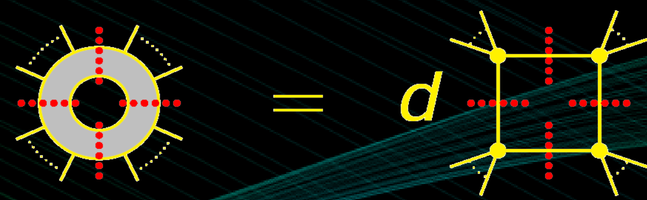
$$C_2(y,t) = A_1(t,y)A_2(t,y)$$

$$B_2(y,t) \equiv C_2(y,t) - \sum box - \sum triangle$$

- Extract the coefficient with a projection

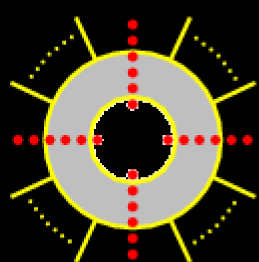
$$b_0 = \frac{1}{20}\sum_{j=0}^{4}\left[B_2\left(0, t_0\, e^{2\pi i j/5}\right) + 3B_2\left(2/3, t_0\, e^{2\pi i j/5}\right)\right]$$

Quadruple cut



•In a quadruple cut, we replace four propagators by delta functions

$$\frac{1}{p_1^2}\frac{1}{p_2^2}\frac{1}{p_3^2}\frac{1}{p_4^2} \to \delta(p_1^2)\delta(p_2^2)\delta(p_3^2)\delta(p_4^2)$$



$$= \int d^4 l \delta(l_1^2)\delta(l_2^2)\delta(l_3^2)\delta(l_4^4)\mathcal{A}$$

• Solve for the loop momentum and insert [Britto,Cachazo,Feng]

$$d \sim \sum A_1^{\text{tree}}(l^\pm) A_2^{\text{tree}}(l^\pm) A_3^{\text{tree}}(l^\pm) A_4^{\text{tree}}(l^\pm)$$

•Momenta are in general complex

Triple cut

- Momentum parametrization with two massless four vectors [del Aguila,Ossola,Papadopoulos,Pittau;Forde]

$$l^\mu = \alpha_1 k_1^\mu + \alpha_2 k_2^\mu + \frac{\alpha_3}{2}\langle k_1|\gamma^\mu|k_2] + \frac{\alpha_4}{2}\langle k_2|\gamma^\mu|k_1]$$

- The three delta functions fix three of the coefficients

$$l^\mu = \tilde{K}_1^\mu + \tilde{K}_2^\mu + \frac{t}{2}\left\langle \tilde{K}_1|\gamma^\mu|\tilde{K}_2\right] + \frac{1}{2t}\left\langle \tilde{K}_2|\gamma^\mu|\tilde{K}_1\right]$$

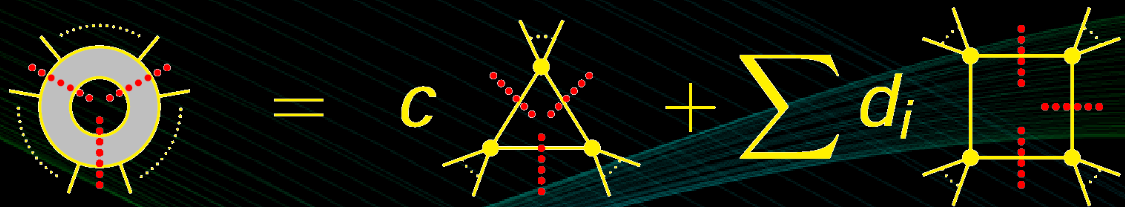
- Generic form of the triple cut is a function of *t*

$$C(t) = A_1(t)A_2(t)A_3(t)$$

$$= \frac{c_{-3}}{t^3} + \frac{c_{-2}}{t^2} + \frac{c_{-1}}{t} + c_0 + c_1 t + c_2 t^2 + c_3 t^3 + \sum_{poles} \frac{d_i}{\xi_i(t-t_i)}$$

- The triangle coefficient is given by c_0
- We want a numerical way to extract the coefficient c_0

Triple cut



- Poles in *t* originate from additional propagators going on-shell

$$(l_i - K)^2 \to (t - t_i)\xi_i$$

- Subtracted triple cut is a function of *t* [OPP]

$$T(t) \equiv C(t) - \sum \frac{d_i}{\xi_i(t - t_i)}$$

$$T(t) = \frac{c_{-3}}{t^3} + \frac{c_{-2}}{t^2} + \frac{c_{-1}}{t} + c_0 + c_1 t + c_2 t^2 + c_3 t^3$$

- Use projection to get the coefficient

$$c_0 = \frac{1}{2p+1} \sum_{j=-p}^{p} T_3(t_0 e^{2\pi i j/(2p+1)})$$