(Towards) CSW / MHV in D-dimensions

and

the S-Matrix Equivalence theorem

P. Mansfield DURHAM

Ettle, Fu, Fudger, Morris, Xiao

0902.1906 0703286 0511264

CSW / MHV alternative Feynman rules QCD

tree-level

conjectured Cachazo, Svrcek, Witten 0403047

proven Britto, Cachazo, Feng, Witten 0501052

Risager

loops

Bedford, Brandhuber, Spence, Travaglini 0407214

0410280, 0412108

Lagrangian derivations

Twistors Boels, Mason, Skinner 0507269

0510262 0604040 0702035

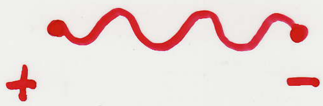
Canonical transformation Gorsky, Rosly 0510111

PM

MHV Rules

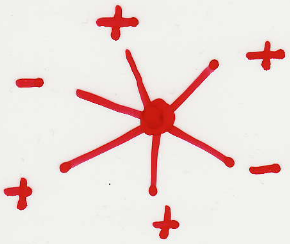
Witten Cachazo
Svrček 0403047

New Feynman Rules proven at <u>tree-level</u>.



propagator

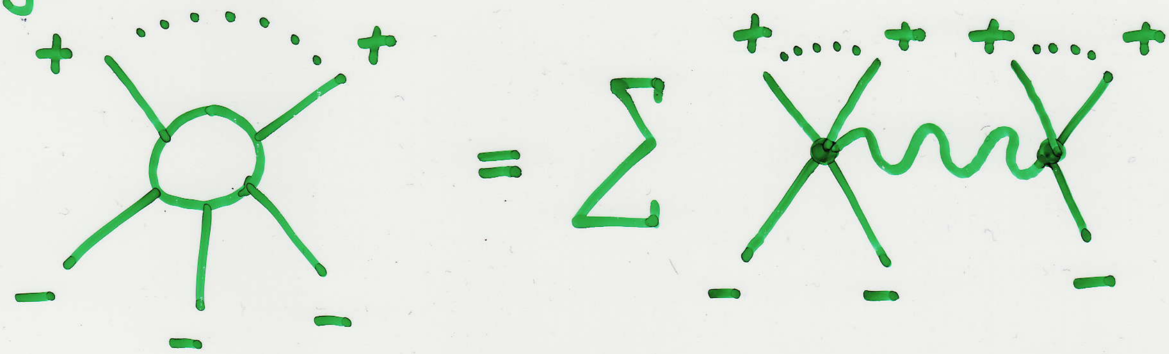
$$\frac{1}{p^2}$$



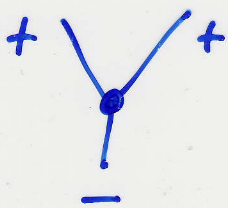
vertices

= Parke – Taylor amplitudes

e.g.



Many fewer diagrams than usually because no



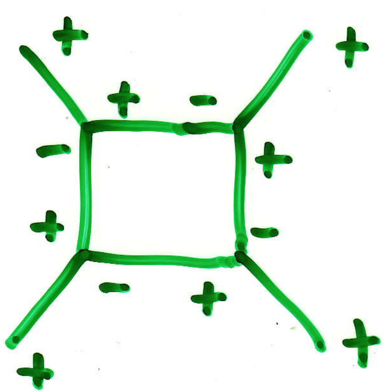
Loops

Regulate $\quad 4 \to D = 4 - \epsilon$

Problem : origin of all + one-loop amplitude

- Anomaly
- Extra terms in Lagrangian
- Something else

Does this spoil simplicity of CSW beyond tree-level ?



Yang-Mills in 4 dimensions

$$ds^2 = d(t-x_3)\,d(t+x_3) - d(x_1+ix_2)\,d(x_1-ix_2)$$

$$= dx^0\,dx^{\bar{0}} - dz\,d\bar{z}$$

gauge $A_{\bar{0}} = 0$ quantisation surface $x^0 = \text{const.}$

A_0 non-dynamical

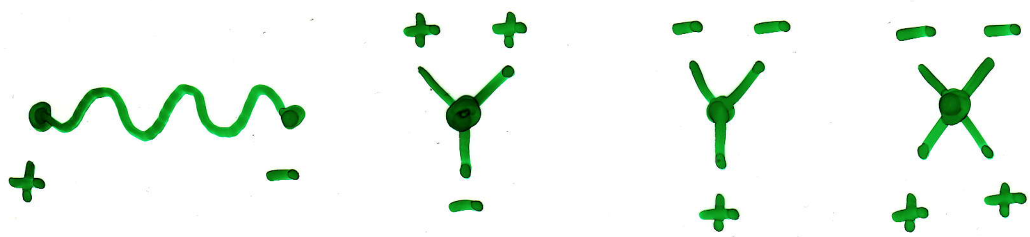
$$\mathcal{L}_{LC} =$$

$$\frac{4}{g^2}\,\mathrm{tr}\left(A\,\partial_0\partial_{\bar{0}}\bar{A} - [\partial_{\bar{z}}+\bar{A},\partial_{\bar{0}}A]\,\frac{1}{\partial_{\bar{0}}^2}[\partial_z+A,\partial_{\bar{0}}\bar{A}]\right)$$

$A = A_z$ +ve helicity field

$\bar{A} = A_{\bar{z}}$ −ve helicity field

$$\mathcal{L}_{LC} = \mathcal{L}_2^{+-} + \mathcal{L}^{++-} + \mathcal{L}^{--+} + \mathcal{L}^{--++}$$



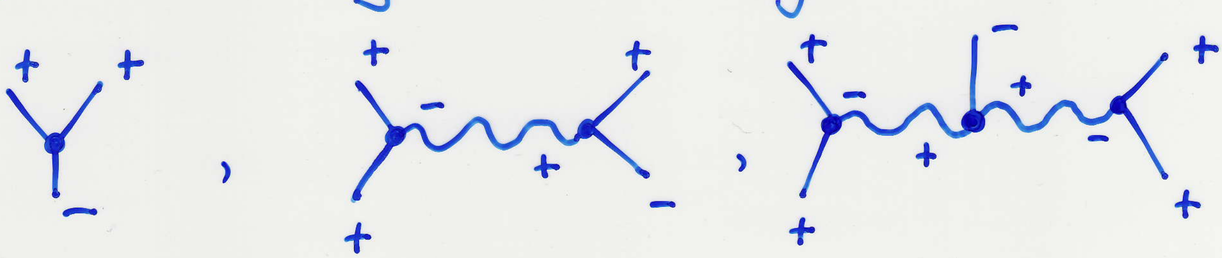
$\frac{1}{p^2}$

↑

not present in MHV

Truncating to $\mathcal{L}_2^{+-} + \mathcal{L}^{++-}$ gives a free theory at tree-level.

generate Feynman diagrams:



...

These are diagrams of One-Rest+ amplitudes in untruncated theory, and these vanish on shell.

Motivates change of variables to free theory

$$\mathcal{L}_2^{+-}[A, \bar{A}] + \mathcal{L}^{++-}[A, \bar{A}]$$

$$= \mathcal{L}_2^{+-}[B, \bar{B}]$$

$$\mathcal{L}_2^{+-}[A,\bar{A}] + \mathcal{L}^{++-}[A,\bar{A}] = \mathcal{L}_2^{+-}[B,\bar{B}]$$

- $B = B[A]$
- local in x^0 (non-local on quantis'n surface)
- Canonical $\Rightarrow$ Jacobian $= 1$

$\partial_{\bar{0}} \bar{A}$ momentum conjugate to A

so $$\partial_{\bar{0}} \bar{A} = \int_{x^0 = \text{const}} \frac{\delta B}{\delta A} \, \partial_{\bar{0}} \bar{B}$$

$\bar{A}$ linear in $\bar{B}$

Substitute into remaining part of $\mathcal{L}_{LC}$

$$\mathcal{L}^{--+}[A,\bar{A}] + \mathcal{L}^{--++}[A,\bar{A}] =$$

$$\mathcal{L}'^{--+}[B,\bar{B}] + \mathcal{L}'^{--++}[B,\bar{B}] +$$
$$\mathcal{L}'^{--+++}[B,\bar{B}] + .. + \mathcal{L}'^{--+\dots+}[B,\bar{B}] + ..$$

Parke Taylor vertices

Transformation

Fourier transform dependence on position in quantisation surface

$$B(x^0, \underline{p}) = A(x^0, \underline{p}) +$$

$$\sum_{n=2}^{\infty} \int_{\underline{k}_1 \dots \underline{k}_n} \delta^3(\underline{p} - \Sigma_r \underline{k}_r)\, \Gamma_n\, A(x^0, \underline{k}_1) \dots A(x^0, \underline{k}_n)$$

$$\Gamma_n =$$

$$i^n\, p_{\bar{0}}^{\,n-1} \Big/ \Big((p, k_1)(p, k_1 + k_2) \dots (p, k_1 + \dots + k_{n-1}) \Big)$$

$$(p, k) \equiv p_{\bar{0}} k_{\bar{z}} - p_{\bar{z}} k_{\bar{0}}$$

Γ_n independent of $p_{\bar{\bar{z}}}$, $k_{\bar{\bar{z}}}$

Regulate $4 \to D$

Dimensional reduction

- Aspects which remain 4-dim.

  $E_\mu^\pm$, gauge invariance ,

  p_μ of "physical" gluons

- Momenta of virtual gluons become D-dimensional.

  Internal lines , vertices

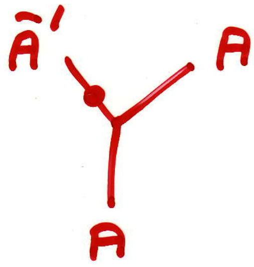
Only effect on $\mathcal{L}_{LC}$ is on free part :

$$\mathcal{L}_2^{+-} \longrightarrow$$

$$\frac{4}{g^2} \, \mathrm{tr} \, A \left(\partial_0 \partial_{\bar{0}} - \sum_{i=1}^{D/2-1} \partial_{z_i} \partial_{\bar{z}_i} \right) \bar{A}$$

Can still transform to free theory

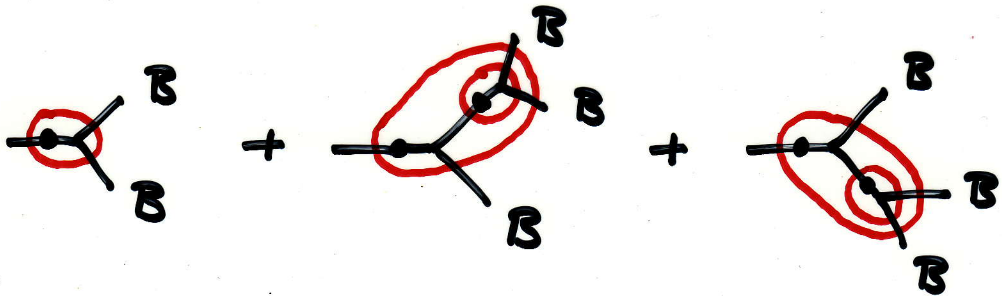
$$\mathcal{L}_2^{+-}[A,\bar{A}] + \mathcal{L}^{++-}[A,\bar{A}] = \mathcal{L}_2^{+-}[B,\bar{B}]$$

$\bar{A}\,\partial^2 A \quad + $ 

$= \quad \bar{B}\,\partial^2 B$

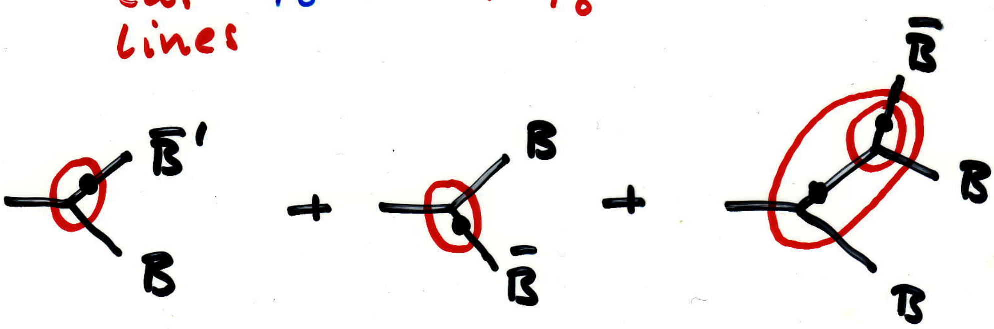
$$\bar{A}' \equiv \frac{\partial \bar{A}}{\partial x^{\bar{0}}}$$

canonical coord. transf^n $\quad A = A[B]$ local in x^0 $\quad \bar{B}' = \int \frac{\delta A}{\delta B}\,\bar{A}'$

$A = B +$ 

$+ \dots$

 denotes $\displaystyle\sum_{\substack{\text{cut}\\ \text{lines}}} \frac{1}{\frac{P^2}{P_{\bar{0}}}}$ ← independent of P_0

$\bar{A}' = \bar{B}' +$ 

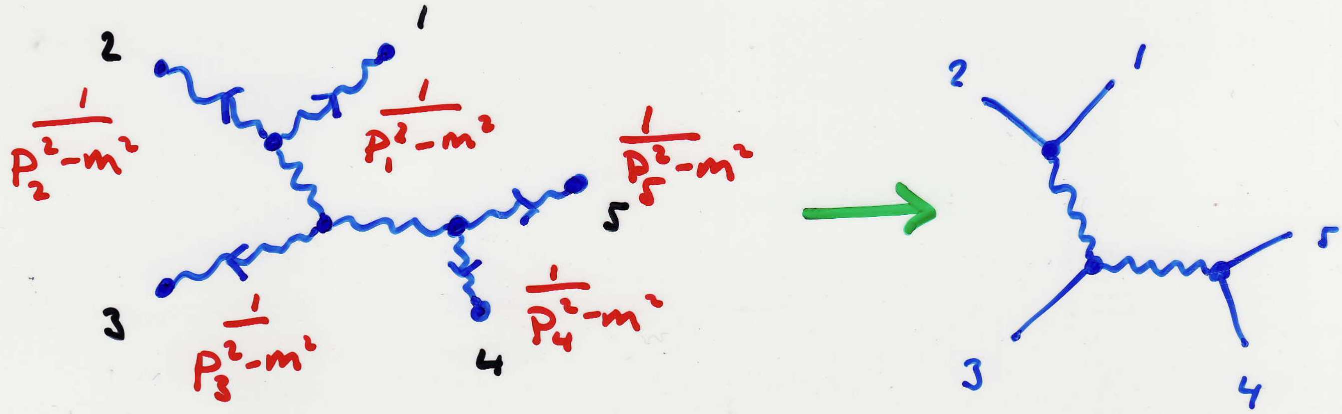
$+ \dots$

$B, \bar{B}$ generate same amplitudes as $A, \bar{A}$?

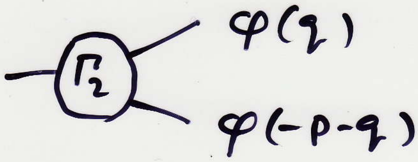
S-matrix equivalence for scalar fields

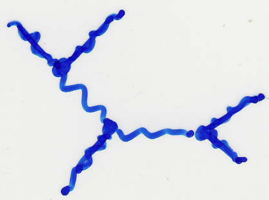
$$\mathcal{A}_n(p_1 .. p_n) = \lim_{p_n^2 \to m^2} (p_1^2 - m^2) .. (p_n^2 - m^2) \underbrace{\langle \varphi(p_1) .. \varphi(p_n) \rangle}_{\int \mathcal{D}\varphi \, e^{i\int d^4x \, (\partial\varphi)^2 - V(\varphi)} \varphi(p_1) .. \varphi(p_n)} Z$$

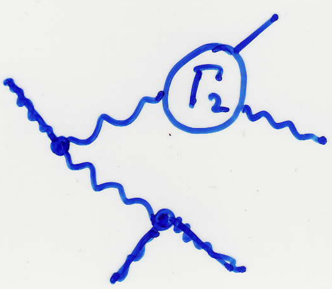
LSZ

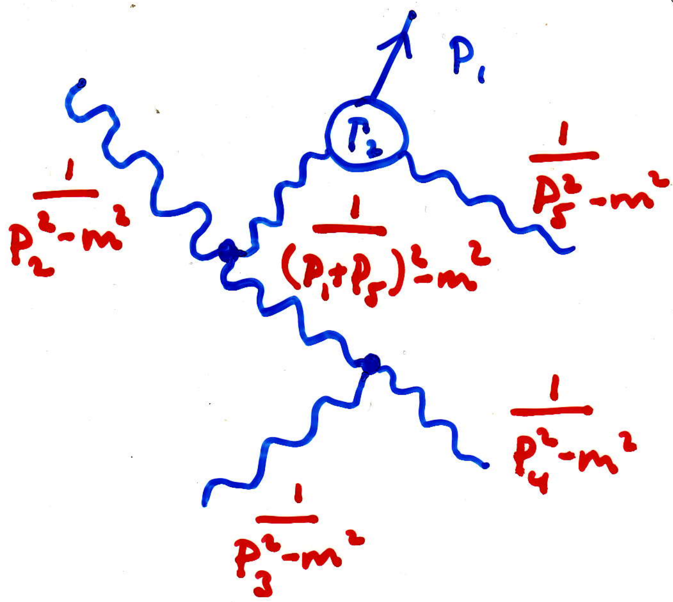


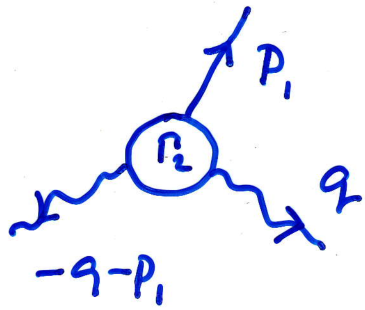
New field: $\psi(p) = \varphi(p) + \int \Gamma_2(p, q, -p-q) \, \varphi(q) \, \varphi(-p-q) \, d^4q + \ldots$

$\psi(p) = \varphi(p) +$  $+ \ldots$

$\langle \psi(p_1) .. \psi(p_n) \rangle =$  $+ ..$

$+$  $+ ..$

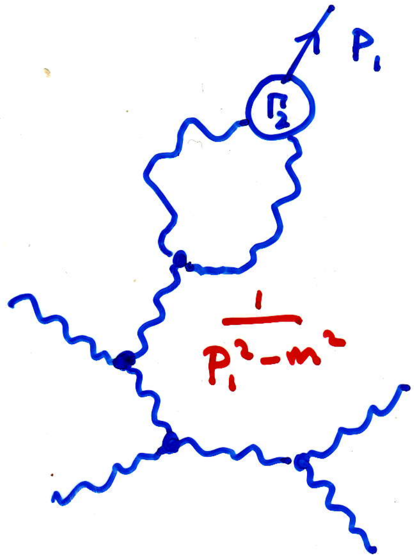




$$\frac{\Gamma_2(p_1, q)}{\left((q+p_1)^2-m^2\right)\left(q^2-m^2\right)}$$

Cannot cancel $p_1^2-m^2$ unless:

- Γ_2 contains $1/(p_1^2-m^2)$ non-local
- Couples to diagram in which momenta q & $-q-p_1$ flow together



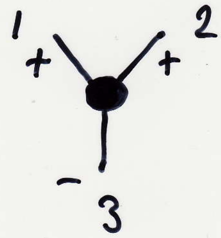
multiple of original diagram

effect eliminated by normalisation of field

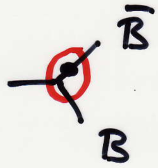
Transformation $A, \bar{A} \to B, \bar{B}$ is local in x^0, so Γ_n is independent of p_0.

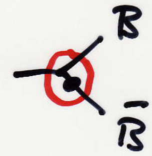
How can Γ_n contain $\dfrac{1}{p_0 p_{\bar{0}} - p_z p_{\bar{z}}}$ to cancel p^2 ?

But $A, \bar{A}$ do not give same amplitudes as $B, \bar{B}$

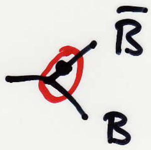
e.g. 

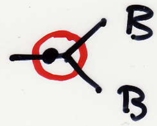
$$p_1^2\, p_2^2\, p_3^2 \langle \bar{A}(p_1)\, \bar{A}(p_2)\, A(p_3) \rangle$$

$= p_1^2\, p_2^2\, p_3^2 \langle \big(\bar{B}(p_1) +$ 

$+$ 

$+ \ldots \big)$

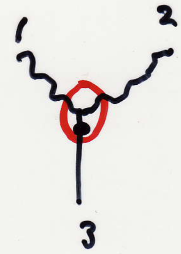
$\big(\bar{B}(p_2) +$ 

$+ \ldots \big) \big(B(p_3) +$ 

$+ \ldots \big) \rangle$

$= p_1^2\, p_2^2\, p_3^2 \Big($ 

$+$ 

$+$ 

$\Big)$

$$\frac{1}{p_2^2}\; \frac{1}{p_3^2}\; \frac{V_{++-}}{p_{1\bar{0}}}\; \frac{1}{\dfrac{p_1^2}{p_{1\bar{0}}} + \dfrac{p_2^2}{p_{2\bar{0}}} + \dfrac{p_3^2}{p_{3\bar{0}}}}$$

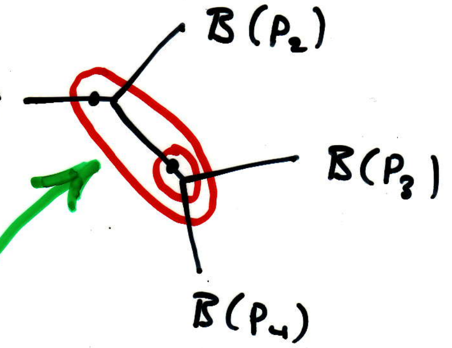
$$p_1^2\, \cancel{p_2^2}\, \cancel{p_3^2}\; \frac{1}{\cancel{p_2^2}\, \cancel{p_3^2}}\; V_{++-}\; \frac{1}{p_1^2 + p_{1\bar{0}}\left(\dfrac{p_2^2}{p_{2\bar{0}}} + \dfrac{p_3^2}{p_{3\bar{0}}} \right)}$$

$$\xrightarrow[p_2,\, p_3 \text{ on-shell}]{} \quad V_{++-}$$

Potentially spoils utility of CSW because we need to include translation factors A → B in amplitude calculations

Which amplitudes are affected?

When do translation factors generate $\frac{1}{p^2}$ to cancel p^2 in LSZ?

$A(p_1) = B(p_1) + \cdot\cdot + 1$  $+ \cdot\cdot$

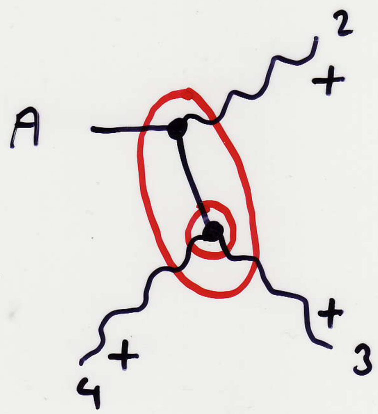
$$\frac{1}{\frac{p_1^2}{p_{1\bar{o}}} + \frac{p_2^2}{p_{2\bar{o}}} + \frac{p_3^2}{p_{3\bar{o}}} + \frac{p_4^2}{p_{4\bar{o}}}}$$

to generate $\frac{1}{p_1^2}$ need $\frac{p_2^2}{p_{2\bar{o}}} + \frac{p_3^2}{p_{3\bar{o}}} + \frac{p_4^2}{p_{4\bar{o}}} = 0$

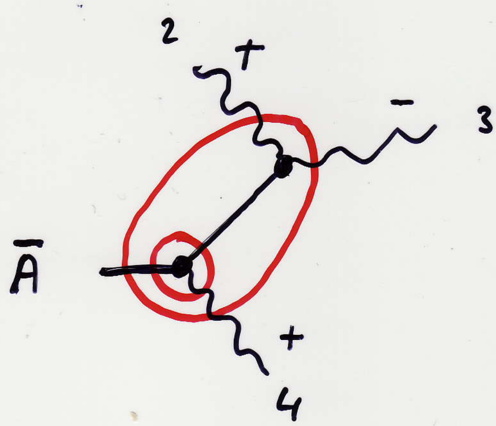
When does $P_2^2/P_{2\bar{0}} + P_3^2/P_{3\bar{0}} + P_4^2/P_{4\bar{0}} = 0$?

At tree-level :

- For $P_2^2 = P_3^2 = P_4^2 = 0$ 2, 3, 4 are <u>external</u> lines



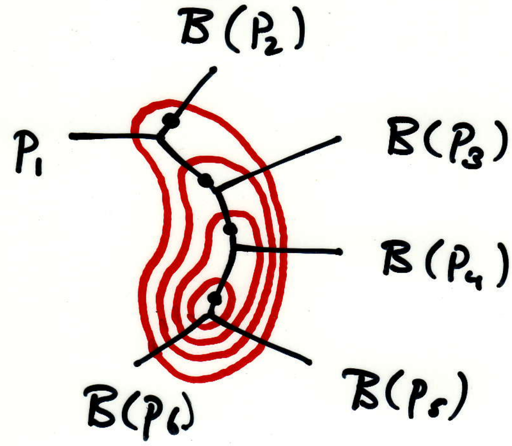
contribute only to $\mathcal{A}^{\text{tree}}(-++\ldots+)$



- 2, 3, or 4 off-shell but there is cancellation — requires special choice of external momenta, not generic

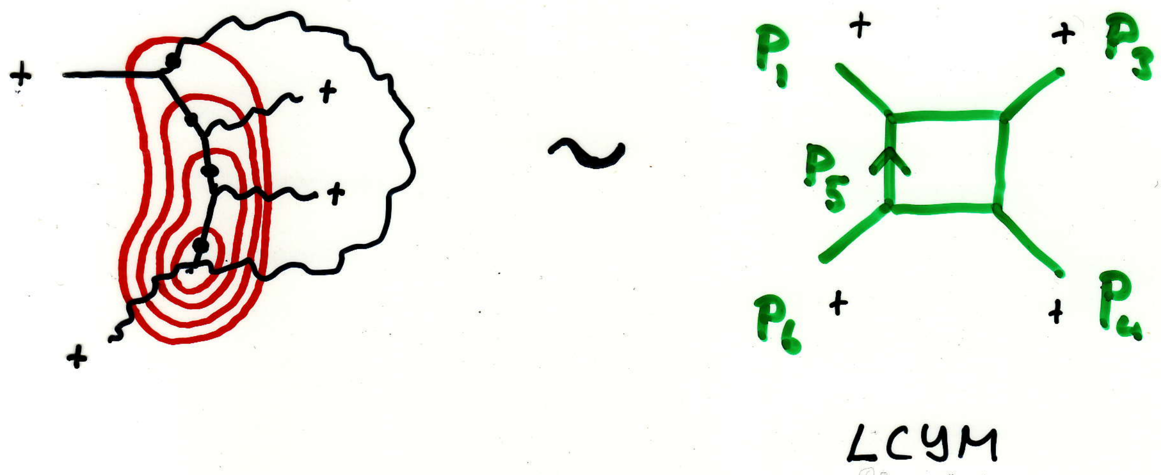
When does $p_2^2/p_{2\bar{0}} + \ldots + p_n^2/p_{n\bar{0}} = 0$?

One-loop

• e.g. $\bar{A}(p_1) = \ldots +$  $+ \ldots$

when $p_2 = -p_5$ loop

p_3, p_4, p_6 on-shell

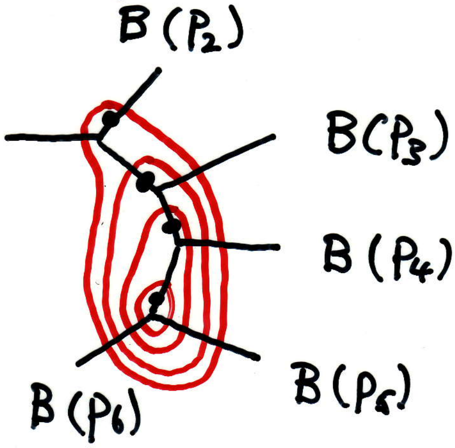


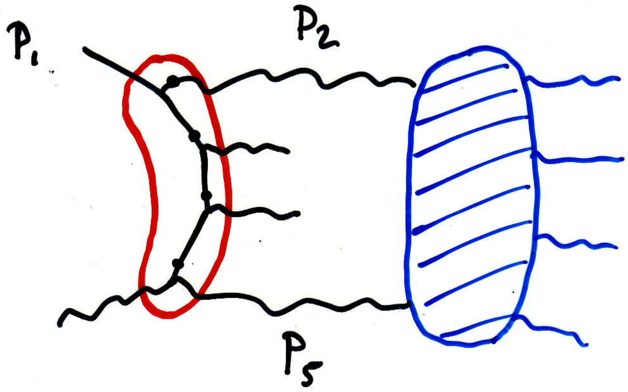
only affects $\mathcal{A}^{1\text{-loop}}(++\ldots++)$

When does $p_2^2 / p_{2\bar{0}} + \ldots + p_n^2 / p_{n\bar{0}} = 0$?

One-loop

inside loop integral

• e.g. $\bar{A}(p_1) = \ldots + p_1$  $+ \ldots$



p_2, p_5 loop

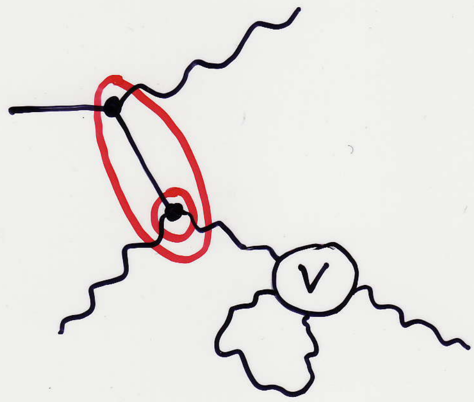
p_3, p_4, p_6 on-shell

$$\int d^D p_2 \quad \frac{1}{p_2^2} \quad \frac{1}{p_5^2} \quad \frac{1}{\frac{p_1^2}{p_{1\bar{0}}} + \frac{p_2^2}{p_{2\bar{0}}} + \frac{p_5^2}{p_{5\bar{0}}}} \quad f(p_2)$$

$$\sim \frac{1}{p_1^2} \quad ?$$

- Propagator dressed

A



absorb into normalis^n of field in LSZ formula

$B\bar{B}$ give same amplitudes as $A\bar{A}$ in LSZ formula except for :

- Tree-level $\mathcal{A}(-++..++)$ (which vanishes for 4-dim gluons)

- One-loop $\mathcal{A}(++..++)$ which is missed by MHV rules.

Conclusion

- Even in D dimensions can transform

$$\mathcal{L}_2^{+-}[A,\bar{A}] + \mathcal{L}^{-++}[A,\bar{A}] = \mathcal{L}_2^{+-}[B,\bar{B}]$$

- $B, \bar{B}$ give same amplitudes as $A, \bar{A}$ <u>except</u>

  tree-level $\mathcal{A}(++\ldots++-)$

  one-loop $\mathcal{A}(++\ldots++)$

  — known

  — computable from translation factors

- **Next step : D-dimensional vertices**