CSW rules for massive particles

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— IPPP Durham —

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Based on

R.Boels, CS arXiv:0712.3409 [hep-th], Phys.Lett.B662:80-86,2008

R.Boels, CS arXiv:0805.1197 [hep-th], JHEP 0807:007,2008

CS, arXiv:0809.1442 [hep-ph], PRD78:085030,2008.

Since 2003: New methods (initially) for massless QCD amplitudes

- CSW rules: MHV diagrams (Cachazo, Svrček, Witten 04)
- BCFW rules: on shell recursion (Britto, Cachazo, Feng/Witten, 04/05)

Generalization of new methods to massive particles?

$\surd$ BCFW applied to massive scalars/quarks/vector bosons

(Badger et.al; 05, Forde, Kosower 05; Ozeren, Stirling 06; CS, S.Weinzierl 07, Hall 07)

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Overview

- Derivations of CSW rules, CSW rules for massive scalars
- Applications: proof of BCFW, one-loop rational part
- Massive quarks and SUSY

Maximally Helicity Violating amplitudes: (Parke-Taylor 1986)

$$A_n(g_1^+,\ldots,g_i^-,\ldots,g_j^-,\ldots g_n^+) = \mathrm{i}2^{n/2-1}\frac{\langle ij\rangle^4}{\langle 12\rangle\langle 23\rangle\ldots\langle n1\rangle}$$

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MHV diagrams (CSW rules): (Cachazo, Svrček, Witten 2004)

All QCD amplitudes from MHV vertices

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- Scalar propagators $\frac{\mathrm{i}}{k^2}$ connecting $+$ and $-$ labels

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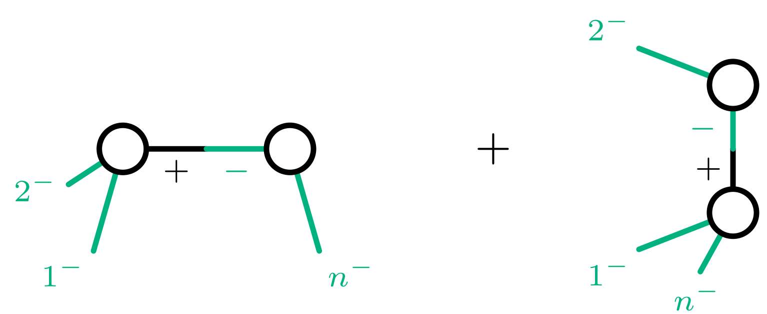
Example: NMHV amplitudes $A(g_1^-, g_2^-, g_3^+, \ldots g_n^-)$:

- Distribute negative helicities over $d = n^- - 1 = 2$ MHV vertices



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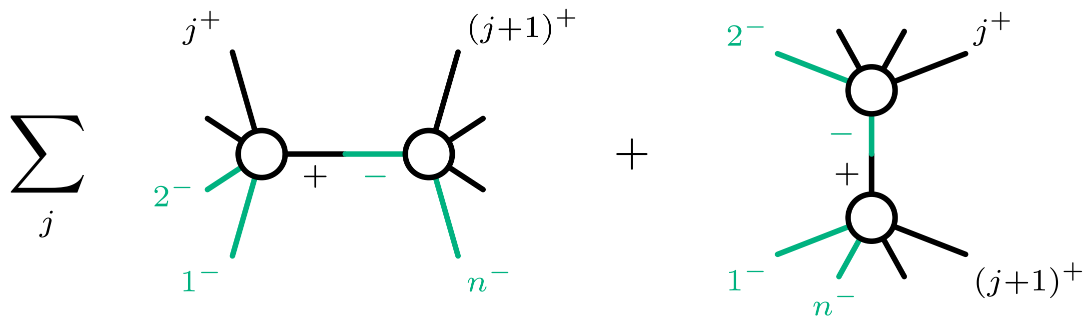
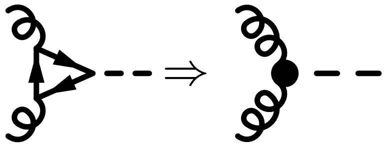


- Distribute positive helicities $\Rightarrow 2(n-3)$ diagrams

External massive particles

Effective Higgs-gluon coupling (Dixon, Glover, Khoze 04)

$$\mathcal{L}_{\text{eff}} = \frac{\alpha_s}{6\pi v} \int dx^4 H \, \text{tr}[F_{\mu\nu} F^{\mu\nu}]$$



MHV vertex for complex scalar $H = \phi + \phi^\dagger$

$$V_{\text{CSW}}(\phi, g_1^+ \dots g_i^- \dots g_j^- \dots g_n^+) = \mathrm{i}2^{n/2-1} \frac{\alpha_s}{6\pi v} \frac{\langle ij \rangle^4}{\langle 12 \rangle \dots \langle (n-1)n \rangle \langle n1 \rangle}$$

External W/Z bosons (Bern, Forde, Kosower, Mastrolia 04)

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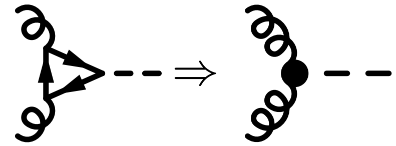

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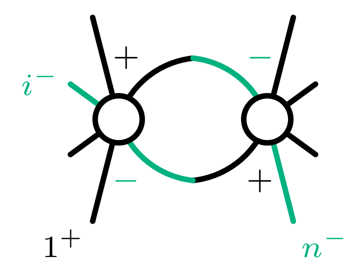
MHV rules for one-loop amplitudes

- "cut constructable" part of amplitudes

  (Bedford, Brandhuber, Spence, Travaglini; Quigly, Rozali)

- Where is all-plus amplitude $A(g_1^+, \dots g_2^+)$?



Can CSW rules be extended to massive, coloured particles?

Massive colored Scalar (R.Boels, CS 07/08)

- Toy model for massive quarks
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No obvious extension of CSW rules

- nonvanishing amplitudes with only positive helicity gluons
- explicit expression does not look twistoresque:

$$A(\bar{\phi}_1^+, g_2^+, \ldots, \phi_n^-) = \frac{i2^{n/2-1} m^2 \langle 2 + | \Pi_{j=3}^{n-2} (y_{1,j} - \not{k}_j \not{k}_{1,j-1}) | (n-1) - \rangle}{y_{1,2} \ldots y_{1,n-2} \langle 23 \rangle \langle 34 \rangle \ldots \langle (n-2)(n-1) \rangle}$$

$$k_{1,j} = \sum_1^j k_j, \qquad y_{1,j} = k_{1,j}^2 - m^2$$

(Ferrario, Rodrigo, Talavera 06)

$\Rightarrow$ **need methods to derive CSW rules**

Generalized BCFW recursion (Risager 05)

Field-redefinition in light-cone QCD (Gorsky, Rosly 05; Mansfield 05)

- Eliminate non-physical degrees of freedom from Lagrangian

$\Rightarrow$ Lagrangian for positive/negative helicity gluons only

- Eliminate non MHV vertices by canonical transformation

$\Rightarrow$ generates tower of MHV vertices

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Gauge theory on twistor space (Mason 05, Boels, Mason, Skinner 06/07)

- Twistor space $(\pi_A, \mu^{\dot{A}}) \sim \lambda(\pi_A, \mu^{\dot{A}}) \Rightarrow$ three complex d.o.f.
- Action for gauge fields on twistor space:
  $(B_0(\pi, x), B_{\dot{A}}(\pi, x)), (\bar{B}_0(\pi, x), \bar{B}_{\dot{A}}(\pi, x))$
- Extended gauge freedom
- "CSW gauge" $\eta^{\dot{A}} B_{\dot{A}} = 0$: reproduces CSW rules

Light-cone decomposition of gauge field

$$A_\pm = \frac{1}{\sqrt{2}}(A_0 \mp A_3), \qquad A_{z/\bar{z}} = \frac{1}{\sqrt{2}}(-A_1 \pm \mathrm{i}A_2)$$

A_z, $A_{\bar{z}}$ ⇔ positive/negative helicity states:

$$(\epsilon^+)_z = (\epsilon^-)_{\bar{z}} = 1 \qquad \text{(for } q \sim (1,0,0,1)\text{)}$$

Lagrangian in light-cone gauge $A_+ = 0$

$$\mathcal{L} = \mathrm{tr}\left[2\partial_- A_z \partial_+ A_{\bar{z}} + \frac{1}{2} F_{z\bar{z}} F_{z\bar{z}} + (\partial_+ A_-)^2 + 2A_- \left(\partial_+ \partial_i A_i + \mathrm{i}g[\partial_+ A_i, A_i]\right)\right]$$

Canonical system:

$$\mathcal{L} = \dot{A}_z \Pi_{A_z} + \ldots \text{ with } \dot{A}_z = \partial_- A_z,\ \Pi_{A_z} = \delta\mathcal{L}/\delta\dot{A}_z = \partial_+ A_{\bar{z}}$$

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Use EOM for A_- $\Rightarrow$ Lagrangian for physical fields A_z, $A_{\bar{z}}$

$$\mathcal{L}^{(2)} + \mathcal{L}^{(3)}_{++-} + \mathcal{L}^{(3)}_{+--} + \mathcal{L}^{(4)}_{++--}$$

$\Rightarrow$ all interactions but $\mathcal{L}^{(3)}_{++-}$ of MHV type

Relation of light-cone to spinor-helicity formalism

$$|p+\rangle = \frac{2^{1/4}}{\sqrt{p_+}} \begin{pmatrix} -p_z \\ p_+ \end{pmatrix}, \qquad |p-\rangle = \frac{2^{1/4}}{\sqrt{p_+}} \begin{pmatrix} p_+ \\ p_{\bar{z}} \end{pmatrix}$$

Spinor products:

$$\langle pq\rangle = \frac{\sqrt{2}}{\sqrt{p_+}\sqrt{q_+}}(p_z q_+ - p_+ q_z), \qquad [qp] = \frac{\sqrt{2}}{\sqrt{p_+}\sqrt{q_+}}(p_{\bar{z}} q_+ - p_+ q_{\bar{z}})$$

spinors independent of p_-: defines off-shell continuation

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Cubic MHV vertex from light-cone Lagrangian: $\big(\langle\eta+| \sim (1,0)\big)$

$$\mathcal{L}^{(3)}_{+--} = 4ig\,\mathrm{tr}\left[\partial_+ A_z \left[A_{\bar{z}}, \frac{\partial_z}{\partial_+} A_{\bar{z}}\right]\right]$$

$$\Rightarrow V(g_1^-, g_2^-, g_3^+) = 2i\frac{k_{3+}}{k_{1+}k_{2+}}(k_{2z}k_{1+} - k_{2+}k_{1z}) = \sqrt{2}i\,\langle 12\rangle\,\frac{[3\eta]^2}{[1\eta][2\eta]} = \sqrt{2}i\frac{\langle 12\rangle^3}{\langle 23\rangle\,\langle 31\rangle}$$

(Extension to arbitrary η: "spacecone gauge", Chalmers, Siegel 98)

Canonical transformation $(A_z, \partial_+ A_{\bar{z}}) \Rightarrow (B, \partial_+ \bar{B})$ to eliminate $\mathcal{L}^{(3)}_{++-}$.

$\Rightarrow$ generates MHV-type vertices: (Mansfield 05)

$$\mathcal{L}^{(3)}_{++-} + \mathcal{L}^{(3)}_{+--} + \mathcal{L}^{(4)}_{++--} \Rightarrow \sum_n \mathcal{L}^{(n)}_{+\cdots+--}$$

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Generating function:

$$G[A_z, \Pi_B] = \int dy B[A_z(y)]\Pi_B(y)$$

Transformations of the fields and momenta:

$$B = \frac{\delta G}{\delta \Pi_B} \qquad \Pi_{A_z} = \partial_+ A_{\bar{z}}(x) = \frac{\delta G}{\delta A_z} = \int d^3y \frac{\delta B(x_-, \vec{y})}{\delta A_z(x_-, \vec{x})} \partial_+ \bar{B}(x_-, \vec{y})$$

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Explicit solution (Ettle,Morris 06)

$$A_z(p) = \sum_{n=1}^{\infty} \int \prod_{i=1}^{n} \widetilde{dk_i} \; \frac{(g\sqrt{2})^{n-1} \langle \nu p \rangle^2}{\langle \nu 1 \rangle \langle 12 \rangle \ldots \langle (n-1)n \rangle \langle \nu n \rangle} B(k_1) \ldots B(k_n)$$

Similar solution for $\partial_+ A_{\bar{z}} \sim \sum_n B_1 \ldots \partial_+ \bar{B} \ldots B_n$

Application to massive scalars (R. Boels, CS 07)

- Lagrangian in light-cone gauge

$$\mathcal{L}^{(2)}(\bar{\phi}\phi) + \mathcal{L}^{(3)}(\bar{\phi}A_z\phi) + \mathcal{L}^{(3)}(\bar{\phi}A_{\bar{z}}\phi) + \mathcal{L}^{(4)}(\bar{\phi}A_z A_{\bar{z}}\phi) + \mathcal{L}^{(4)}(\bar{\phi}\phi\bar{\phi}\phi)$$

- eliminate $\mathcal{L}^{(3)}(\bar{\phi}A_z\phi)$ by transformation for massless scalars

$$\phi(p) = \sum_{n=1}^{\infty} \int \prod_{i=1}^{n} \widetilde{dk_i} \, \frac{(g\sqrt{2})^{n-1} \langle \nu n \rangle}{\langle \nu 1 \rangle \langle 12 \rangle \ldots \langle (n-1) n \rangle} B(k_1) \ldots B(k_{n-1}) \xi(k_n)$$

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- but mass term not invariant:

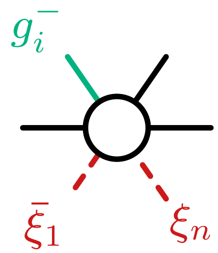
$$-m^2 \bar{\phi}(p)\phi(-p) = \sum_{n=2}^{\infty} \int \prod_{i=1}^{n} \widetilde{dp_i} \mathcal{V}_{1,\ldots,n} \, \bar{\xi}(k_1) B(k_2) \ldots B(k_{n-1}) \xi(k_n)$$

$\Rightarrow$ new CSW-vertex $\quad \mathcal{V}_{1,\ldots,n} = (g\sqrt{2})^{n-2} \dfrac{-m^2 \langle 1n \rangle}{\langle 12 \rangle \ldots \langle (n-1) n \rangle}$

Same result using Twistor Yang-Mills approach

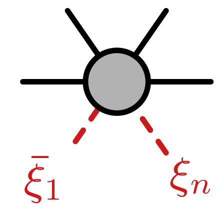
Vertices: (R.Boels, CS, 07)

massless MHV vertices



$$= i2^{n/2-1} \frac{\langle in\rangle^2 \langle 1i\rangle^2}{\langle 12\rangle \ldots \langle (n-1)n\rangle \langle n1\rangle}$$

new vertex $\sim m^2$



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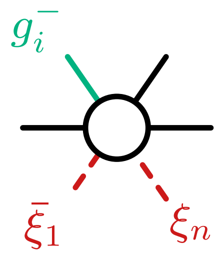

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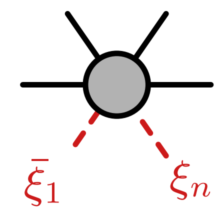

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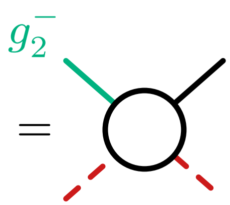
Comments:

- Not an "on-shell" method
- only holomorphic vertices $\Rightarrow$ twistor interpretation
- Number of vertices not fixed by g^- legs:

$A_4(\bar{\xi}_1, g_2^-, g_3^+, \xi_4) =$



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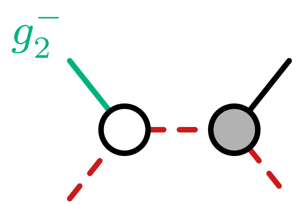

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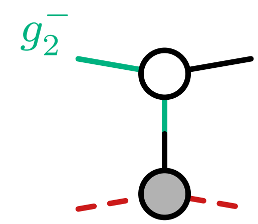
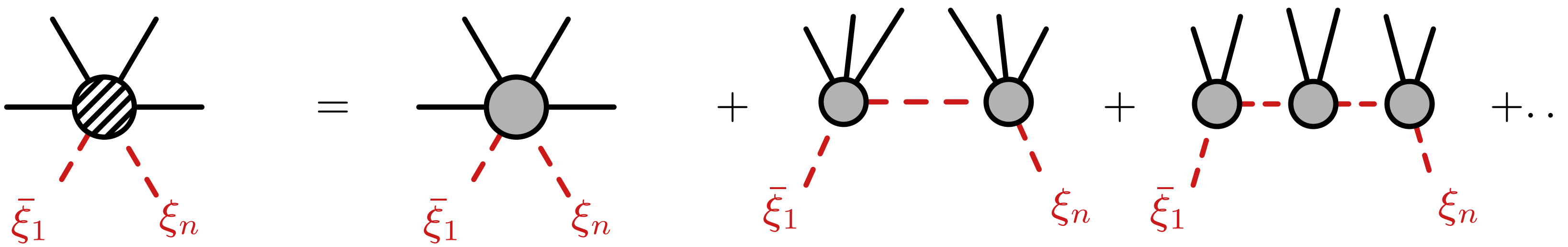


Scalar amplitudes with positive helicity gluons:



Leading piece for $m \to 0$ from single vertex:

$$A_n(\bar\xi_1, g_2^+ ..., , \xi_n) = \mathrm{i}2^{n/2-1} \frac{-m^2 \langle 1n\rangle}{\langle 12\rangle \ldots \langle (n-1)n\rangle} + \mathcal{O}(m^2)$$

$$= \mathrm{i}2^{n/2-1} \frac{m^2 \langle 2+|\not k_1 \not k_n|(n-1)-\rangle}{2(k_1\cdot k_2)2(k_n \cdot k_{n-1}) \langle 23\rangle \ldots \langle (n-2)(n-1)\rangle} + \mathcal{O}(m^2)$$

(agrees with Bern, Dixon, Dunbar, Kosower 96)

Condition for BCFW recursion

$$A(z) \to 0 \text{ for } z \to \infty$$

with $|i'+\rangle = |i+\rangle + z\,|j+\rangle \quad |j'-\rangle = |j-\rangle + z\,|i-\rangle$¡

Allowed shifts:

- (i^+, j^-): $A(z) \sim \frac{1}{z}$ from powercounting; (BCFW 05)
  diagrammatical proof (Draggiotis et.al.; Vaman, Yao; 05)
- (i^+, j^+), (i^-, j^-): from CSW diagrams (BCFW 05)
  auxiliary shift (Badger,Glover,Khoze,Svrček 05)
  Lagrangian analysis (Arkani-Hamed, Kaplan 08)

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Simpler proof of (g_i^+, g_j^+) shift
for amplitudes with massive scalars from CSW rules, e.g.: (Boels, CS)

$$V_{\text{CSW}}(\bar{\xi}_1 \ldots g_i^{+\prime} \ldots g_j^{+\prime} \ldots, \xi_n) = \frac{-m^2 \langle 1n \rangle}{\ldots \langle (i-1) i' \rangle \langle i'(i+1) \rangle \ldots} \sim \begin{cases} z^{-2} & j \neq i \pm 1 \\ z^{-1} & j = i \pm 1 \end{cases}$$

Rational part of one-loop amplitudes in MHV formalism

- 4-D regulator in light-cone QCD (Brandhuber et.al. 07)
- D-dim MHV vertices + careful application of LSZ (Ettle et.al 07)
- SUSY decomposition of gluon one-loop amplitudes:

$$A^{\mathcal{N}=0}_{\text{gluon}} = A^{\mathcal{N}=4} - 4A^{\mathcal{N}=1}_{\text{chiral}} + A^{\mathcal{N}=0}_{\text{scalar}}$$

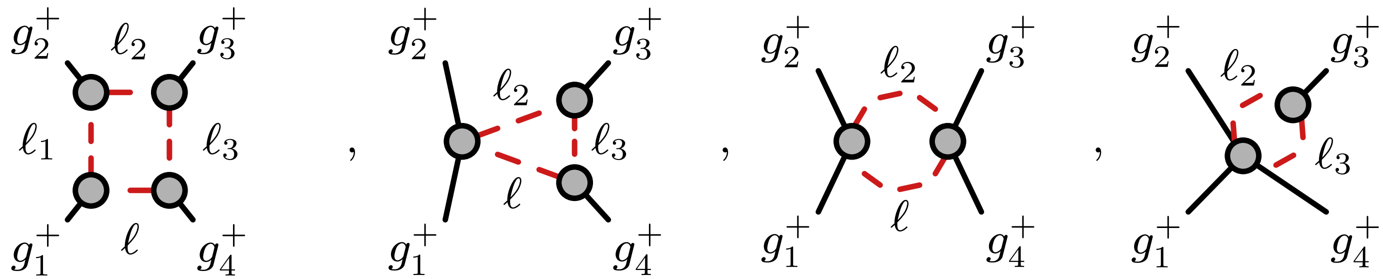
no rational part in SUSY pieces $\Rightarrow$ need scalar in $4-2\epsilon$ dim.

$$\ell_D^2 = \ell^2 + \ell_{-2\epsilon}^2 \equiv \ell^2 - \mu^2$$

$$\int \frac{d^D\ell}{(2\pi)^D} f(\ell_D^2) = \int \frac{d^{-2\epsilon}\mu}{(2\pi)^{-2\epsilon}} \frac{d^4\ell}{(2\pi)^4} f(\ell^2 - \mu^2) \quad \text{(Bern, Morgan 95)}$$

$\Rightarrow$ use CSW rules for massive scalar (Boels, CS 08; Glover, Williams 08)

Topologies for $A(g_1^+, g_2^+, g_3^+, g_4^+)$ in massive CSW rules:



Three-point vertex for g_1^+ vanishes for $|\eta-\rangle = |1-\rangle \Rightarrow$ box vanishes

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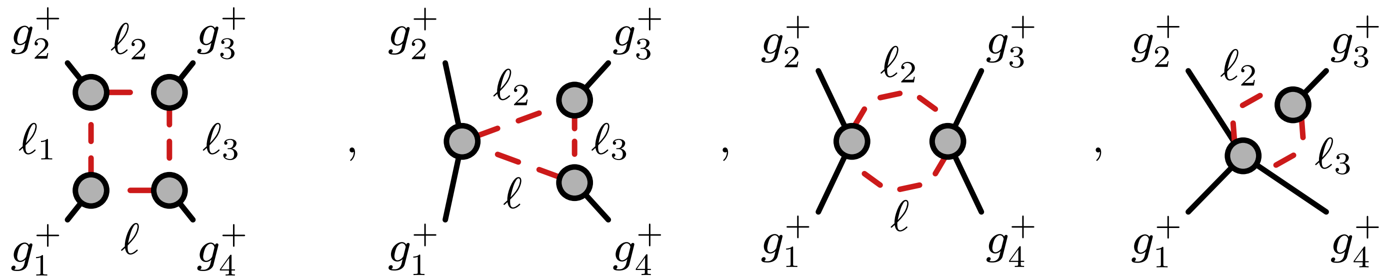
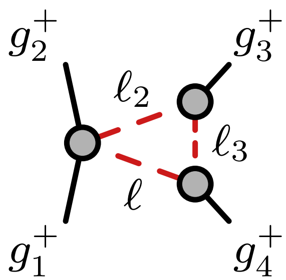


Three-point vertex for g_1^+ vanishes for $|\eta-\rangle = |1-\rangle$ $\Rightarrow$ box vanishes

Example for triangle: $d(\ell_i) = \ell_i^2 - \mu^2$



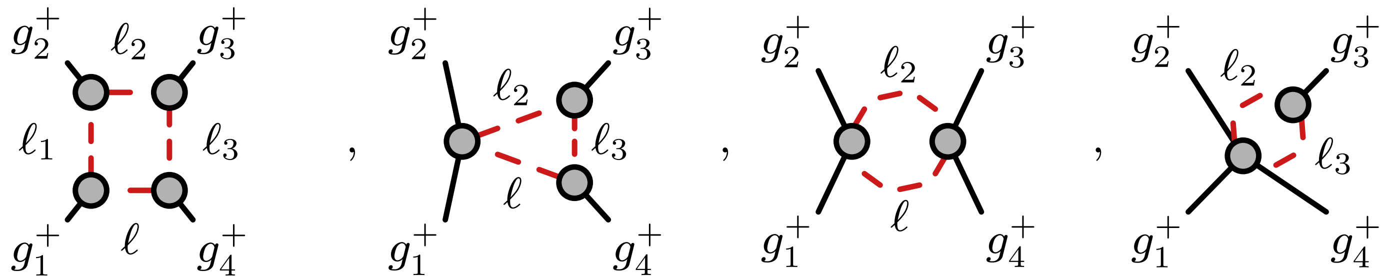
$$= \int \frac{d^D\ell}{(2\pi)^D} \frac{4\mu^6 \langle \ell\ell_2\rangle \langle \ell_2\ell_3\rangle \langle \ell_3\ell\rangle}{d(\ell)d(\ell_2)d(\ell_3) \langle \ell 1\rangle \langle 12\rangle \langle 2\ell_2\rangle \langle \ell_2 3\rangle \langle 3\ell_3\rangle \langle \ell_3 4\rangle \langle 4\ell\rangle}$$

$$= \frac{4[12]}{\langle 12\rangle \langle 34\rangle} \int \frac{d^D\ell}{(2\pi)^D} \frac{\mu^4}{d(\ell)d(\ell_2)d(\ell_3)2(\ell\cdot k_1)} \frac{-\mu^2 \langle 1-|\not{k}_3\not{k}_4|1-\rangle}{\langle 3-|\not{\ell}_3|1-\rangle \langle 4-|\not{\ell}|1-\rangle}$$

Cancel spurious poles using (Brandhuber, Spence, Travaglini 06)

$$\mu^2 \langle 1+|\not{k}_3\not{k}_4|1-\rangle = [34] \langle 1+|\not{\ell}_3|3+\rangle \langle 4-|\not{\ell}_4|1-\rangle + \sim d(\ell_{i-1}), d(\ell_i), d(\ell_{i+1})$$

Topologies for $A(g_1^+, g_2^+, g_3^+, g_4^+)$ in massive CSW rules:



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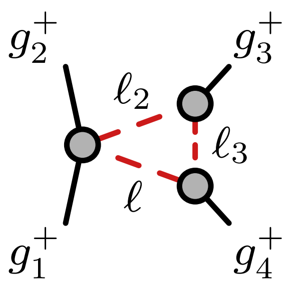
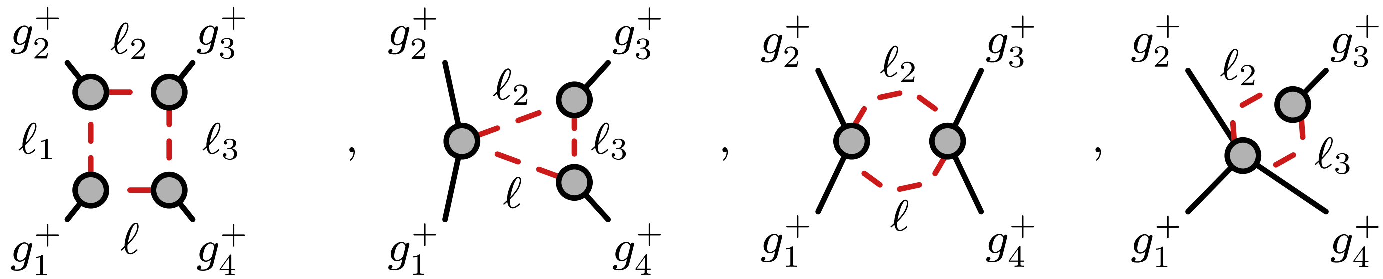


$$= \frac{-4[12][34]}{\langle 12\rangle \langle 34\rangle} \int \frac{d^D\ell}{(2\pi)^D} \frac{\mu^4}{d(\ell)d(\ell_2)d(\ell_3)2(\ell\cdot k_1)} + \text{bubbles}$$

CSW rules for massive scalars

Topologies for $A(g_1^+, g_2^+, g_3^+, g_4^+)$ in massive CSW rules:



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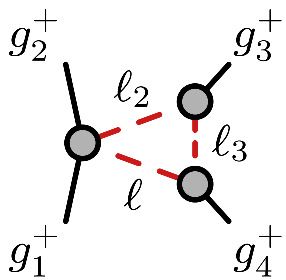

$$= \frac{-4[12][34]}{\langle 12\rangle \langle 34\rangle} \int \frac{d^D\ell}{(2\pi)^D} \frac{\mu^4}{d(\ell)d(\ell_2)d(\ell_3)2(\ell\cdot k_1)} + \text{bubbles}$$

Adding other diagrams: bubbles cancel $\Rightarrow$ known result:

$$A_4(g_1^+, g_2^+, g_3^+, g_4^+) = \frac{4[12][34]}{\langle 12\rangle \langle 34\rangle} \int \frac{d^4\ell}{(2\pi)^4} \frac{d^{-2\epsilon}\mu}{(2\pi)^{-2\epsilon}} \frac{\mu^4}{(\ell^2-\mu^2)(\ell_2^2-\mu^2)(\ell_3^2-\mu^2)(\ell_4^2-\mu^2)}$$

Similar pattern for higher point amplitudes, g^-! (Glover, Williams 08)

CSW rules for massive Quarks

- Some amplitudes related to scalars by SUSY (CS, S.Weinzierl 06)
- Explicit derivation using canonical transformation

(Ettle, Morris, Xiao; CS 08)

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Spinors for massive quarks (Kleiss, Stirling 85;...; CS, S.Weinzierl 05)

$$u(\pm, \eta) = \frac{1}{\langle p^\flat \pm | \eta \mp \rangle} (\not{p} + m) | \eta \mp \rangle$$

with light cone projection

$$p^\flat = p - \frac{p^2}{2p \cdot \eta} \eta$$

Eigenstates of projectors $(1 \pm \not{s} \gamma^5)$ with spin vector

$$s^\mu = \frac{p^\mu}{m} - \frac{m}{(p \cdot \eta)} \eta^\mu$$

"Helicity" amplitudes depend on η!

Embed QCD with massive quark $Q = \binom{\psi_+}{\bar\psi_-}$ in $N = 1$ SQCD

$\Rightarrow$ two complex scalars $\phi_\pm$ as superpartners:

$$\Psi_+ = (\varphi_+, \psi_+, F_+) \qquad , \qquad \overline{\Psi}_- = (\overline{\varphi}_-, \overline{\psi}_-, \overline{F}_-)$$

Transformations of helicity states $\quad ((\bar\phi_\pm)^\dagger = \phi_\mp) \quad$ (CS, S.Weinzierl, 06)

$$\delta_\kappa \phi^- = [\kappa k] Q^- + m \frac{[\eta\kappa]}{[\eta k]} Q^+ \qquad \delta_\kappa \phi^+ = \langle \kappa k \rangle\, Q^+ + m \frac{\langle \eta\kappa \rangle}{\langle \eta k \rangle} Q^-$$

$$\delta_\kappa Q^+ = [k\kappa] \phi^+ + m \frac{\langle \eta\kappa \rangle}{\langle \eta k \rangle} \phi^- \qquad \delta_\kappa Q^- = \langle k\kappa \rangle\, \phi^- + m \frac{[\eta\kappa]}{[\eta k]} \phi^+$$

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Simplify for $|\kappa\pm\rangle \propto |\eta\pm\rangle \Rightarrow$ similar to massless case!

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Simplify for $|\kappa\pm\rangle \propto |\eta\pm\rangle$ $\Rightarrow$ similar to massless case!

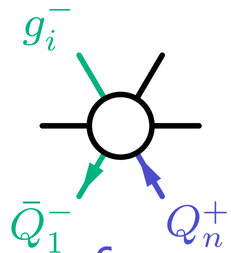
SUSY relations to scalar amplitudes:

$$A(\bar{Q}_1^+, 2^+, \dots, Q_n^-) = \frac{\langle n\eta \rangle}{\langle \eta 1 \rangle} A(\bar{\phi}_1^+, 2^+, \dots, \phi_n^-)$$

$$A(\bar{Q}_1^-, 2^+, \dots, Q_n^-) = \frac{\langle 1n \rangle}{m} A(\bar{\phi}_1^+, 2^+, \dots, \phi_n^-)$$

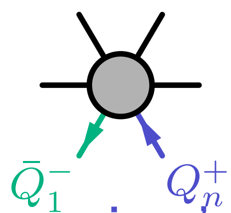
Vertices from canonical transformation (Ettle, Morris, Xiao 08; CS 08)

MHV-vertex: (+ 4-quark MHV vertex)



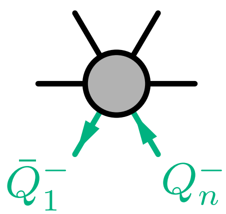
$$= \mathrm{i}2^{n/2-1}\frac{\langle 1i\rangle^3\langle in\rangle}{\langle 12\rangle\ldots\langle (n-1)n\rangle\langle n1\rangle}$$

Vertex from transformation of mass term $\sim m^2$:

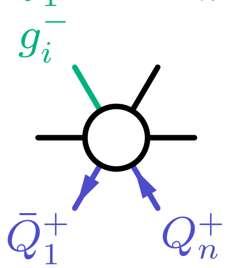


$$= \mathrm{i}2^{n/2-1}m^2\frac{\langle \eta 1\rangle\langle 1n\rangle}{\langle 12\rangle\ldots\langle (n-1)n\rangle\langle n\eta\rangle}$$

'Helicity flip vertices' $\sim m$: (+ 4-quark flip vertex)



$$= -\mathrm{i}2^{n/2-1}m\,\frac{\langle 1n\rangle^2}{\langle 12\rangle\ldots\langle (n-1)n\rangle}\,,$$



$$= -\mathrm{i}2^{n/2-1}m\,\frac{\langle 1i\rangle\langle in\rangle}{\langle 12\rangle\ldots\langle (n-1)n\rangle}\frac{\langle \eta i\rangle^2}{\langle \eta 1\rangle\langle \eta n\rangle}\,,$$

Derivation of CSW rules

using canonical transformation

Extension to massive particles

- new CSW vertices for massive scalars and quarks
- SUSY-relations of massive quarks to massive scalars

Applications

- rational part of QCD amplitudes
- Methods can be applied to effective Higgs-gluon coupling (Boels, CS 08), CSW rules for EW currents,...