Building a GOLEM for the LHC

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April 1, 2009

Gauge and string amplitudes workshop

IPPP Durham, England

Content:

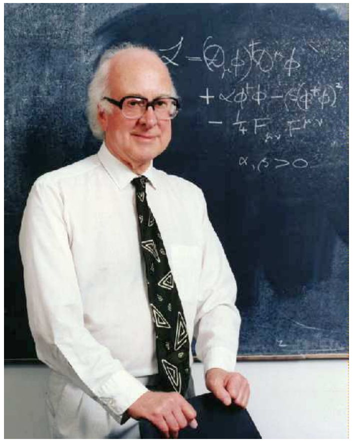
- Motivation: LHC @ NLO
- Framework for one-loop amplitudes: the GOLEM project
- The process $qq \longrightarrow b\bar{b}b\bar{b}$
- Unitarity vs. Feynman diagrammatic methods
- Summary

"...religious battle between Feynmanians and Unitarians..."

Joey Huston

The advent of the LHC era

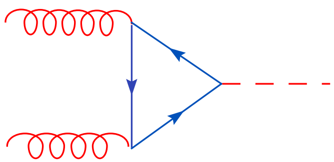
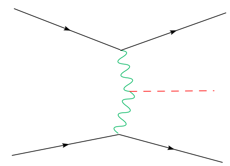
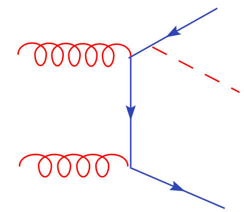
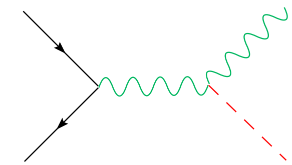
LHC: Large Hadron Collider at CERN, $\sqrt{s} = 14$ TeV, data expected soon !



What do we expect?

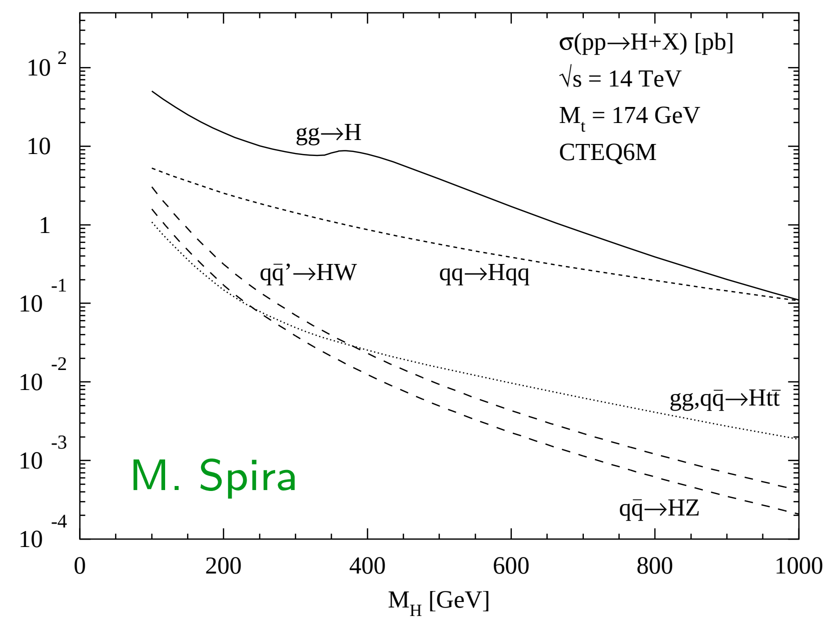
- test Higgs mechanism
  - $\longrightarrow$ SM Higgs boson: 114.4 GeV $< m_H < 185$ GeV (!)
  - $\longrightarrow$ $V(H) = \frac{1}{2} M_H^2 H^2 + \lambda_3\ H^3 + \lambda_4\ H^4$
    SM: $\lambda_4 = \lambda_3 / v = 3\ M_H^2 / v^2$
- explore physics beyond the Standard Model
  - $\longrightarrow$ SM $\subset$ "Extra Dimensions", "Little Higgs", "Strong interaction" Model
    SM $\subset$ MSSM $\subset$ SUSY GUT $\subset$ Supergravity $\subset$ Superstring $\subset$ $\mathcal{M}$-Theory
  - $\longrightarrow$ BSM something around 1 TeV (?)
- nothing ?!
  - $\longrightarrow$ hint of a hidden sector (?)
  - $\longrightarrow$ hint of strong interactions in the e.w. sector (?)

S+B for the Higgs boson

Signal:

- Decays: $H \to \gamma\gamma$, $H \to WW^{(*)}$, $H \to ZZ^{(*)}$, $H \to \tau^+\tau^-$
- $pp \to H + 0, 1, 2$ jets Gluon Fusion
- $pp \to Hjj$ Weak Boson Fusion
- $pp \to H + t\bar{t}$
- $pp \to H + W, Z$



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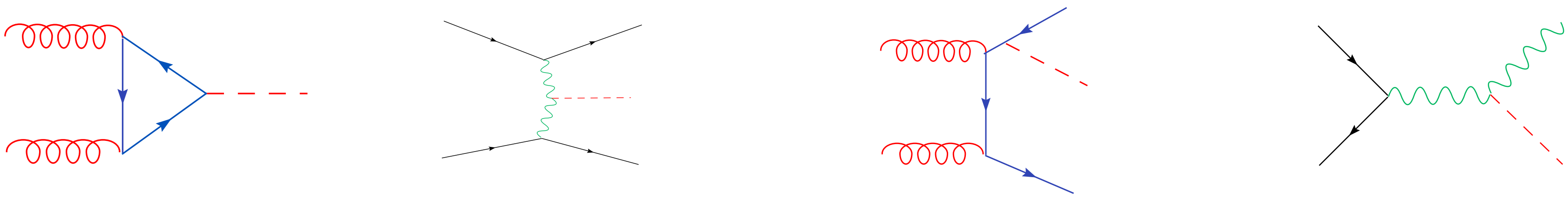

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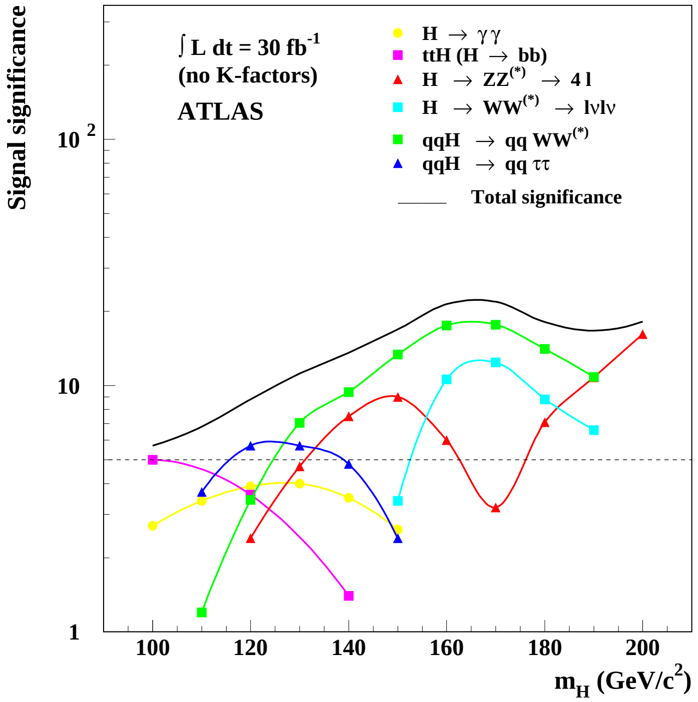
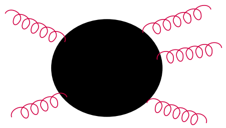
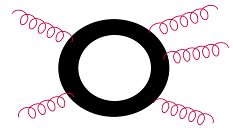
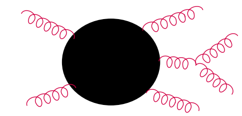


Backgrounds:

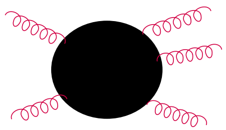
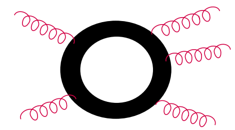
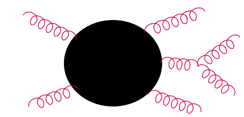
- $pp \to \gamma\gamma + 0, 1, 2\,\text{jets}$
- $pp \to WW^*, ZZ^* + 0, 1, 2\,\text{jets}$
- $pp \to t\bar{t} + 0, 1, 2\,\text{jets}$
- $pp \to V +$ up to $3\,\text{jets}$ $\quad (V = \gamma, W, Z)$
- $pp \to VVV + 0, 1\,\text{jet}$

Framework for NLO calculations

$\mathcal{M}_{\rm LO}$:  $\mathcal{M}_{\rm NLO,virtual}$:  $\mathcal{M}_{\rm NLO,real}$: 

$$
\begin{aligned}
\sigma &= \sigma_{LO} + \sigma_{NLO} \\
\sigma_{LO} &= \int dPS_N \, \frac{1}{2s} \, \mathcal{O}_N(\{p_j\}) \; |\mathcal{M}_{\rm LO}|^2 \\
\sigma_{NLO} &= \int dPS_N \, \frac{1}{2s} \, \alpha_s \, \Big(\mathcal{O}_N(\{p_j\}) \, \Big[\mathcal{M}_{\rm LO} \mathcal{M}^*_{\rm NLO,V} + \mathcal{M}^*_{\rm LO} \mathcal{M}_{\rm NLO,V} \Big] \\
&\qquad + \int dPS_1 \, \mathcal{O}_{N+1}(\{p_j\}) \; |\mathcal{M}_{\rm NLO,R}|^2 \Big)
\end{aligned}
$$

Framework for NLO calculations

$\mathcal{M}_{\mathrm{LO}}$:  $\mathcal{M}_{\mathrm{NLO,virtual}}$:  $\mathcal{M}_{\mathrm{NLO,real}}$: 

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\end{aligned}
$$

- For IR-safe observables, $\mathcal{O}_{N+1} \underset{IR}{\rightarrow} \mathcal{O}_N$, IR divergences cancel
- NLO $\rightarrow$ logarithmic scale dependences cancel, first sensible QCD approximation
- IR subtraction: e.g. dipole method à la Catani, Seymour (massless); Dittmaier, Trocsanyi, Weinzierl, Phaf (massive).
- automated dipole subtraction: Gleisberg, Krauss (2007); Seymour, Tevlin (2008); Hasegawa, Moch, Uwer (2008); Frederix, Gehrmann, Greiner (2008).
- Bottleneck: virtual corrections

Status QCD@NLO for LHC:

$2 \to 2:$ everything you want (see e.g. MCFM by Campbell/Ellis)

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$2 \to 3$: before 2005:

- $pp \to jjj$, $pp \to \gamma\gamma\, j$, $pp \to Vjj$
- $pp \to Hjj$ [WBF], $pp \to Hjj$ [GF], $pp \to Ht\bar{t}$

after 2005:

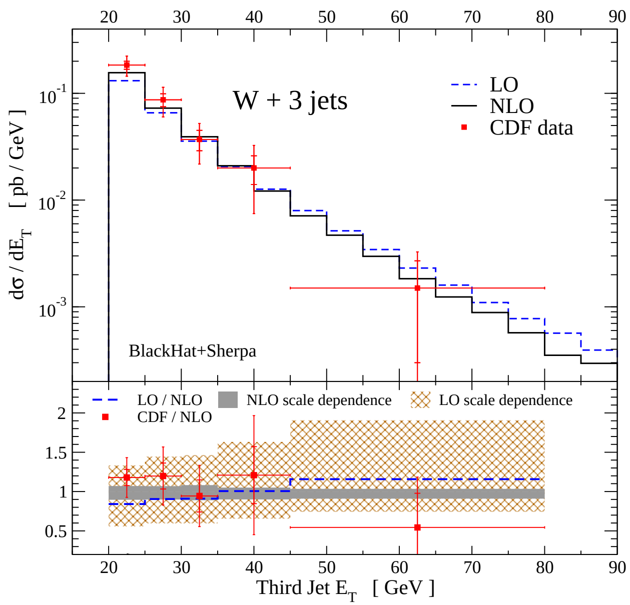
- $pp \to HHH$ (2005)
- $pp \to VVjj$ [WBF] (2006)
- $pp \to ZZZ$, $pp \to t\bar{t}j$, $pp \to WWj$ (2007)
- $pp \to VVV$, $pp \to b\bar{b}V$, $pp \to t\bar{t}Z$ (2008)

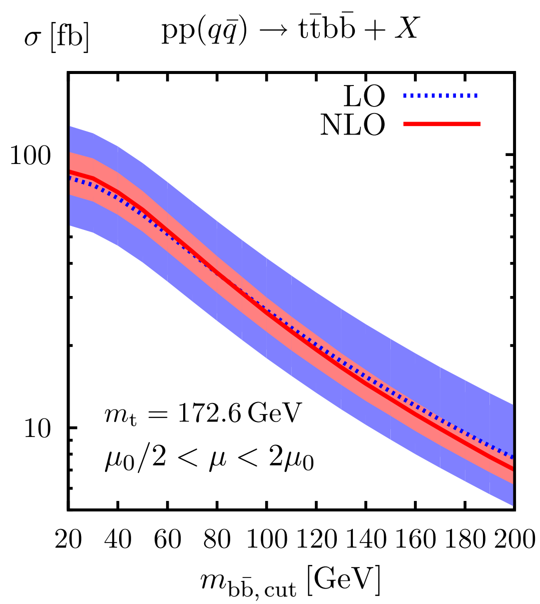
Many people involved: Andersen, Berger, Bern, Binoth, Bredenstein, Britto, Campbell, Dawson, del Duca, Denner, Dittmaier, Dixon, Ellis, Febres Cordero, Feng, Figy, Forde, Giele, Gleisberg, Glover, Guillet, Forde, Ita, Jager, Kallweit, Karg, Kosower, Kunszt, Lazopoulos, Mahmoudi, Maitre, Mastrolia, McElmurry, Melnikov, Miller, Nagy, Oleari, Orr, Ossola, Papadopoulos, Petriello, Pittau, Pozzorini, Reina, Sanguinetti, Smillie, Soper, Uwer, Wackeroth, Weinzierl, Zanderighi, Zeppenfeld,...and many others

Status QCD@NLO for LHC:

$2 \to 4:$ Complete LHC cross sections under construction!

- 6 gluon amplitude (1994-2006) cut-construction, ...
- 6 photon amplitude (2007) cut-construction, Feynman diagrams, OPP, numerical
- $N \leq 20$ gluon amplitudes evaluated (2008) Giele, Zanderighi
- $q\bar{q} \to b\bar{b}t\bar{t}$ (2008) Bredenstein, Denner, Dittmaier, Pozzorini
- $pp \to Wjjj$ (leading colour) (2009) $\to$ talks Kunszt, Zanderighi; Maitre





Blackhat collab., hep-ph:0902.2760 Bredenstein et. al., hep-ph:0807.1248

The GOLEM project

General One Loop Evaluator for Matrix elements

- Evaluation of 1-loop amplitudes bottleneck for LHC@NLO
- Combinatorial complexity $\leftrightarrow$ numerical instabilities
  $\Rightarrow$ switching between algebraic/numerical representations
- Aim: Automated evaluation of numerically stable one-loop amplitudes for multi-leg processes

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- Aim: Automated evaluation of numerically stable one-loop amplitudes for multi-leg processes
- The GOLEM team: T.B., A. Guffanti, J.Ph. Guillet, G. Heinrich, S. Karg, N. Kauer, E. Pilon, T. Reiter, G. Sanguinetti

Helicity management

Helicity amplitudes $\rightarrow$ work only with physical degrees of freedom
Spinor-helicity formalism: work with Weyl spinors for massless fermions

$$|k^{\pm}\rangle = \Pi^{\pm} u(k) \quad , \quad \langle k^{\pm}| = \bar{v}(k)\Pi^{\mp} \quad , \quad \not{k}|k^{\pm}\rangle = 0$$

spinor products: $\langle kq \rangle = \langle k^- | q^+ \rangle \quad , \quad [kq] = \langle k^+ | q^- \rangle$

massless gluon/photon (two helicity states, axial gauge: $\varepsilon^{\pm} \cdot k = \varepsilon^{\pm} \cdot q = 0$)

$$\varepsilon^{+}_{\mu}(k,q) = \frac{1}{\sqrt{2}} \frac{\langle q^- | \gamma_\mu | k^- \rangle}{\langle qk \rangle} \quad , \quad \varepsilon^{-}_{\mu}(k,q) = \frac{1}{\sqrt{2}} \frac{\langle q^+ | \gamma_\mu | k^+ \rangle}{[kq]}$$

$\Rightarrow$ Compact representations for tree and loop helicity amplitudes !

Note: for N-gluon amplitude 2^N helicity amplitudes (exponential growth)

Colour management:

widely used: colour flow representation for $SU(N_C)$
'tHooft (1974); Maltoni, Paul, Stelzer, Willenbrock (2001)

$$
\begin{aligned}
i f^{abc} T^c_{ik} &= T^a_{ij} T^b_{jk} - T^b_{ij} T^a_{jk} \\
T^a_{ik} T^a_{jl} &= \frac{1}{2}\left(\delta^i_l \delta^j_k - \frac{1}{N_C} \delta^i_k \delta^j_l \right)
\end{aligned}
$$

maps to colour basis (N = # gluons + #quark lines)

$$
\mathcal{A} = \sum_{\sigma \in S_N} \mathcal{A}_\sigma |c_\sigma\rangle \quad , \quad |c_\sigma\rangle = \delta^{j_{\sigma(1)}}_{i_1} \delta^{j_{\sigma(2)}}_{i_2} \ldots \delta^{j_{\sigma(N)}}_{i_N}
$$

for N-gluon amplitude $(N-2)!$ independent colour states (factorial growth)

Feynman diagrammatic approach:

$$\mathcal{A}^{c,\lambda}(p_j, m_j) = \sum_{\alpha}^{\#graphs} \sum_{\sigma \in S_N} \mathcal{G}^{\{\lambda\}}_{\alpha,\sigma} |c_\sigma\rangle$$

$$\mathcal{G}^{\{\lambda\}}_{\alpha} = \int \frac{d^n k}{i\pi^{n/2}} \frac{\mathcal{N}^{\{\lambda\}}}{D_1 \dots D_N} = \sum_R \mathcal{N}^{\{\lambda\}}_{\mu_1,\dots,\mu_R} \, I_N^{\mu_1 \cdots \mu_R}(p_j, m_j)$$

$$I_N^{\mu_1 \cdots \mu_R}(p_j, m_j) = \int \frac{d^n k}{i\pi^{n/2}} \frac{k^{\mu_1} \dots k^{\mu_R}}{D_1 \dots D_N} \, , \; D_j = (k - r_j)^2 - m_j^2, \; r_j = p_1 \cdots + p_j$$

- Passarino-Veltman: $\rightarrow 1/\det(G)^R$, $G_{ij} = 2\, r_i \cdot r_j$ induce numerical problems
- projection on helicity amplitudes reduces $2\, k \cdot r_j = D_N - D_j + r_j \cdot r_j$
- Lorentz Tensor Integrals $\rightarrow$ form factor representation à la Davydychev
- Reduction in Feynman parameter space

$$I_N^{\mu_1 \cdots \mu_R} = \sum \tau^{\mu_1 \cdots \mu_R}(r_{j_1}, \dots, r_{j_r}, g^m) \, I_N^{n+2m}(j_1, \dots, j_r)$$

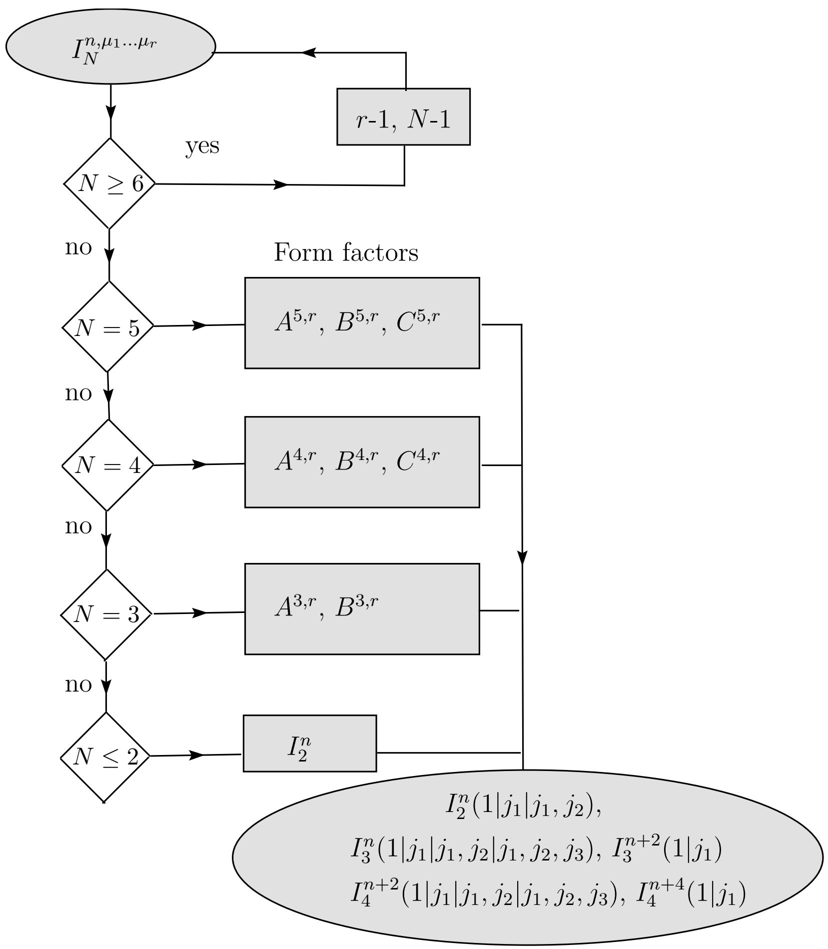
$$I_N^D(j_1, \dots, j_r) = (-1)^N \Gamma(N - \frac{D}{2}) \int_0^\infty d^N z \, \delta(1 - \sum_{l=1}^N z_l) \frac{z_{j_1} \dots z_{j_r}}{(-\frac{1}{2} z \cdot \mathcal{S} \cdot z)^{N-D/2}}$$

$$\mathcal{S}_{ij} = (r_i - r_j)^2 - m_i^2 - m_j^2$$

Schematic overview of N-point tensor integral reduction

T.B., J.P. Guillet, G. Heinrich (2000); T.B., Guillet, Heinrich, Pilon, Schubert (2005).

- works for general N
- no inverse Gram determinants
- isolation of IR divergences simple
- tractable expressions
- form factors for $N \leq 6$ implemented in Fortran95 code "Golem95"
- optional reduction to scalar integrals
- evaluation of rational terms



$$I^{n,n+2}_{N=3,4}(j_1,\ldots,j_R) \quad \sim \quad \int_0^1 \prod_{i=1}^{4} dz_i \, \delta(1-\sum_{l=1}^{4} z_l) \, \frac{z_{j_1}\ldots z_{j_r}}{(-\frac{1}{2}\, z\cdot\mathcal{S}\cdot z - i\delta)^{3-n/2}}$$

Form factors and the Golem95 library

$$I_N^{\mu_1\cdots\mu_r}(S) =$$

$$\sum_{l_1\cdots l_R\in S} \left[r_{l_1\cdot}^{\cdot}\cdots r_{l_R\cdot}^{\cdot}\right]^{\{\mu_1\cdots\mu_R\}} A_{l_1\ldots,l_R}^{N,R}(S)$$

$$+ \sum_{l_1\cdots l_{R-2}\in S} \left[g^{\cdot\cdot} r_{l_1\cdot}^{\cdot}\cdots r_{l_{R-2}\cdot}^{\cdot}\right]^{\{\mu_1\cdots\mu_R\}} B_{l_1\ldots,l_{R-2}}^{N,R}(S)$$

$$+ \sum_{l_1\cdots l_{R-4}\in S} \left[g^{\cdot\cdot} g^{\cdot\cdot} r_{l_1\cdot}^{\cdot}\cdots r_{l_{R-4}\cdot}^{\cdot}\right]^{\{\mu_1\cdots\mu_r\}} C_{l_1\ldots,l_{R-4}}^{N,R}(S)$$

- use set notation for identifying integrals and kinematics: $S = \{1, \ldots, N\}$
- all form factors coded in Golem95 $(N \leq 6, m_j^2 = 0)$
- Code + instructions + demos: http://lappweb.in2p3.fr/lapth/Golem/golem95.html
- for $N > 6$: $I_N^R(S) \to \sum_{j\in S} C(j)\, I_{N-1}^{R-1}(S/\{j\})$

Form factors and the Golem95 library

$$I_5^{\mu_1\,\mu_2}(S) = g^{\mu_1\,\mu_2}\,B^{5,2}(S) + \sum_{l_1,l_2\in S} r_{l_1}^{\mu_1}\,r_{l_2}^{\mu_2}\,A_{l_1\,l_2}^{5,2}(S)$$

$$B^{5,2}(S) = -\frac{1}{2}\sum_{j\in S} b_j\,I_4^{n+2}(S\setminus\{j\})$$

$$A_{l_1 l_2}^{5,2}(S) =$$

$$\sum_{j\in S}\left(\mathcal{S}^{-1}{}_{j\,l_1}\,b_{l_2} + \mathcal{S}^{-1}{}_{j\,l_2}\,b_{l_1} - 2\,\mathcal{S}^{-1}{}_{l_1\,l_2}\,b_j + b_j\,\mathcal{S}^{\{j\}-1}{}_{l_1\,l_2}\right)\,I_4^{n+2}(S\setminus\{j\})$$

$$+\frac{1}{2}\sum_{j\in S}\sum_{k\in S\setminus\{j\}}\left[\mathcal{S}^{-1}{}_{j\,l_2}\,\mathcal{S}^{\{j\}-1}{}_{k\,l_1} + \mathcal{S}^{-1}{}_{j\,l_1}\,\mathcal{S}^{\{j\}-1}{}_{k\,l_2}\right]\,I_3^n(S\setminus\{j,k\})$$

- algebraic separation of IR poles, contained in 3-point integrals
- option to compile in double/quadruple precision
- exceptional kinematics: numerical evaluation one-dimensional integral representations available
- caches avoid multiple evaluation of same object

Implementation of amplitude evaluation in a nutshell

Preparation:

- Diagram generation: `QGRAF` P. Nogueira, `FeynArts 3.2` T. Hahn
- Perform colour algebra
- Determine integral basis
- Project on helicity amplitudes

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From here two independent set-ups:

a) Symbolic reduction to scalar integrals based on FORM and MAPLE/MATHEMATICA

- $\mathcal{M}^{\{\lambda\}} \to C_{box} I_4^{d=6} + C_{tri} I_3^{d=4-2\epsilon} + C_{bub} I_2^{d=4-2\epsilon} + C_{tad} I_1^{d=4-2\epsilon} + \mathcal{R}$
- automated method to evaluate $\mathcal{R}$ T.B., Guillet, Heinrich (2006)
- introduces $1/\det G$ but allows to apply symbolic simplifications

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b) Convert to form factor representation, link to `Fortran95` library "golem95"

- $\mathcal{M}^{\{\lambda\}} \to C_{box}^{j_1 j_2 j_3} I_4^{n+2,n+4}(j_1,j_2,j_3) + C_{tri}^{j_1 j_2 j_3} I_3^{n,n+2}(j_1,j_2,j_3) + \ldots$
- In numerically critical phase space regions:
  – use one-dimensional integral representations for $I_{N=3,4}^{n+2,n+4}(j_1,j_2,j_3)$

Computations with GOLEM:

some recent applications ...

- $gg \to W^*W^* \to l\nu l'\nu'$, GG2WW code
  http://hepsource.sourceforge.net/programs/GG2WW
  T.B., M. Ciccolini, M. Krämer, N. Kauer (2006)
- $gg \to HH, HHH$
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- $pp \to Hjj$ GF/WBF NLO interference $\mathcal{O}(\alpha^2\alpha_s^3)$
  J.R. Andersen, T.B., G. Heinrich, J. Smillie (2007)
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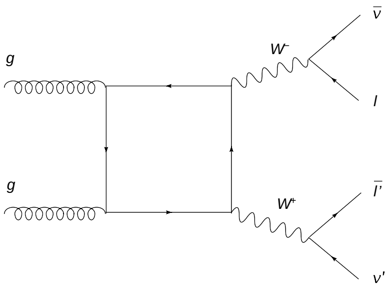

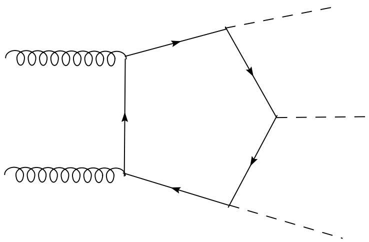

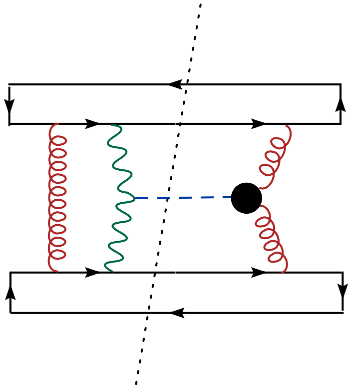

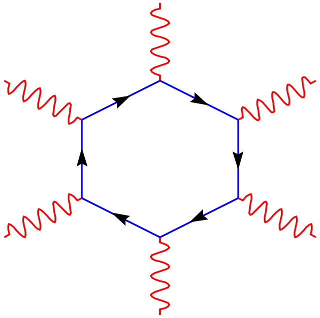

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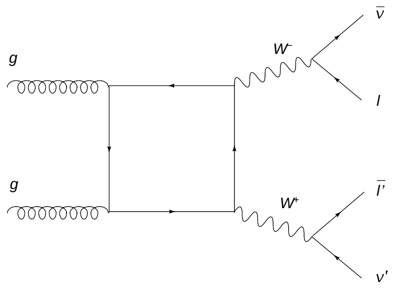
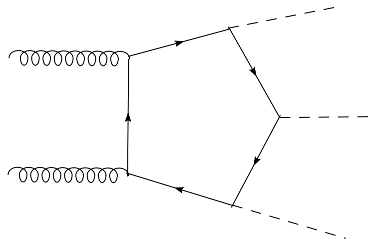
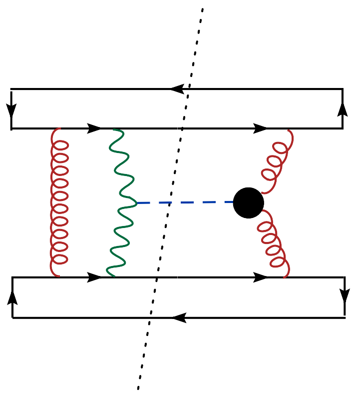
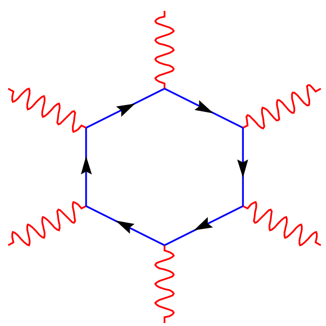






... and ongoing work:

- $gg \to Z^*Z^*, \gamma^*Z^*, \gamma^*\gamma^* \to l\bar{l}l'\bar{l}'$, GG2ZZ code
- $pp \to WWj, ZZj$, $gg \to WWg, ZZg$
- $pp \to bbbb$

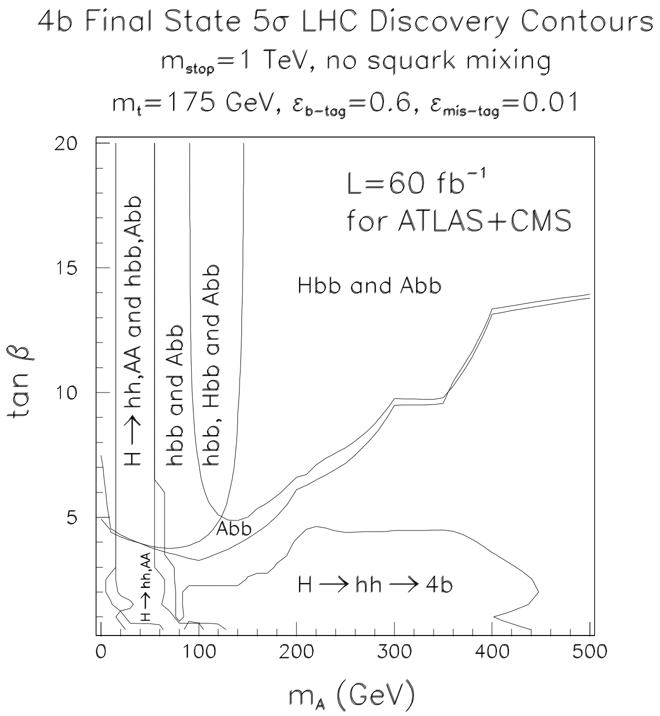


The process $pp \to b\bar{b}b\bar{b}$ at NLO QCD

Motivation: Higgs search in two Higgs doublet models/MSSM for large $\tan\beta$

Dai, Gunion, Vega 1995/1996; Richter-Was, Froidveaux 1997;
Lafaye, Miller, Muhlleitner, Moretti 2000

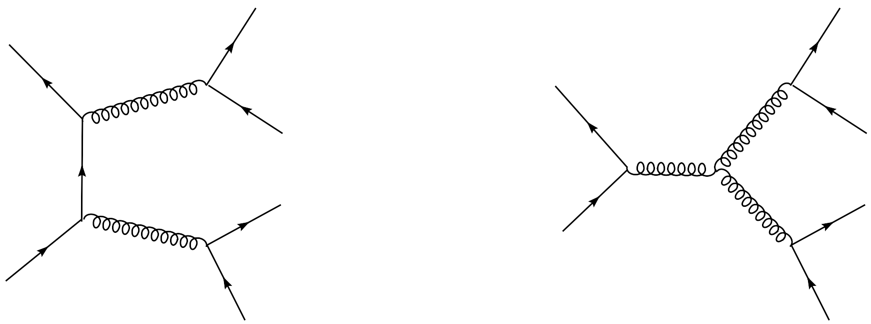
- "$b\bar{b}\tau^+\tau^-$, $b\bar{b}b\bar{b}$ may provide only access to two of the three neutral Higgs bosons"
- "explicit calculation of K-factors needed."
- included in the Les Houches wish-list 2007



Dai et. al. Phys. Lett. B387 (1996)

Structure of the amplitude

- 2 initial states: $q\bar{q} \to b\bar{b}b\bar{b}$, $gg \to b\bar{b}b\bar{b}$
- $\mathcal{A}(q\bar{q} \to b\bar{b}b\bar{b}) = \mathcal{A}(q\bar{q} \to b\bar{b}b'\bar{b}') - \mathcal{A}(q\bar{q} \to b\bar{b}'b'\bar{b})$
- two helicity amplitudes needed: $\mathcal{A}^{++++++}$, $\mathcal{A}^{++++--}$
- six different colour structures: $\mathcal{A} = \sum_{j=1,6} |c_j\rangle \mathcal{A}_j$



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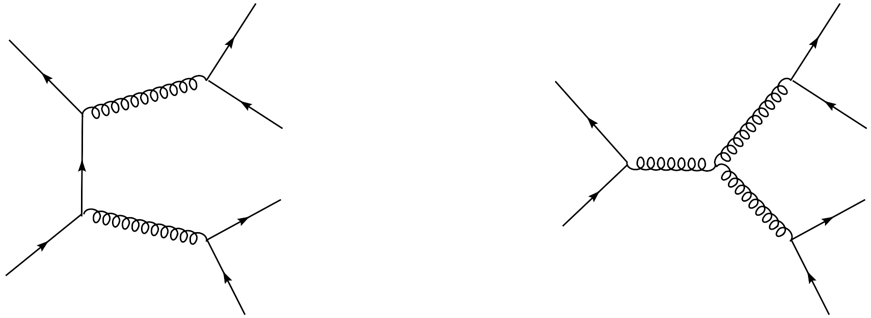
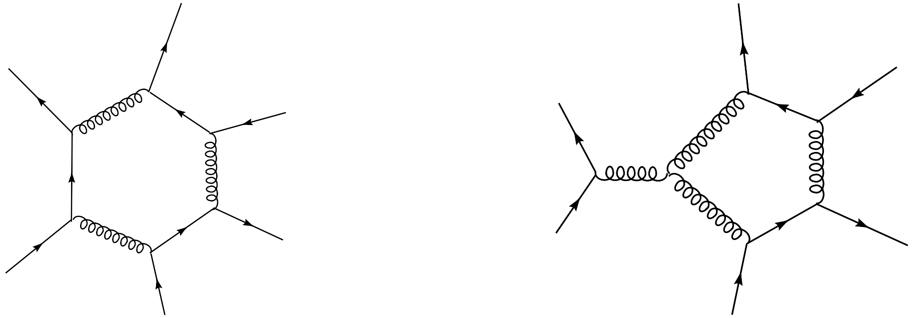


- Virtual corrections $q\bar{q} \to b\bar{b}b'\bar{b}'$
  $\sim$ 250 diagrams, 25 pentagon and 8 hexagon diagrams, 8 independent scales



Evaluation of loop amplitude

- Diagram generation: QGRAF P. Nogueira, FeynArts 3.2 T. Hahn
- Colour algebra [colour flow decomposition] $\Rightarrow \mathcal{A} = \sum_n \sum_c \mathcal{G}_{nc} |c\rangle$

$$\mathcal{G}^{\lambda_1\lambda_1\lambda_3\lambda_3\lambda_5\lambda_5}_{\ldots} = \sum_{\alpha\beta\gamma} \int \frac{dk^n}{i\pi^{n/2}} T^{\alpha\beta\gamma}_{\ldots}(k, \{p_j\}) \langle 2^{\lambda_1} | \Gamma^{(1)}_\alpha | 1^{\lambda_1} \rangle \langle 3^{\lambda_3} | \Gamma^{(2)}_\beta | 4^{\lambda_3} \rangle \langle 5^{\lambda_5} | \Gamma^{(3)}_\gamma | 6^{\lambda_5} \rangle$$

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- Helicity projection
  e.g. for $\mathcal{A}^{++++++}$ multiply with $\frac{\langle 1^+|3|2^+\rangle\langle 4^+|\mu|3^+\rangle\langle 6^+|\mu|5^+\rangle}{2[13]\langle 32\rangle[46]\langle 53\rangle} = 1$

$$\mathcal{G}_{\dots}^{++++++} = \sum_{\alpha\beta\gamma} \int \frac{dk^n}{i\pi^{n/2}} T_{\dots}^{\alpha\beta\gamma}(k,\{p_j\}) \frac{\mathrm{tr}^+(132\Gamma_\alpha^{(1)})\mathrm{tr}^+(4\hat{\mu}3\Gamma_\beta^{(2)})\mathrm{tr}^+(6\hat{\mu}5\Gamma_\gamma^{(3)})}{2[13]\langle 32\rangle[46]\langle 53\rangle}$$

- γ_5 + dim. reg. $\Rightarrow$ 'tHooft-Veltman Scheme and dimension splitting rules
  $k_j = \hat{k}_j$, $k = \hat{k} + \tilde{k}$, $\gamma = \hat{\gamma} + \tilde{\gamma}$, $\{\gamma_5, \hat{\gamma}\} = 0$, $[\gamma_5, \tilde{\gamma}] = 0$

Evaluation of loop amplitude

Strategy 1: Form factor representation

- $$\int \frac{dk^D}{i\pi^{D/2}} \frac{k_1^\mu \dots k_R^\mu}{(k+r_1)^2 \dots (k+r_N)^2} \to A_N^R, B_N^R, C_N^R \times [g^{\cdot\cdot} \dots r^{\cdot} \dots]^{\{\mu_1 \dots \mu_R\}}$$

  leads to

$$\mathcal{A}^{\{\lambda\}} = \exp(i\phi) \sum \text{Form Factors} \otimes \prod_j \text{tr}_j^{\pm}(\{p_l\})$$

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  leads to

  $$\mathcal{A}^{\{\lambda\}} = \exp(i\phi) \sum \text{Form Factors} \otimes \prod_j \text{tr}_j^{\pm}(\{p_l\})$$

- export to `Fortran 95` code
- link with `Golem95` library
- all steps are fully automated
- $\prod \text{tr}_j^{\pm}(\{p_l\}) \sim \text{Polynomials}(\{\langle ij \rangle, [kl]\})$ evaluated numerically using `Golem95`

Evaluation of loop amplitude

Strategy 2: Master integral representation

- symbolic evaluation with FORM
- irreducible scalar products canceled algebraically
- at most rank 1 6-point functions, rank 3 5-point functions
- symbolic reduction to master integrals $I_4^{D=6}$, $I_3^{D=4-2\epsilon}$, $I_2^{D=4-2\epsilon}$, $\mathcal{R}$ leads to

$$\mathcal{A}^{\{\lambda\}} = \exp(i\phi) \sum \text{Polynomials}(\{s_{ij}\}) \otimes \text{Master integrals}$$

Evaluation of loop amplitude

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- symbolic evaluation with `FORM`
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$$\mathcal{A}^{\{\lambda\}} = \exp(i\phi) \sum \text{Polynomials}(\{s_{ij}\}) \otimes \text{Master integrals}$$

- automated simplification of polynomial coefficients with `MAPLE`
- not as efficient as strategy 1 (room for improvement!)
- used as independent check

Renormalization and IR-structure

$$|\mathcal{A}_{LO+V}\rangle = |\mathcal{A}_0\rangle + \alpha_s|\mathcal{A}_1\rangle = \sum_l \Big(|\mathcal{A}_{0,l}\rangle + \alpha_s|\mathcal{A}_{1,l}\rangle\Big) \times |c_l\rangle$$

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$$\begin{aligned}
\langle\mathcal{A}_{LO+V}|\mathcal{A}_{LO+V}\rangle &= \sum_{kl}\langle\mathcal{A}_{0,k}|\mathcal{A}_{0,l}\rangle\langle c_l|c_k\rangle\Big(1 - \frac{1}{\epsilon}\frac{2\beta_0}{\pi}\alpha_s\Big) \\
&\quad + \alpha_s \sum_{kl} \left(\langle\mathcal{A}_{0,k}|\mathcal{A}_{1,l}\rangle + \langle\mathcal{A}_{1,k}|\mathcal{A}_{0,l}\rangle\right)\langle c_l|c_k\rangle \\
&\quad - \alpha_s \sum_{kl}\langle\mathcal{A}_{0,k}|\mathcal{A}_{0,l}\rangle\langle c_l|\mathbf{I}(\epsilon)|c_k\rangle
\end{aligned}$$

$$\begin{aligned}
\langle c_j|\mathbf{I}(\epsilon)|c_k\rangle &= \frac{1}{2\pi}\frac{1}{\Gamma(1-\epsilon)}\frac{\mathcal{V}_q}{C_F}\sum_{I,J}\ \langle c_j|\mathbf{T}_I\cdot\mathbf{T}_J|c_k\rangle\left(\frac{4\pi\mu^2}{2p_I\cdot p_J}\right)^{\epsilon} \\
\mathcal{V}_q &= C_F\left(\frac{1}{\epsilon^2} - \log^2(\alpha) + \frac{3}{2}\frac{1}{\epsilon} + \frac{3}{2}(\alpha - 1 - \log(\alpha)) + 5 - \frac{\pi^2}{2}\right)
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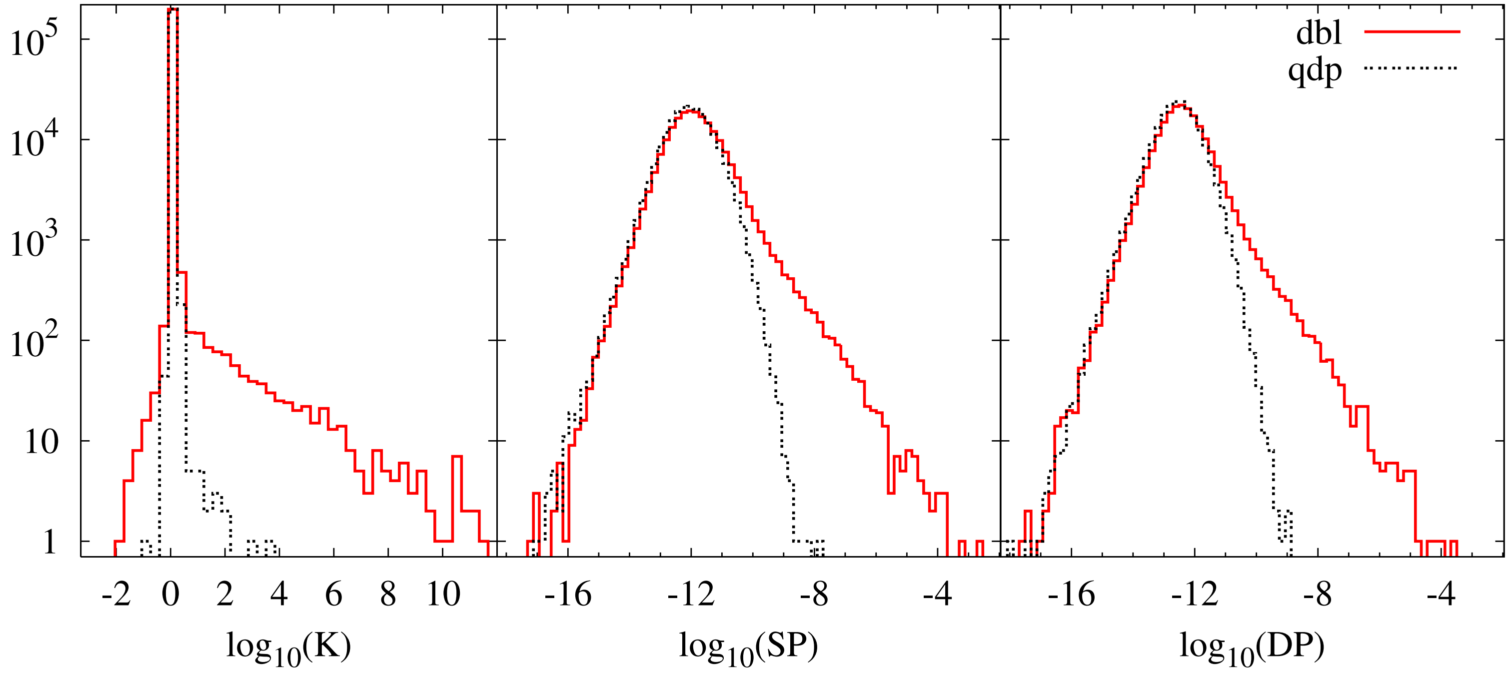
$$\begin{aligned}\langle c_j|\mathbf{I}(\epsilon)|c_k\rangle \quad &= \quad \frac{1}{2\pi}\frac{1}{\Gamma(1-\epsilon)}\frac{\mathcal{V}_q}{C_F}\sum_{I,J}\ \langle c_j|\mathbf{T}_I\cdot\mathbf{T}_J|c_k\rangle\left(\frac{4\pi\mu^2}{2p_I\cdot p_J}\right)^{\epsilon} \\ \mathcal{V}_q \quad &= \quad C_F\left(\frac{1}{\epsilon^2} - \log^2(\alpha) + \frac{3}{2}\frac{1}{\epsilon} + \frac{3}{2}(\alpha - 1 - \log(\alpha)) + 5 - \frac{\pi^2}{2}\right)\end{aligned}$$

- cancellation of pole parts non-trivial check of implementation
- GOLEM aims to provide the finite combination $|\mathcal{A}_{LO+V}|^2$
- we use "α"-improved version of dipole subtraction method (Z. Nagy), $\alpha = 0.1$

Numerical precision of virtual correction evaluation

- evaluation of 200.000 random phase space points
- cuts: $\eta < |2.5|$, $\Delta R > 0.4$, $p_T > 25$ GeV

$$K = \frac{|\mathcal{A}_{LO+V}|^2}{|\mathcal{A}_{LO}|^2}$$



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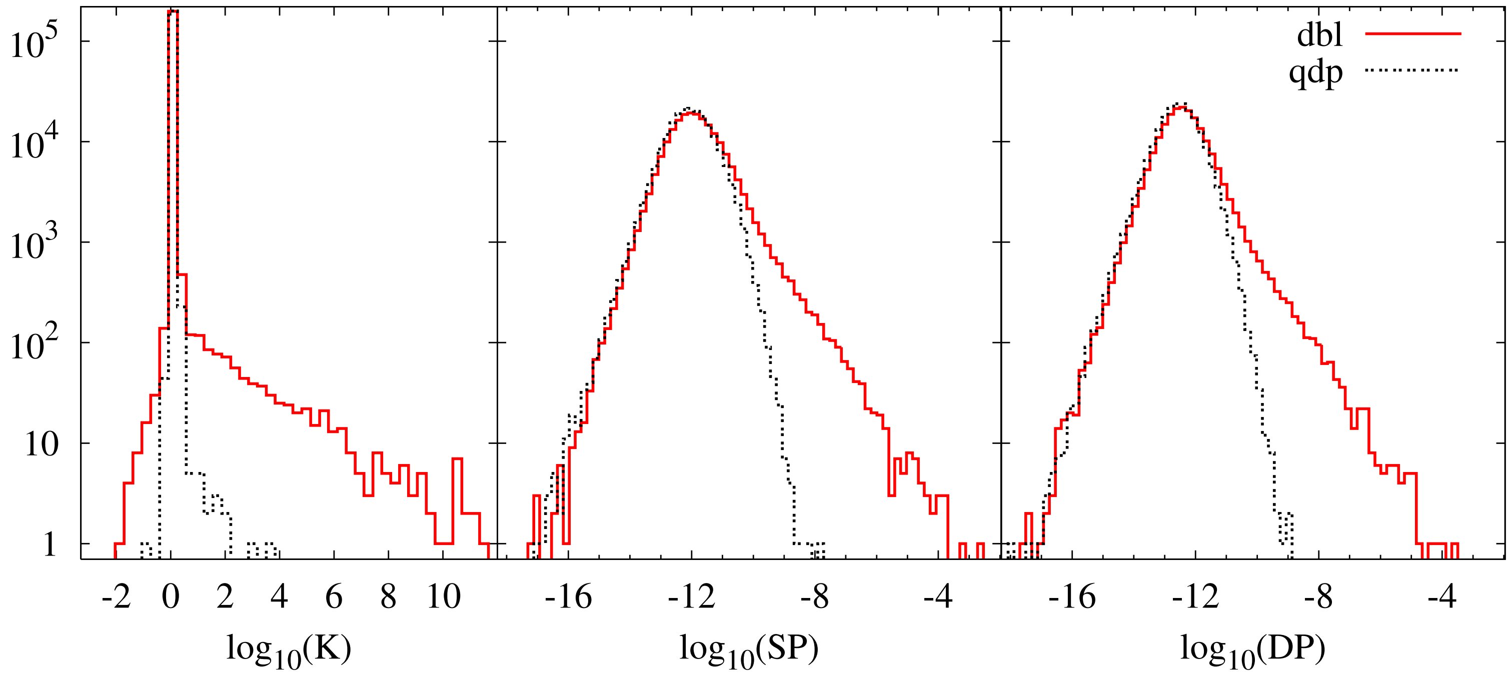


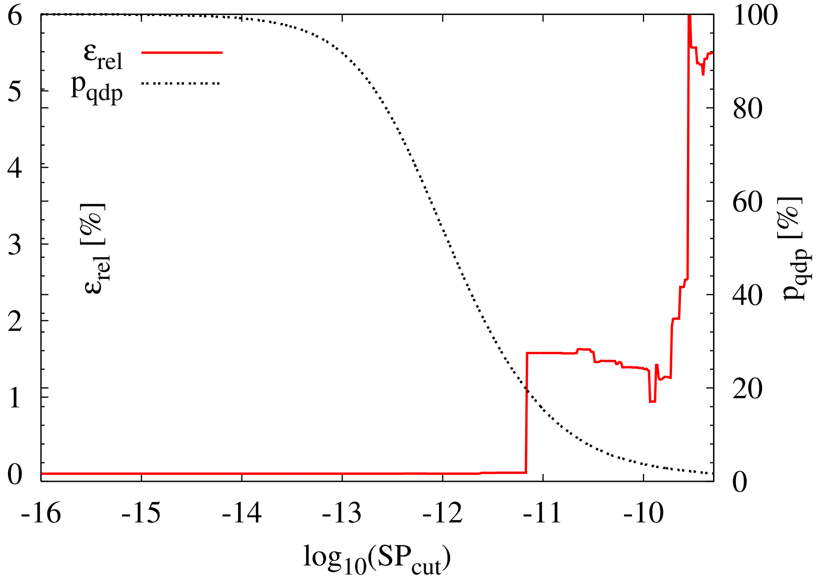
- imperfect numerical cancellations lead to inaccuracies
- dangerous for adaptive integration methods
- size of local K-factor good indicator for numerical problem

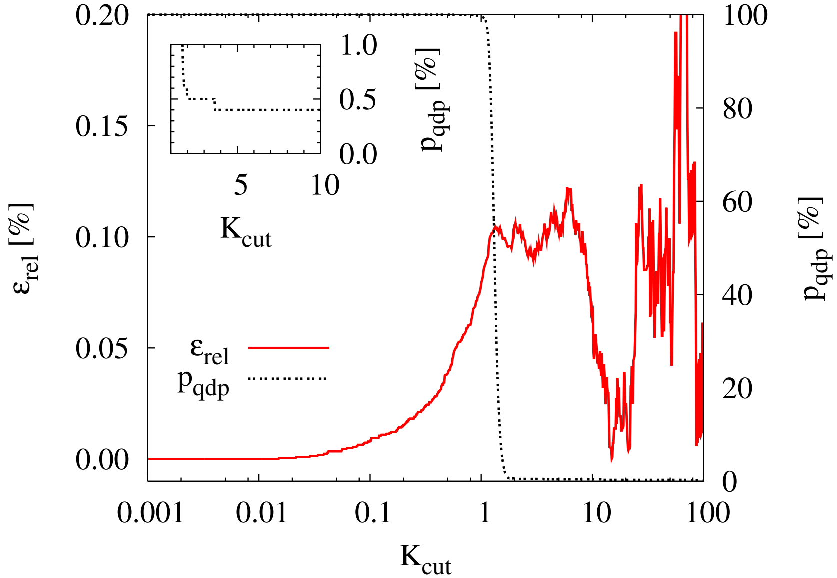
Criterion for double/quad precision

Switch to quadruple precision if

- single pole (SP) cancellation better than SP_{CUT}
- K factor is larger than K_{CUT}

$$\epsilon_{rel} = \frac{\sigma(SP/K{-}cut) - \sigma(quad.prec)}{\sigma(quad.prec)}$$





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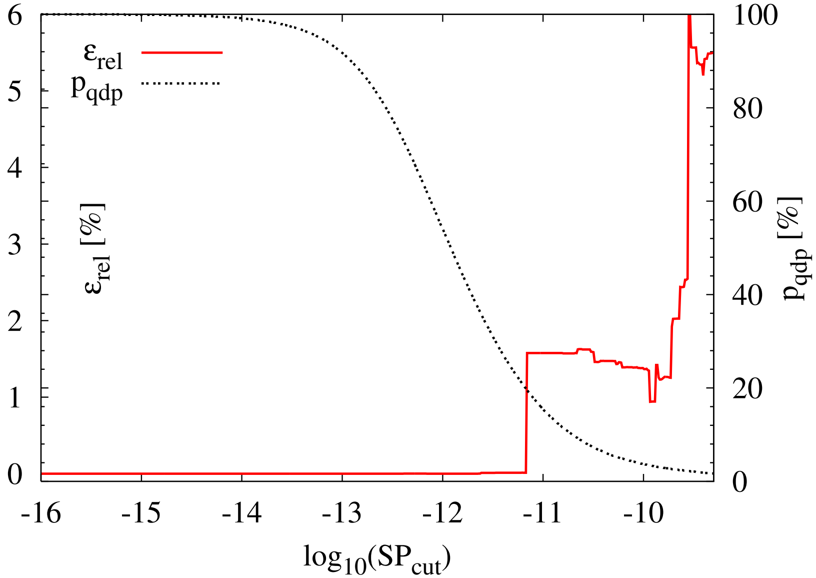

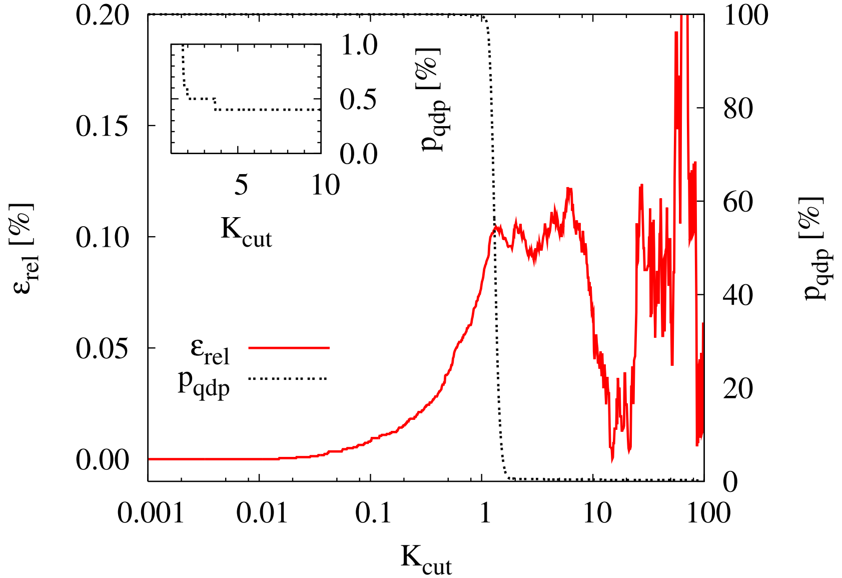

Statistical precision better than 1 % needs:

- SP criterion: needs 20 % of points in quadruple precision
- K-factor criterion: needs less than 1% !

GOLEM integration strategy

Step 1:

- generate unweighted event sample from $\sigma_{LO} \sim |\mathcal{A}_{LO}|^2$
- sort event into histograms

$$
\begin{aligned}
\sigma_{LO} &= \int d\vec{x}\, f_0(\vec{x}) = \frac{1}{N} \sum_{j=1}^{N} f_0(x_j) \\
&= \sigma_{LO} \int d\vec{y} = \frac{\sigma_{LO}}{N} \sum_{j=1}^{N} 1 \\
\langle \mathcal{O} \rangle_{LO} &= \frac{\sigma_{LO}}{N} \sum_{j=1}^{N} \chi(E_j) \;, \quad \chi(E_j) = \begin{cases} 1, & E_j \in \mathcal{O} \\ 0, & \text{else} \end{cases}
\end{aligned}
$$

GOLEM integration strategy

Step 2:

- reweight each event, E_j, by local K-factor: $K = f_1/f_0$
- no destructive interference with phase space integration !

$$\begin{aligned}
\sigma_{LO+virtual} &= \int d\vec{x} f_1(\vec{x}) \\
&= \sigma_{LO} \int d\vec{y}\, K(\vec{y})\,, \\
\langle \mathcal{O} \rangle_{LO+virt.} &= \frac{\sigma_{LO}}{N} \sum_{j=1}^{N} \chi(E_j) K(E_j)
\end{aligned}$$

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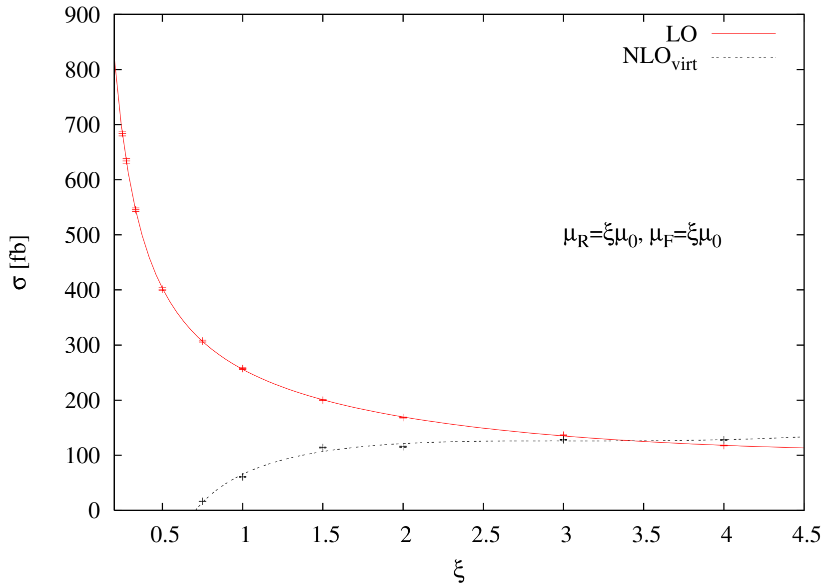
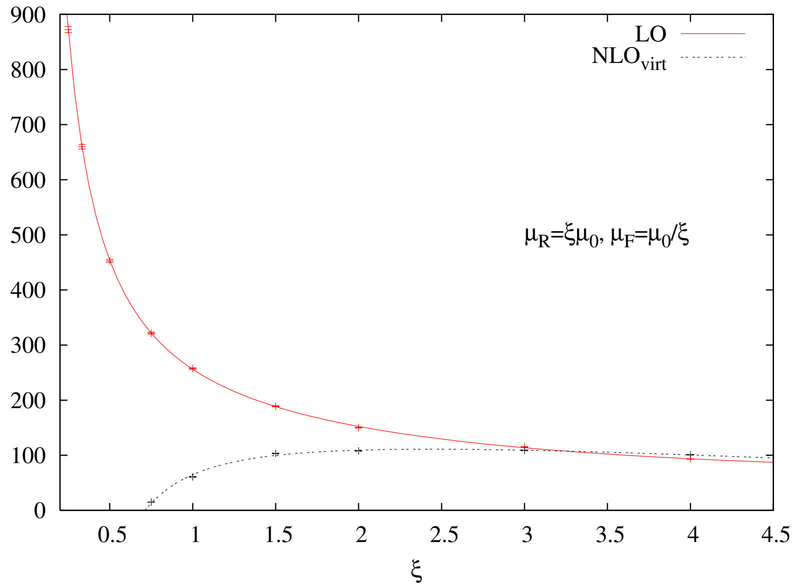
$$\langle \mathcal{O} \rangle_{LO+virt.} = \frac{\sigma_{LO}}{N} \sum_{j=1}^{N} \chi(E_j) K(E_j)$$

- GOLEM: UV/IR subtracted one-loop amplitudes $\Rightarrow K(E_j)$
- Need in addition: tree-level matrix element generator like Madgraph, Whizard, Sherpa,...
- including dipole subtraction, e.g. MadDipole,TevJet,Sherpa,...

Scale variations $\sigma_{NLO,Virtual}(q\bar{q} \rightarrow b\bar{b}b\bar{b})$

Standard scale choice: $\mu_R = \mu_F = \sum_{j=1}^{4} p_{T\,j}/4$





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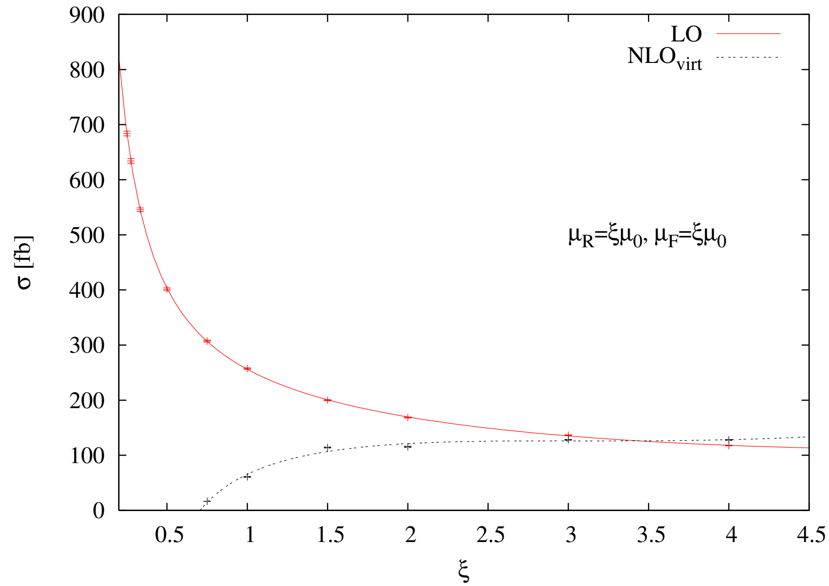
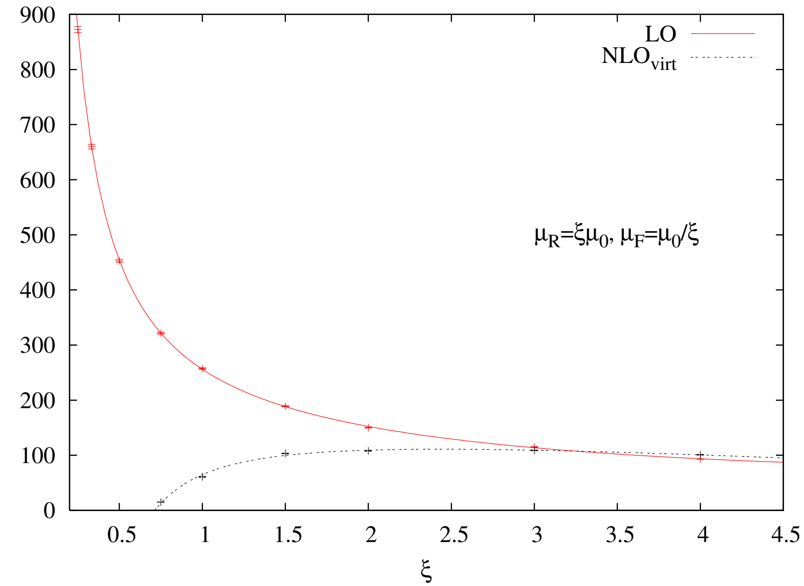
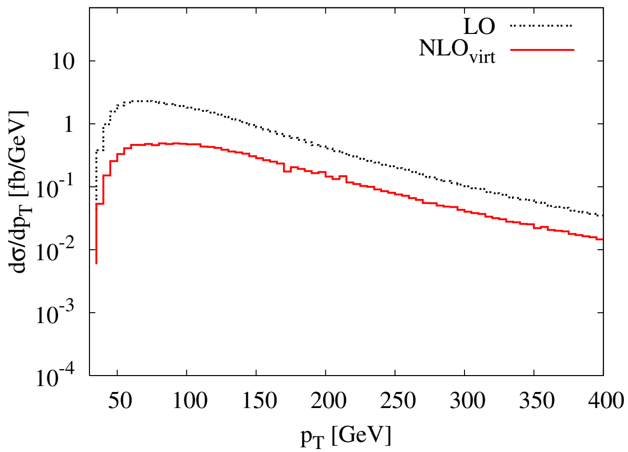
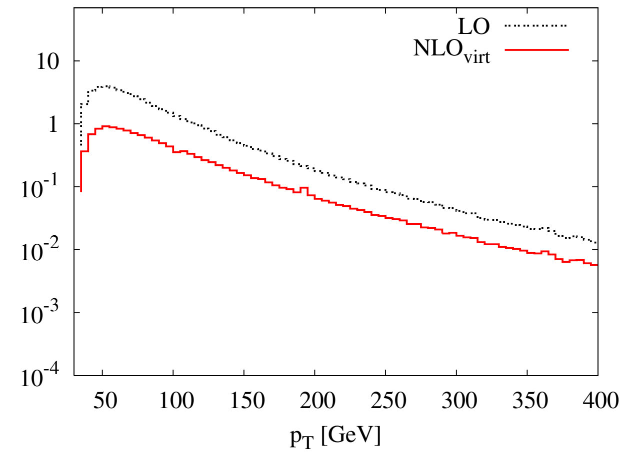


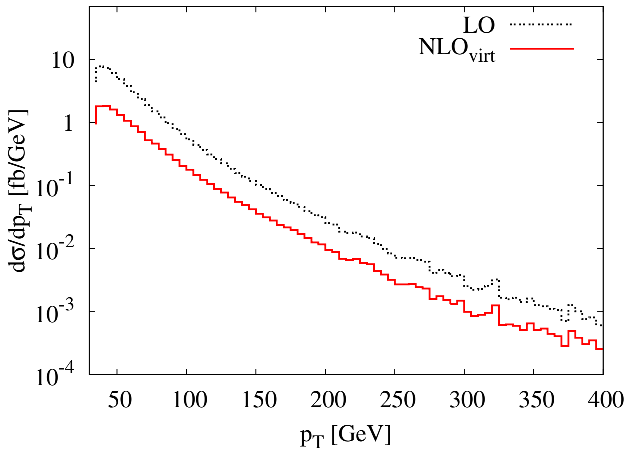


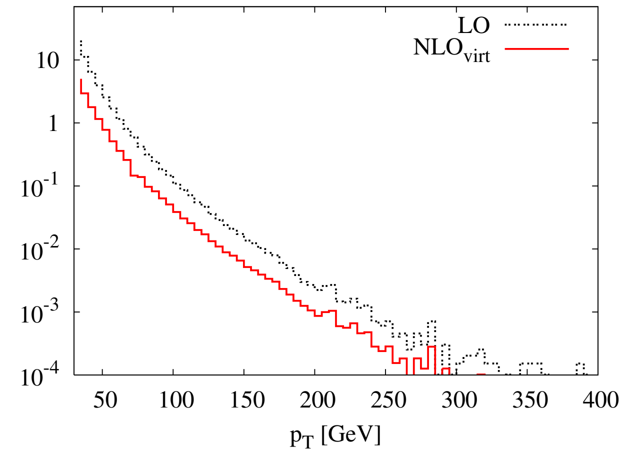
- Dipole subtraction done only in relevant part of phase space [Z. Nagy] $\to \alpha = 0.1$
- real emission contribution not yet added, further compensation of $\alpha_s \log(\mu_F)$
- under construction using Whizard, MadDipole

p_T distributions of leading, subleading, etc. b-jet

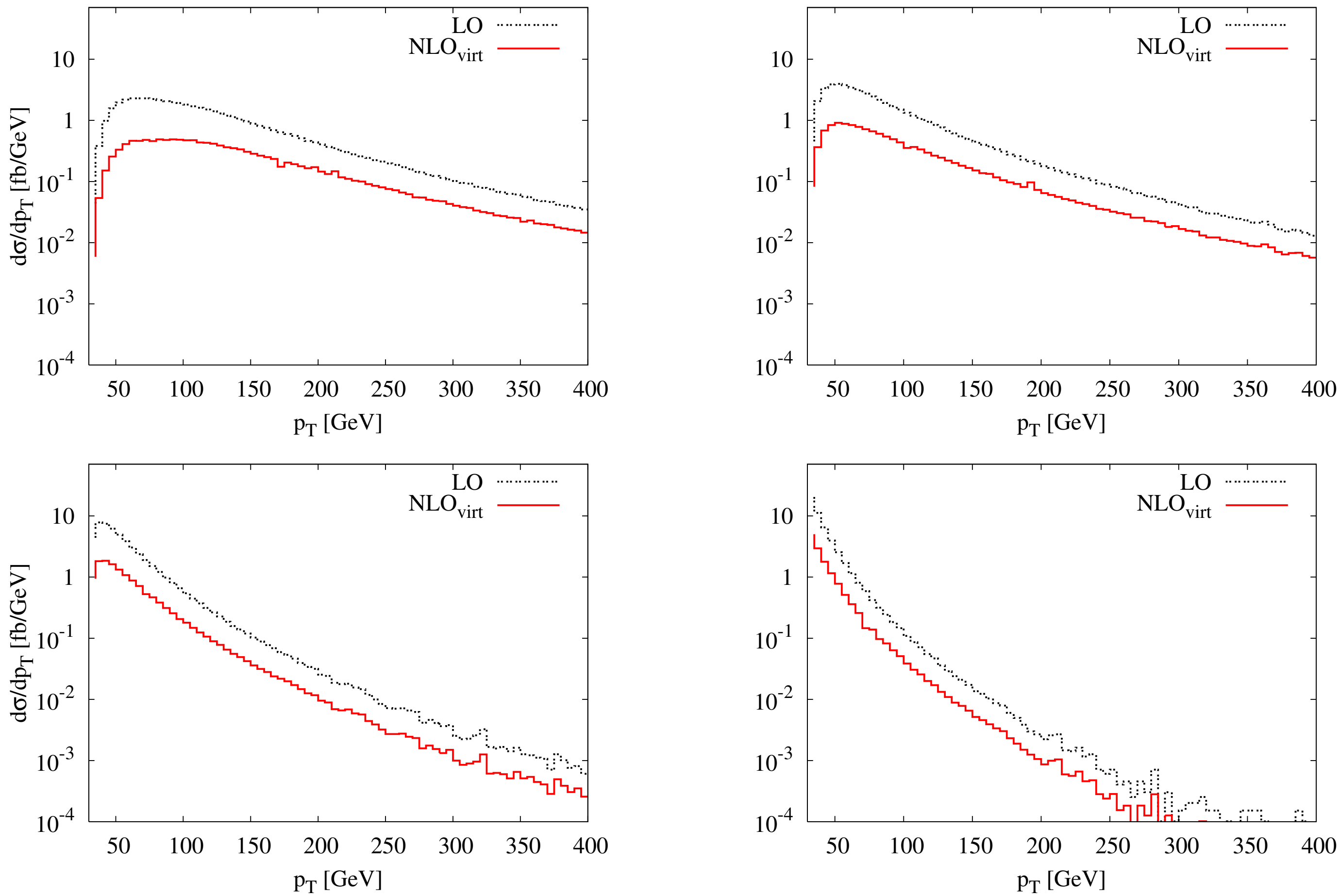






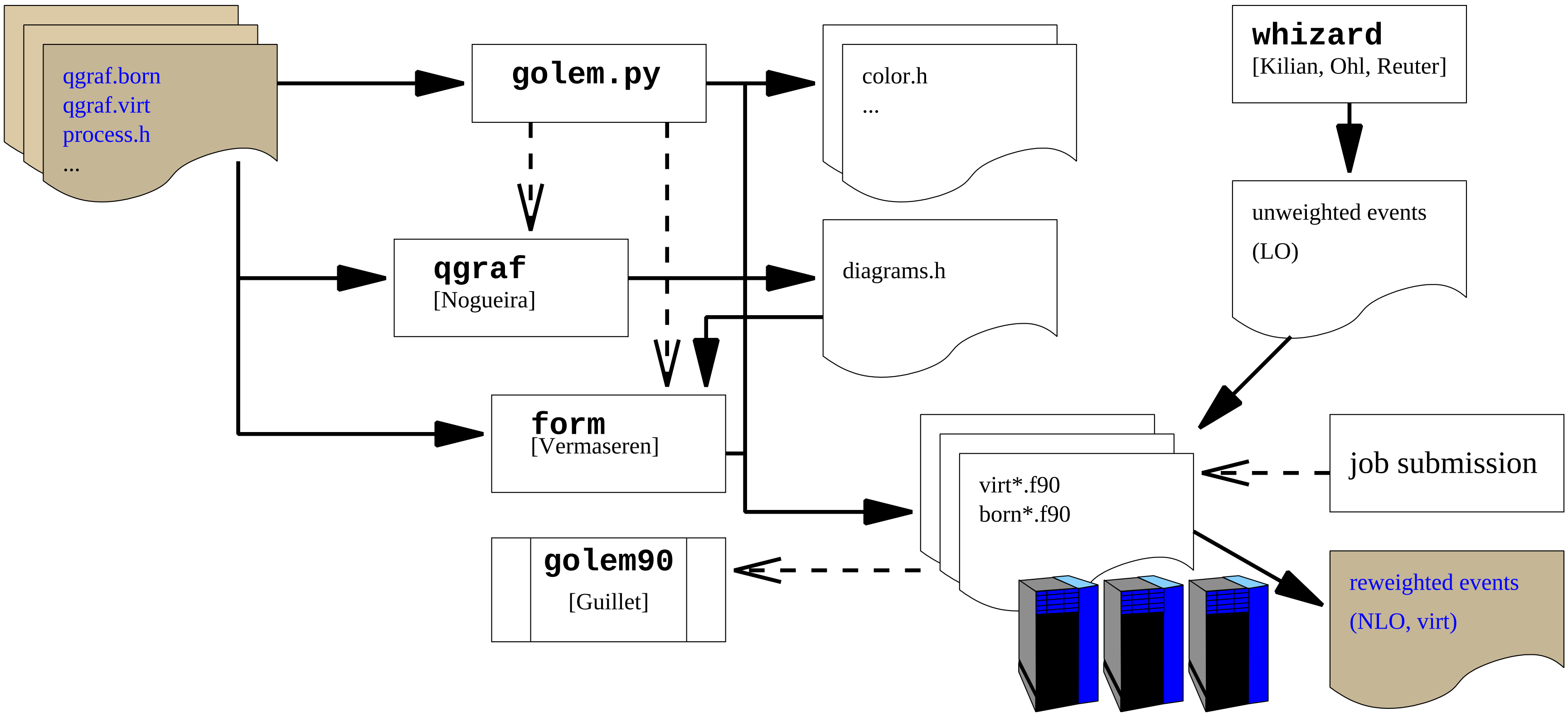


p_T distributions of leading, subleading, etc. b-jet



- Distributions obtained by binning histograms from event Ntuples
- `GOLEM` takes care for NLO reweighting
- real emission contribution not yet included

Flow chart of computation



(from Thomas Reiter's PhD thesis hep-ph:0903.0947)

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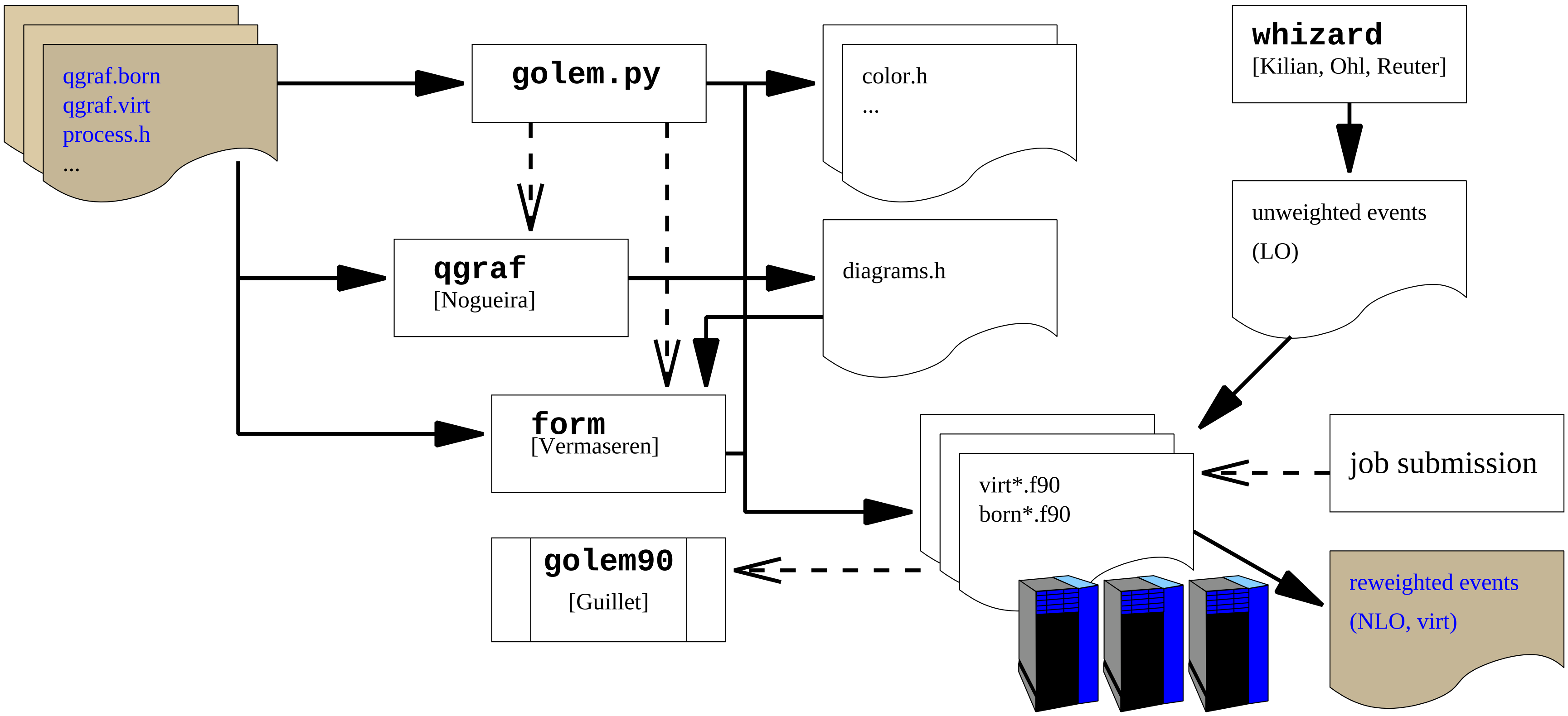

(from Thomas Reiter's PhD thesis hep-ph:0903.0947)

- T. Reiter: “performance debate is overrated”, “problem is embarrassingly parallel”
- reweighting of events done in parallel on Edinburgh (ECDF) cluster
- General set-up for NLO computations, to be used for other processes

Performance Feynman diagrams vs. unitarity based methods

"...religious battle between Feynmanians and Unitarians..."

Joey Huston

Look at colour ordered multi-gluon amplitudes:

- unitarity based method $\sim \tau_{\text{Tree}} \times \tau_{\#\text{ cuts}} \sim N^9$
- Feynman diagrams $\sim \tau_{\#\text{Formfactors}} \times \#\text{Diagrams} \sim \Gamma(N)\, 2^N$

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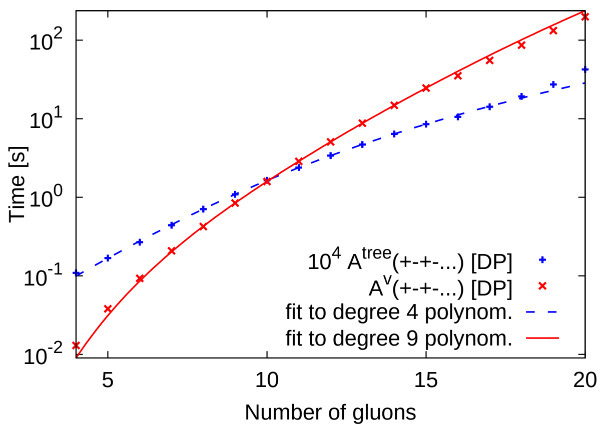

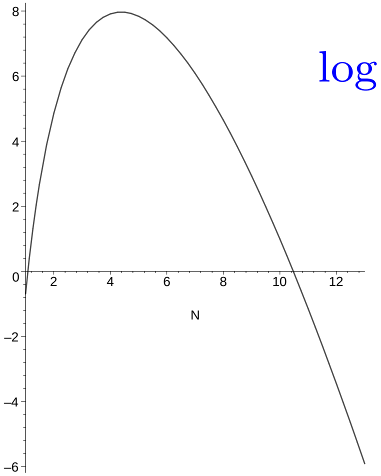

$\log[N^9 \,/\, (\Gamma(N)\, 2^N)]$

Giele, Zanderighi (2008), hep-ph:0805.2152

- asymptotic behaviour not relevant for LHC region $N \leq 8$
- for LHC both methods can/will do the job!

Summary

LHC phenomenology needs and deserves predictions at the NLO level

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NLO multileg processes still challenging, lot of progress

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- ... improvements in Feynman diagram calculations
- ... for LHC both methods can/will do the job!

The issue is automation, public & experimentalist friendly code !!!

- `Blackhat`, `Rocket`, `CutTools`, `SANC`, `GRACE`, `GOLEM`, `FormCalc`, ...
- NLO virtual corrections are just reweighting exercise of event NTUPLES
- problem is "embarrassingly" parallel
- combine with parton showers $\Rightarrow$ MC with NLO precision!
- topic of the Les Houches workshop in June 2009 !

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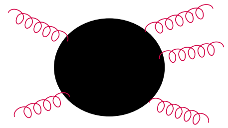
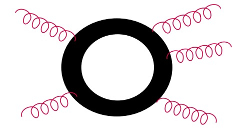
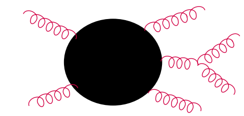
The issue is automation, public & experimentalist friendly code !!!

- `Blackhat, Rocket, CutTools, SANC, GRACE, GOLEM, FormCalc, ...`
- NLO virtual corrections are just reweighting exercise of event NTUPLES
- problem is "embarrassingly" parallel
- combine with parton showers $\Rightarrow$ MC with NLO precision!
- topic of the Les Houches workshop in June 2009 !

Outlook: standardisation of NLO computations

$\mathcal{A}^{\{\lambda\}}_{\mathrm{LO}}$:  $\mathcal{A}^{\{\lambda\}}_{\mathrm{NLO,virtual}}$:  $\mathcal{A}^{\{\lambda\}}_{\mathrm{NLO,real}}$: 

Philosophy and vision:

- no standalone NLO computations, instead transportable modules
- necessary: use same conventions for colour/helicity
- amplitude^2 representations from different groups/methods may be used interchangeably
- $\Rightarrow$ database with one-loop matrix-elements in common format
- multi-precision libraries guarantee numerical accuracy

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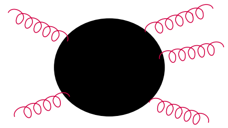
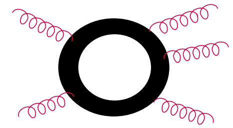
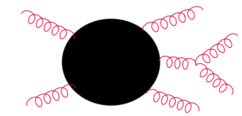
$\mathcal{A}^{\{\lambda\}}_{\text{LO}}$:  $\mathcal{A}^{\{\lambda\}}_{\text{NLO,virtual}}$:  $\mathcal{A}^{\{\lambda\}}_{\text{NLO,real}}$: 

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- tree matrix elements $2 \to N$, $2 \to N+1$ can be done with public generators (should include IR subtraction method)
- matrix elements may be merged with parton shower (compatibility issue with IR subtraction)
- NLO $\Rightarrow$ experimentalists should simply reweight partonic event NTUPLES

We need to start discussion on NLO standardisation!