Generalized D-dimensional unitarity

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Based on collaboration with R. K. Ellis, W. Giele, K. Melnikov and G. Zanderighi

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Introduction

- Fully automated cross section calculations for LHC in SM and BSM

  at the tree-level the problem is solved
  ALPGEN, MADGRAPH,COMIX,... $d\sigma_n^{(0)} \approx |M_n^{(0)}|^2 d\Phi_{n-2}$, n=3...12,...

- At LHC tree level is not enough, NLO precision is mandatory

$$d\sigma_n^{(1)} \approx |M_n^{(0)}|^2 d\Phi_{n-2} + 2\mathrm{Re}(M_n^{(0)\dagger} M_n^{(1)}) d\Phi_{n-2} + |M_{n+1}^{(0)}|^2 d\Phi_{n-1}$$

- This talk: theoretical developments on generalized unitarity
  - efficient automatic calculation of $M_n^{(1)}$, n=4,...6,...20,...

- Important new results for $d\sigma_n^{(1)}$, talks by G. Zanderighi, D. Maitre

The Unitarity Method: successful P-algorithm at NLO

Bern, Dixon, Kosower (1994-…)

gauge theory one-loop amplitudes from tree amplitudes
review on analytic results (BDK, 2007)

i) BDK theorem: SUSY gauge theories have no rational parts
applications to N=1,N=4 SYM also multi-loops
ii) Impressive QCD results: e.g. $e^+ + e^-$ annihilation to four jets in NLO (1998)

series of nifty tricks: analytic results, only four-dimensional states on cut lines, spinor helicity formalism, rational part is obtained from soft and collinear limits, triple cuts, SUSY identities etc.

DIFFICULTIES in QCD applications

i) Reduction of cut tensor integrals (Passarino-Veltman,Neerven-Vermaseren)
ii) The cut lines are treated in four dimensions (no rational parts)
iii) Only double cuts have been applied. Usefulness of triple cut.

Constraints from Unitarity: $M^\dagger - M = -iM^\dagger M$

$$A_n^{\text{one loop}} = \sum_j c_{n,j} \mathcal{I}_j \qquad\qquad \text{Im} A_n^{\text{one loop}} = \sum_j c_{n,j} \text{Im}\mathcal{I}_j$$

$$A_n = A_{\text{cut}-\text{constructible}} + \mathcal{R}$$

* factorization structure on the cuts

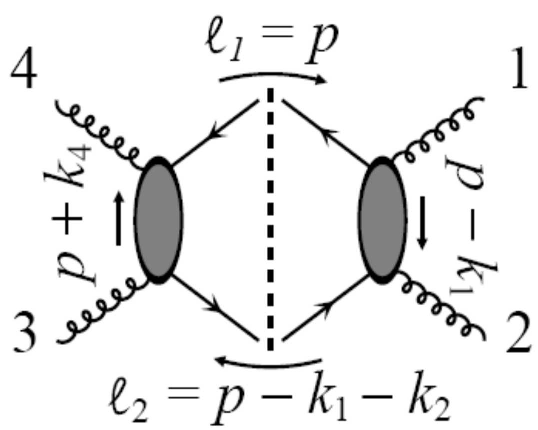
* discontinuity given by tree amplitudes

* how to get the real part? program in the 1960's faltered

* how to get the coefficients c_j efficiently?

Constraints from Unitarity: $M^\dagger - M = -iM^\dagger M$

Imaginary part from tree amplitudes, iterative in coupling



$$A_n = A_{\text{cut-constructible}} + \mathcal{R}$$

$$\text{-}i\,\text{Disc}\,A_4(1,2,3,4)\Big|_{s-\text{cut}} = \int \frac{d^4p}{(2\pi)^4}\,2\pi\delta^{(+)}(\ell_1^2 - m^2)\,2\pi\delta^{(+)}(\ell_2^2 - m^2) \times A_4^{\text{tree}}(-\ell_1, 1, 2, \ell_2)\,A_4^{\text{tree}}(-\ell_2, 3, 4, \ell_1)\,,$$

polynomial terms extracted by calculating cuts in $D = 4 - 2\epsilon$ dimensions

van Neerven (1986), Bern, Morgan (1995), Bern, Dixon, Kosower (1996), Dixon Tasi95

It appeared to be a hard task to get the full ϵ dependence of

Towards systematic treatment

INSPIRATION FROM TWISTOR FORMULATION

(Witten, 2003, Santa Barbara Workshop 2004)

i) Generalized unitarity , complex four momenta (Britto, Cachazo, Feng)

ii) New tree level recursion relations (Britto,Cachazo, Feng, Witten)

iii) New loop level recursion relations for rational parts (Bern, Dixon, Koswer, ...)

iv) Reduction with spinor integration in D=4 (Britto, Cachazo, Feng, Mastrolia).

v) Reduction with spinor integration (scalars in the loop) in D-dimension

(Anastasiou, Britto, Feng, Kunszt, Mastrolia)

2006: formalism is ready for numerical implementations !

Decomposing one-loop N-point amplitudes in terms of master integrals

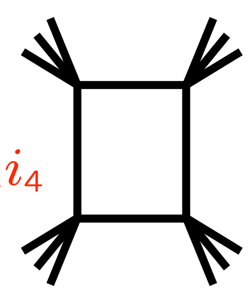
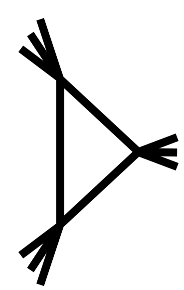
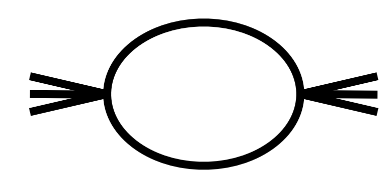
$$\mathcal{A}_N(p_1,p_2,\ldots,p_N) = \sum_{1\le i_1<i_2<i_3<i_4\le N} d_{i_1i_2i_3i_4}(p_1,p_2,\ldots,p_N)I_{i_1i_2i_3i_4}$$

$$+ \sum_{1\le i_1<i_2<i_3\le N} c_{i_1i_2i_3}(p_1,p_2,\ldots,p_N)I_{i_1i_2i_3}$$

$$+ \sum_{1\le i_1<i_2\le N} b_{i_1i_2}(p_1,p_2,\ldots,p_N)I_{i_1i_2}$$

$$+ \sum_{1\le i_1\le N} a_{i_1}(p_1,p_2,\ldots,p_N)I_{i_1}$$

$$I_{i_1\cdots i_M} = \int [d\,l]\,\frac{1}{d_{i_1}\cdots d_{i_M}}$$

$$\mathcal{A}_N(\{p_i\}) = \sum d_{i_1i_2i_3i_4}$$

$$+\sum c_{i_1i_2i_3}$$

$$+\sum b_{i_1i_2}$$


FDHS scheme: coefficients d and c are independent from ε

Rational part is generated by the order ε part of b_{ij}

New powerful alternative approaches

NEW REDUCTION METHODS :

Aquila, Ossola, Papadopoulos, Pittau ,2006,HP2

an alternative to Passarino-Veltman (1979) reduction

- systematic algebraic reduction at the integrand level
- integrand is decomposed by partial fractioning
  into linear combination of terms with 4-,3-,2,-1 denominator factors
- numerical implementation is based on Feynman diagrams

New powerful alternative approaches

ALTERNATIVE IMPLEMENTATION :

OPP reduction allows efficient numerical implementation of calculating loop amplitudes from tree amplitudes : cut-constructible part of the 6-gluon power low increase of the computer time (Ellis Giele, Kunszt, 2007).

UNIFIED METHOD FOR CALCULATING THE CUT-CONSTRUCTIBLE AND RATIONAL PARTS:

To get the rational parts we need to use tree amplitudes in D=6,8 integer dimensions with four and five dimensional complex cut loop momenta

(Giele, Kunszt, Melnikov 2008)

Four independent numerical implementations

1) Rocket (Fortran 90) (Ellis, Giele, Melnikov, Zanderighi, Kunszt)
many gluons, t+tbar+3gluon, q+qbar+(W)+n-gluon, q+qbar+Q+Qbar+W+1gluon

2) Black Hat (C++) (Berger,Bern,Dixon, Febres Cordero, Forde, Gleisberg,Ita, Kosower, Maitre)
many gluons, q+qbar+W+3gluon (leading color), q+qbar+Q+Qbar+W+1gluon (leading color)

3) Lazopoulos (C++) many gluons

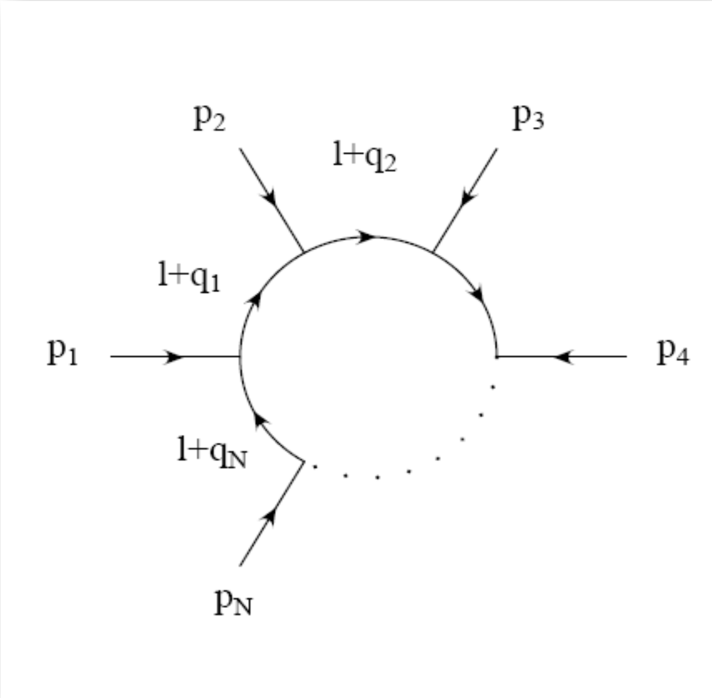
4) Giele, Winter (C++) many gluons

All the four codes implemented OPP and D-dimensional unitarity for the rational part

5) van Hameren, Papadopoulos, Pittau (C++) Feynman diagram based

OPP method to determine the coefficient of scalar integrals in D=4 dimension in terms of tree amplitudes

The unintegrated one-loop amplitude is linear combination of quadro-, triple-,double-,single-pole and polynomial terms



partial decomposition for the integrand

$$\mathcal{A}_N(p_1,p_2,\ldots,p_N;l) = \frac{\mathcal{N}(p_1,p_2,\ldots,p_N;l)}{d_1 d_2 \cdots d_N} =$$

$$\sum_{1\le i_1<i_2<i_3<i_4\le N} \frac{\bar{d}_{i_1 i_2 i_3 i_4}(l)}{d_{i_1}d_{i_2}d_{i_3}d_{i_4}} + \sum_{1\le i_1<i_2<i_3\le N} \frac{\bar{c}_{i_1 i_2 i_3}(l)}{d_{i_1}d_{i_2}d_{i_3}} + \sum_{1\le i_1<i_2\le N} \frac{\bar{b}_{i_1 i_2}(l)}{d_{i_1}d_{i_2}} + \sum_{1\le i_1\le N} \frac{\bar{a}_{i_1}(l)}{d_{i_1}}$$

Parameterization of the loop momentum

The loop momenta is decomposed in terms of VN basis vectors

we define: **a set of dual momenta** v_i **,** $v_i p_j = \delta_{ij}$
and : **a set of orthogonal unit vectors** $n_i,\quad n_i p_j = 0$

$$l^\mu = V_R^\mu + \sum_{i=1}^{D_P} \frac{1}{2}(d_i - d_{i-1})\, v_i^\mu + \sum_{i=1}^{D_T} \alpha_i\, n_i^\mu \;, \qquad V_R^\mu = -\frac{1}{2}\sum_{i=1}^{D_P}\Big((q_i^2 - m_i^2) - (q_{i-1}^2 - m_{i-1}^2)\Big)\, v_i^\mu$$

$$v_i^\mu(k_1,\ldots,k_{D_P}) \equiv \frac{\delta^{k_1\ldots k_{i-1}\mu k_{i+1}\ldots k_{D_P}}_{k_1\ldots k_{i-1}k_i k_{i+1}\ldots k_{D_P}}}{\underbrace{\Delta(k_1,\ldots,k_{D_P})}_{\text{Gram-determinant}}} \;, \qquad \delta^{\mu_1\mu_2\cdots\mu_R}_{\nu_1\nu_2\cdots\nu_R} = \begin{vmatrix} \delta^{\mu_1}_{\nu_1} & \delta^{\mu_1}_{\nu_2} & \ldots & \delta^{\mu_1}_{\nu_R} \\ \delta^{\mu_2}_{\nu_1} & \delta^{\mu_2}_{\nu_2} & \ldots & \delta^{\mu_2}_{\nu_R} \\ \vdots & \vdots & & \vdots \\ \delta^{\mu_R}_{\nu_1} & \delta^{\mu_R}_{\nu_2} & \ldots & \delta^{\mu_R}_{\nu_R} \end{vmatrix} \;,$$

Van Neerven-Vermaseren: reduction at the integrand level

Parametrization of the numerators

$$\mathcal{A}_N(l) = \sum_{1\le i_1<i_2<i_3<i_4\le N} \frac{\bar{d}_{i_1 i_2 i_3 i_4}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} + \sum_{1\le i_1<i_2<i_3\le N} \frac{\bar{c}_{i_1 i_2 i_3}(l)}{d_{i_1} d_{i_2} d_{i_3}} + \sum_{1\le i_1<i_2\le N} \frac{\bar{b}_{i_1 i_2}(l)}{d_{i_1} d_{i_2}} + \sum_{1\le i_1\le N} \frac{\bar{a}_{i_1}(l)}{d_{i_1}}$$

parametric integral over the loop momentum

18 structures but only 3 non-vanishing integrals

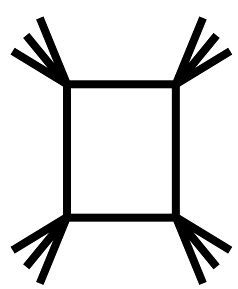
$$\bar{d}_{ijkl}(l) \equiv \bar{d}_{ijkl}(n_1 \cdot l) = d_{ijkl} + \tilde{d}_{ijkl}\, s_1 \;,\; s_i = n_i \cdot l$$

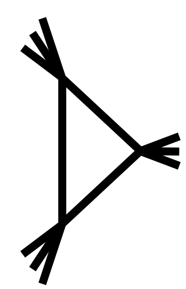
$$\bar{c}_{ijk}(l) = c_{ijk}^{(0)} + c_{ijk}^{(1)} s_1 + c_{ijk}^{(2)} s_2 + c_{ijk}^{(3)}(s_1^2 - s_2^2) + s_1 s_2 (c_{ijk}^{(4)} + c_{ijk}^{(5)} s_1 + c_{ijk}^{(6)} s_2)$$

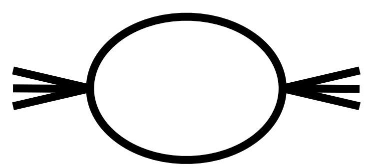
$$\bar{b}_{ij}(l) = b_{ij}^{(0)} + b_{ij}^{(1)} s_1 + b_{ij}^{(2)} s_2 + b_{ij}^{(3)} s_3 + b_{ij}^{(4)}(s_1^2 - s_3^2) + b_{ij}^{(5)}(s_2^2 - s_3^2) + b_{ij}^{(6)} s_1 s_2 + b_{ij}^{(7)} s_1 s_3 + b_{ij}^{(8)} s_2 s_3$$

$$\int [d\,l]\,\frac{\bar{d}_{ijkl}(l)}{d_i d_j d_k d_l} = \int [d\,l]\,\frac{d_{ijkl} + \tilde{d}_{ijkl}\,n_1 \cdot l}{d_i d_j d_k d_l} = d_{ijkl} \int [d\,l]\,\frac{1}{d_i d_j d_k d_l} = d_{ijkl} I_{ijkl}\ ,$$

Scalar integrals: QCD package (Ellis, Zanderighi)







loop momenta on the cut $d_j = 0$

1. Quadrupole cut $d_i=d_j=d_k=d_l=0$ (two solutions)

$$l^\mu = V_4^\mu + \alpha_1\, n_1^\mu$$

Complex valued loop momenta

$$l_\pm^\mu = V_4^\mu \pm i\,\sqrt{V_4^2 - m_l^2} \times n_1^\mu$$

2. Triple cut, infinite number of solutions (on a circle circle)

$$l^\mu = V_3^\mu + \alpha_1 n_1^\mu + \alpha_2 n_2^\mu$$

$$l_{\alpha_1\alpha_2}^\mu = V_3^\mu + \alpha_1\, n_1^\mu + \alpha_2\, n_2^\mu;\;\; \alpha_1^2 + \alpha_2^2 = -(V_3^2 - m_k^2)$$

3. Double cut, infinite number of solutions (on a “sphere”)

$$l^\mu = V_2^\mu + \alpha_1 n_1^\mu + \alpha_2 n_2^\mu + \alpha_3 n_3^\mu$$

$$l_{\alpha_1\alpha_2\alpha_3}^\mu = V_2^\mu + \alpha_1\, n_1^\mu + \alpha_2\, n_2^\mu + \alpha_3\, n_3^\mu;\;\; \alpha_1^2 + \alpha_2^2 + \alpha_3^2 = -(V_2^2 - m_j^2)\ .$$

The parameters are fixed by linear algebraic equations

generalized unitarity: the residues are taken with (complex) "cut loop momenta"

$$\mathrm{Res}_{ij\cdots k}\left[F(l)\right] \equiv \left[d_i(l) d_j(l) \cdots d_k(l) F(l)\right]\Big|_{l=l_{ij\cdots k}} .$$

$$\bar{d}_{ijkl}(l) = \mathrm{Res}_{ijkl}\left(\mathcal{A}_N(l)\right)$$ $d_i=d_j=d_k=d_l=0$ two solutions

$$\bar{c}_{ijk}(l) = \mathrm{Res}_{ijk}\left(\mathcal{A}_N(l) - \sum_{l\neq i,j,k} \frac{\bar{d}_{ijkl}(l)}{d_i d_j d_k d_l}\right)$$ $d_i=d_j=d_k=0$ infinite # of solutions

$$\bar{b}_{ij}(l) = \mathrm{Res}_{ij}\left(\mathcal{A}_N(l) - \sum_{k\neq i,j} \frac{\bar{c}_{ijk}(l)}{d_i d_j d_k} - \frac{1}{2!}\sum_{k,l\neq i,j} \frac{\bar{d}_{ijkl}(l)}{d_i d_j d_k d_l}\right)$$ $d_i=d_j=0$ infinite # of solutions

unitarity: the residues factorize into the products of tree amplitudes

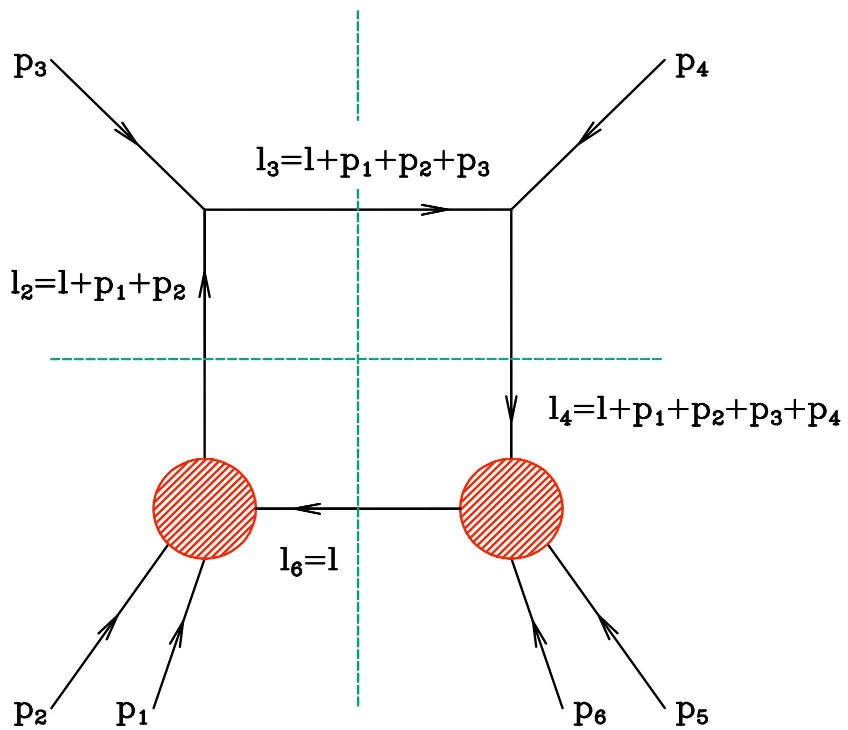
we fully reconstruct the integrand in terms of product of tree amplitudes

in combination with the s_j factors and denominator factors, no Feynman diagrams

The box residue

$$\text{Res}_{2346}\Big(\mathcal{A}_6(l^\pm)\Big) = \mathcal{M}_4^{(0)}(l_6^\pm; p_1, p_2; -l_2^\pm) \times \mathcal{M}_3^{(0)}(l_2^\pm; p_3; -l_3^\pm)\mathcal{M}_3^{(0)}(l_3^\pm; p_4; -l_4^\pm)$$

$$\times\mathcal{M}_4^{(0)}(l_4^\pm; p_5, p_6; -l_6^\pm) \quad = \quad \bar{d}_{ijkl}(l) = d_{ijkl} + \tilde{d}_{ijkl}\, l \cdot n_1$$



$$d_{ijkl} = \frac{\text{Res}_{ijkl}\Big(\mathcal{A}_N(l^+)\Big) + \text{Res}_{ijkl}\Big(\mathcal{A}_N(l^-)\Big)}{2}$$

$$\tilde{d}_{ijkl} = \frac{\text{Res}_{ijkl}\Big(\mathcal{A}_N(l^+)\Big) - \text{Res}_{ijkl}\Big(\mathcal{A}_N(l^-)\Big)}{2i\sqrt{V_4^2 - m_l^2}}$$

Unitarity in D-dimension: uniform treatment of the cut constructible and rational parts (GKM)

Two sources of D-dependence **(Bern, Dixon, Kosower, Wang,01.01.00)**

i) spin-polarization states live in D_s.

ii) loop momentum component live in D. (D_s>D)

$$\mathcal{A}_{(D,D_s)}(\{p_i\},\{J_i\}) = \int \frac{d^D l}{i(\pi)^{D/2}} \frac{\mathcal{N}^{(D_s)}(\{p_i\},\{J_i\};l)}{d_1 d_2 \cdots d_N}.$$

We can calculate the D_s dependence before carrying out the integral over the loop momentum

$$\sum_{i=1}^{D_s-2} e_\mu^{(i)}(l) e_\nu^{(i)}(l) = -g_{\mu\nu}^{(D_s)} + \frac{l_\mu b_\nu + b_\mu l_\nu}{l \cdot b},$$

$$l^2 = \bar{l}^2 - \tilde{l}^2 = l_1^2 - l_2^2 - l_3^2 - l_4^2 - \sum_{i=5}^{D} l_i^2$$

Two key features

Dependence on D_s is linear

$$\mathcal{N}^{(D_s)}(l) = \mathcal{N}_0(l) + (D_s - 4)\mathcal{N}_1(l)$$

full D_s dependence

- Choose two integer values $D_s = D_1$ and $D_s = D_2$ to reconstruct the full D_s dependence.
- Suitable for numerical implementation
- $D_s = 4-2\varepsilon$ 't Hooft Veltman scheme, $D_s = 4$ FDHS (Bern, Koswer)
- for closed fermion loops $\mathcal{N}^{D_s}(l) = 2^{(D_s-4)/2}\mathcal{N}_0(l)$

The loop momentum effectively has only 4+1 component

$$\mathcal{N}(l) = \mathcal{N}(\tilde{l}, \mu), \qquad l^2 = \tilde{l}^2 - \mu^2$$

maximum 5 unitarity constraints: pentagon cuts

Loop integrals are in $D < D_s$ **dimensions** $D = 4 - 2\epsilon$

OPP reduction is well defined for any integer D_s and D dimensions

- **We need to carry out the analytic continuation to $D = 4 - 2\epsilon$ only at the evaluation of the scalar integral functions.**

- **In D dimensions the loop momenta allow for**

  i) penta poles,
  ii) new structures in the numerators
  iii) four new non-vanishing integrals

$$
\frac{\mathcal{N}^{(D_s)}(l)}{d_1 d_2 \cdots d_N} = \sum_{[i_1|i_5]} \frac{\overline{e}^{(D_s)}_{i_1 i_2 i_3 i_4 i_5}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4} d_{i_5}} + \sum_{[i_1|i_4]} \frac{\overline{d}^{(D_s)}_{i_1 i_2 i_3 i_4}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}}
$$

$$
+ \sum_{[i_1|i_3]} \frac{\overline{c}^{(D_s)}_{i_1 i_2 i_3}(l)}{d_{i_1} d_{i_2} d_{i_3}} + \sum_{[i_1|i_2]} \frac{\overline{b}^{(D_s)}_{i_1 i_2}(l)}{d_{i_1} d_{i_2}} + \sum_{[i_1|i_1]} \frac{\overline{a}^{(D_s)}_{i_1}(l)}{d_{i_1}} .
$$

New structures and new integrals

$$\bar{e}^{(D_s)}_{ijkmn}(l) = e^{(D_s,(0))}_{ijkmn}$$

no new scalar integrals

$$\bar{d}^{\mathrm{FDH}}_{ijkn}(l) = d^{(0)}_{ijkn} + d^{(1)}_{ijkn} s_1 + (d^{(2)}_{ijkn} + \cancel{d^{(3)}_{ijkn}} s_1) s_e^2 + d^{(4)}_{ijkn} s_e^4,$$

two new scalar integrals

$$\bar{c}^{\mathrm{FDH}}_{ijk}(l) = \ldots + c^{(7)}_{ijk} \cancel{s_1} s_e^2 + c^{(8)}_{ijk} \cancel{s_2} s_e^2 + c^{(9)}_{ijk} s_e^2,$$

one new scalar integrals

$$\bar{b}^{\mathrm{FDH}}_{ij}(l) = \ldots + b^{(9)}_{ij} s_e^2$$

one new scalar integrals

$$s_e^2 = -\sum_{i=5}^{D} (l \cdot n_i)^2 = -\sum_{i=5}^{D} (\tilde{l} \cdot n_i)^2$$

One-loop amplitudes up to terms of order ε

One loop amplitudes as sum of cut-constructible and rational parts:

$$\mathcal{A}_N = \mathcal{A}_N^{CC} + R_N .$$

The cut constructible part is as before (EGK):

$$\mathcal{A}_N^{CC} = \sum_{[i_1|i_4]} \tilde{d}^{(0)}_{i_1 i_2 i_3 i_4} \, I^{(4-2\epsilon)}_{i_1 i_2 i_3 i_4} + \sum_{[i_1|i_3]} c^{(0)}_{i_1 i_2 i_3} \, I^{(4-2\epsilon)}_{i_1 i_2 i_3} + \sum_{[i_1|i_2]} b^{(0)}_{i_1 i_2} \, I^{(4-2\epsilon)}_{i_1 i_2} + \sum_{i_1=1}^{N} a^{(0)}_{i_1} \, I^{(4-2\epsilon)}_{i_1} ,$$

The rational part is new (GKM):

$$R_N = -\sum_{[i_1|i_4]} \frac{d^{(4)}_{i_1 i_2 i_3 i_4}}{6} + \sum_{[i_1|i_3]} \frac{c^{(7)}_{i_1 i_2 i_3}}{2} - \sum_{[i_1|i_2]} \left(\frac{(q_{i_1} - q_{i_2})^2}{6} - \frac{m^2_{i_1} + m^2_{i_2}}{2} \right) b^{(9)}_{i_1 i_2} ,$$

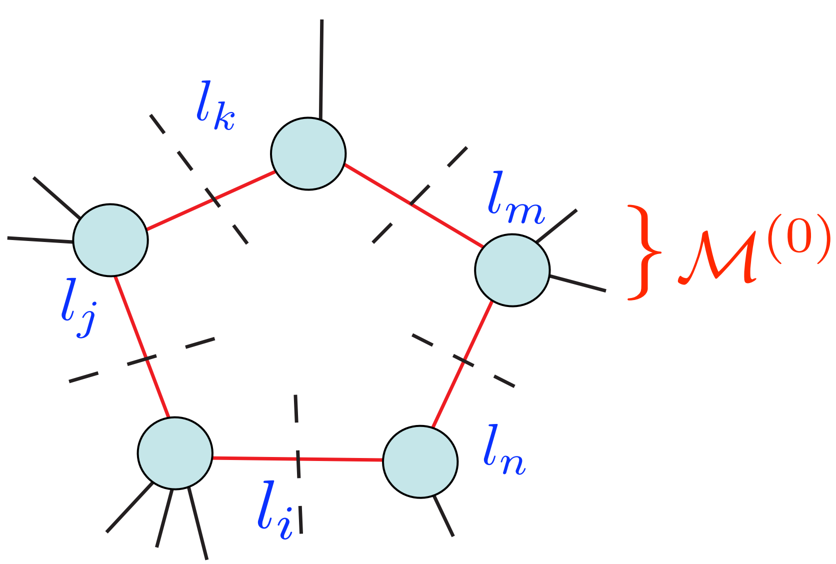
The residues are sum over the products of tree amplitudes in D=6 and 8 dimensions

$$\bar{e}^{(D_s)}_{i_1\cdots i_5}(\ell) = \mathrm{Res}_{i_1\cdots i_5}\left[\mathcal{A}_N^{(D_s)}(\ell)\right] \equiv d_{i_1}(\ell)\cdots d_{i_5}(\ell)\,\mathcal{A}_N^{(D_s)}(\ell)\Big|_{d_{i_1}(\ell)=\cdots=d_{i_5}(\ell)=0}$$

the residues are products of tree amplitudes of D_s dimensions with complex on-shell D=5 loop momenta l summed over helicities

$$\mathrm{Res}_{i_1\cdots i_M}\left[\mathcal{A}_N^{(D_s)}(\ell)\right] = \sum_{\{\lambda_1,\ldots,\lambda_M\}=1}^{D_s-2}\left\{\prod_{k=1}^{M}\mathcal{M}^{(0)}\left(\ell_{i_k}^{(\lambda_k)};p_{i_k+1},\ldots,p_{i_{k+1}};-\ell_{i_{k+1}}^{(\lambda_{k+1})}\right)\right\}$$

sum is over internal polarization states



$$\ell_{i_k} = \ell + q_{i_k} - q_{i_M}$$

Numerical evaluations of many gluon amplitudes

- **Choose D_1=5 and D_2=6:** $\mathcal{A}^{\mathrm{FDH}} = 2\mathcal{A}_{(D,D_s=5)} - \mathcal{A}_{(D,D_s=6)}$

- **colorless primitive amplitudes are calculated with colorless Berends-Giele recursion relations in D_{s1} and D_{s2} dimensions;**
- **numerical evaluation in maple (GKM,6g)**
- **and in ROCKET (W. Giele,G. Zanderighi,20g)**

Tests, CPU time (N^9), numerical stability

i) known analytic results (Bern,Dixon,Kosower,)
ii) known semi-numerical results (IBP) (Ellis, Giele, Zanderighi)
iii) soft, collinear limits
iv) results by Black Hat, Lazopoulos; Giele, Winter

D-dimensional unitary algorithm for massive fermions (EGKM)

Application to ggtt and gggtt

- We have to choose even values for D_s

$$\mathcal{A}^{\text{FDH}} = 2\mathcal{A}_{(D,D_s=6)} - \mathcal{A}_{(D,D_s=8)}$$

- Pentagon, box, triangle ,bubble and tadpole cuts
- The treatment of bubble and tadpole cuts is more subtle:
  i) light-like bubbles, tadpoles
  ii) (1,n-1) partitioning of the n-legs has to be included
  unitarity has difficulty with self-energy insertions on external legs
- Particles of different flavors: more sophisticated bookkeeping
- Color and “flavor ordered” primitive amplitudes
- More master integrals (use QCDLoop, Ellis, Zanderighi)

Dirac spinors in 6 dimensions

gamma-matrices in $D_s = 4$ $\{\gamma^0, \gamma^1, \gamma^2, \gamma^3, \gamma^5\}$

gamma-matrices in $D_s = 6$ $\Gamma^0 = \begin{pmatrix} \gamma^0 & 0 \\ 0 & \gamma^0 \end{pmatrix}, \quad \Gamma^{i=1,2,3} = \begin{pmatrix} \gamma^i & 0 \\ 0 & \gamma^i \end{pmatrix}, \quad \Gamma^4 = \begin{pmatrix} 0 & \gamma^5 \\ -\gamma^5 & 0 \end{pmatrix}, \quad \Gamma^5 = \begin{pmatrix} 0 & i\gamma^5 \\ i\gamma^5 & 0 \end{pmatrix}$

Dirac spinors in D_s dimensions

$$u^{(s)}(l,m) = \frac{(l_\mu \Gamma^\mu + m)}{\sqrt{l_0 + m}} \eta^{(s)}_{D_s}, \quad s = 1, \ldots, 2^{D_s/2-1} . \quad \text{in } D_s = 4: \quad \eta_4^{(1)} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \eta_4^{(2)} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix},$$

in $D_s = 6$, they are constructed recursively

$$\eta_6^{(1)} = \begin{pmatrix} \eta_4^{(1)} \\ \mathbf{0} \end{pmatrix}, \quad \eta_6^{(2)} = \begin{pmatrix} \eta_4^{(2)} \\ \mathbf{0} \end{pmatrix}, \quad \eta_6^{(3)} = \begin{pmatrix} \mathbf{0} \\ \eta_4^{(1)} \end{pmatrix}, \quad \eta_6^{(4)} = \begin{pmatrix} \mathbf{0} \\ \eta_4^{(2)} \end{pmatrix} .$$

conjugate spinors:

$$\bar{u}^{(s)}(l,m) = \bar{\eta}^{(s)}_{D_s} \frac{(l_\mu \Gamma^\mu + m)}{\sqrt{l_0 + m}}$$

l_μ is not conjugated

The full ϵ-dependence is trivially obtained

$$\int \frac{\mathrm{d}^D l}{(i\pi)^{D/2}} \frac{s_e^2}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} = -\frac{D-4}{2} I^{D+2}_{i_1 i_2 i_3 i_4},$$

$$\int \frac{\mathrm{d}^D l}{(i\pi)^{D/2}} \frac{s_e^4}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} = \frac{(D-2)(D-4)}{4} I^{D+4}_{i_1 i_2 i_3 i_4},$$

$$\int \frac{\mathrm{d}^D l}{(i\pi)^{D/2}} \frac{s_e^2}{d_{i_1} d_{i_2} d_{i_3}} = -\frac{(D-4)}{2} I^{D+2}_{i_1 i_2 i_3},$$

$$\int \frac{\mathrm{d}^D l}{(i\pi)^{D/2}} \frac{s_e^2}{d_{i_1} d_{i_2}} = -\frac{(D-4)}{2} I^{D+2}_{i_1 i_2}.$$

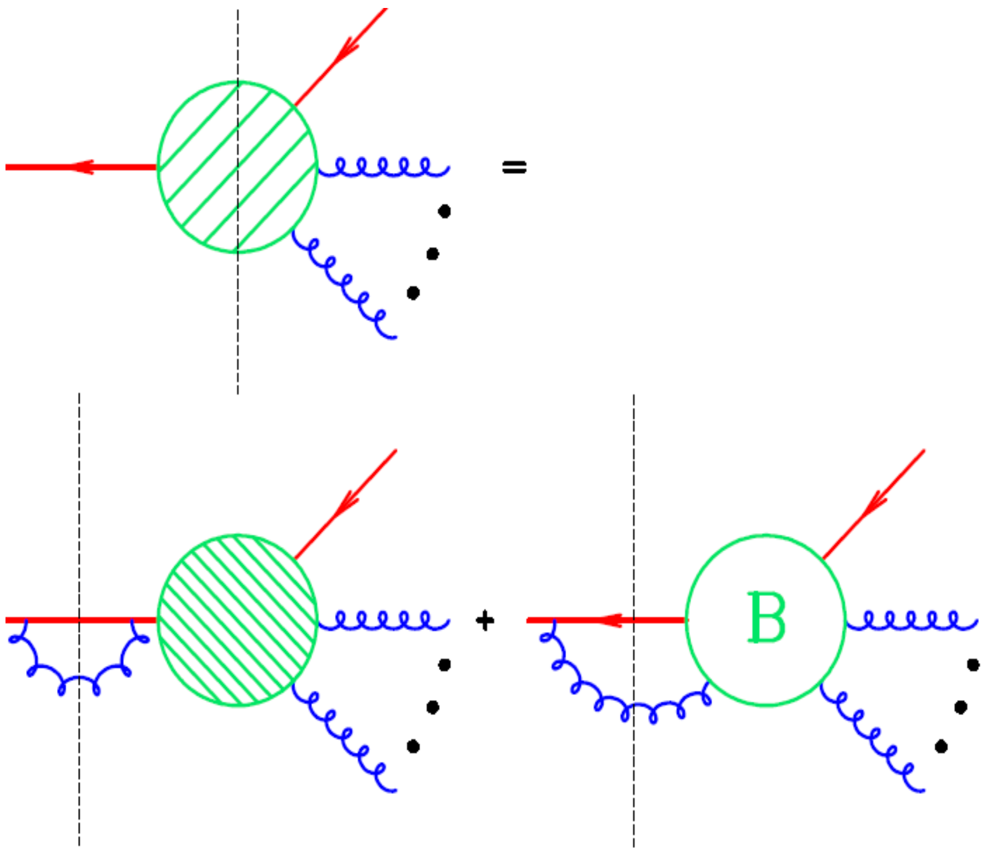
$$\lim_{D\to 4} \frac{(D-4)}{2} I^{(D+2)}_{i_1 i_2 i_3 i_4} = 0,$$

$$\lim_{D\to 4} \frac{(D-4)(D-2)}{4} I^{(D+4)}_{i_1 i_2 i_3 i_4} = -\frac{1}{3},$$

$$\lim_{D\to 4} \frac{(D-4)}{2} I^{(D+2)}_{i_1 i_2 i_3} = \frac{1}{2},$$

$$\lim_{D\to 4} \frac{(D-4)}{2} I^{(D+2)}_{i_1 i_2} = -\frac{m_{i_1}^2 + m_{i_2}^2}{2} + \frac{1}{6}\left(q_{i_1} - q_{i_2}\right)^2.$$

Self-energy on external massive fermion leg



For massless line: vanishing contributions

Tree amplitude on the right hand side is not well defined

$$\mathrm{Res}\left[\mathcal{A}^{[1]}(t, g_1, \ldots, g_n, \bar{t})\right] \sim \sum_{\mathrm{states}} \mathcal{A}^{[0]}(t, g^*, \bar{t}^*) \times \mathcal{A}^{[0]}(t^*, g^*, g_1, \ldots, g_n, \bar{t}) \ ,$$

$$\mathcal{A}^{[0]}(t^*, g^*, g_1, \ldots, g\quad, \bar{t}) = \frac{R(t^*, g^*, g_1, \ldots, g\quad, \bar{t})}{(p_{\ *} + p_{\ *})^2 - m^2} + B(t^*, g^*, g_1, \ldots, g\quad, \bar{t}).$$

Self-energy contribution, gauge invariance and generic conflict with unitarity

Feynman diagram calculation:

i) one particle reducible self-energy corrections on external legs are discarded

ii) Their effects are included by wave-function renormalization constants (Z_2)

Follow the same path:

i) discard the term in the tree amplitude generating one particle reducible diagrams

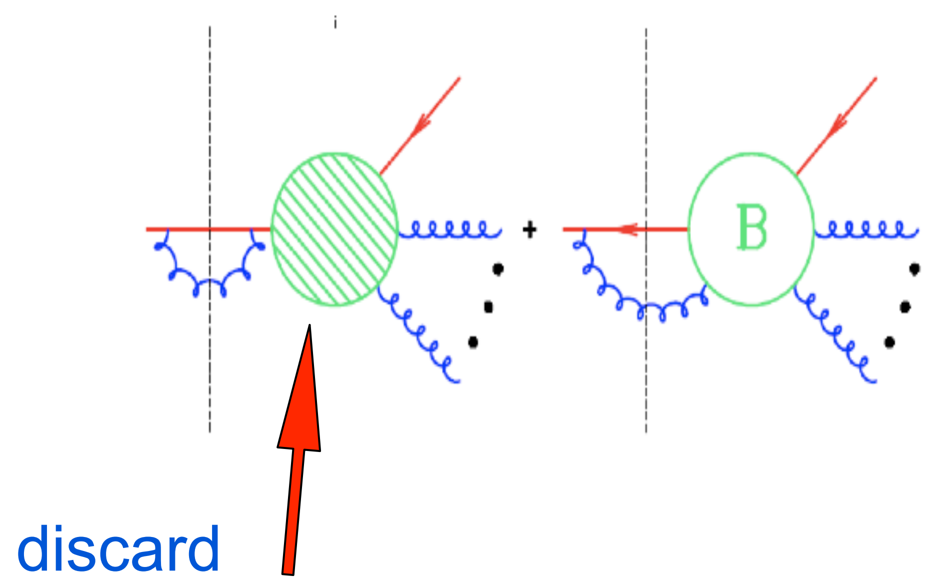
BG recursion relations can accommodate it by truncating the recursive steps

ii) It is taken into account by adding later wave function renormalization

The remaining part of the amplitude (B) is not gauge invariant

iii) **The gauges used to calculate Z_2 and B must be the same**

It mildly violates "unitarity ": sum over non-physical states



Numerical evaluation of the primitive amplitudes for ttgg and ttggg

INPUT

i) Born primitive amplitudes are calculated using BG recursion

tadpole cuts: N+1,N+2 leg tree amplitudes

ii) calculate renormalized one loop primitive amplitudes

Z_2, Z_m factors + mass counter term diagrams (restores gauge invariance)

iii) test: correct soft collinear limits, + traditional calculation

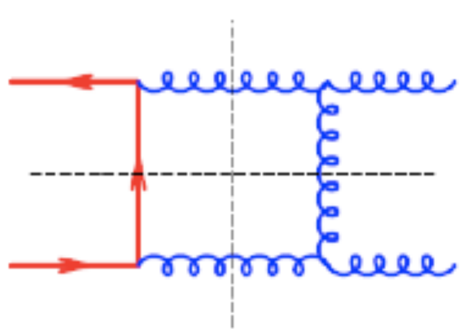
iv) Master integral input from QCDLoop

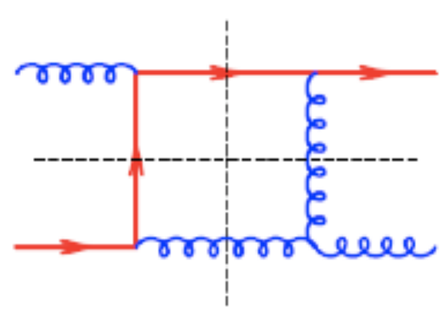
Results for the tree and loop primitive amplitudes

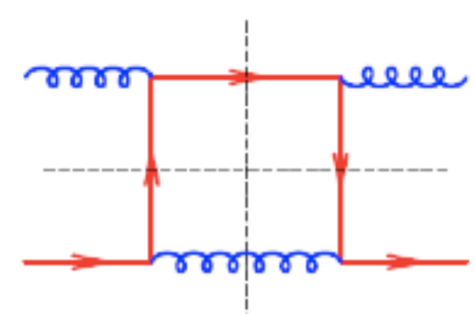
Computer time for ggtt to gggtt scales the same way as in case of only gluons

Amplitude	tree	c^{cut}	c
$+_{\bar t}, +_t, +_3, +_4, +_5$	-0.000533-0.000137 i	9.584144+6.530925 i	51.8809+6.543042 i
$+_{\bar t}, -_t, +_3, -_4, +_5$	-0.004540 + 0.018665 i	19.65913-11.77003 i	23.00306-9.699584 i
$+_{\bar t}, +_t, -_3, +_4, -_5$	-0.004726+ 0.014201 i	33.15950-1.832717 i	33.71943 -3.142751 i
$+_{\bar t}, -_t, -_3, +_4, +_5$	0.045786 + 0.010661 i	22.84043-6.540697 i	23.03114-7.313041 i
$+_{\bar t}, +_3, +_t, +_4, +_5$	0.000182 + 0.001369 i	6.517366-1.277070 i	19.37656+7.563101 i
$+_{\bar t}, +_3, -_t, -_4, +_5$	0.0467366-0.006020 i	19.440997-7.639466 i	20.93024-9.936409 i
$+_{\bar t}, -_3, +_t, +_4, -_5$	0.019275 -0.0732138 i	15.31910 -3.9278496 i	15.176306-4.102803 i
$+_{\bar t}, -_3, -_t, +_4, +_5$	-0.018203-0.111312 i	24.13158+1.431596 i	24.70002+1.018096 i
$+_{\bar t}, +_3, +_4, +_t, +_5$	0.00060-0.001377 i	13.13854+6.157043 i	10.13113+13.83997 i
$+_{\bar t}, +_3, -_4, -_t, +_5$	-0.047199-0.021516 i	23.90539 -2.168867 i	22.905695-4.284617 i
$+_{\bar t}, -_3, +_4, +_t, -_5$	-0.015110+0.063118 i	13.54258-7.800591 i	13.50273-8.018127 i
$+_{\bar t}, -_3, +_4, -_t, +_5$	-0.048800+ 0.112645 i	21.77602+ 2.078051 i	22.52784+1.424481 i
$+_{\bar t}, +_3, +_4, +_5, +_t$	-0.000252+0.000144 i	-10.35085+45.26276 i	-98.81384+52.81712 i
$+_{\bar t}, +_3, -_4, +_5, -_t$	0.0050023+0.008871 i	23.944473+2.862220 i	20.92683-0.968026 i
$+_{\bar t}, -_3, +_4, -_5, +_t$	0.000561-0.004105 i	-2.987822-42.00048 i	-3.834451-43.67103 i
$+_{\bar t}, -_3, +_4, +_5, -_t$	0.021216-0.011994 i	19.72995-2.120128 i	20.94996-1.684734 i

Fortran77 code, CPU with N^9 low







Comment on color treatment

Tree level:

BG recursion relations for colorless ordered amplitudes
different color basis (T-basis, F-basis, mixed basis, color-flow basis

recursion relations for color dressed amplitude

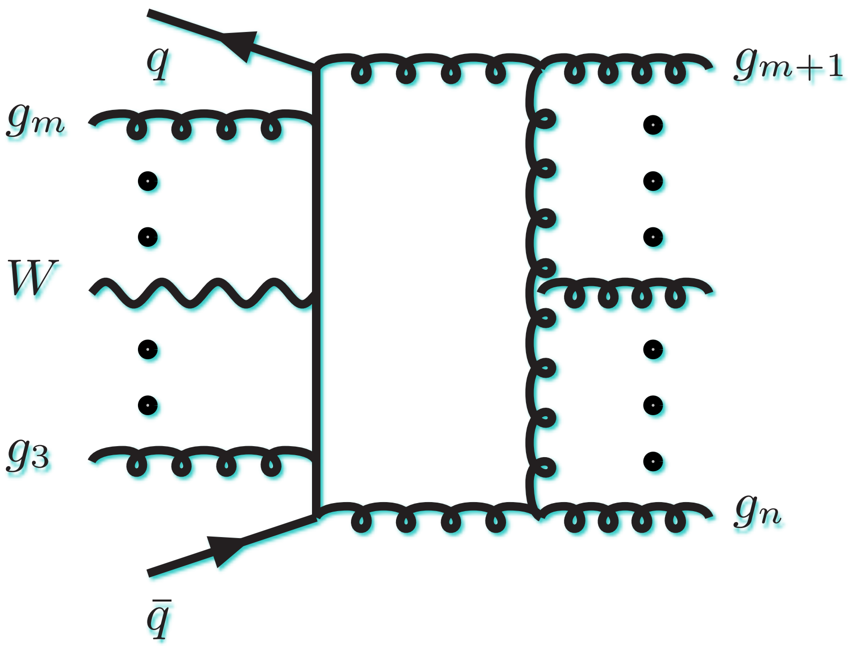
One loop:

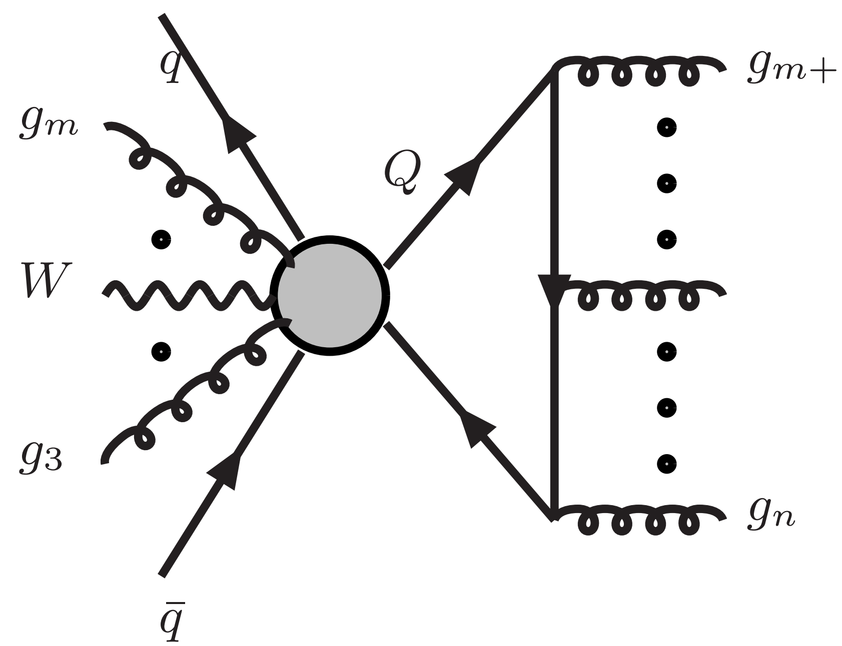
Decomposition to color and flavor ordered partial amplitudes
It becomes cumbersome for increasing number of flavor

Use of color dressed tree amplitudes in generalized unitarity?

Comment on color treatment

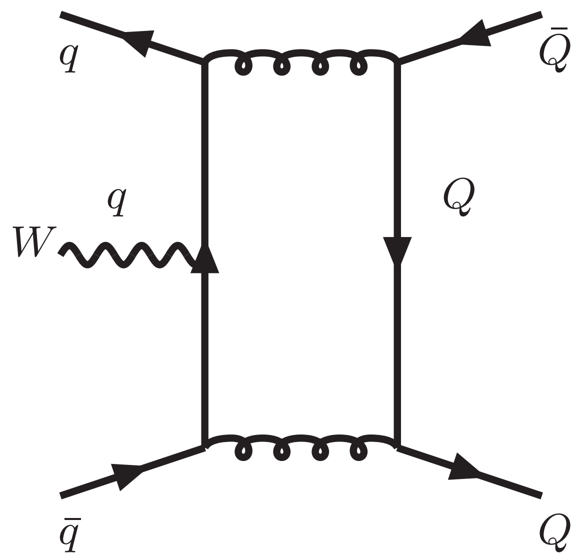
Parent diagrams for primitive amplitudes



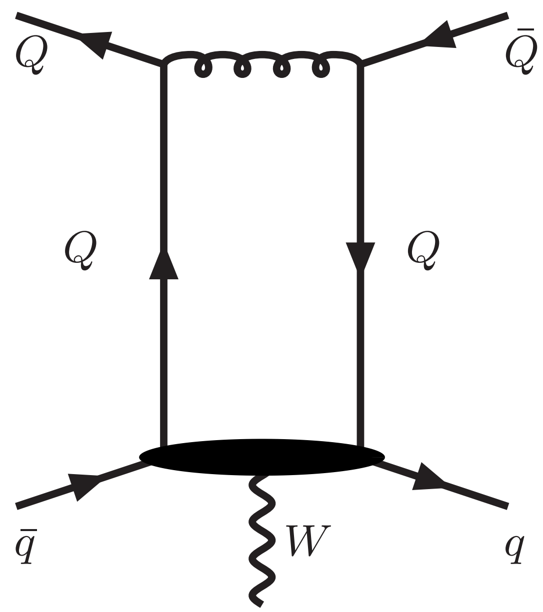


Parent diagrams for four quarks

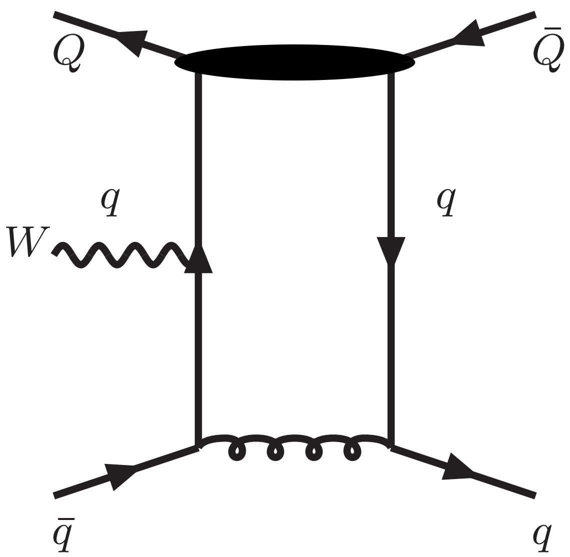
proliferation of ordered partial amplitudes



$a)$

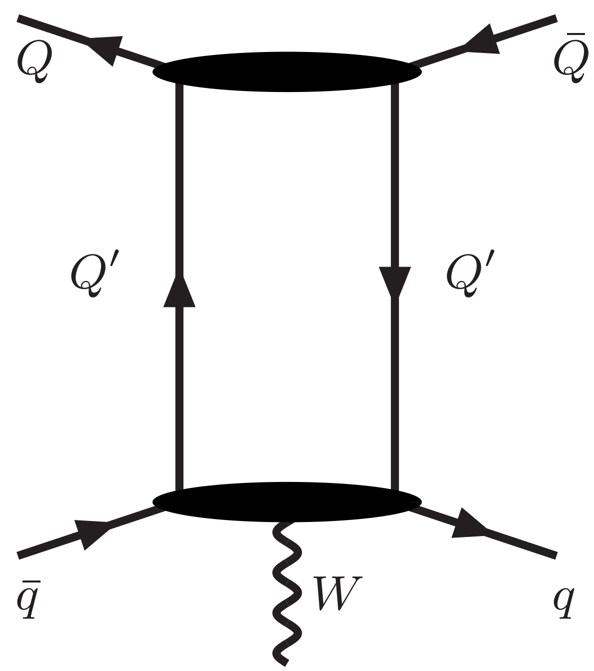


$c)$



$b)$

less then 5 propagators
in the loop



Closed fermion loop

Concluding remarks

- We know how to calculate the real part of an amplitude from the imaginary part using generalized unitarity at one-loop order for any quantum field theory.
- Cut-constructible part and rational part is treated uniformly
- A significant number of one-loop virtual amplitudes are implemented in F90 program set (Rocket, see talk of Giulia).
  First results for W+3jet cross-sections are encouraging
- The Rocket and Black Hat : can be developed to fully automated NLO generators for SM and BSM up to 7 (?) leg processes
- Generalized unitarity appears to be well suited to developed general, fully automated user friendly NLO codes (NLO extensions of ALPGEN, MADGRAPH,COMIX, etc.)

Outlook

✤ improve the speed of the codes

✤ improve the level of automation

- Efficient automated phase space integration
- Automation of real emission: dipole subtraction (SHERPA)
- Explore other approaches for automated subtraction schemes for real emission (POWHEG)
- Explore the use of color-dressed amplitudes