D-dimensional unitarity at work

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Based on work done with Keith Ellis, Walter Giele, Zoltan Kunszt, Kirill Melnikov

Motivation

Interest per se:

- shed light on all order properties of highly symmetric gauge theories
- insight in the structure of gauge theories in and beyond large N_c limit
- hope for a better understanding of full QCD

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Practical:

- NLO calculations crucial for the LHC programme
- bottleneck at NLO are virtual corrections
- aim is to be able to do N-leg one-loop calculations for a general process (generic spins and masses) $\Rightarrow$ e.g. Alpgen@NLO

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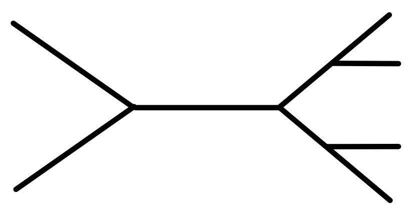
- tree graph rates with N+1 partons
  → soft/collinear divergences



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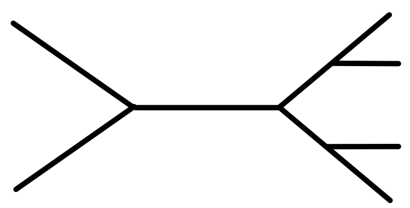
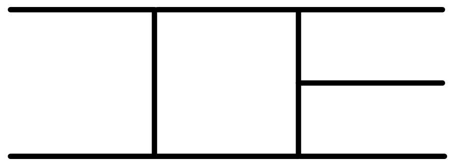


- virtual correction to N-leg process
  → divergence from loop integration



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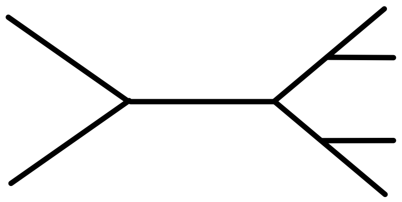

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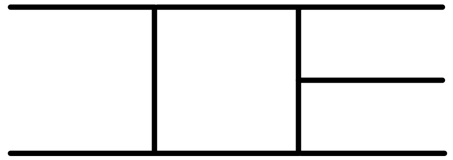

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A full N-particle NLO calculation requires:

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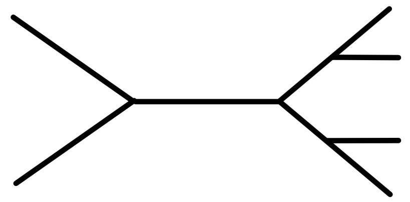
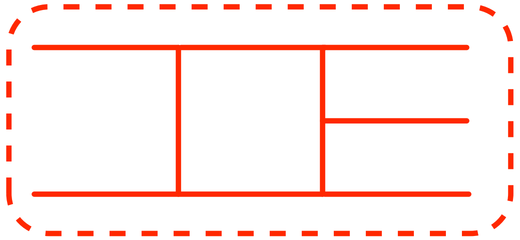
- [x] set of subtraction terms



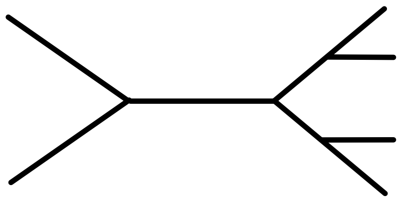


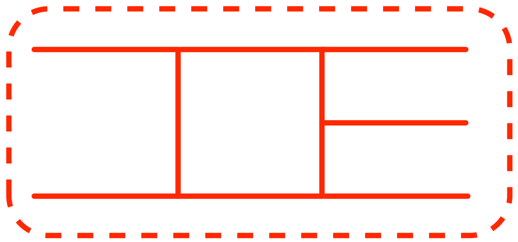
bottleneck

Ingredients for NLO

A full N-particle NLO calculation requires:

- ☑ tree graph rates with N+1 partons
  ➜ soft/collinear divergences
- ☐ virtual correction to N-leg process
  ➜ divergence from loop integration
- ☑ set of subtraction terms





bottleneck

Tree level (real correction) and subtraction terms are fully understood and automated ⇒ concentrate on the virtual contribution in the following

Automated subtraction:

Gleisberg, Krauss '07; TeVJet [public] Seymour, Tevlin '08; Hasegawa, Moch, Uwer '08

Traditional approaches to NLO

- draw all possible Feynman diagrams (use automated tools)
- write one-loop amplitudes as ∑ (coefficients × tensor integrals)
- automated (PV-style) reduction of tensor integrals to scalar ones

Most 2→3 and the first 2→4 LHC processes [pp→Hjj, WWj, WWW, ttj, qq→ttbb ...] computed this way

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Many new ideas recently. I will talk about generalized unitarity and show its *simplicity, generality, efficiency, and thus suitability for automation*

D-dimensional unitarity

We just heard a comprehensive overview of generalized unitarity with an accurate historical perspective by Zoltan Kunszt

In the following I will concentrate on practical aspects:

numerical implementation, efficiency, performance, applications, results

References:

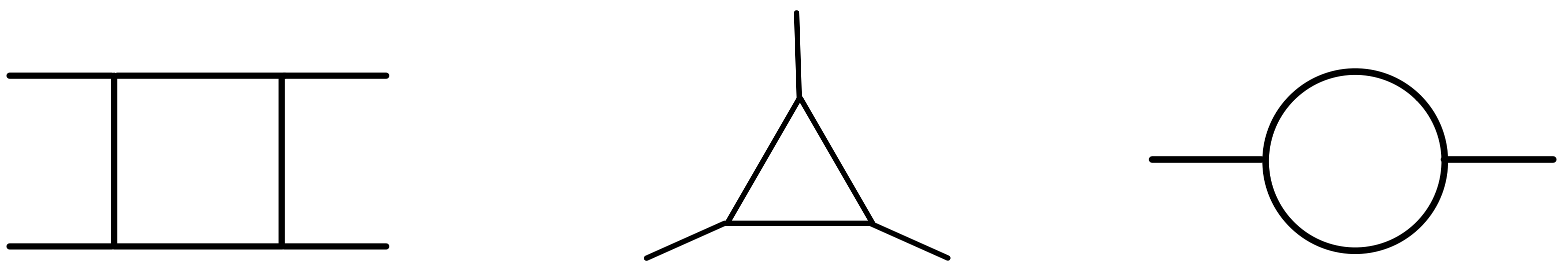
- Ellis, Giele, Kunszt '07 *[Unitarity in D=4]*
- Giele, Kunszt, Melnikov '08 *[Unitarity in D≠4]*
- Giele & GZ '08 *[All oneloop N-gluon amplitudes]*
- Ellis, Giele, Melnikov, Kunszt '08 *[Massive fermions, ttggg amplitudes]*
- Ellis, Giele, Melnikov, Kunszt, GZ '08 *[W+5p oneloop amplitudes]*
- Ellis, Melnikov, GZ '09 *[W+3jets]*

These papers heavily rely on previous work

- Bern, Dixon, Kosower '94 *[Unitarity, oneloop from trees]*
- Ossola, Pittau, Papadopoulos '06 *[OPP]*
- Britto, Cachazo, Feng '04 *[Generalized cuts]*
- *[....]*

Decomposition of the one-loop amplitude

$$\mathcal{A}_N^D = \sum_{[i_1|i_4]} d^D_{i_1 i_2 i_3 i_4} I^{(D)}_{i_1 i_2 i_3 i_4} + \sum_{[i_1|i_3]} c^D_{i_1 i_2 i_3} I^{(D)}_{i_1 i_2 i_3} + \sum_{[i_1|i_2]} b^D_{i_1 i_2} I^{(D)}_{i_1 i_2} \; {}^{*}$$



Remarks:

- higher point function reduced to boxes + vanishing terms
- coefficients depend on D (i.e. on ϵ) $\Rightarrow$ rational part
- box, triangles and bubble integrals all known analytically

['t Hooft & Veltman '79; Bern, Dixon Kosower '93, Duplancic & Nizic '02; Ellis & GZ '08, public code ⇒ http://www.qcdloop.fnal.gov]

* if non-vanishing masses: tadpole term; notation: $[i_1|i_m] = 1 \leq i_1 < i_2 \ldots < i_m \leq N$

Cut-constructable part

Start from

$$\mathcal{A}_N^{\text{cut}} = \sum_{[i_1|i_4]} d_{i_1 i_2 i_3 i_4} I^{(D)}_{i_1 i_2 i_3 i_4} + \sum_{[i_1|i_3]} c_{i_1 i_2 i_3} I^{(D)}_{i_1 i_2 i_3} + \sum_{[i_1|i_2]} b_{i_1 i_2} I^{(D)}_{i_1 i_2} = \int \frac{d^D l}{i(\pi)^{D/2}} \mathcal{A}_N^{\text{cut}}(l)$$

with

$$I^D_{i_1 \cdots i_M} = \int \frac{d^D l}{i(\pi)^{D/2}} \frac{1}{d_{i_1} \cdots d_{i_M}}$$

Look at the integrand

$$\mathcal{A}_N^{\text{cut}}(l) = \sum_{[i_1|i_4]} \frac{\bar{d}_{i_1 i_2 i_3 i_4}}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} + \sum_{[i_1|i_3]} \frac{\bar{c}_{i_1 i_2 i_3}}{d_{i_1} d_{i_2} d_{i_3}} + \sum_{[i_1|i_1]} \frac{\bar{b}_{i_1 i_2}}{d_{i_1} d_{i_2}}$$

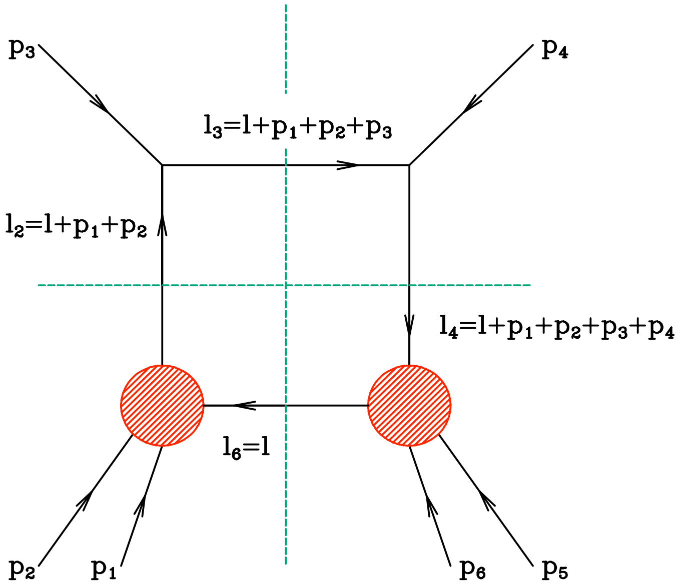
Get cut numerators by taking residues: i.e. set inverse propagator = 0
In D=4 up to 4 constraints on the loop momentum (4 onshell propagators) ⇒ get up to box integrals coefficients

Construction of the box residue

Four cut propagators are onshell
⇒ the amplitude factorizes into 4 tree-level amplitudes

Residues at the poles [⇒ coefficients at the poles $\bar{d}_{ijkl}(l^\pm)$]

$$\begin{aligned}\mathrm{Res}_{ijkl}\Big(\mathcal{A}_N(l^\pm)\Big) &= \mathcal{M}^{(0)}(l_i^\pm; p_{i+1}, \ldots, p_j; -l_j^\pm) \times \mathcal{M}^{(0)}(l_j^\pm; p_{j+1}, \ldots, p_k; -l_k^\pm) \\ &\times \mathcal{M}^{(0)}(l_k^\pm; p_{k+1}, \ldots, p_l; -l_l^\pm) \times \mathcal{M}^{(0)}(l_l^\pm; p_{l+1}, \ldots, p_i; -l_i^\pm)\end{aligned}$$



Need full loop momentum dependence of the coefficients: $\bar{d}_{ijkl}(l)$

Construction of the box residue

p_1, p_2, p_3 span the physical space. The dependence on loop momentum enters only through component in the orthogonal, trivial space (n_1)

$$\overline{d}_{ijkl}(l) \equiv \overline{d}_{ijkl}(n_1 \cdot l)$$

Use

$$(n_1 \cdot l)^2 \sim n_1^2 = 1$$

Then the maximum rank is one and the most general form is

$$\overline{d}_{ijkl}(l) = d^{(0)}_{ijkl} + d^{(1)}_{ijkl}\, l \cdot n_1$$

Using the two solutions of the unitarity constraint one obtains

$$\begin{aligned} d^{(0)}_{ijkl} &= \frac{\mathrm{Res}_{ijkl}\Big(\mathcal{A}_N(l^+)\Big) + \mathrm{Res}_{ijkl}\Big(\mathcal{A}_N(l^-)\Big)}{2} \\ d^{(1)}_{ijkl} &= \frac{\mathrm{Res}_{ijkl}\Big(\mathcal{A}_N(l^+)\Big) - \mathrm{Res}_{ijkl}\Big(\mathcal{A}_N(l^-)\Big)}{2i\sqrt{V_4^2 - m_l^2}} \end{aligned}$$

For triangle, bubble and tadpole coefficients proceed in the same way

Final result: cut-constructable part

Spurious terms integrate to zero

$$\int [d\,l]\ \frac{\overline{d}_{ijk}(l)}{d_i d_j d_k d_l} = d^{(0)}_{ijkl} \int [d\,l]\ \frac{1}{d_i d_j d_k d_l} = d_{ijkl} I_{ijkl}$$

$$\int [d\,l]\ \frac{\overline{c}_{ijk}(l)}{d_i d_j d_k} = c^{(0)}_{ijk} \int [d\,l]\ \frac{1}{d_i d_j d_k} = c_{ijk} I_{ijk}$$

$$\int [d\,l]\ \frac{\overline{b}_{ij}(l)}{d_i d_j} = b^{(0)}_{ijk} \int [d\,l]\ \frac{1}{d_i d_j} = b_{ij} I_{ij}$$

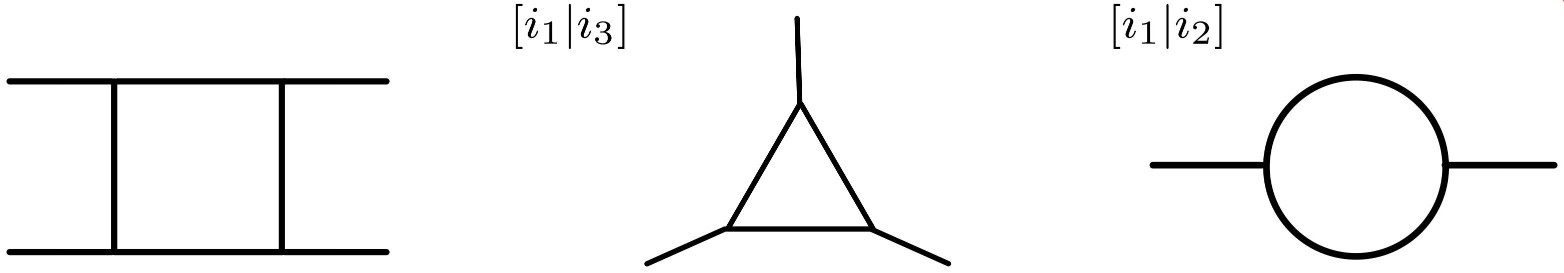
The final result for the cut constructable part then reads

$$\mathcal{A}_N^{\rm cut} = \sum_{[i_1|i_4]} d^{(0)}_{i_1 i_2 i_3 i_4} I^{(D)}_{i_1 i_2 i_3 i_4} + \sum_{[i_1|i_3]} c^{(0)}_{i_1 i_2 i_3} I^{(D)}_{i_1 i_2 i_3} + \sum_{[i_1|i_2]} b^{(0)}_{i_1 i_2} I^{(D)}_{i_1 i_2}$$

Full one-loop virtual amplitudes

Cut constructable part can be obtained by taking residues in D=4

$$\mathcal{A}_N = \sum_{[i_1|i_4]} \left(d_{i_1 i_2 i_3 i_4} \; I^{(D)}_{i_1 i_2 i_3 i_4} \right) + \sum_{[i_1|i_3]} \left(c_{i_1 i_2 i_3} \; I^{(D)}_{i_1 i_2 i_3} \right) + \sum_{[i_1|i_2]} \left(b_{i_1 i_2} \; I^{(D)}_{i_1 i_2} \right) + \mathcal{R}$$



Want rational part: need to think about D ≠ 4

Generic D dependence

Two sources of D dependence

dimensionality of loop momentum D

nr. of spin eigenstates/ polarization states D_s

Keep D and D_s distinct

$$\mathcal{A}^D \Rightarrow \mathcal{A}^{(D,D_s)}$$

Two key observations

1. External particles in D=4 ⇒ no preferred direction in the extra space

$$\mathcal{N}(l) = \mathcal{N}(l_4, \tilde{l}^2) \qquad \tilde{l}^2 = -\sum_{i=5}^{D} l_i^2$$

$\mathcal{N}$: numerator function

☛ in arbitrary D up to 5 constraints ⇒ get up to pentagon integrals

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☛ in arbitrary D up to 5 constraints ⇒ get up to pentagon integrals

2. Dependence of $\mathcal{N}$ on D_s is linear (or almost...) as it appears from closed loops of contracted metrics

$$\mathcal{N}^{D_s}(l) = \mathcal{N}_0(l) + (D_s - 4)\mathcal{N}_1(l)$$

☛ evaluate at any D_{s1}, D_{s2} ⇒ get $\mathcal{N}_0$ and $\mathcal{N}_1$, i.e. full $\mathcal{N}$

Choose D_{s1}, D_{s2} integer ⇒ suitable for numerical implementation

[D_s = 4 - 2ϵ ‘t-Hooft-Veltman scheme, D_s = 4 FDH scheme]

Practically: pentagon cuts

$$\frac{\mathcal{N}^{(D_s)}(l)}{d_1 d_2 \cdots d_N} = \sum_{[i_1|i_5]} \frac{\overline{e}^{(D_s)}_{i_1 i_2 i_3 i_4 i_5}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4} d_{i_5}} + \sum_{[i_1|i_4]} \frac{\overline{d}^{(D_s)}_{i_1 i_2 i_3 i_4}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} + \sum_{[i_1|i_3]} \frac{\overline{c}^{(D_s)}_{i_1 i_2 i_3}(l)}{d_{i_1} d_{i_2} d_{i_3}} + \sum_{[i_1|i_2]} \frac{\overline{b}^{(D_s)}_{i_1 i_2}(l)}{d_{i_1} d_{i_2}} + \sum_{[i_1|i_1]} \frac{\overline{a}^{(D_s)}_{i_1}(l)}{d_{i_1}}$$

Pentagon residue:

$$\overline{e}^{(D_s)}_{ijkmn}(l_{ijkmn}) = \mathrm{Res}_{ijkmn}\left(\frac{\mathcal{N}^{(D_s)}(l)}{d_1 \cdots d_N}\right) \quad \Leftrightarrow \quad d_i(l_{ijkmn}) = \cdots = d_n(l_{ijkmn}) = 0$$

Solution:

$$l^{\mu}_{ijkmn} = V_5^{\mu} + \sqrt{\frac{-V_5^2 + m_n^2}{\alpha_5^2 + \cdots + \alpha_D^2}} \left(\sum_{h=5}^{D} \alpha_h n_h^{\mu}\right) \quad \forall \alpha_i$$

$$\mathrm{Res}_{ijkmn}\left(\frac{\mathcal{N}^{(D_s)}(l)}{d_1 \cdots d_N}\right) = \sum \mathcal{M}(l_i; p_{i+1}, \ldots, p_j, -l_j) \times \mathcal{M}(l_j; p_{j+1}, \ldots, p_k; -l_k)$$

$$\times \mathcal{M}(l_k; p_{k+1}, \ldots, p_m; -l_m) \times \mathcal{M}(l_m; p_{m+1}, \ldots, p_n; -l_n) \times \mathcal{M}(l_n; p_{n+1} \ldots, p_i; -l_i)$$

V_5: function of the 4 inflow momenta

n_i: span trivial space, $\perp$ to physical one

Most general parameterization:

$$\bar{e}^{D_s}_{ijkmn}(l) = \bar{e}^{D_s}_{ijkmn}(l_{ijlmn}) \equiv \bar{e}^{Ds,(0)}_{ijkmn}$$

Practically: box cuts

Box residue:

$$\bar{d}^{(D_s)}_{ijkn}(l) = \mathrm{Res}_{ijkn}\left(\frac{\mathcal{N}^{(D_s)}(l)}{d_1\cdots d_N} - \sum_{[i_1|i_5]}\frac{e^{(D_s,(0))}_{i_1i_2i_3i_4i_5}}{d_{i_1}d_{i_2}d_{i_3}d_{i_4}d_{i_5}}\right) \quad \Leftrightarrow \quad d_i(l_{ijkm}) = \cdots = d_n(l_{ijkm}) = 0$$

Solution:

$$l^\mu_{ijkn} = V_4^\mu + \sqrt{\frac{-V_4^2 + m_n^2}{\alpha_1^2 + \alpha_5^2 + \cdots + \alpha_D^2}}\left(\alpha_1 n_1^\mu + \sum_{h=5}^{D}\alpha_h n_h^\mu\right) \qquad \forall \alpha_i$$

V_4: function of the 3 inflow momenta

n_i: span trivial space, ⊥ to physical one

$$\mathrm{Res}_{ijkm}\left(\frac{\mathcal{N}^{(D_s)}(l)}{d_1\cdots d_N}\right) = \sum \mathcal{M}(l_i;p_{i+1},\ldots,p_j,-l_j)\times\mathcal{M}(l_j;p_{j+1},\ldots,p_k;-l_k)$$
$$\times\mathcal{M}(l_k;p_{k+1},\ldots,p_m;-l_m)\times\mathcal{M}(l_m;p_{m+1},\ldots,p_n;-l_i)$$

Most general parameterization of quadrupole cut:

$$\bar{d}_{ijkn}(l) = d^{(0)}_{ijkn} + d^{(1)}_{ijkn}s_1 + (d^{(2)}_{ijkn} + d^{(3)}_{ijkn}s_1)s_e^2 + d^{(4)}_{ijkn}s_e^4$$

$$s_1 = l\cdot n_1$$
$$s_e \equiv -\sum_{i=5}^{D}(l\cdot n_i)^2$$

➥ make 5 choices of α_i and solve for the 5 coefficients

Triangles and bubbles: same procedure with appropriate changes

Putting it all together

$$\frac{\mathcal{N}^{(D_s)}(l)}{d_1 d_2 \cdots d_N} = \sum_{[i_1|i_5]} \frac{\overline{e}^{(D_s)}_{i_1 i_2 i_3 i_4 i_5}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4} d_{i_5}} + \sum_{[i_1|i_4]} \frac{\overline{d}^{(D_s)}_{i_1 i_2 i_3 i_4}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} + \sum_{[i_1|i_3]} \frac{\overline{c}^{(D_s)}_{i_1 i_2 i_3}(l)}{d_{i_1} d_{i_2} d_{i_3}} + \sum_{[i_1|i_2]} \frac{\overline{b}^{(D_s)}_{i_1 i_2}(l)}{d_{i_1} d_{i_2}} + \sum_{[i_1|i_1]} \frac{\overline{a}^{(D_s)}_{i_1}(l)}{d_{i_1}}$$

Combine the two evaluations:

$$\mathcal{A}^{\mathrm{FDH}} = \left(\frac{D_2 - 4}{D_2 - D_1}\right) \mathcal{A}_{(D, D_s = D_1)} - \left(\frac{D_1 - 4}{D_2 - D_1}\right) \mathcal{A}_{(D, D_s = D_2)}$$

Need to evaluate loop integration, use:

$$\int \frac{\mathrm{d}^D l}{(i\pi)^{D/2}} \frac{s_e^2}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} = \frac{D-4}{2} I^{D+2}_{i_1 i_2 i_3 i_4} \to 0$$

$$\int \frac{\mathrm{d}^D l}{(i\pi)^{D/2}} \frac{s_e^4}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} = \frac{(D-2)(D-4)}{4} I^{D+4}_{i_1 i_2 i_3 i_4} \to -\frac{1}{6}$$

$$\int \frac{\mathrm{d}^D l}{(i\pi)^{D/2}} \frac{s_e^2}{d_{i_1} d_{i_2} d_{i_3}} = \frac{(D-4)}{2} I^{D+2}_{i_1 i_2 i_3} \to \frac{1}{2}$$

$$\int \frac{\mathrm{d}^D l}{(i\pi)^{D/2}} \frac{s_e^2}{d_{i_1} d_{i_2}} = \frac{(D-4)}{2} I^{D+2}_{i_1 i_2} \to \frac{m_{i1}^2 + m_{i2}^2}{2} - \frac{1}{6}\left(q_{i1}^2 - q_{i2}^2\right)^2$$

$$\int \frac{\mathrm{d}^D l}{(i\pi)^{D/2}} \frac{s_i}{d_{i_1} \cdots d_{i_N}} = 0$$

Final result

Full one-loop amplitude:

$$\mathcal{A}_{(D)} = \sum_{[i_1|i_5]} e^{(0)}_{i_1 i_2 i_3 i_4 i_5} \; I^{(D)}_{i_1 i_2 i_3 i_4 i_5}$$
$$+ \sum_{[i_1|i_4]} \left(d^{(0)}_{i_1 i_2 i_3 i_4} \; I^{(D)}_{i_1 i_2 i_3 i_4} - \frac{D-4}{2} d^{(2)}_{i_1 i_2 i_3 i_4} \; I^{(D+2)}_{i_1 i_2 i_3 i_4} + \frac{(D-4)(D-2)}{4} d^{(4)}_{i_1 i_2 i_3 i_4} \; I^{(D+4)}_{i_1 i_2 i_3 i_4} \right)$$
$$+ \sum_{[i_1|i_3]} \left(c^{(0)}_{i_1 i_2 i_3} \; I^{(D)}_{i_1 i_2 i_3} - \frac{D-4}{2} c^{(9)}_{i_1 i_2 i_3} \; I^{(D+2)}_{i_1 i_2 i_3} \right) + \sum_{[i_1|i_2]} \left(b^{(0)}_{i_1 i_2} \; I^{(D)}_{i_1 i_2} - \frac{D-4}{2} b^{(9)}_{i_1 i_2} \; I^{(D+2)}_{i_1 i_2} \right)$$

Cut-constructable:

$$\mathcal{A}_N^{CC} = \sum_{[i_1|i_4]} d^{(0)}_{i_1 i_2 i_3 i_4} \; I^{(4-2\epsilon)}_{i_1 i_2 i_3 i_4} + \sum_{[i_1|i_3]} c^{(0)}_{i_1 i_2 i_3} \; I^{(4-2\epsilon)}_{i_1 i_2 i_3} + \sum_{[i_1|i_2]} b^{(0)}_{i_1 i_2} \; I^{(4-2\epsilon)}_{i_1 i_2}$$

Rational part:

$$R_N = -\sum_{[i_1|i_4]} \frac{d^{(4)}_{i_1 i_2 i_3 i_4}}{6} + \sum_{[i_1|i_3]} \frac{c^{(9)}_{i_1 i_2 i_3}}{2} - \sum_{[i_1|i_2]} \left(\frac{(q_{i_1} - q_{i_2})^2}{6} - \frac{m_{i_1}^2 + m_{i_2}^2}{2} \right) \; b^{(9)}_{i_1 i_2}$$

Vanishing contributions: $\mathcal{A} = \mathcal{O}(\epsilon)$

The F90 Rocket program

Rocket science!

Eruca sativa =Rocket=roquette=arugula=rucola
Recursive unitarity calculation of one-loop amplitudes

First step: use only 3 and 4-gluon vertices ⇒ pure gluonic amplitudes

Input: arbitrary number of gluons and their arbitrary helicities (+/-)

Output: (un)-renormalized virtual amplitude in FDH or t'HV scheme

[Giele & GZ '08]

Automated one-loop

Issues:

- checks of the results
- numerical instabilities at special points
- numerical efficiency: how fast is the algorithm? scaling of time with N
- practicality: computation of realistic LHC processes

Checks on the results

- poles

$$A_{\rm v} = c_\Gamma \left(\frac{N}{\epsilon^2} + \frac{1}{\epsilon} \left(\sum_{i=1}^{N} \ln \frac{-s_{i,i+1}}{\mu^2} - \frac{11}{3} \right) \right) A_{\rm tree}$$

NB: single pole checks the coefficients of two-point functions, which because of subtraction terms are sensitive to higher-point coefficients

Checks on the results

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NB: single pole checks the coefficients of two-point functions, which because of subtraction terms are sensitive to higher-point coefficients

- there is an infinite solutions of the unitarity constraints, results are independent of the specific choice

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- checks with some known analytical results
  (all N=6, finite and MHV amplitudes for larger N)

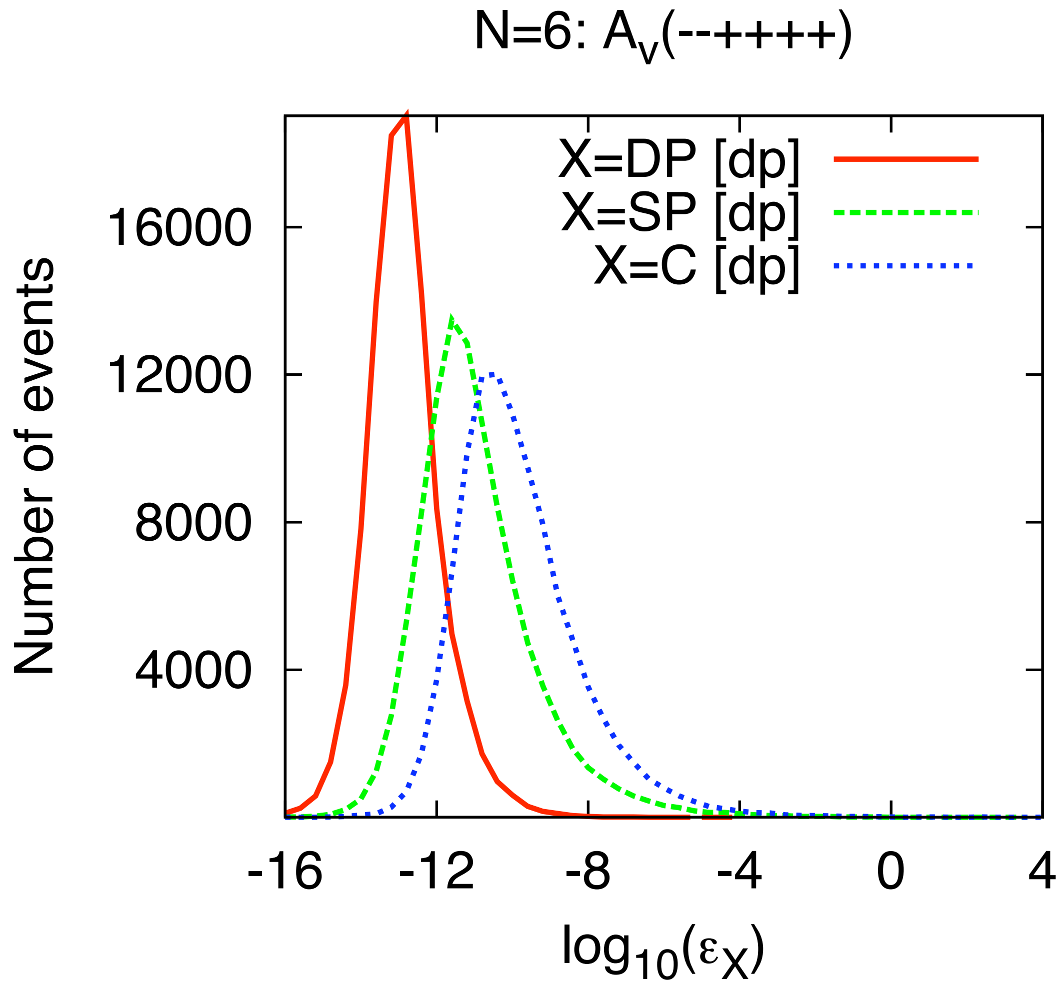
Accuracy

Define:

$$\varepsilon_C = \log_{10} \frac{|A_N^{\mathrm{v,unit}} - A_N^{\mathrm{v,anly}}|}{|A_N^{\mathrm{v,anly}}|}$$

similar for ε_{DP} and ε_{SP}

Based on 10^5 flat phase space points with minimal cuts



- peak position of
  - double pole: $10^{-12.8}$
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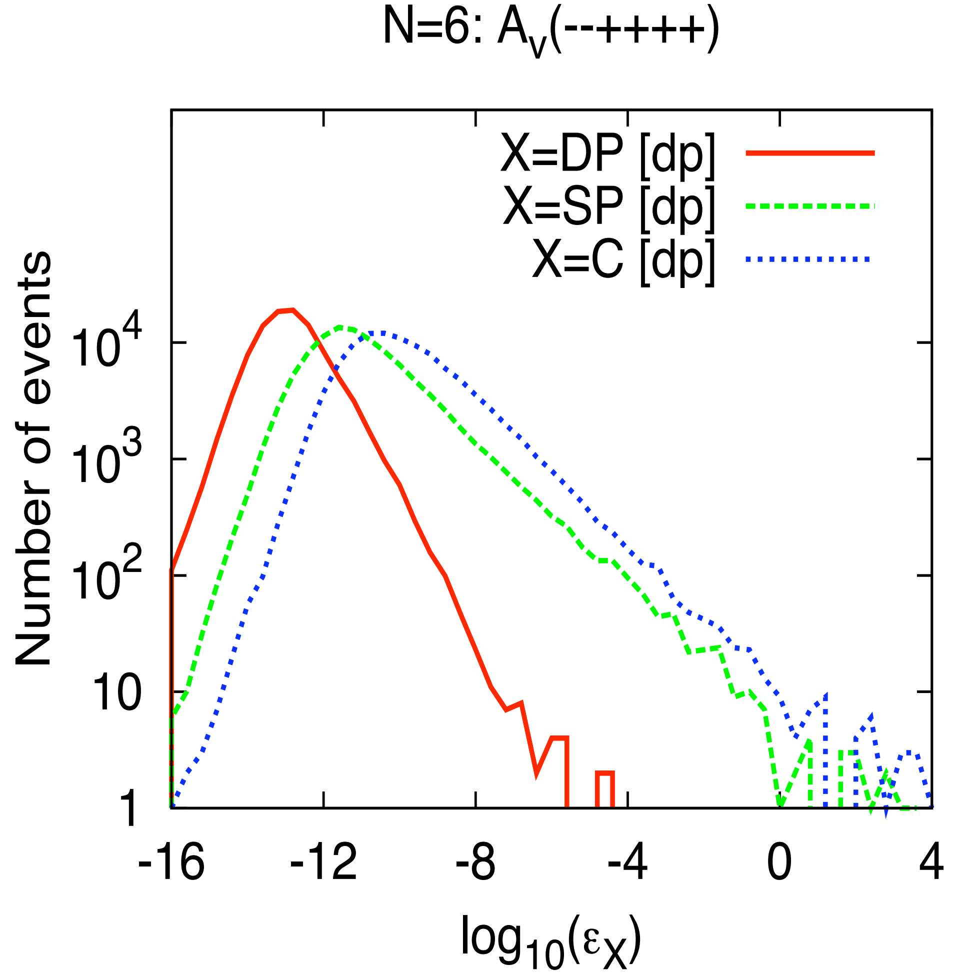

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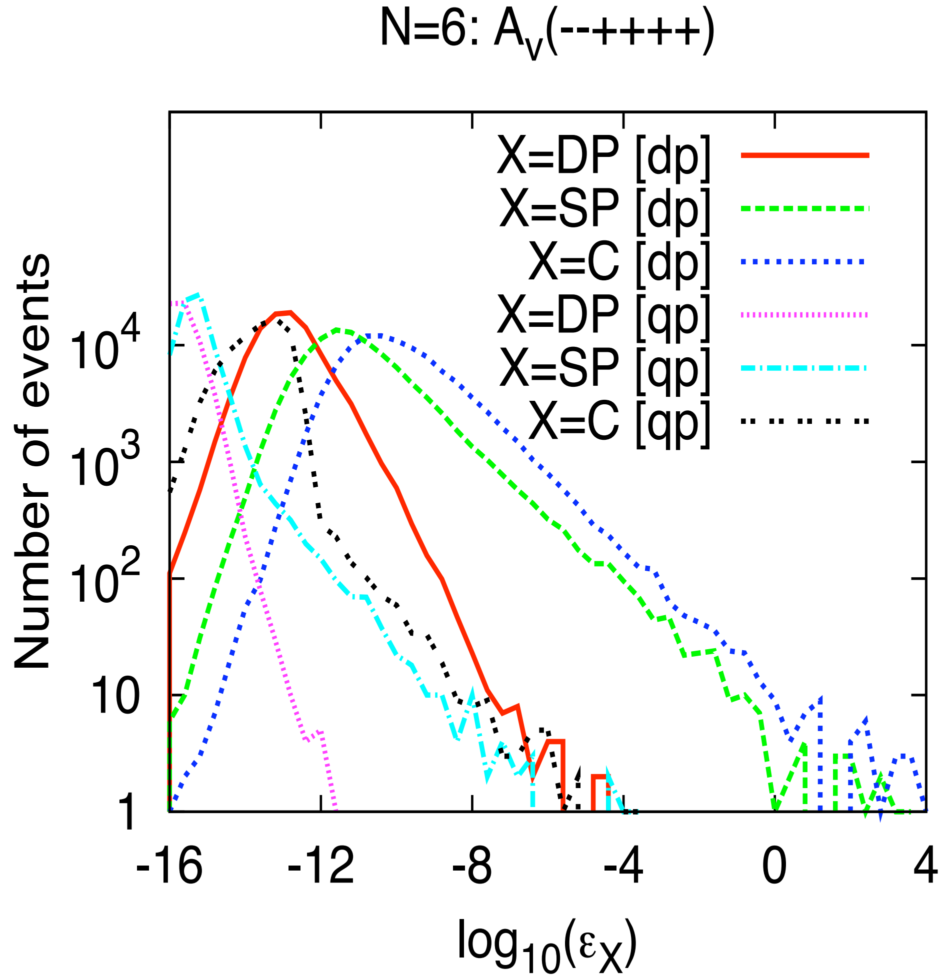

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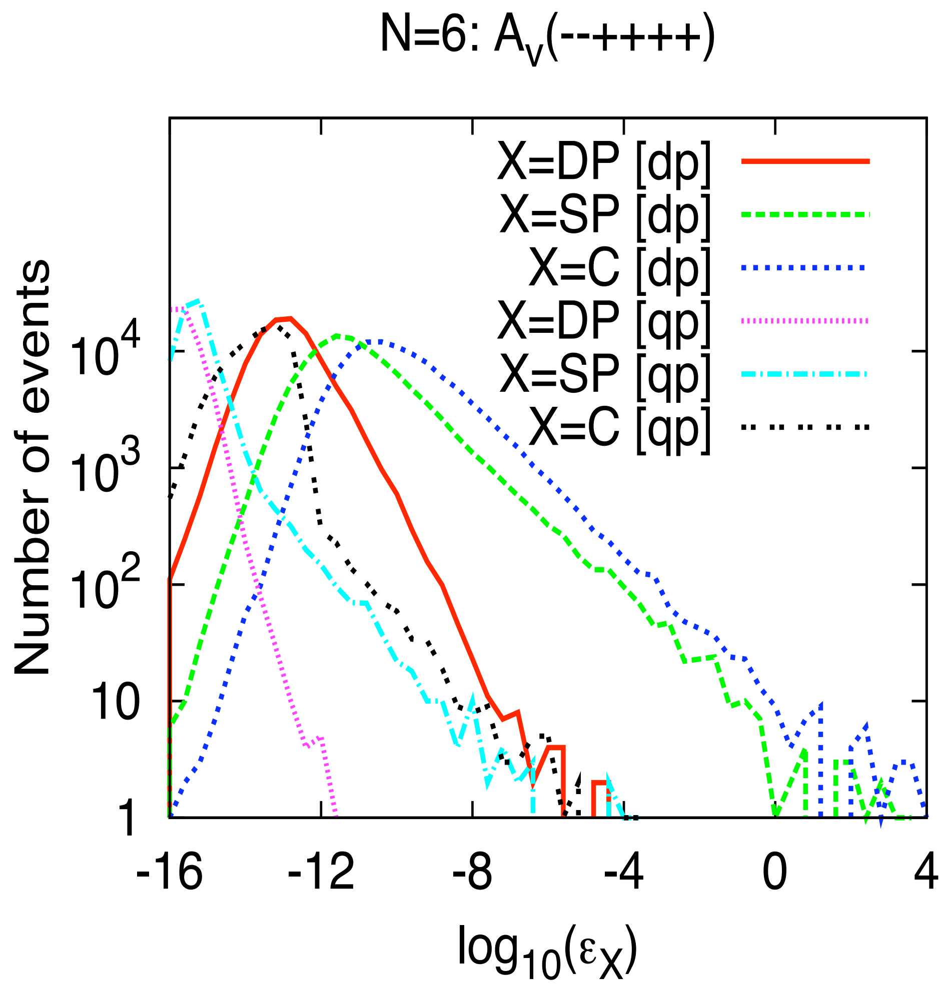

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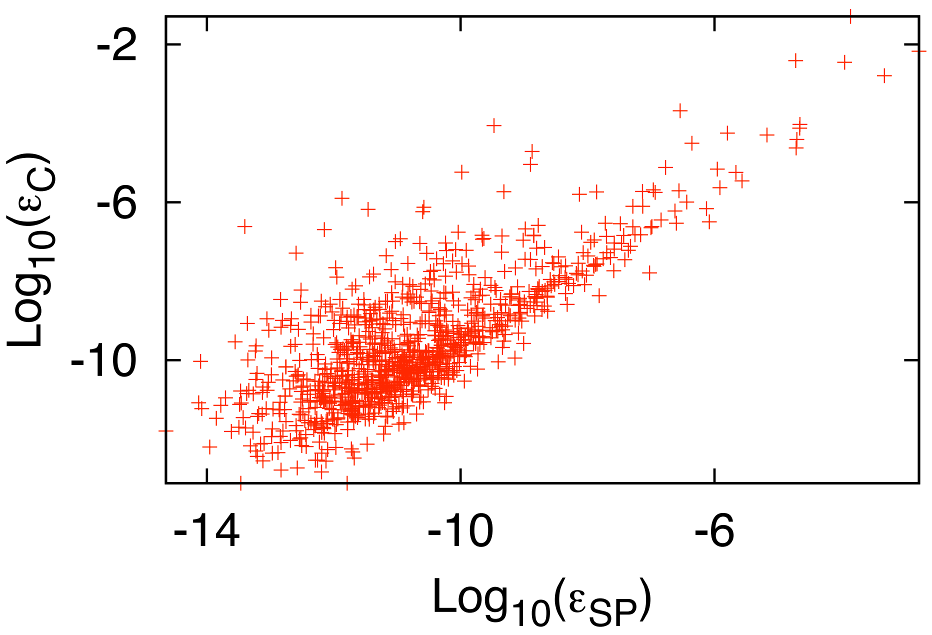
Same picture holds increasing the number of gluons: N=7,8,9,10,11, ...

Finding instabilities

1) Correlation in the accuracy of single pole and constant part

⇒ if the accuracy on the poles is worse than X use higher precision

But this does not check the rational part



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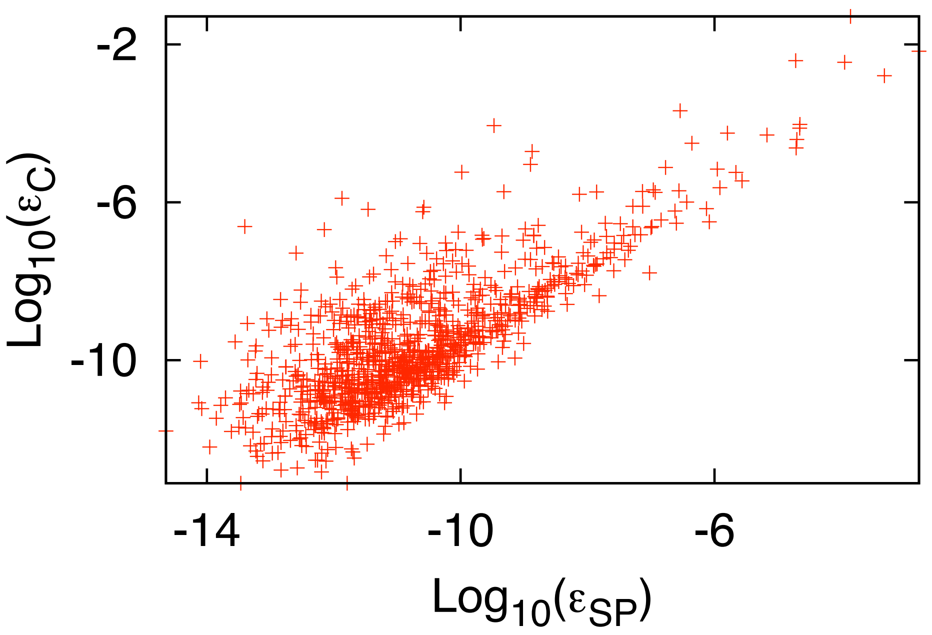


II) How good is the system of equations solved ?

Look at how well residues are reconstructed using the coefficients.

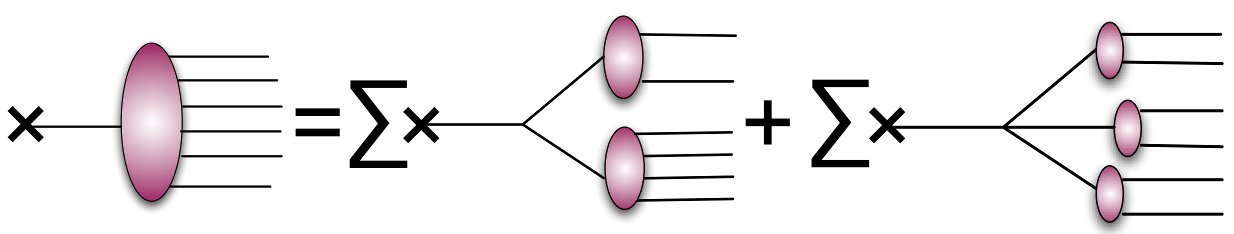
Practically: choose a random loop momentum and for a given cut

- compute the residue as linear combination of coefficients
- compute the residue directly

⇒ if the results differ more than X use higher precision

Tree: 3 methods beyond Feynman

✓ Berends-Giele relations: compute helicity amplitudes recursively using off-shell currents



Berends, Giele '88

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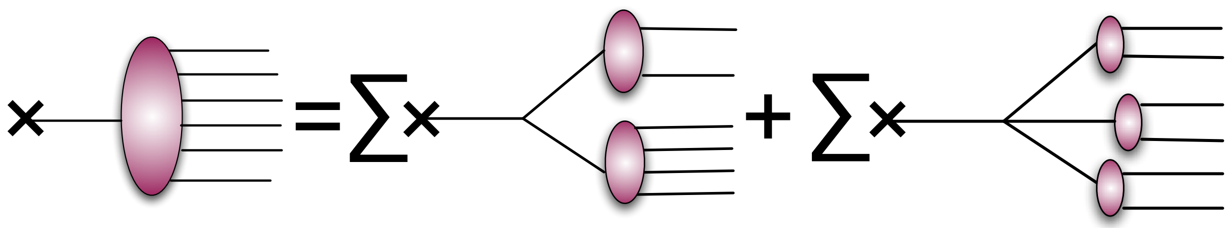
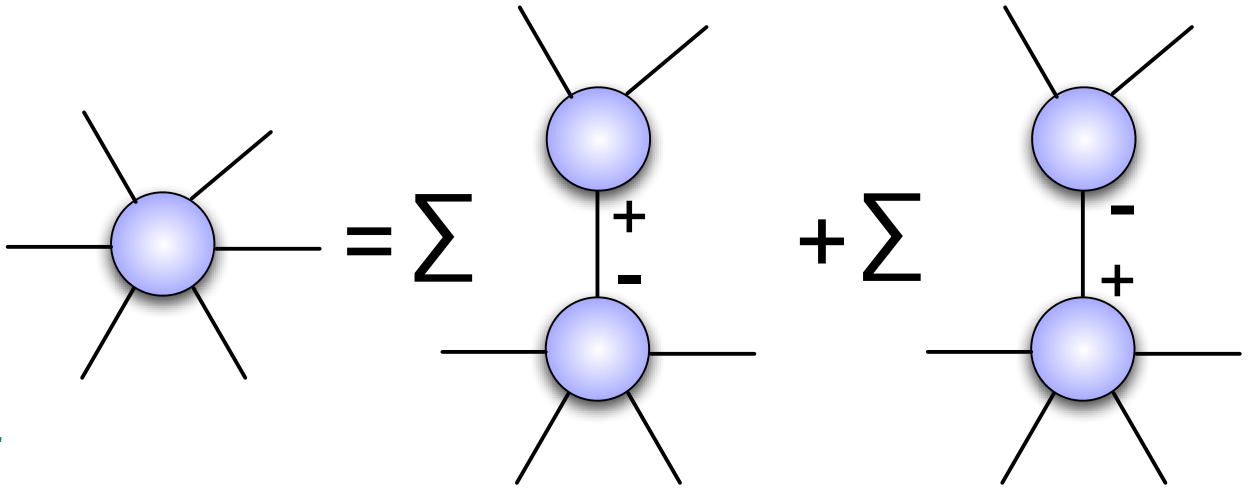


Berends, Giele '88

✓ BCF relations: compute helicity amplitudes via on-shell recursions (use complex momentum shifts)



Britto, Cachazo, Feng '04

Tree: 3 methods beyond Feynman

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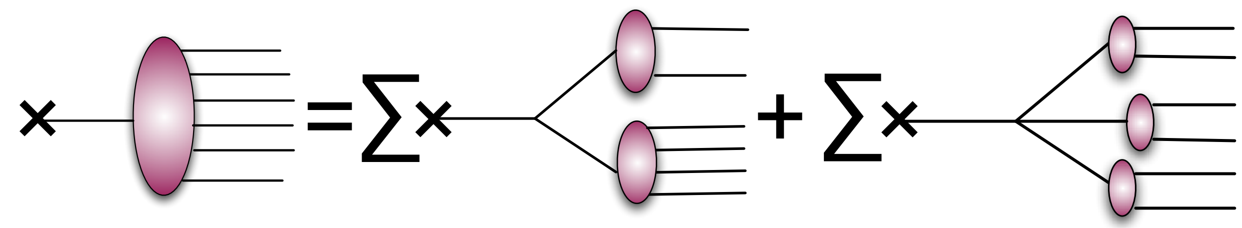

Berends, Giele '88

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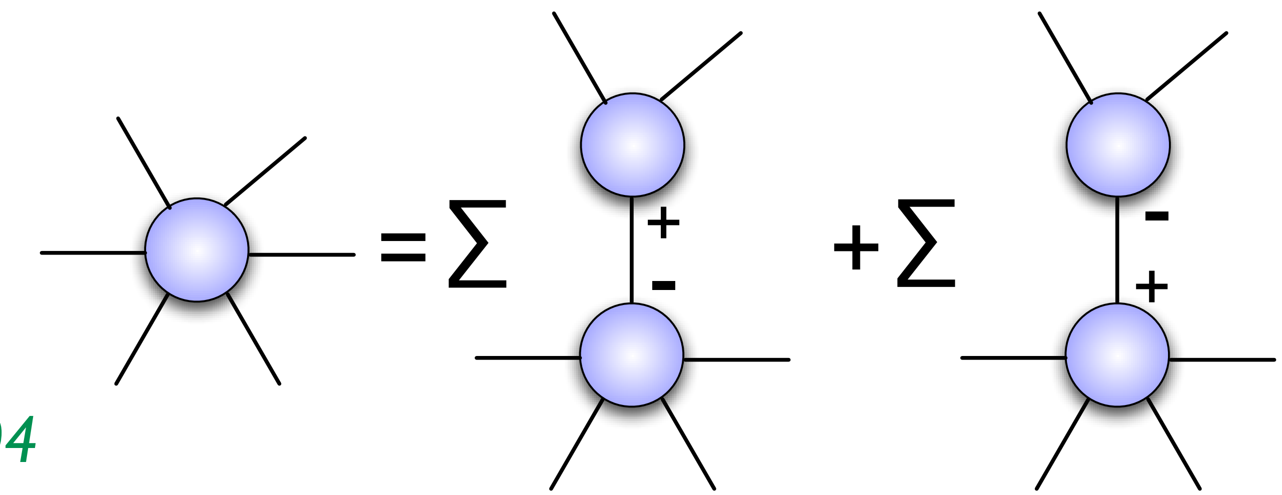
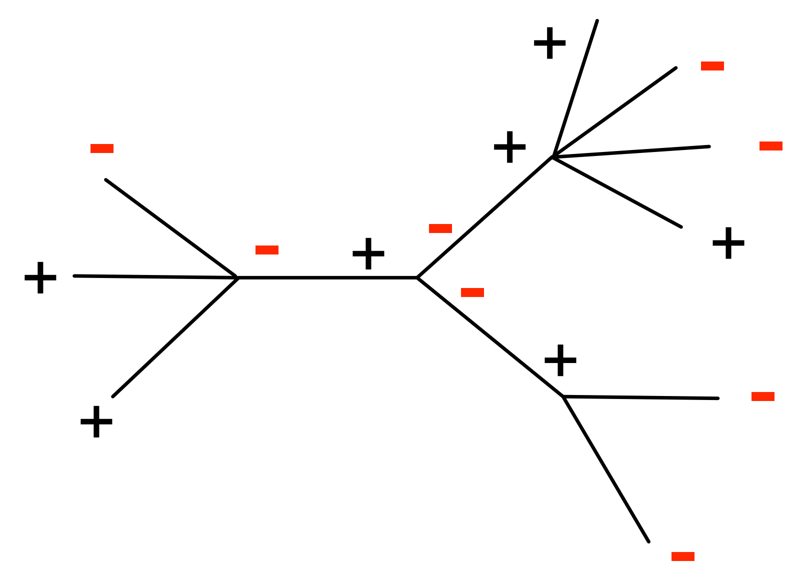


Britto, Cachazo, Feng '04

✓ <u>CSW relations:</u> compute helicity amplitudes by sewing together MHV amplitudes [- - + + ... +]



Cachazo, Svrcek, Witten '04

Numerical performance

Time [s] for 2 → n gluon amplitudes for 10^4 points

Duhr et al. '06
also Dinsdale et al. '06

Final state	BG	BCF	CSW
2g	0.28	0.33	0.26
3g	0.48	0.51	0.55
4g	1.04	1.32	1.75
5g	2.69	7.26	5.96
6g	7.19	59.1	30.6
7g	23.7	646	195
8g	82.1	8690	1890
9g	270	127000	29700
10g	864	-	-

☛ numerical superiority of Berends-Giele recursion for large N

Time dependence

Constructive implementation of BG tree-level amplitudes (or recursive with caching)

$$\tau_{\rm tree} = \binom{N}{3} E_3 + \binom{N}{4} E_4 \propto N^4$$

E_3 (E_4) → time for the evaluation of a 3 (4) gluon vertex

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Number of tree level amplitudes needed at one-loop

$$n_{\text{tree}} = \left\{(D_{s1}-2)^2 + (D_{s2}-2)^2\right\} \times \left(5\, c_{5,\max}\binom{N}{5} + 4\, c_{4,\max}\binom{N}{4} + 3\, c_{3,\max}\binom{N}{3} + 2\, c_{2,\max}\left[\binom{N}{2} - N\right]\right)$$

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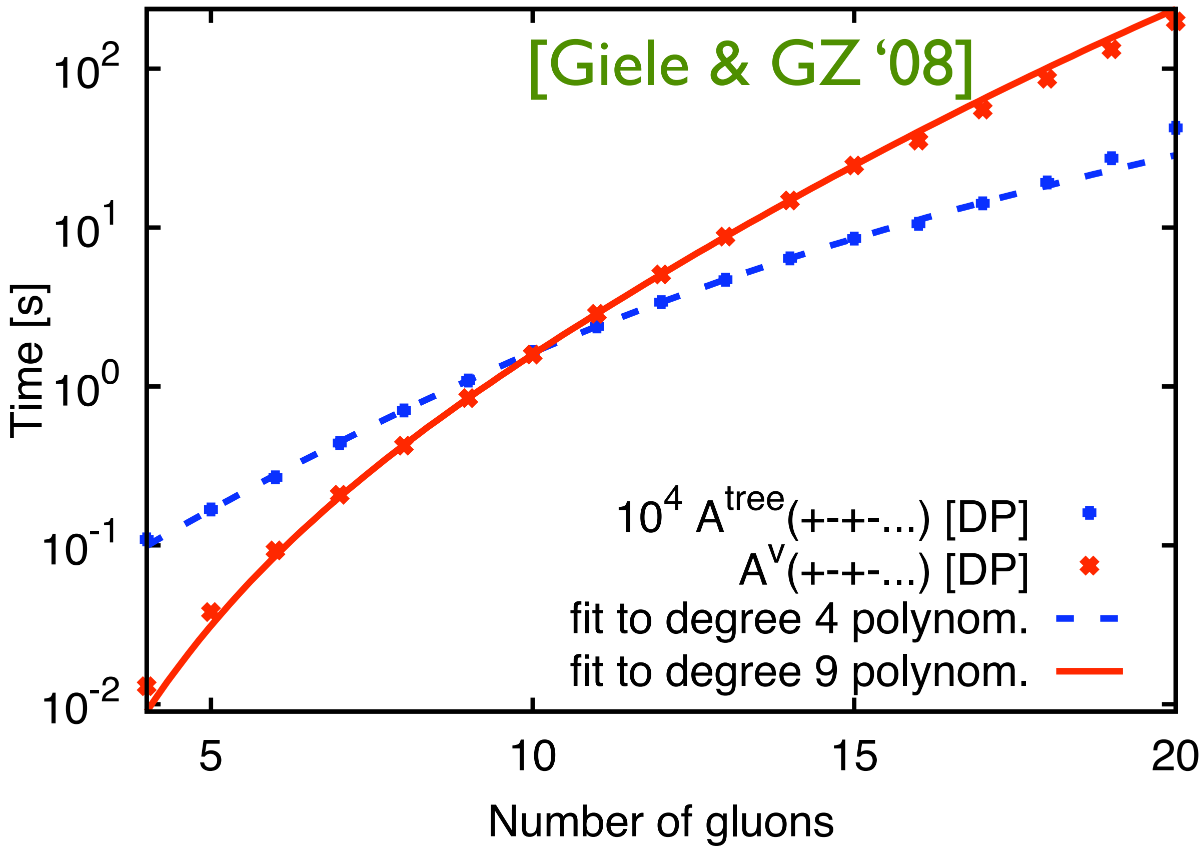
☛ $\tau_{\rm one-loop,N} \sim n_{\rm tree} \cdot \tau_{\rm tree,N} \propto N^9$

[to be compared with factorial growth!]

Time dependence at one-loop up to N=20



☛ time ∝ N^9 as expected

☛ independent of the helicity configuration

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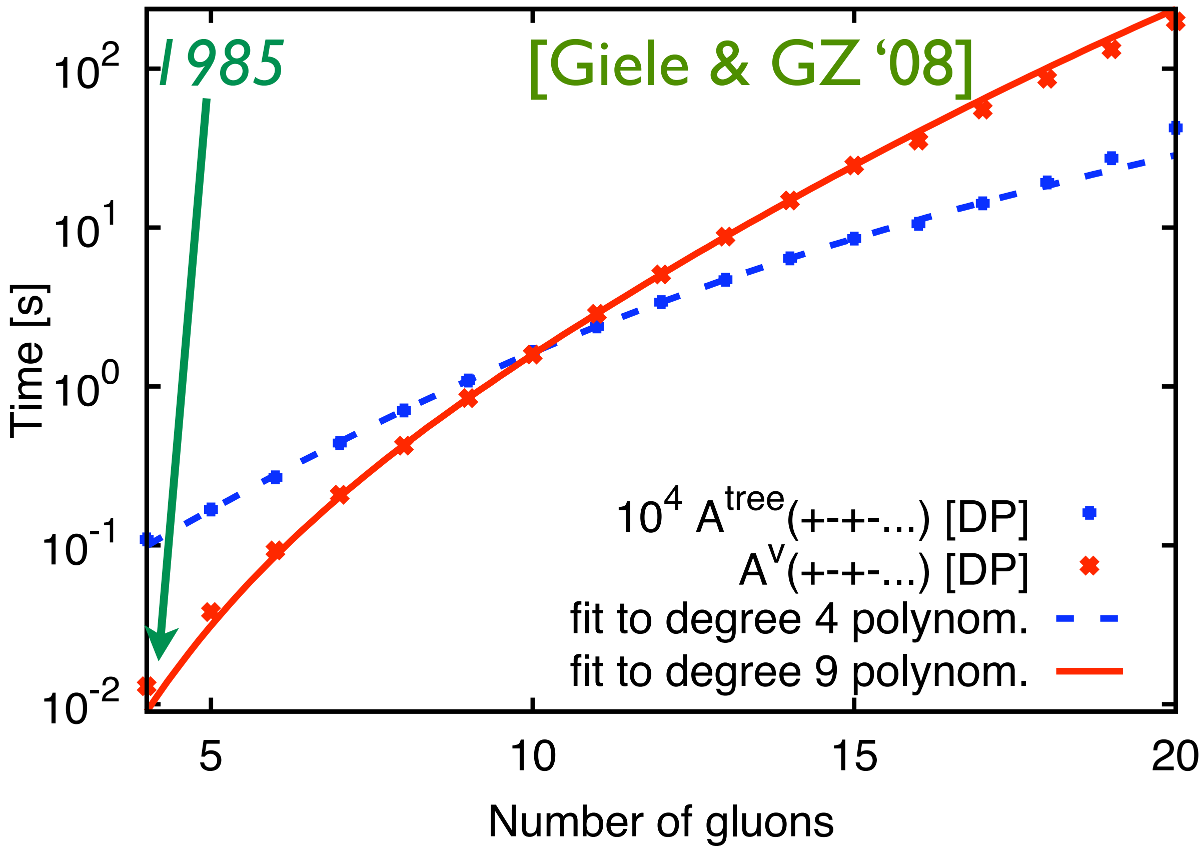

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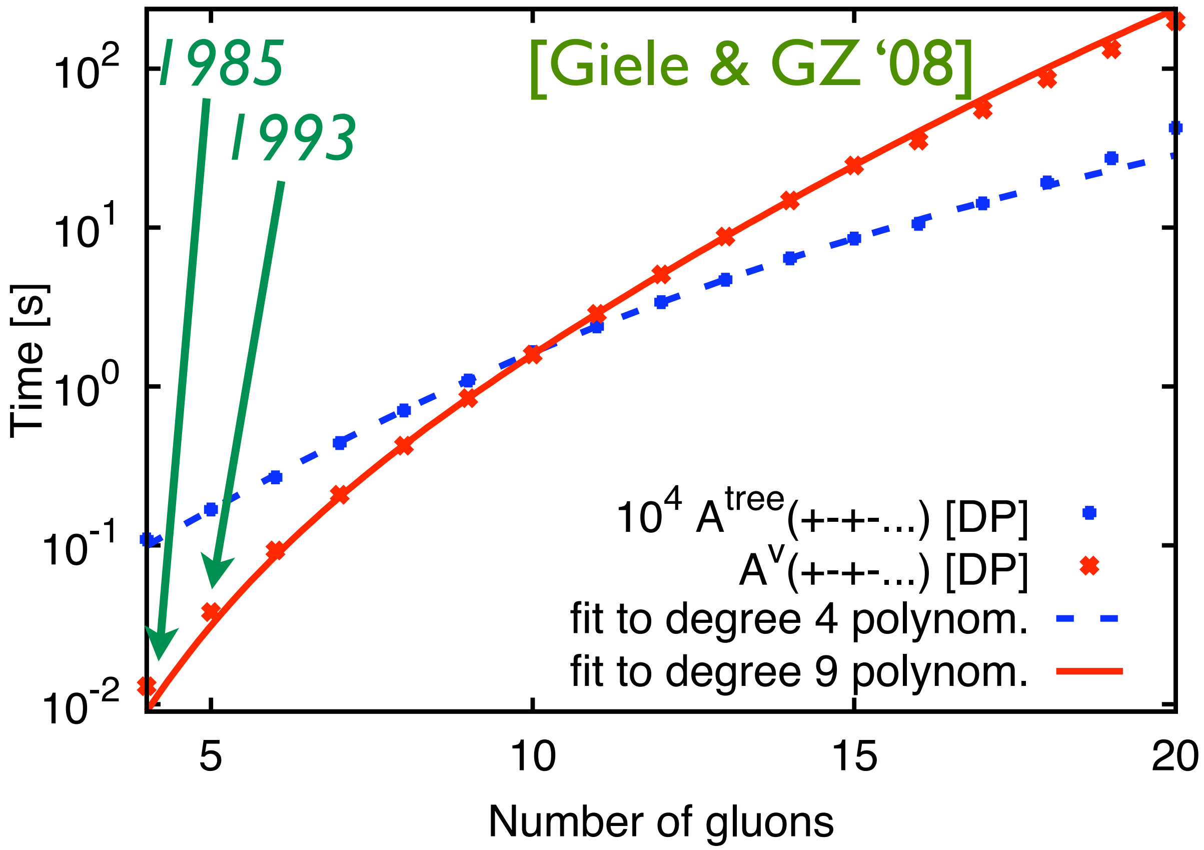

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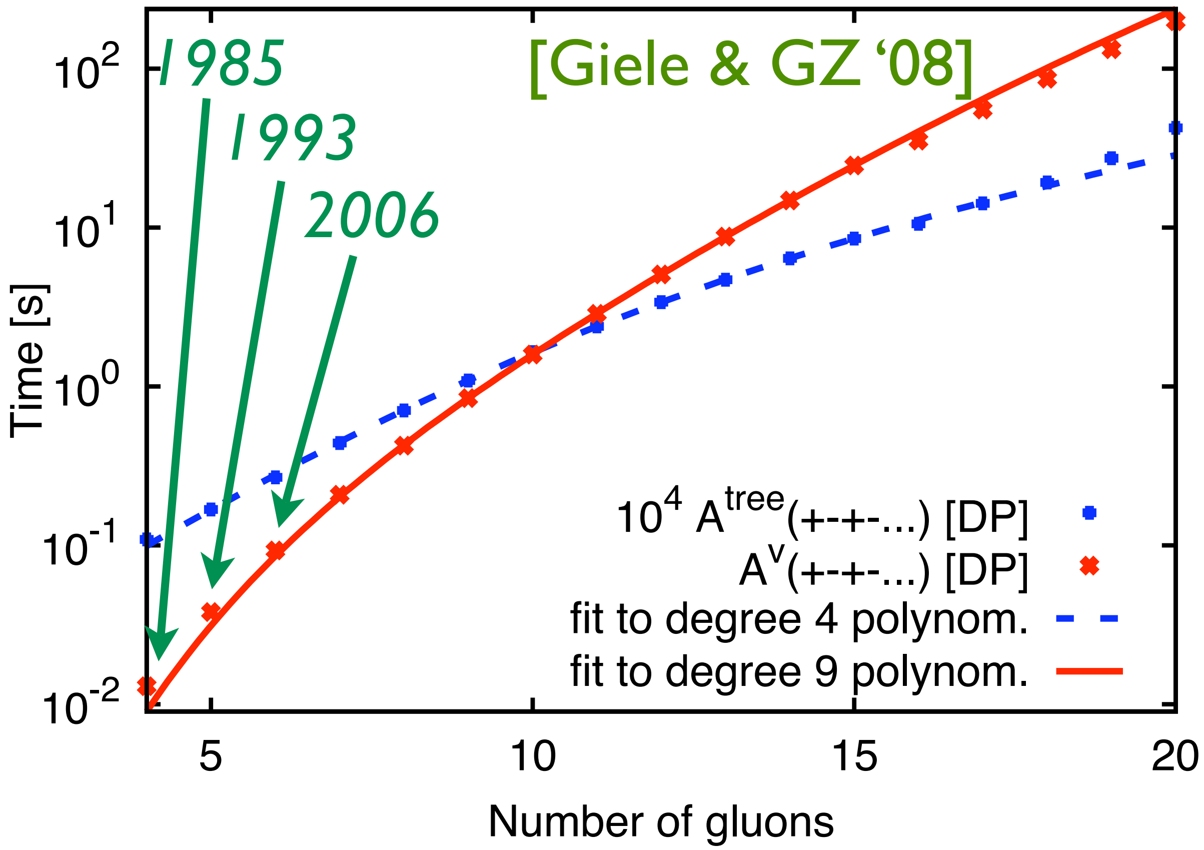

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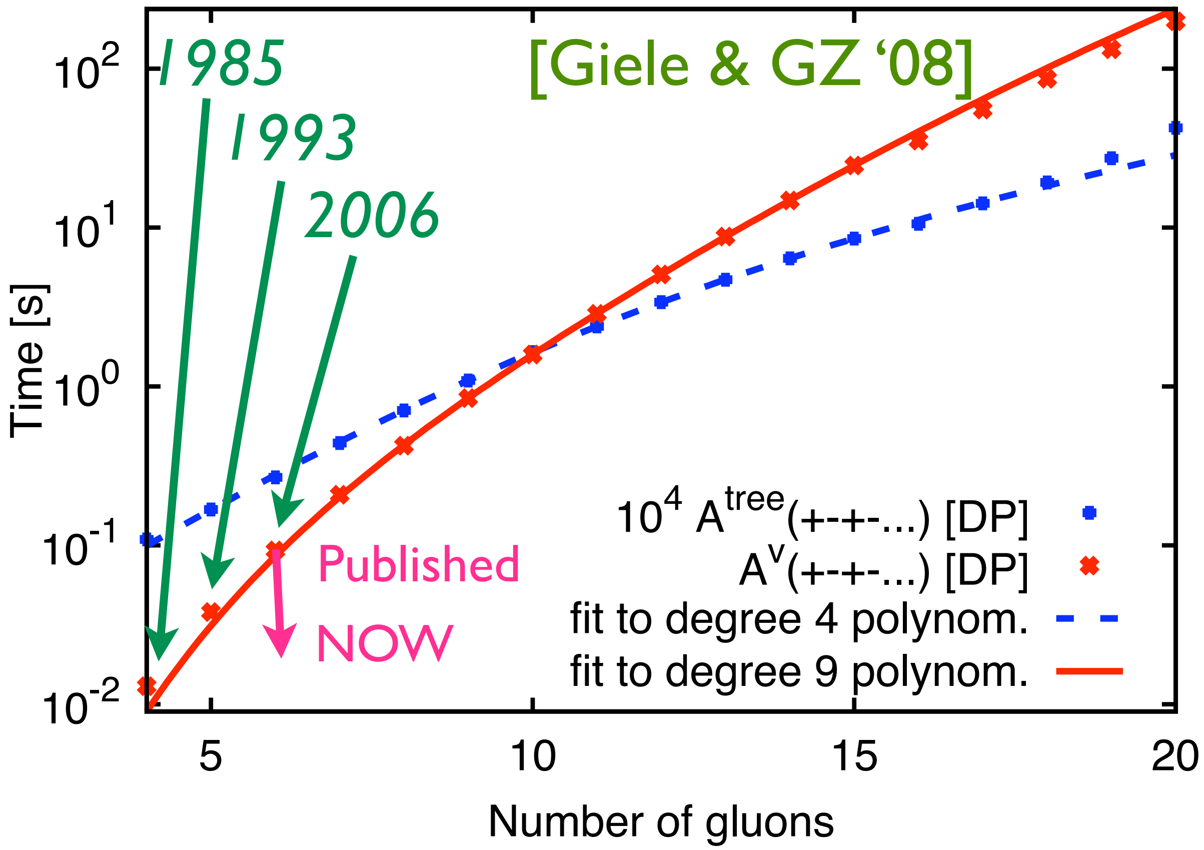

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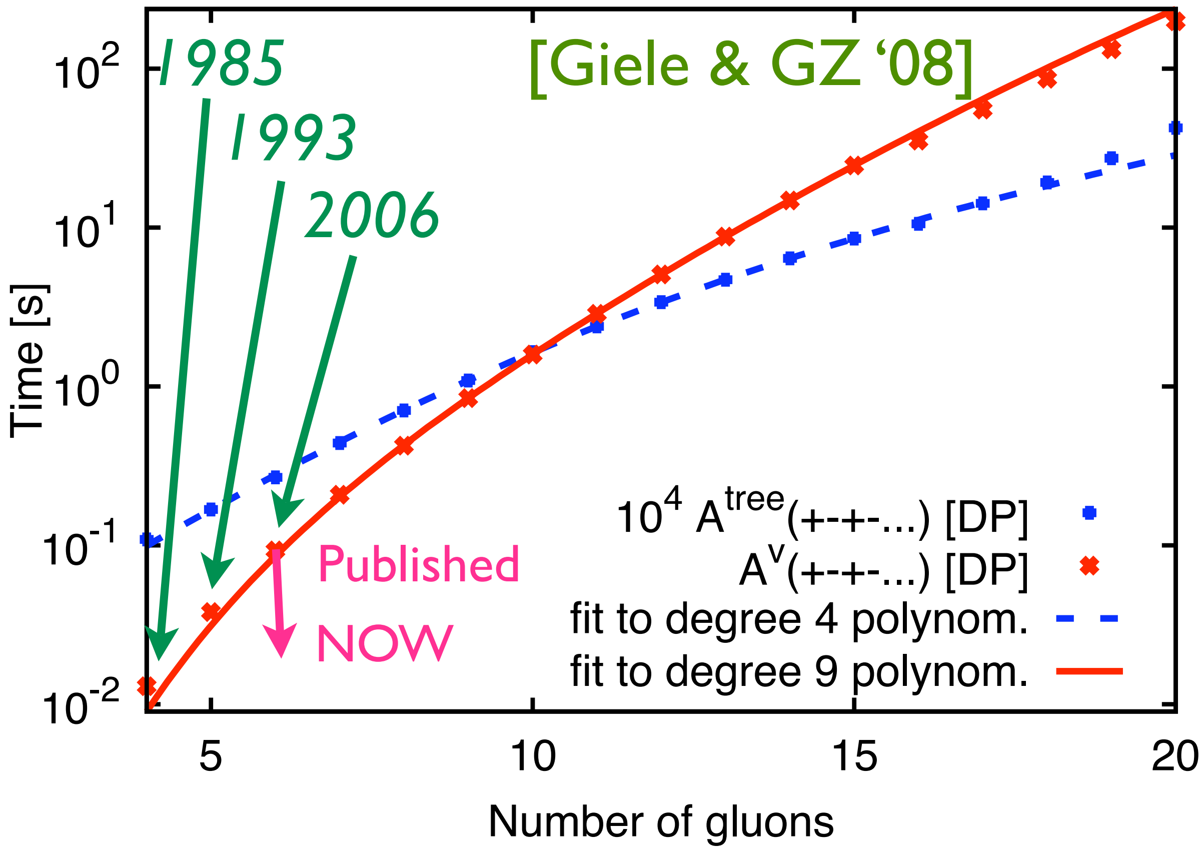

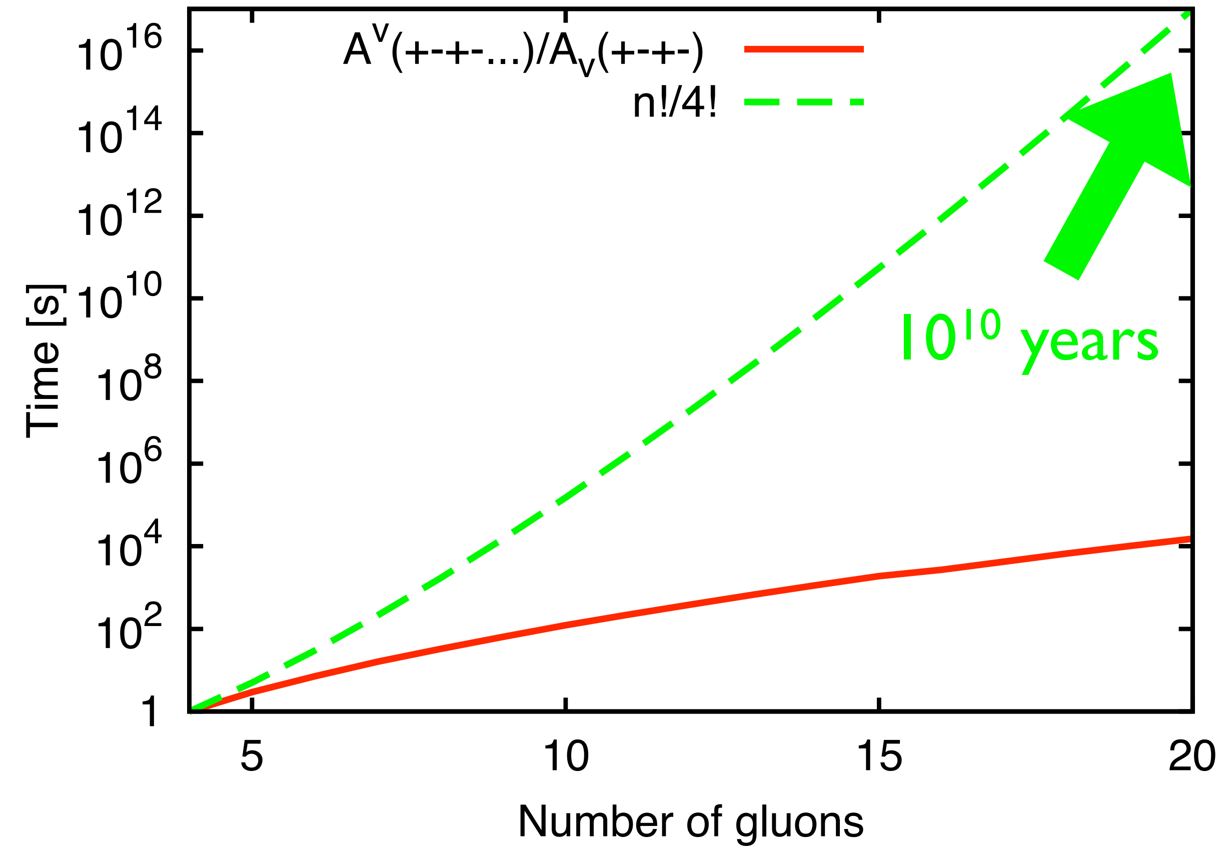

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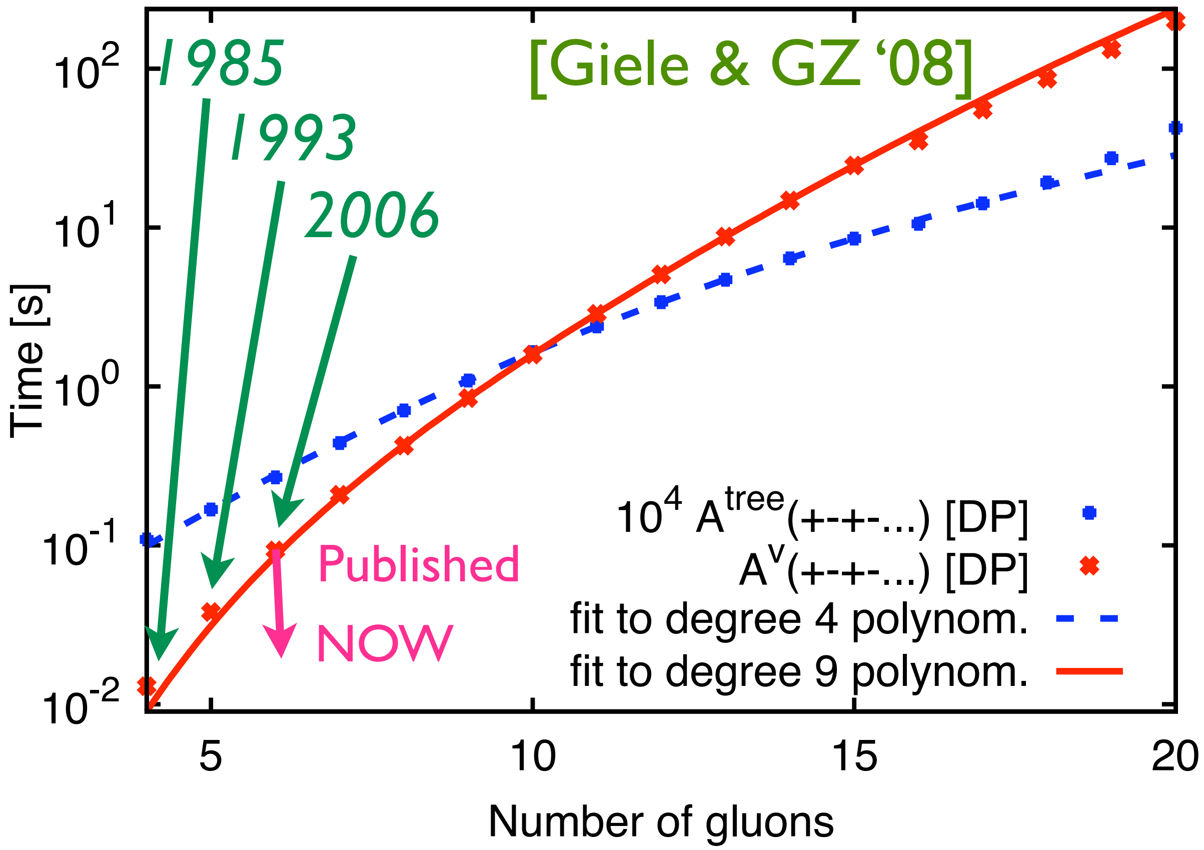

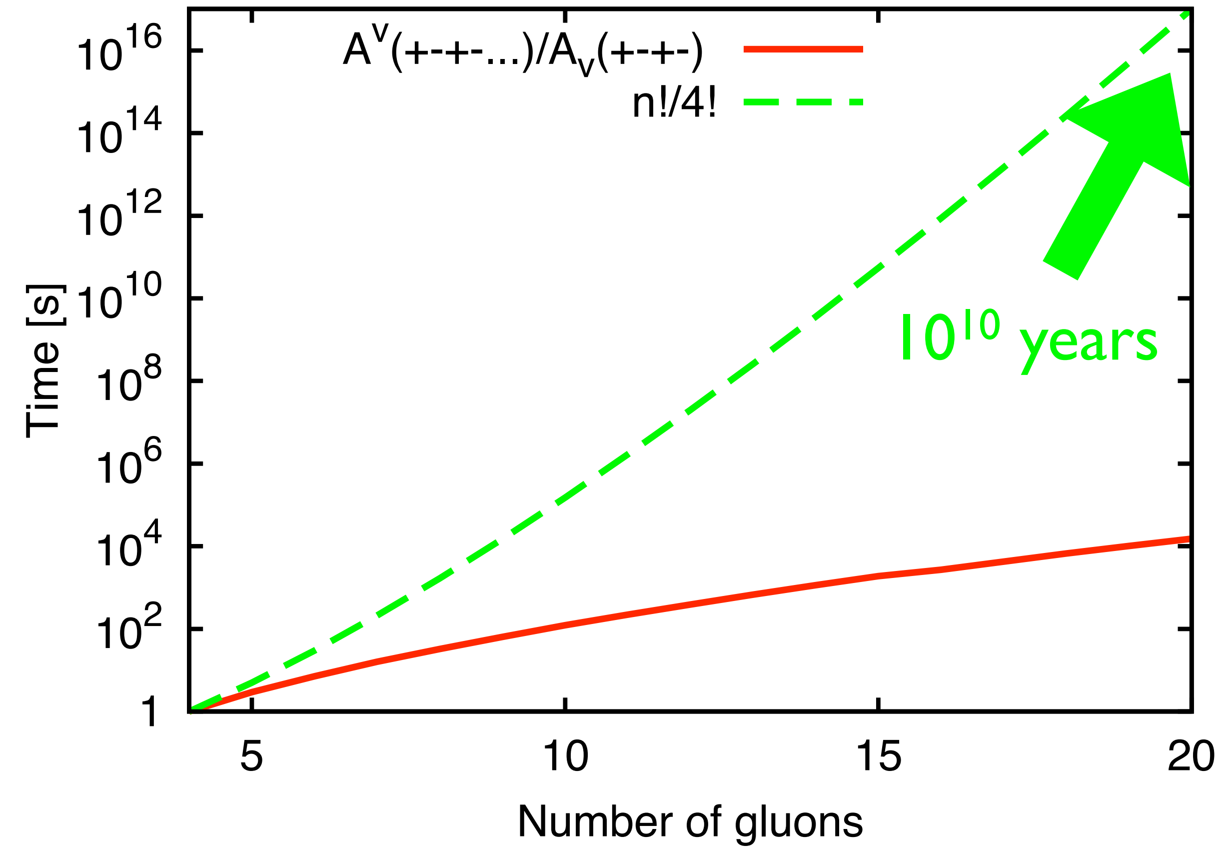

☛ time $\propto N^9$ as expected

☛ independent of the helicity configuration

☛ compare with factorial growth...

Comparison with other methods: time roughly comparable

Berger, Bern, Cordero, Dixon, Forde, Ita, Kosower, Maitre '08
Giele & Winter '09
Lazopoulos '09

Sample results at fixed points

Rocket can compute any N-gluon amplitude with arbitrary helicities, consider e.g. 15* gluon momenta random generated:

$$
\begin{aligned}
p_1 &= (-7.500000000000000, 7.500000000000000, 0.000000000000000, 0.000000000000000)\\
p_2 &= (-7.500000000000000, -7.500000000000000, 0.000000000000000, 0.000000000000000)\\
p_3 &= (0.368648489648050, 0.161818085189973, 0.125609635286264, -0.306494430207942)\\
p_4 &= (0.985841964092509, -0.052394238926518, -0.664093578996812, 0.726717923425790)\\
p_5 &= (1.470453194926588, -0.203016239158633, 0.901766792550452, -1.143605551298596)\\
p_6 &= (2.467058579094687, -1.840106401193462, 0.715811527707121, 1.479189075734789)\\
p_7 &= (0.566021478235079, -0.406406330753485, -0.393435666409983, -0.020556861225509)\\
p_8 &= (0.419832726637289, -0.214182754609525, 0.074852807863799, -0.353245414886707)\\
p_9 &= (2.691168687878469, 1.868400546247601, 1.850615607221259, -0.571568175905795)\\
p_{10} &= (1.028090983779864, -0.986442664896249, -0.193408556327968, 0.215627155388572)\\
p_{11} &= (1.377779821947130, -0.155359745837053, -1.074009172530291, -0.848908054184264)\\
p_{12} &= (1.432526153404585, 0.621168997409793, -0.290964068761809, 1.257624811911176)\\
p_{13} &= (0.335532948820133, 0.244811479043329, 0.138986808214636, 0.182571538348285)\\
p_{14} &= (1.085581415795683, 0.330868645896313, -0.756382142822373, -0.704910635118478)\\
p_{15} &= (0.771463555739934, 0.630840621587917, -0.435349992994295, 0.087558618018677)
\end{aligned}
$$

* up to N=20 given in 0805.2152

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Helicity amplitude	c_Γ/ϵ^2	c_Γ/ϵ	1
$\lvert A_{15}^{\text{tree}}(++++\dots)\rvert$	-	-	0
$\lvert A_{15}^{\text{v,unit}}(++++\dots)\rvert$	0	0	1.07572071884782
$\lvert A_{15}^{\text{v,anly}}(++++\dots)\rvert$	0	0	1.07572071880769
$\lvert A_{15}^{\text{tree}}(-+++\dots++)\rvert$	-	-	0
$\lvert A_{15}^{\text{v,unit}}(-+++\dots++)\rvert$	0	0	0.181194659968483
$\lvert A_{15}^{\text{v,anly}}(-+++\dots++)\rvert$	0	0	0.181194659846677
$\lvert A_{15}^{\text{tree}}(--+++\dots++)\rvert$	-	-	7.45782101450887
$\lvert A_{15}^{\text{v,unit}}(--++\dots++)\rvert$	111.867315217633	586.858955605213	1810.13038312828
$\lvert A_{15}^{\text{v,anly}}(--++\dots++)\rvert$	111.867315217633	586.858955605213	1810.13038312852
$\lvert A_{15}^{\text{tree}}(-+-\dots+-)\rvert$	-	-	$5.851039428822597 \cdot 10^{-3}$
$\lvert A_{15}^{\text{v,unit}}(-+-\dots+-)\rvert$	$8.776559143021942 \cdot 10^{-2}$	0.460420629357800	1.52033417713680
$\lvert A_{15}^{\text{v,anly}}(-+-\dots+-)\rvert$	$8.776559143233895 \cdot 10^{-2}$	0.460420661976678	N.A.
$\lvert A_{15}^{\text{tree}}(+-+\dots-+)\rvert$	-	-	$5.851039428822597 \cdot 10^{-3}$
$\lvert A_{15}^{\text{v,unit}}(+-+\dots-+)\rvert$	$8.776559143021942 \cdot 10^{-2}$	0.460420565320471	1.52960647292231
$\lvert A_{15}^{\text{v,anly}}(+-+\dots-+)\rvert$	$8.776559143233895 \cdot 10^{-2}$	0.460420661976678	N.A.

* Mahlon '93; Bern et al '05; ** Forde, Kosower '05

* up to N=20 given in 0805.2152

First LHC application: W + 3 jets

Why W+3 jets?

1. W+3jets measured at the Tevaton, but LO varies by more that a factor 2 under reasonable changes in scales

	$W^{\pm}$, TeV	W^{+}, LHC	W^{-}, LHC
σ [pb], $\mu = 40$ GeV	74.0 ± 0.2	783.1 ± 2.7	481.6 ± 1.4
σ [pb], $\mu = 80$ GeV	45.5 ± 0.1	515.1 ± 1.1	316.7 ± 0.7
σ [pb], $\mu = 160$ GeV	29.5 ± 0.1	353.5 ± 0.8	217.5 ± 0.5

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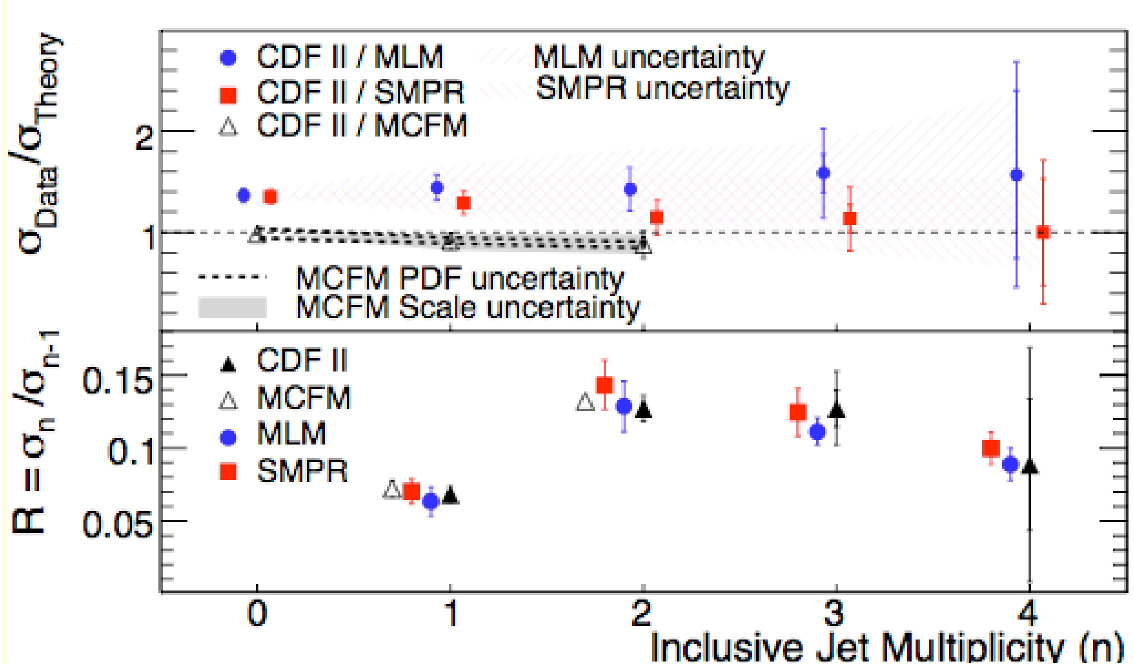
II. measurements at the Tevaton for W +njets with n=1,2: data is described well by NLO QCD
$\Rightarrow$ verify this for 3 and more jets



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IV. Calculation highly non-trivial optimal testing ground

$$0 \rightarrow \bar{u}\, d\, g\, g\, g\, W^+$$ → 1203 +104 Feynman diagrams

$$0 \rightarrow \bar{u}\, d\, \bar{Q}\, Q\, g\, W^+$$ → 258 +18 Feynman diagrams

Primitive amplitudes

For practical reason

want amplitudes where external particles are ordered

At tree level

color ordered $\Rightarrow$ momentum ordered external particles

At one-loop level

color ordered generic amplitude $\not\Rightarrow$ momentum ordered external particles

Solution

decompose color ordered amplitudes into primitive amplitudes. Colored particles are then ordered, but color blind ones not.

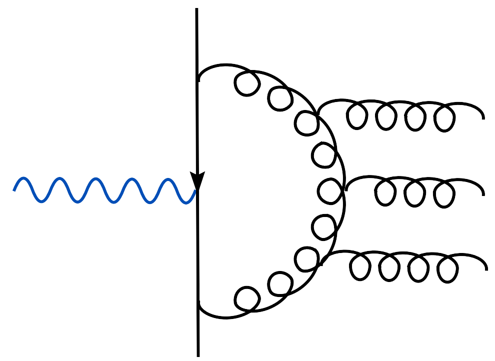
Bern, Dixon, Kosower '94

Primitives: sample color structures

Leading color | Fermion loops | (SSS)Subleading

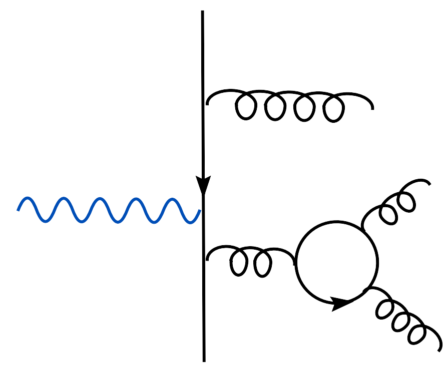
2-quark 3-gluon

Leading color:



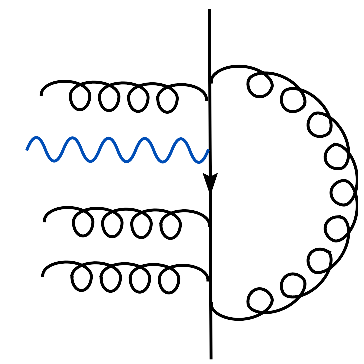
$$\mathrm{LC} \equiv (N_c^2 - 1)N_c^3$$

Fermion loops:



$$\mathrm{LC} \cdot \frac{n_f}{N_c}$$

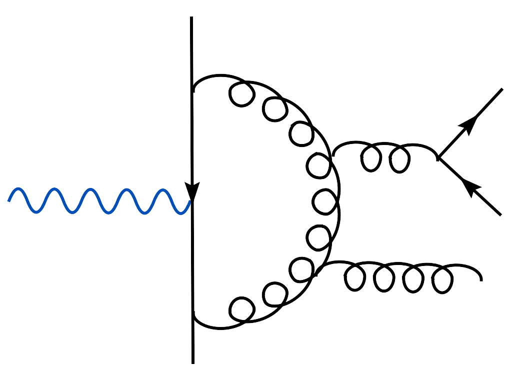
(SSS)Subleading:



$$\mathrm{LC} \cdot \frac{1}{N_c^3}$$

4-quark 1-gluon

Leading color:



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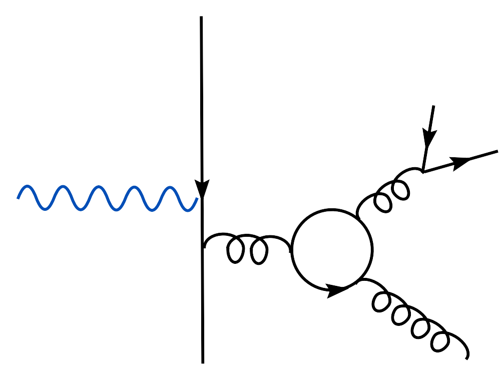
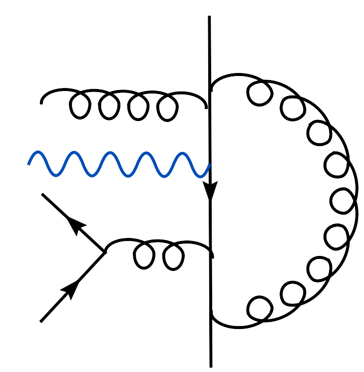


$$\mathrm{LC} \cdot \frac{n_f^2}{N_c^2}$$

(SSS)Subleading:



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Rules of the game

Procedure:

- order all SU(3) particles & allow all orderings of colorless particles

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Explicitly for W+3jets:

❶ ❷ ❸ ❹ ❺

$u_1 \; g_2 \; g_3 \; g_4 \; d_5 + W$

Rules of the game

Procedure:

- order all SU(3) particles & allow all orderings of colorless particles
- draw the parent diagram so that the loop is in the fixed position compared to the external fermion line [L/R]

Explicitly for W+3jets:

❶ ❷ ❸ ❹ ❺

u_1 g_2 g_3 g_4 d_5 + W

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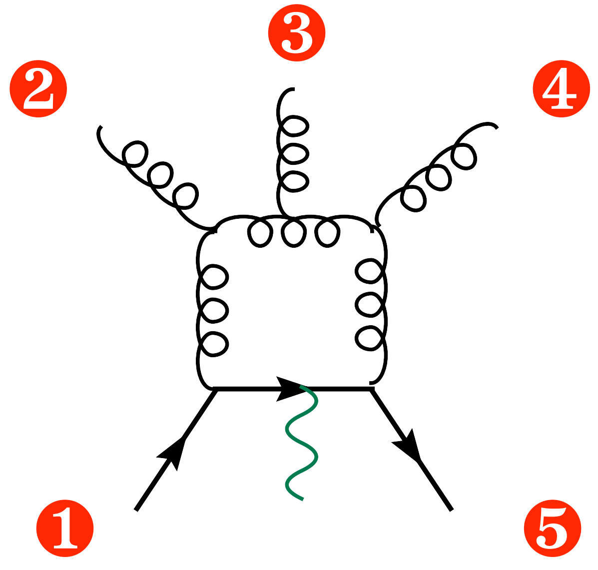

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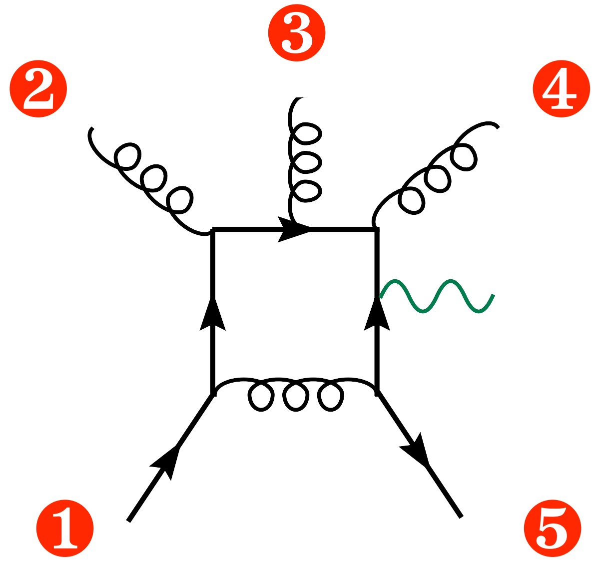

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Explicitly for W+3jets:

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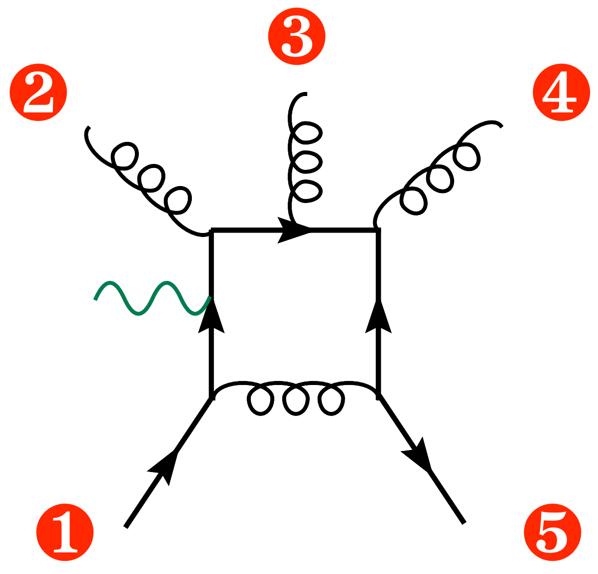
$u_1 \; g_2 \; g_3 \; g_4 \; d_5 + W$



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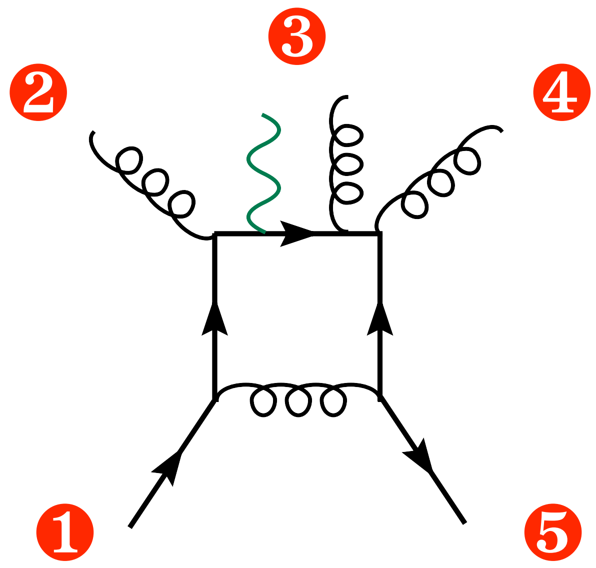
❶ ❷ ❸ ❹ ❺

u_1 g_2 g_3 g_4 d_5 + W



Rules of the game

Procedure:

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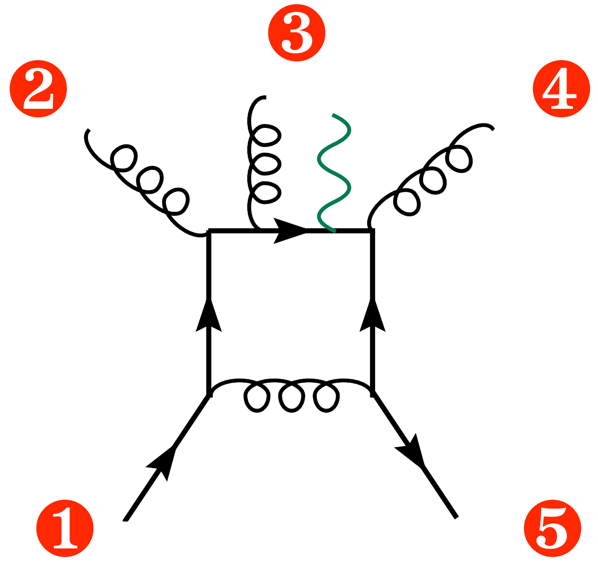
Explicitly for W+3jets:

❶ ❷ ❸ ❹ ❺

$u_1 \ g_2 \ g_3 \ g_4 \ d_5 + W$



Rules of the game

Procedure:

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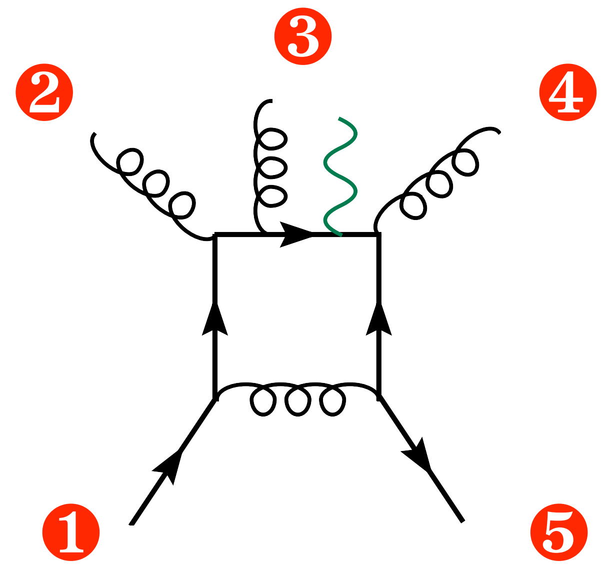
- draw the parent diagram so that the loop is in the fixed position compared to the external fermion line [L/R]
- N-point case: parent must be 1PI N-point

Explicitly for W+3jets:

❶ ❷ ❸ ❹ ❺

u_1 q_2 g_3 q_4 d_5 + W



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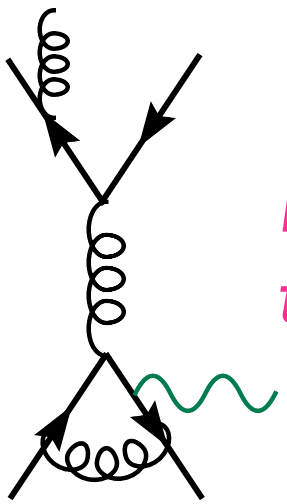
Explicitly for W+3jets:

❶ ❷ ❸ ❹ ❺

$u_1 \quad q_2 \quad g_3 \quad q_4 \quad d_5 + W$



How does this work?

Rules of the game

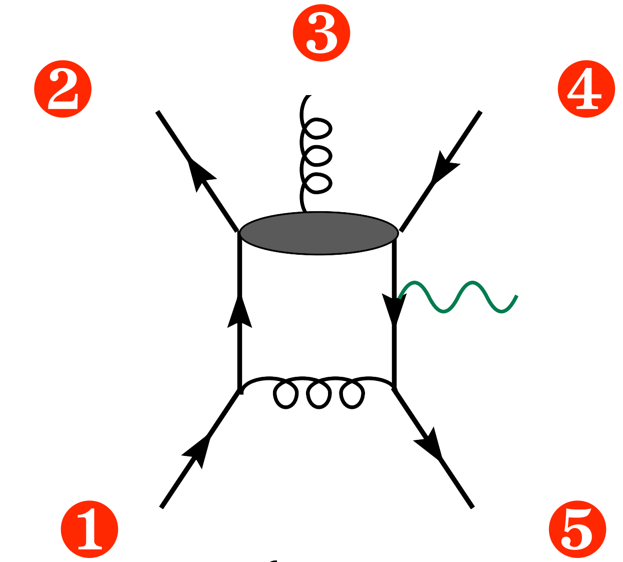
Procedure:

- order all SU(3) particles & allow all orderings of colorless particles
- draw the parent diagram so that the loop is in the fixed position compared to the external fermion line [L/R]
- N-point case: parent must be 1PI N-point, use dummy lines if needed

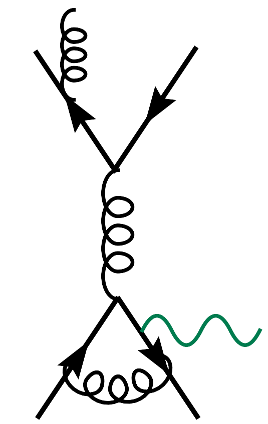
Explicitly for W+3jets:

❶ ❷ ❸ ❹ ❺

u_1 q_2 g_3 q_4 d_5 + W



Refers e.g. to:



Rules of the game

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- draw the parent diagram so that the loop is in the fixed position compared to the external fermion line [L/R]
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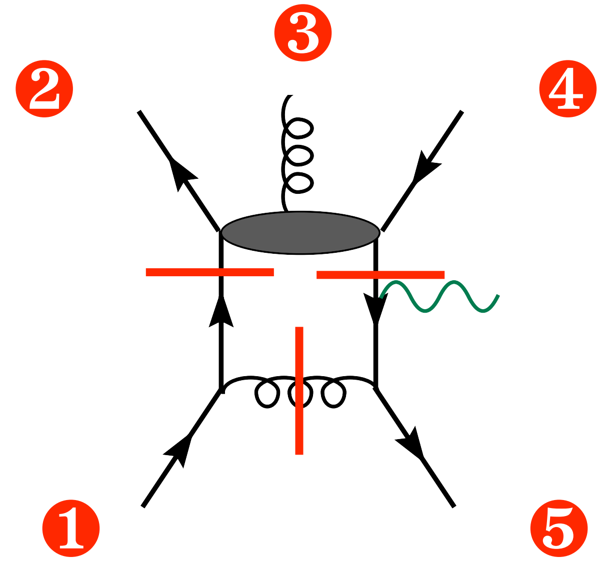
- consider all cuts and throw away those involving dummy lines

Explicitly for W+3jets:

❶ ❷ ❸ ❹ ❺

$u_1 \quad q_2 \quad g_3 \quad q_4 \quad d_5 + W$



✓ *accept*

Rules of the game

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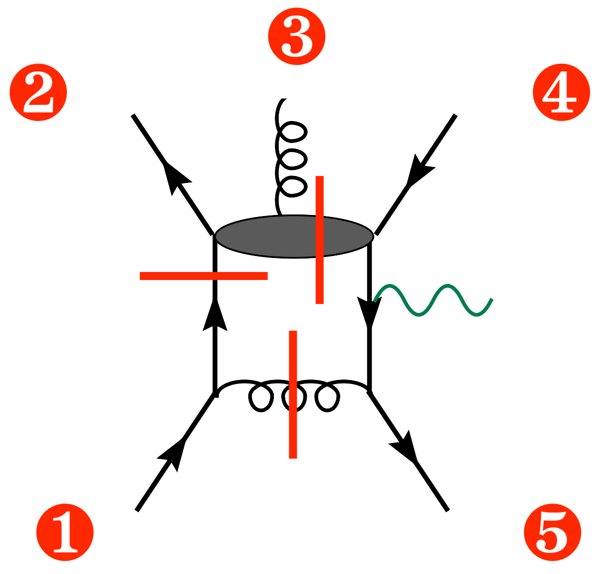
u_1 q_2 g_3 q_4 d_5 + W



✗ *reject*

Rules of the game

Procedure:

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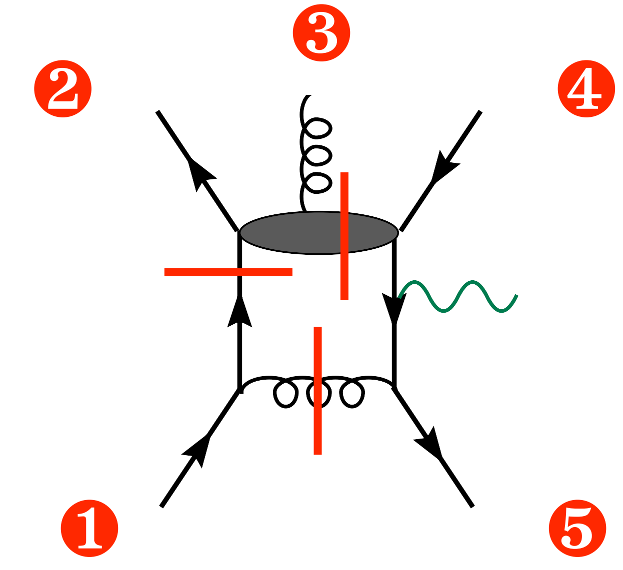
- draw the parent diagram so that the loop is in the fixed position compared to the external fermion line [L/R]
- N-point case: parent must be 1PI N-point, use dummy lines if needed
- consider all cuts and throw away those involving dummy lines
- process each cut use standard D-dimensional unitarity

Explicitly for W+3jets:

❶ ❷ ❸ ❹ ❺

u_1 q_2 g_3 q_4 $d_5 + W$



✗ *reject*

Rules of the game

Procedure:

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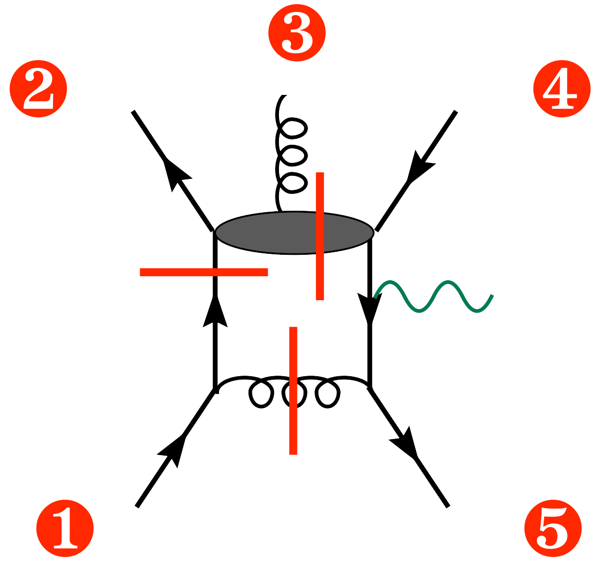
- draw the parent diagram so that the loop is in the fixed position compared to the external fermion line [L/R]
- N-point case: parent must be 1PI N-point, use dummy lines if needed
- consider all cuts and throw away those involving dummy lines
- process each cut use standard D-dimensional unitarity
- tree level amplitudes are computed via color stripped Feynman rules

Explicitly for W+3jets:

❶ ❷ ❸ ❹ ❺

u_1 q_2 g_3 q_4 d_5 + W



✗ *reject*

Sample results

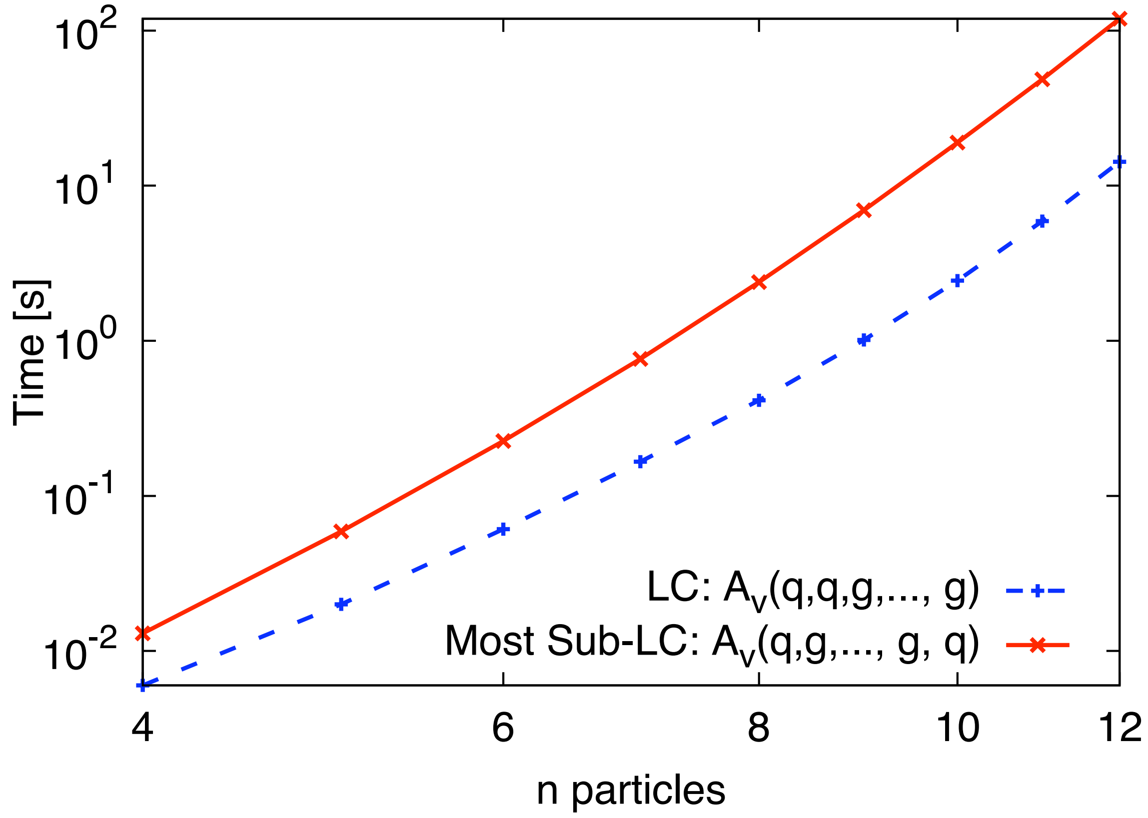
Helicity	$1/\epsilon^2$	$1/\epsilon$	ϵ^0
$A^{\rm tree}(1^+_{\bar q}\, 2^-_q\, 3^+_g\, 4^+_g\, 5^+_g\, 6^+_{\bar l}\, 7^-_l)$			$-0.006873 + i\,0.011728$
$r_L^{[1]}(1^+_{\bar q}\, 2^-_q\, 3^+_g\, 4^+_g\, 5^+_g\, 6^+_{\bar l}\, 7^-_l)$	-4.00000	$-10.439578 - i\,9.424778$	$5.993700 - i\,19.646278$
$A^{\rm tree}(1^+_{\bar q}\, 2^-_q\, 3^+_g\, 4^+_g\, 5^-_g\, 6^+_{\bar l}\, 7^-_l)$			$0.010248 - i\,0.007726$
$r_L^{[1]}(1^+_{\bar q}\, 2^-_q\, 3^+_g\, 4^+_g\, 5^-_g\, 6^+_{\bar l}\, 7^-_l)$	-4.00000	$-10.439578 - i\,9.424778$	$-14.377555 - i\,37.219716$
$A^{\rm tree}(1^+_{\bar q}\, 2^-_q\, 3^-_g\, 4^+_g\, 5^+_g\, 6^+_{\bar l}\, 7^-_l)$			$0.495774 - i\,1.274796$
$r_L^{[1]}(1^+_{\bar q}\, 2^-_q\, 3^-_g\, 4^+_g\, 5^+_g\, 6^+_{\bar l}\, 7^-_l)$	-4.00000	$-10.439578 - i\,9.424778$	$-1.039489 - i\,30.210418$
$A^{\rm tree}(1^+_{\bar q}\, 2^-_q\, 3^-_g\, 4^+_g\, 5^-_g\, 6^+_{\bar l}\, 7^-_l)$			$-0.294256 - i\,0.223277$
$r_L^{[1]}(1^+_{\bar q}\, 2^-_q\, 3^-_g\, 4^+_g\, 5^-_g\, 6^+_{\bar l}\, 7^-_l)$	-4.00000	$-10.439578 - i\,9.424778$	$-1.444709 - i\,26.101951$

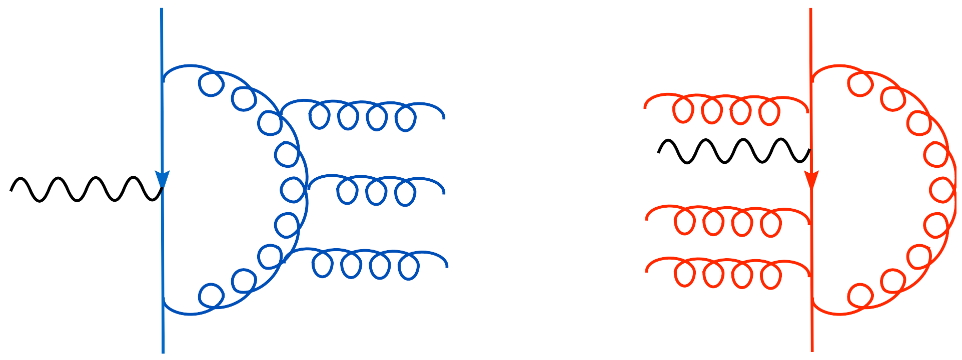
$$r_L^{[j]}(1,2,3,4,5,6,7) = \frac{1}{c_\Gamma}\frac{A_L^{[j]}(1,2,3,4,5,6,7)}{A^{\rm tree}(1,2,3,4,5,6,7)}, \quad c_\Gamma = \frac{\Gamma(1+\epsilon)\Gamma(1-\epsilon)^2}{(4\pi)^{2-\epsilon}\Gamma(1-2\epsilon)},$$

Leading color amplitudes in 0808.0941
[Berger, Bern, Cordero, Dixon, Forde, Ita, Kosower, Maitre]

All amplitudes in 0810.2542
[Ellis, Giele, Kunszt, Melnikov, GZ]

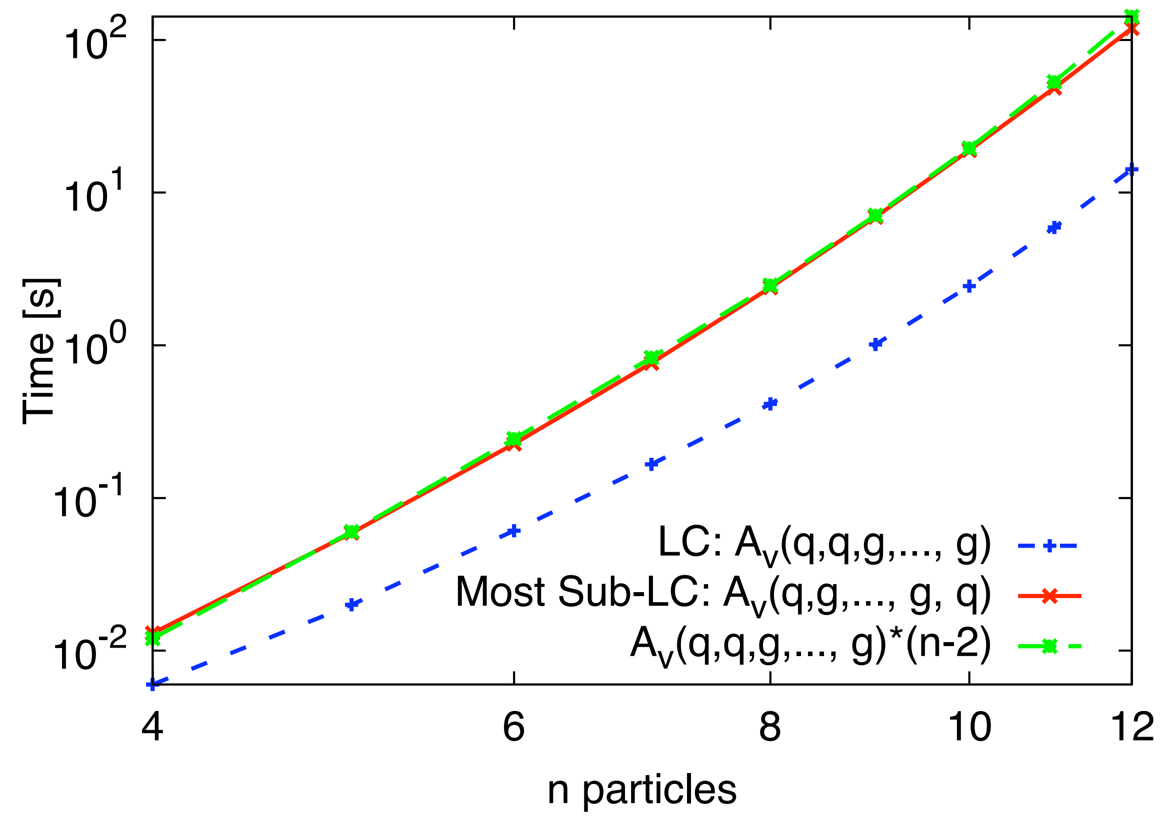
Time dependence of qq + W + n gluons

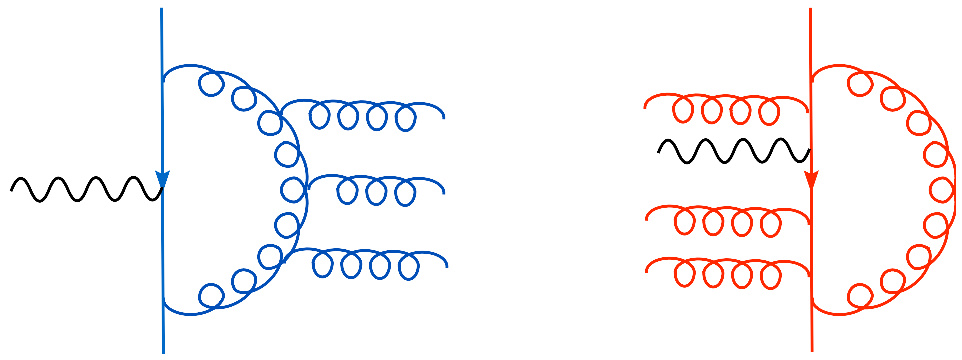




# of cuts: N_{cuts} $N_{\text{cuts}} \cdot (n-2)$

Time dependence of qq + W + n gluons

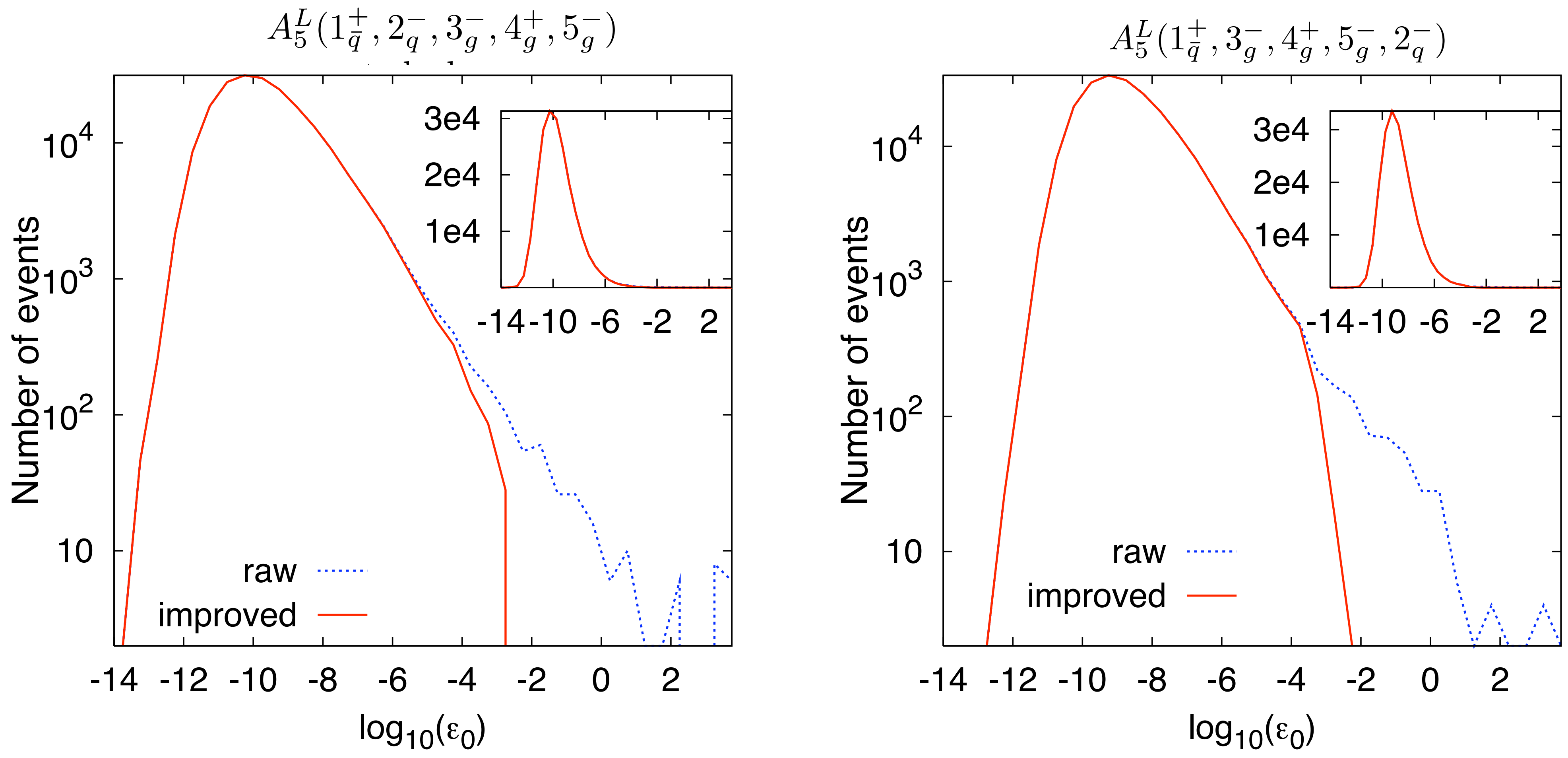




\# of cuts: N_{cuts} $N_{\text{cuts}} \cdot (n-2)$

Similar plots for qq+n-gluons

Instabilities and accuracy



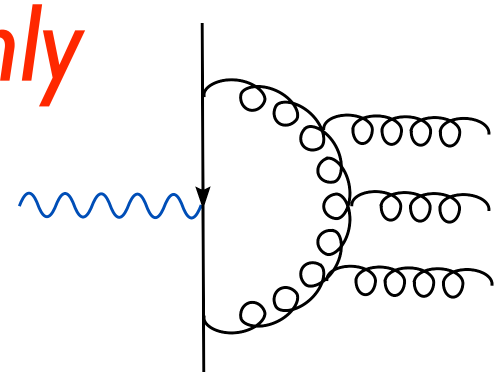
$\Rightarrow$ *All instabilities detected and cured with quadruple precision*

Approximation in first cross-section

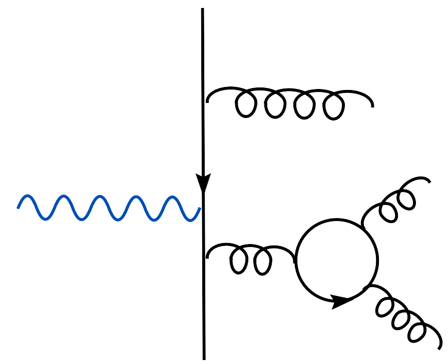
Leading color | **Fermion loops** | **(SSS)Subleading**

2-quark, 3-gluon

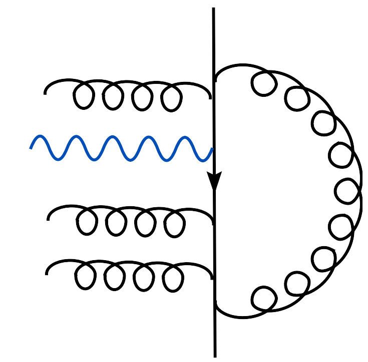
only



$$\mathrm{LC} \equiv (N_c^2 - 1)N_c^3$$

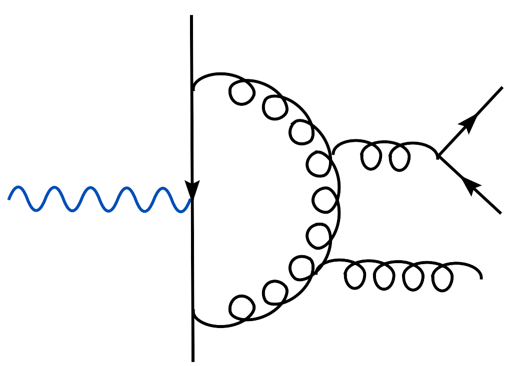


$$\mathrm{LC} \cdot \frac{n_f}{N_c}$$

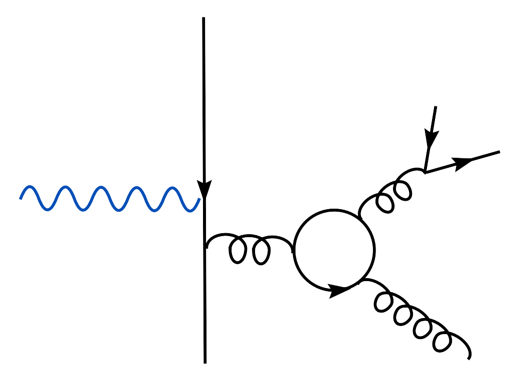


$$\mathrm{LC} \cdot \frac{1}{N_c^3}$$

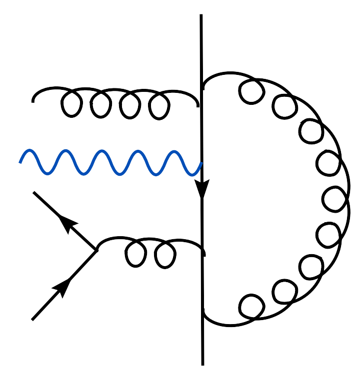
4-quark, 1-gluon



$$\mathrm{LC} \cdot \frac{n_f}{N_c}$$



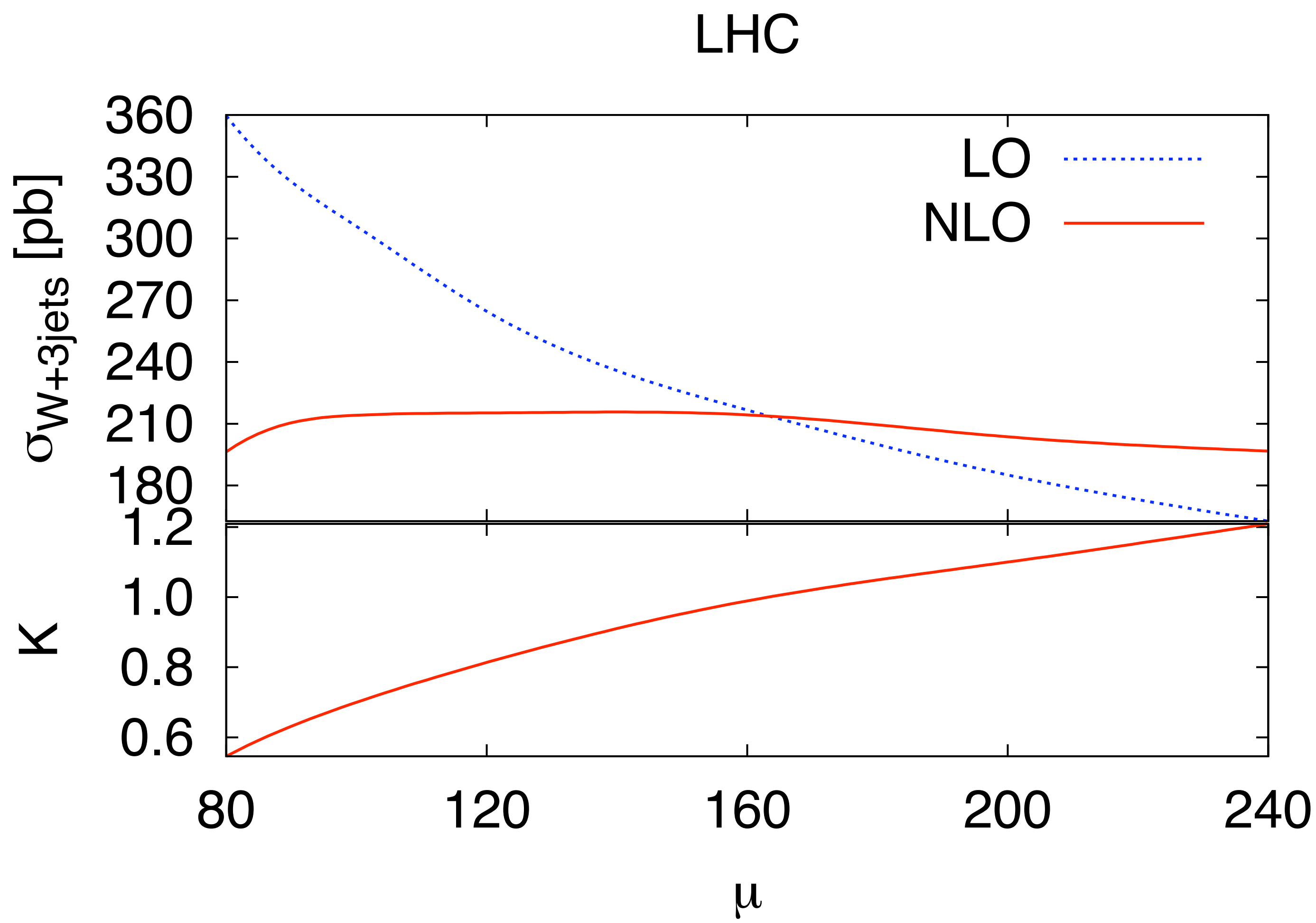
$$\mathrm{LC} \cdot \frac{n_f}{N_c^2}$$



$$\mathrm{LC} \cdot \frac{n_f}{N_c^3}$$

NB: at tree level leading color works very well and 4-quark processes small

Scale variation: W^++3 jets

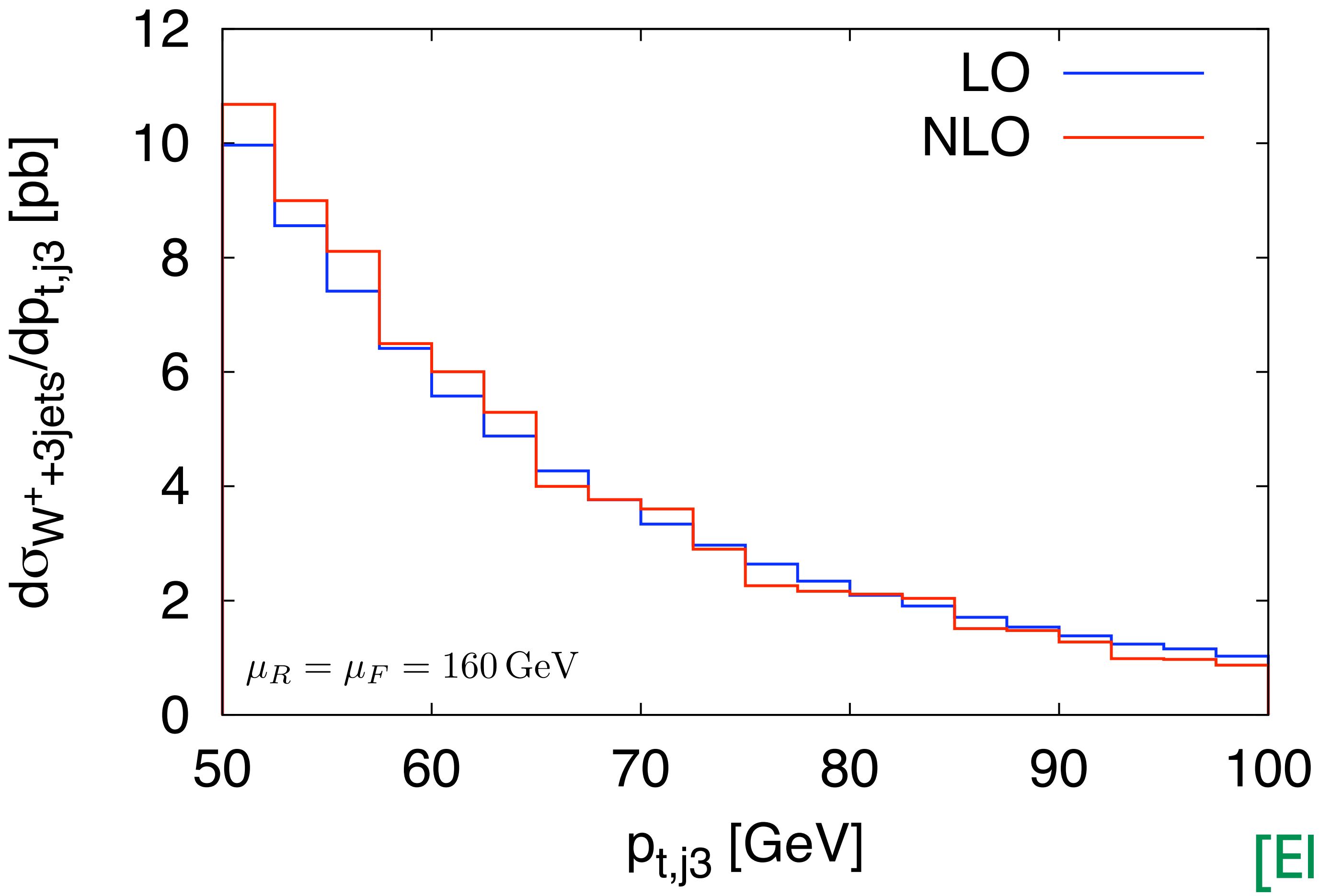


[Cuts and input defined in Ellis, Melnikov, GZ '09]

- remarkable independence of cross-section on unphysical scales at NLO
- LO=NLO at scales ~ 160 GeV
- gross features of W+3jets are similar to W+2jets, however the price one pays for an infelicitous choice of scales is higher now
- similar results at the Tevatron

$p_{T,j3}$ distribution

Transverse momentum of the 3rd hardest jets in inclusive jet sample

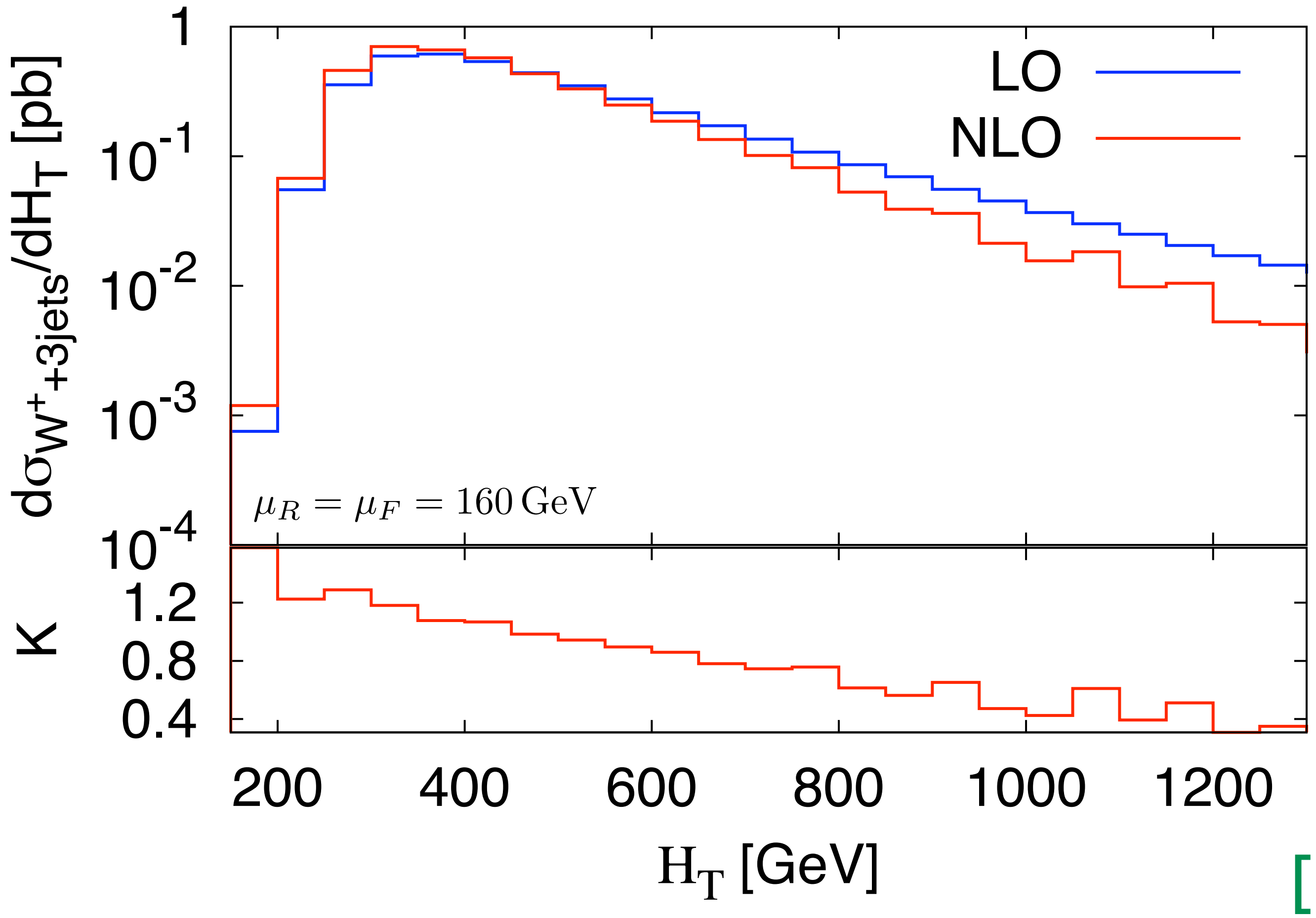


[Ellis, Melnikov, GZ '09]

➡ 3rd hard jet is softer at NLO

H_T distribution

Measure of the overall hardness of the event $H_T = \sum_j E_{\perp,j} + E_\perp^{\rm miss} + E_\perp^e$



[Ellis, Melnikov, GZ '09]

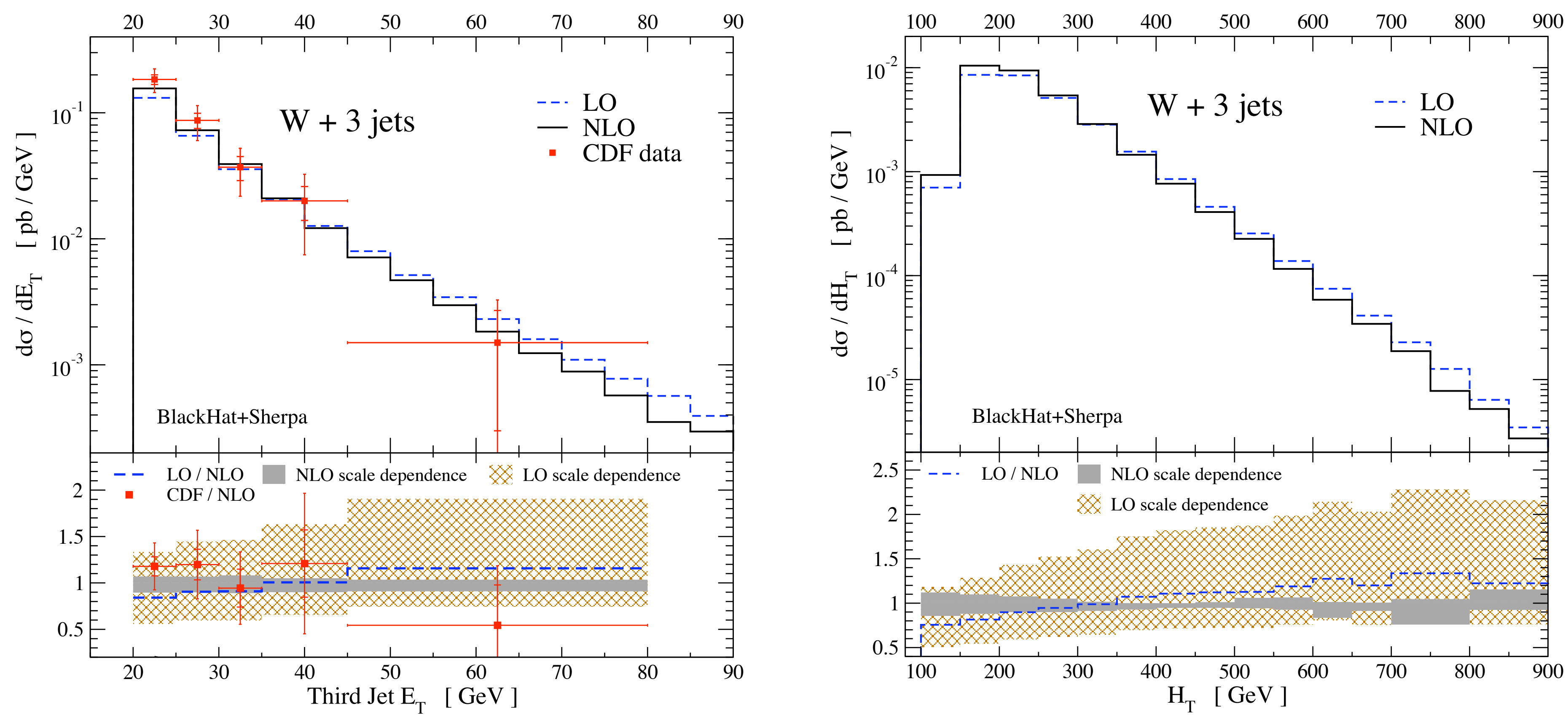
➡ K-factor H_T dependent, spectrum becomes softer at NLO

Second W+3jet calculation

More recently, similar calculation for W+3jets done in Blackhat+Sherpa

C.F. Berger, Z. Bern, L.J. Dixon, F.Febres Cordero, D. Forde, T. Gleisberg, H.Ita, D.A. Kosower, D. Maitre [0902.2760]

Still leading color approximation in virtual (not real), all subprocesses included (but no fermion loops) ⇒ see Daniel Maitre's talk in few mins



number of jets	CDF	LC NLO	NLO
1	53.5 ± 5.6	$58.3^{+4.6}_{-4.6}$	$57.8^{+4.4}_{-4.0}$
2	6.8 ± 1.1	$7.81^{+0.54}_{-0.91}$	$7.62^{+0.62}_{-0.86}$
3	0.84 ± 0.24	$0.826^{+0.049}_{-0.084}$	—

Final remarks

The beauty and robustness of generalized D-dimensional unitarity

- ☺ for tree level amplitudes: make use general Berends-Giele recursion, numerically efficient (large N), general (D, spins, masses)
- ☺ simple method, straightforward to implement/automate
- ☺ universal method (general masses, spins) and unified approach, no "special or extra" cases, no exceptions
- ☺ fast: numerical performance as expected (polynomial)
- ☺ transparent: full control on all parts (can extract specific bits)

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- ☺ transparent: full control on all parts (can extract specific bits)

Maturity reached for cross-sections calculations?
Yes, demonstrated by first explicit calculations
(but still room for further improvements)

(Instead of) Conclusions

Rocket science!

Eruca sativa =Rocket=roquette=arugula=rucola

Recursive unitarity calculation of one-loop amplitudes

On a more general side, the current version of Rocket computes one-loop amplitudes for processes $0 \to\ n$ gluons, $0 \to \bar{q}q + n$ gluons, $0 \to \bar{q}qW + n$ gluons and $0 \to \bar{q}q\bar{Q}QW + 1$ gluon. It is straightforward to extend the program to include similar processes with the Z boson and processes with massive quarks $0 \to \bar{t}t + n$ gluons. This list is a testimony to the power of the method and indicates that the development of automated programs for one-loop calculations may finally be within reach.

[Ellis, Giele, Kunszt, Melnikov, GZ 0810.2542]