Matrix Elements and Shower Matching at Tree level

P. Nason
INFN, Sez. of Milano Bicocca

Outline

Introduction:

- Brief summary on Showers: Collinear factorization, Virtual corrections, Sudakov form factors Shower algorithm, colour assignment

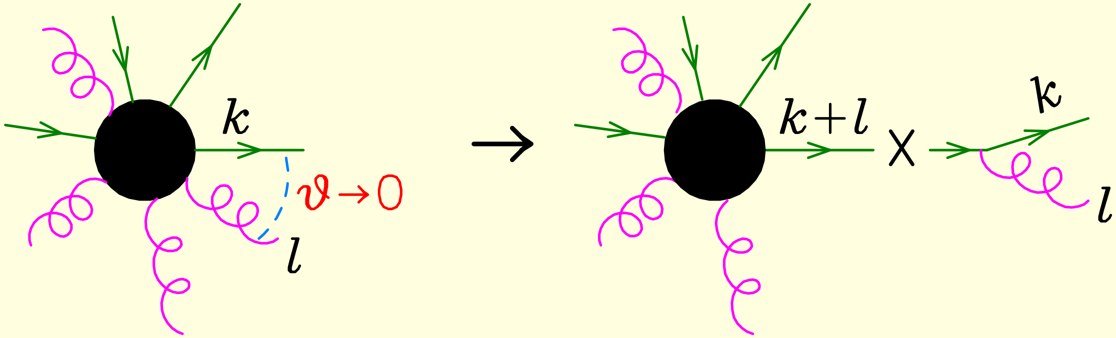
Shower improvements

- Large angle emissions: ME corrections
- Multi-parton matrix elements and showers: CKKW
  - Coherence and angular ordering
  - k_T-cluster multiplicities: how to compute it
  - Detailed prescription for CKKW
  - Interfacing to showers: truncated showers:
- Variants of CKKW
  - MLM matching: equivalence with CKKW
  - What is done in practice

Shower basics: Collinear factorization

QCD emissions are enhanced near the collinear limit

Cross sections factorize near collinear limit



$$|M_{n+1}|^2 d\Phi_{n+1} \Longrightarrow |M_n|^2 d\Phi_n \, \frac{\alpha_s}{2\pi} \frac{dt}{t} P_{q,qg}(z) \, dz \, \frac{d\phi}{2\pi}$$

t : hardness (either virtuality or p_T^2 or $E^2\theta^2$ etc.)

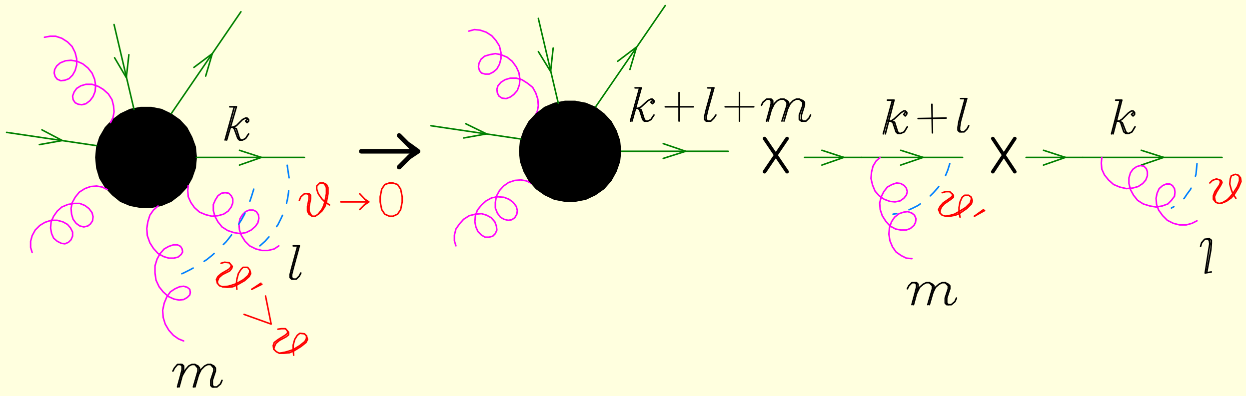
$z = k^0/(k^0 + l^0)$: energy (or $p_{\parallel}$, or p^+) fraction of quark

$P_{q,qg}(z) = C_F \frac{1+z^2}{1-z}$: Altarelli – Parisi splitting function

(ignore $z \to 1$ IR divergence for now)

If another gluon becomes collinear, iterate the previous formula:

$\theta', \theta \to 0$
with $\theta' > \theta$



$$|M_{n+1}|^2 d\Phi_{n+1} \Longrightarrow |M_{n-1}|^2 d\Phi_{n-1} \times \frac{\alpha_s}{2\pi}\frac{dt'}{t'} P_{q,qg}(z')\, dz' \frac{d\phi'}{2\pi} \times \frac{\alpha_s}{2\pi}\frac{dt}{t} P_{q,qg}(z)\, dz \frac{d\phi}{2\pi}\, \theta(t'-t)$$

Collinear partons can be described by a factorized integral ordered in t.
For m collinear emissions:

$$\left(\frac{\alpha_s}{2\pi}\right)^m \int_{\theta_{\rm min}} \frac{d\theta_1}{\theta_1} \int_{\theta_1} \frac{d\theta_2}{\theta_2} \cdots \int_{\theta_{m-1}} \frac{d\theta_m}{\theta_m} \;\propto\; \frac{\log^m \frac{1}{\theta^2_{\rm min}}}{m!} \approx \left(\frac{\alpha_s}{2\pi}\right)^m \frac{\log^m \frac{Q^2}{\Lambda^2}}{m!}$$

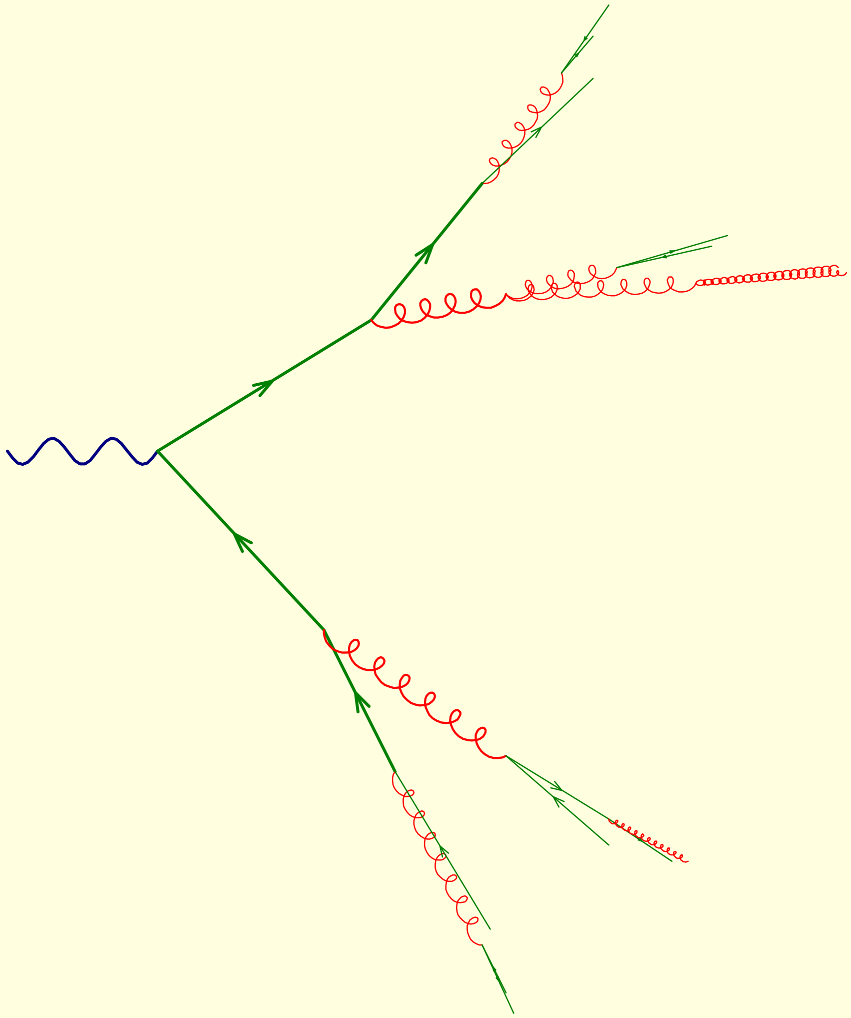
where we have taken $\theta_{\rm min} \approx \Lambda/Q$; (Leading Logs) This is of order 1!
Typical dominant configuration at very high Q^2

Besides $q \to qg$, also $g \to gg$, $g \to q\bar{q}$ come into play.

Typical configurations: intermediate angles of order of geometric average of upstream and downstream angles.

Each angle is $\mathcal{O}(\alpha_s)$ smaller than its upstream angle, and $\mathcal{O}(\alpha_s)$ bigger than its downstream angle.

As relative momenta become smaller α_s becomes bigger, and this picture breaks down.



For a consistent description:
include virtual corrections to same LL approximation

One can show that the effect of virtual corrections is given by

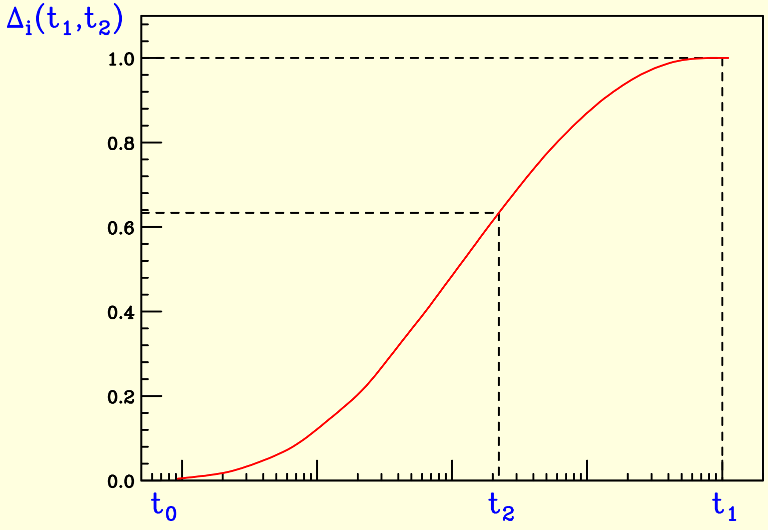
- Let $\alpha(\mu) \Longrightarrow \alpha(t)$ in each vertex, where t is the hardness of the vertex (i.e. hardness of the incoming line)
- For each intermediate line include the factor

$$\Delta_i(t_h, t_l) = \exp\left[-\sum_{(jk)} \int_{t_l}^{t_h} \frac{dt'}{t'} \int dz \, \frac{\alpha_s(t')}{2\pi} \, P_{i,jk}(z) \right]$$

where t_h is the hardness of the vertex originating the line, and t_l is the hardness of the vertex where the line ends.

Sudakov form factor

$$\Delta_i(t_h, t_l) = \exp\left[-\sum_{(jk)} \int_{t_l}^{t_h} \frac{dt'}{t'} \int dz \, \frac{\alpha_s(t')}{2\pi} \, P_{i,jk}(z) \right]$$



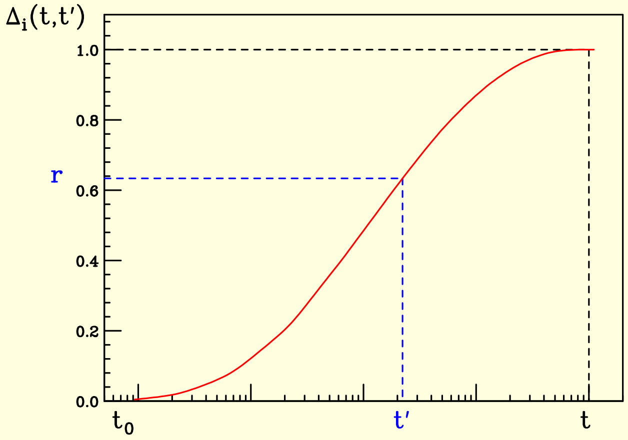
As t_l becomes small the exponent tend to diverge, and $\Delta_i(t_h, t_l)$ approaches 0. In fact, because of $\alpha_s(t)$, we must stop at $t_0 \gtrsim \Lambda_{\rm QCD}$.

Final Recipe

- Consider all tree graphs.
- Assign ordered hardness parameters t to each vertex.
- Include a factor
$$\frac{\alpha_s(t)}{2\pi} P_{i,jk}(z) \frac{dt}{t} dz \frac{d\phi}{2\pi}$$
at each vertex $i \to jk$.
- Include a factor $\Delta_i(t_1, t_2)$ to each internal line with a parton i, from hardness t_1 to hardness t_2.
- Include a factor $\Delta_i(t, t_0)$ on final lines (t_0: IR cutoff)

Most important: the shower recipe can be easily implemented as a computer code!
Shower Algorithm:

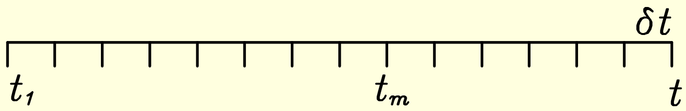
- Generate a uniform random number $0 < r < 1$;
- Solve the equation $\Delta_i(t, t') = r$ for t';
- If $t' < t_0$ stop here (final state line);
- generate z, jk with probability $P_{i,jk}(z)$, and $0 < \phi < 2\pi$ uniformly;
- restart from each branch, with hardness parameter t'.



Probabilistic intepretation: branching probability of line of flavor i

$$dP(t_1,t) = \underbrace{\exp\left[-\sum_{(jk)}\int_t^{t_1}\frac{dt'}{t'}\int dz\,\frac{\alpha_s(t')}{2\pi}P_{i,jk}(z)\right]}_{\Delta(t_1,t)}\frac{\alpha_s(t)}{2\pi}P_{i,jk}(z)\,\frac{dt}{t}\,dz\,\frac{d\phi}{2\pi}$$

break up t_1, t into small subintervals:



$$dP(t_1,t) = \left[\prod_m \left(\underbrace{1-\sum_{(jk)}\frac{\delta t}{t_m}\int dz\,\frac{\alpha_s(t_m)}{2\pi}P_{i,jk}(z)}_{\text{No emission prob. in } t_m,\, t_m+\delta t}\right)\right]\underbrace{\frac{\alpha_s(t)}{2\pi}P_{i,jk}(z)\,\frac{\delta t}{t}\,dz\,\frac{d\phi}{2\pi}}_{\text{emission prob. in } t,\, t+\delta t}$$

So: the probability for the first branching at hardness t is the product of the non-emission probability $\Delta(t_1,\ t)$ in all hardness intervals between t_1 and t, times the emission probability at hardness t.

(more or less) obvious consequences:

- The total branching probability plus the no-branching probability is 1; mathematically

$$\int_{t_0}^{t_1} dP(t_1, t') = \int_{t_0}^{t_1} d\Delta_i(t_1, t') = 1 - \Delta_i(t_1, t_0)$$

- The Sudakov form factor $\Delta_i(t_1, t)$ is the no-branching probability from scale t_1 down to the scale t.

- The branching probability is independent of what happens next (because the total probability of what happens next is 1).

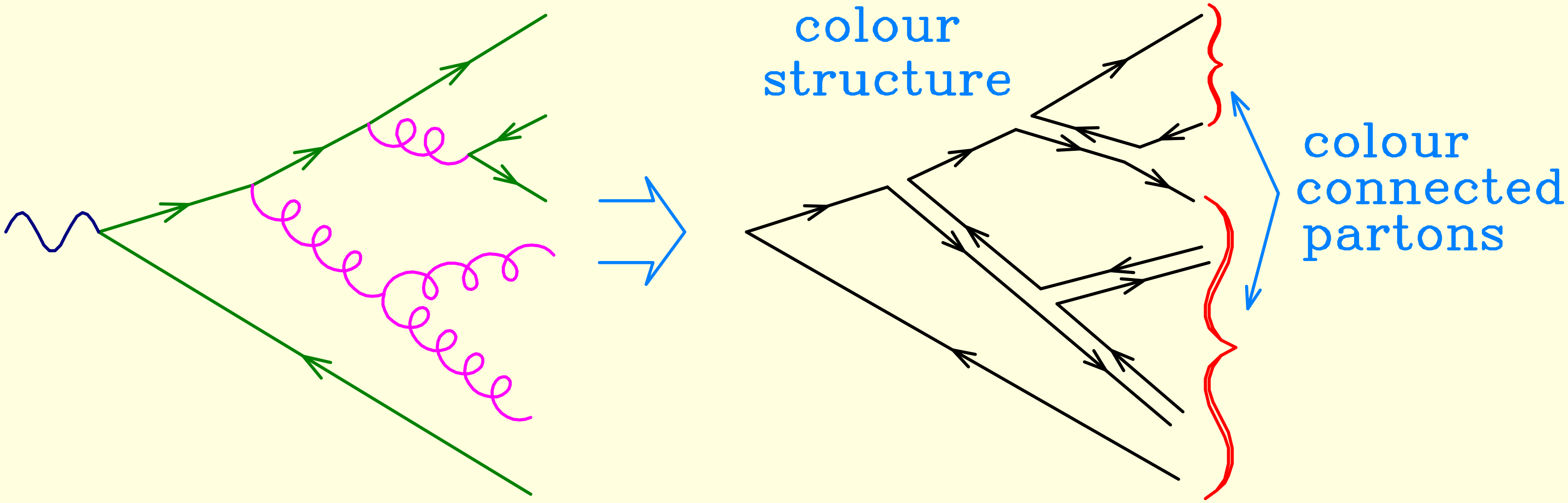
This property is often called unitarity of the shower. It is a consequence of the Kinoshita-Lee-Nauenberg theorem: collinear divergence must cancel in the inclusive cross section.

COLOUR AND HADRONIZATION

SMC's assign colour labels to partons.

Only colour connections are recorded (as in large N limit).

Initial colour assigned according to hard cross section.



Colour assignements are used in the hadronization model.

Most popular models: Lund String Model, Cluster Model.

In all models, color singlect structures are formed out of colour connected partons, and are decayed into hadrons preserving energy and momentum.

Large angle emission

A disturbing feature of SMC's: the hardest jet generated in the shower is not really collinear in about 10% of the events (i.e. $\mathcal{O}(\alpha_s)$). Thus, the gross feature of the event is wrongly described by the SMC in 10% of the cases.
Most SMC deal with this problem, implementing a matrix element correction for the simplest processes:

PYTHIA method: compute the first emission with exact real emission matrix element (and corresponding Sudakov form factor)

HERWIG: at any stage of the shower, if the current radiation is the hardest so far, correct radiation with exact matrix element (soft ME correction) or populate the dead region (hard correction) with exact ME.

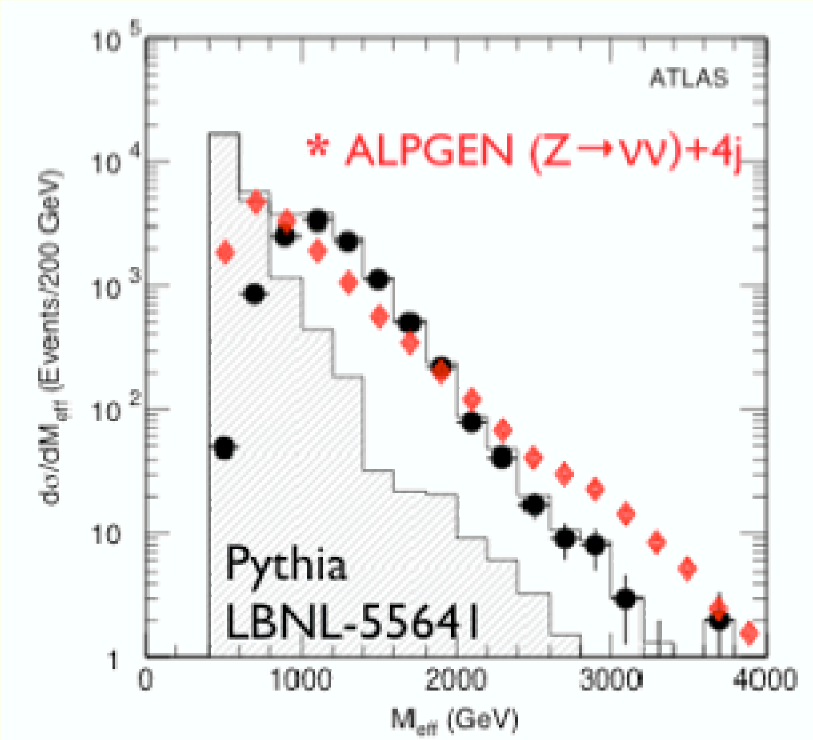
Multi-parton processes: algorithms exist to compute them with high efficiency; problem: how to interface them to SMC's

Multi-parton Matrix Elements

With LHC physics: cannot trust collinear approximation for multi-jet background to complex processes

The use of exact ME is mandatory (Gianotti,Mangano, 05)

$M_{\rm eff}$ distribution for a potential multijet+$E_T^{\rm miss}$ SUSY signal
dark circles: signal
Shaded area: MC background



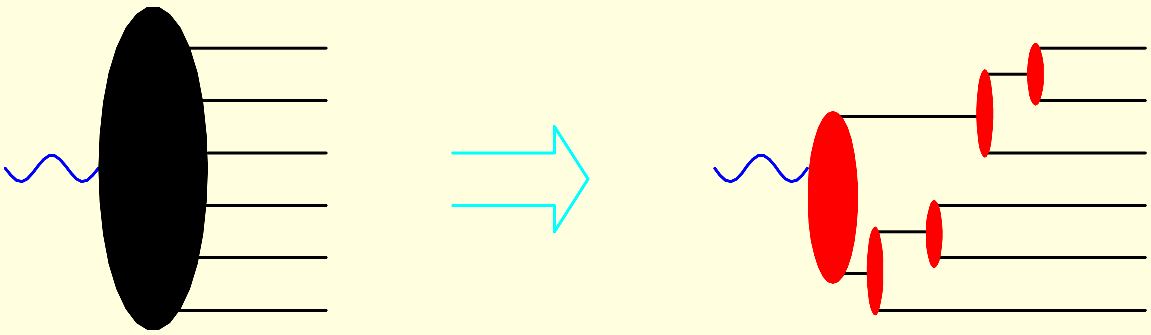
Historical approach: CKKW

Catani, Krauss, Küen, Webber (2001), (in e^+e^- annihilation).

In a nut-shell:

Clusterize ME partons to reconstruct a shower skeleton
(by pairing up particles that yield smallest t recursively)

Red blobs have decreasing t



- Do not allow t below a given cut $t_{\rm cut}$.
- Re-evaluate ME couplings at scales t of vertices in shower skeleton
- Assign Sudakov form factors to the skeleton (as in Shower MC)
- Continue the shower for $t < t_{\rm cut}$ with the Shower MC

CKKW: details

CKKW relies upon the theory of soft-collinear radiation in QCD, through the following steps:

A) Theory of multiple emissions in the soft collinear regions (Mueller, 1981; Ermolaev and Fadin, 1981; Bassetto, Ciafaloni, Marchesini, etc.)

B) k_T-cluster multiplicity calculable at the NLL level in framework A) (Catani, Dokshitzer, Olsson, Turnock and Webber, 1991)

C) k_T-cluster cross section is improved with Sudakov form factors and running α_s (i.e. dominant virtual corrections) from step B)

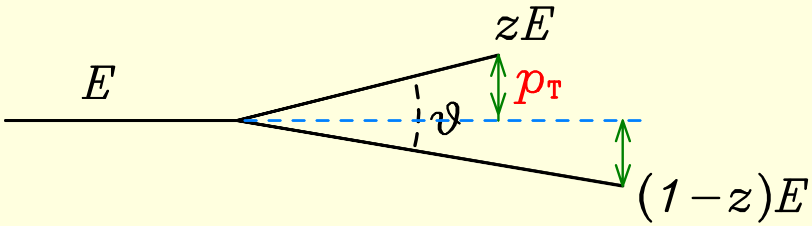
D) Completion of the algorithm with subsequent angular ordered shower in collinear and soft approximation

Soft divergences and double log region

$z \to 1$ ($z \to 0$) region problematic: for $z \to 1$: $P_{qq}, P_{gg} \propto \frac{1}{1-z}$

Choice of hardness variable makes a difference

$$\begin{aligned} \text{virtuality:} \quad t &\equiv E^2 z(1-z)\, \overbrace{\theta^2/2}^{1-\cos\theta} \\ p_T^2\text{:} \quad t &\equiv E^2 z^2 (1-z)^2\, \theta^2 \\ \text{angle:} \quad t &\equiv E^2\, \theta^2 \end{aligned}$$



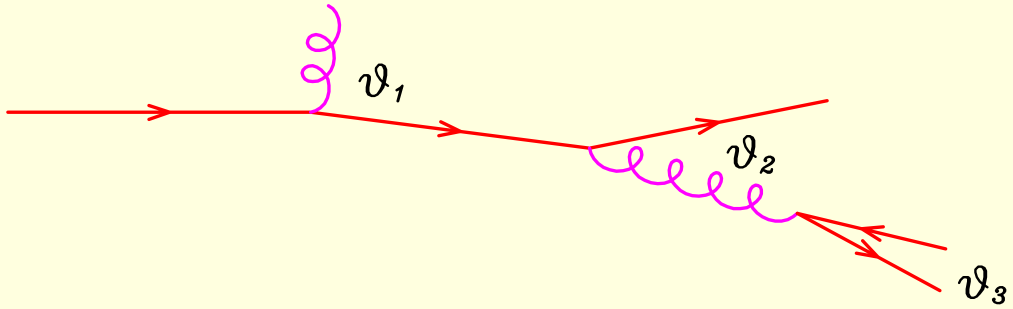
Notice, from the figure, for small p_T: $\theta \approx \frac{p_T}{zE} + \frac{p_T}{(1-z)E} = \frac{p_T}{z(1-z)E}$

$$\underbrace{\int \frac{dt}{t} \int_0^{1-\sqrt{t}/E} \frac{dz}{1-z}}_{\text{virtuality: } 1-z > t/E^2} \approx \frac{\log^2 \frac{t}{E^2}}{4}; \underbrace{\int \frac{dt}{t} \int_0^{1-t/E^2} \frac{dz}{1-z}}_{p_T^2\text{: } (1-z)^2 > t/E^2} \approx \frac{\log^2 \frac{t}{E^2}}{2}; \underbrace{\int \frac{dt}{t} \int_0^{1} \frac{dz}{1-z}}_{\text{angle}} \approx \log t \log \Lambda$$

Sizeable difference in double log structure!

At double log level:
angular ordering is the correct choice (Mueller 1981)

$$\frac{d\theta}{\theta}\frac{\alpha_s(p_T^2)}{2\pi}P(z)\mathrm{dz}$$



$$\theta_1 > \theta_2 > \theta_3 \ldots$$

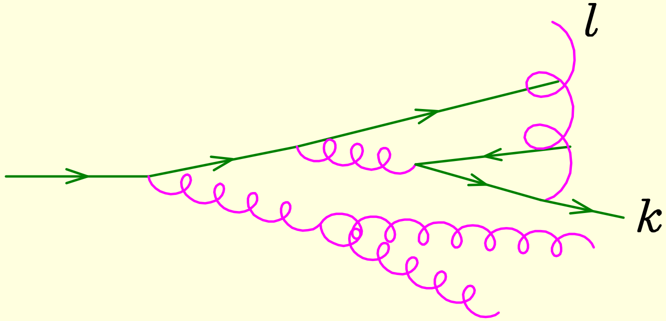
$$p_T^2 = E^2 z^2 (1-z)^2\,\theta^2$$

$\alpha_s(p_T)$ for a correct treatment of charge renormalization in soft region.

$$\Delta_i(t,t') = \exp\left[-\int_{t'}^{t}\frac{dt}{t}\int_{\sqrt{\frac{t_0}{t}}}^{1-\sqrt{\frac{t_0}{t}}} dz\frac{\alpha_s(p_T)}{2\pi}\sum_{(jk)} P_{i,jk}(z)\right]$$

$$\approx \exp\left[-\frac{c_i}{4\pi b_0}\left\{\log\frac{t}{\Lambda^2}\log\frac{\log\frac{t}{\Lambda^2}}{\log\frac{t_0}{\Lambda^2}} - \log\frac{t}{t_0}\right\}_{t'}^{t}\right] \qquad (c_q = C_F, c_g = 2C_A)$$

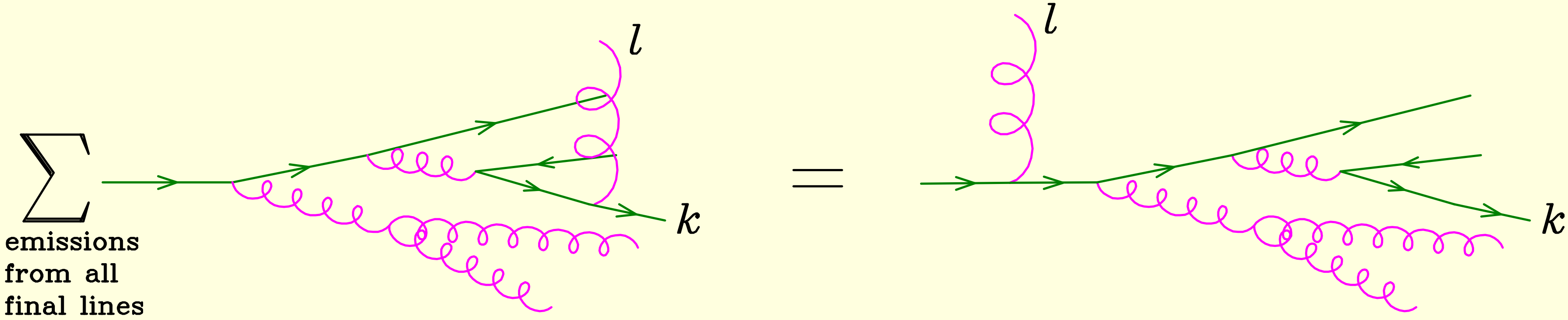
Sudakov damping stronger than any power of t.



With virtuality ordering:

Soft emissions give small virtuality.
At end of shower, large amount of
unrestricted (all angles) soft radiation

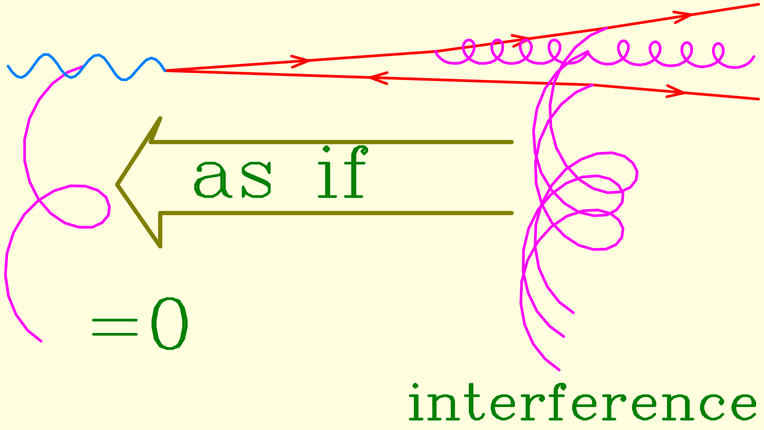
But soft gluons emitted at large angles from final state partons add coherently!



large angle, high energy: already ordered in angle
large angle, small energy: should be reordered by angle;

Thus: order in angle

Look at the example:



Angular ordering accounts

for soft gluon interference.

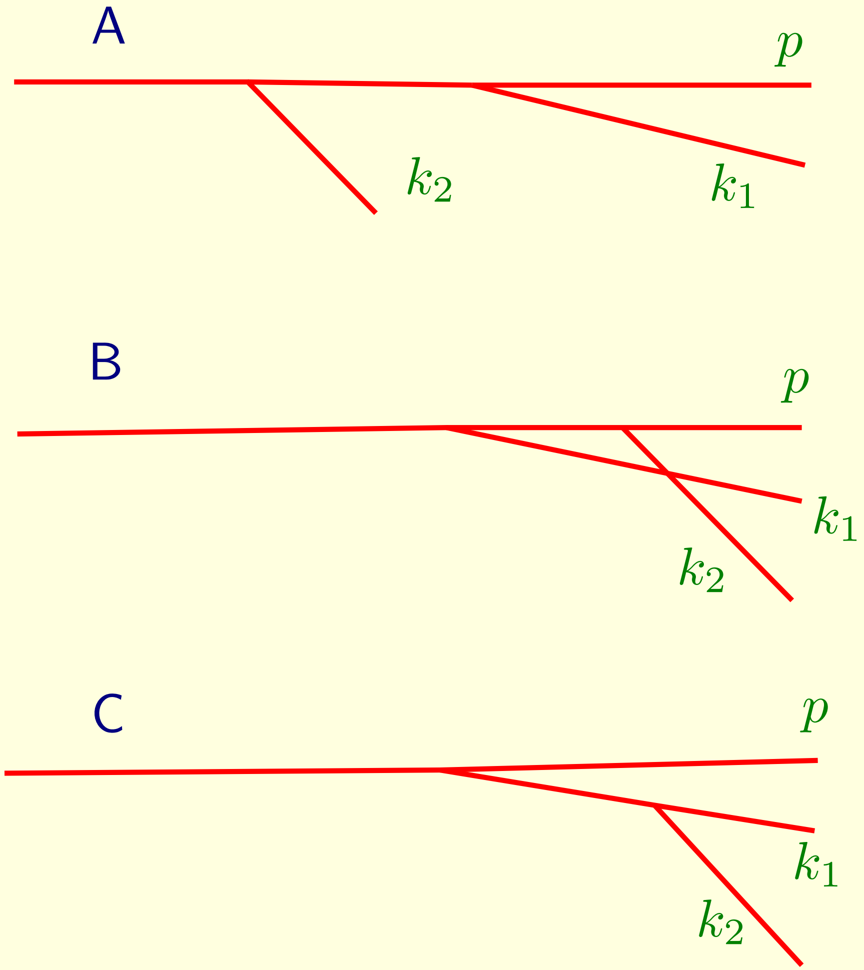
Intensity for photon jets $= 0$

Intensity for gluon jets $= C_A$

instead of $2C_F + C_A$

Consistent with a boosted jet pair, in the case of a photon jet.
In angular ordered SMC large angle soft emission is generated first.
Hardest emission (i.e. highest pt) happens later.

Angular ordering is a non-trivial result; look at double emission



Largest angle gluon k_2 can be inserted in 3 ways;

$(k_2 + p)^2 > (k_1 + p)^2$:
only A contributes;
$(k_2 + p)^2 < (k_1 + p)^2$:
B+C contribute

In the last cases B and C add coherently: total as if k_2 was emitted in A, neglecting the virtuality of the incoming line.

Angular ordering found by Mueller in a 3-loop calculation of soft emissions

The theory of multiple soft emission has been extended from double log accuracy (i.e. only small angle) to large angle emissions. In the large N limit (only planar graphs) this formulation acquires the particularly simple form of an energy ordered dipole cascade.

Normally in soft gluon (and photon) physics, one uses the fact that the insertion of the softest gluon is IR divergent only if it is attatched to external lines. This is not the case if you have collinear singularities! (see previous slide) Dipole cascade is also non-trivial ...
(Fiorani, Marchesini, Reina, 1988)

Is coherence important?

- Eccessive multiplicity growth in virtuality ordered MC ("historical" problem with multiplicity when LEP was turned on)
- Angular ordered MC's (HERWIG) agree with mutliplicity data in e^+e^- annihilation
- Agreement of PYTHIA with multiplicity data was achieved by superimposing an angular ordered veto over the virtuality ordered shower. This amounts to take the interference as being totally destructive. No major differences between PYTHIA and HERWIG are seen if the angular order veto is applied.

So: if the MC does not include coherence correctly, approximate remedies should be found that adjust the main observables

k_T-clusters

Given a set of n particles in an e^+e^- final state, reconstruct jets by pairing up recursively pairs of particles with minimum

$$y_{kl} = 2(1 - \cos\theta_{kl}) \min\left(E_k^2, E_l^2\right)/Q^2.$$

The pair of particles with minimum y_{kl} are combined into a single pseudo-particle, with momentum $p_{kl} = p_k + p_l$ (or any variant of this, like the P or E_0 schemes). Notice:

$$y \approx \frac{p_T^2}{Q^2},$$

since

$$2(1 - \cos\theta) \approx \theta^2, \quad \min\left(E_k, E_l\right) \approx \frac{E_k E_l}{E_k + E_l}\,.$$

k_T-cluster multiplicity can be computed at NLL level using the theory of multiple soft gluon emission.

In the following: how to reproduce the results of Catani, Dokshitzer, Olsson, Turnock and Webber, 1991 using angular ordering

k_T-clusters multiplicity calculation: use angular ordering!

Sudakov form factor as in angular ordered shower, but veto radiation that yields $y > y_{\rm min}$. Introducing: $Q_{\rm min} = \sqrt{y_{\rm min}}\,Q$, $t = \theta E$, $q = k_T = \sqrt{t}\,z(1-z)$

$$\Delta(Q) = \exp\left[-\int_0^{Q^2}\frac{dt}{t}\int dz\,\frac{\alpha_s(q)}{2\pi}P(z)\theta(q - Q_{\rm min})\right]$$

$$= \exp\left[-2\int\frac{dq}{q}\,dz\,\frac{\alpha_s(q)}{2\pi}P(z)\theta(q - Q_{\rm min})\theta(Q\,z(1-z) - q)\right].$$

For example, for P_{qq} (HOMEWORK PROBLEM!):

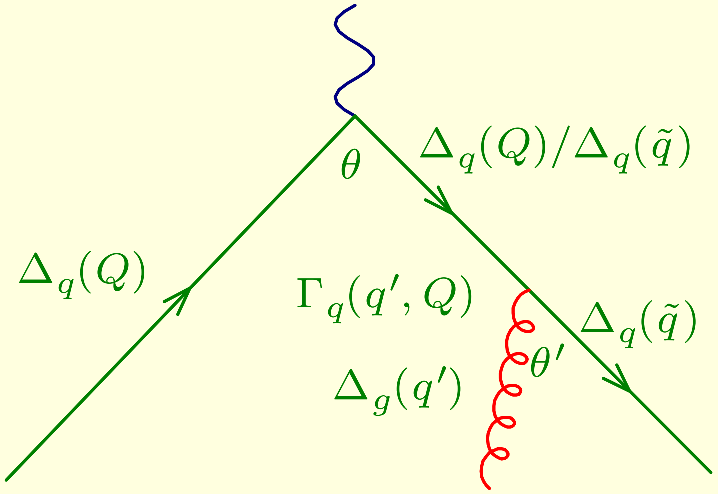
$$\Delta_q(Q) = \exp\left[-\int_{Q_{\rm min}}^{Q}\Gamma_q(q,Q)\,dq\right], \qquad \Gamma_q(q,Q) = \frac{2C_F}{\pi}\frac{\alpha_s(q)}{q}\left(\log\frac{Q}{q} - \frac{3}{4}\right)$$

For P_{gg}, P_{gq}: $\Gamma_g(q,Q) = \frac{2C_F}{\pi}\frac{\alpha_s(q)}{q}\left(\log\frac{Q}{q} - \frac{11}{12}\right)$, $\Gamma_f(q,Q) = \frac{N_F}{3\pi}\frac{\alpha_s(q)}{q}$

and $\Delta_g(Q) = \exp\left[-\int_{Q_{\rm min}}^{Q}[\Gamma_g(q,Q) + \Gamma_f(q,Q)]\,dq\right]$

Thus, the 2-clusters multiplicity is: $\dfrac{\sigma_2}{\sigma_{\rm tot}} = \Delta_q^2(Q)$.

3-clusters: The antiquark line gets a factor $\Delta_q(Q)$ as before.

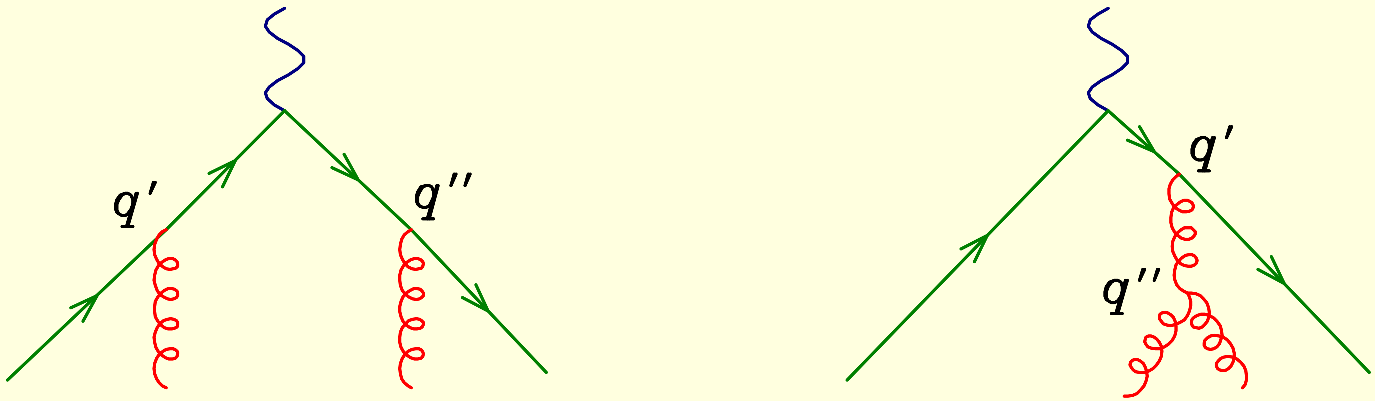


The gluon line from the gluon vertex gets a factor $\Delta_g(q')$, where $q' = \theta' E_g$.
(This uses angular ordering! no gluon radiation with angles $> \theta'$)
The quark line from the gluon vertex gets a factor $\Delta_q(\tilde{q})$, where $\tilde{q} = \theta' E_q$.
The quark line from the photon to the gluon vertex gets a factor:

$$\exp\left[-\int_{\theta'}^{\theta} \frac{d\theta^2}{\theta^2} \int dz \, \frac{\alpha_s(q)}{2\pi} P(z)\theta(q - Q_{\rm min}) \right] \approx \frac{\Delta_q(Q)}{\Delta_q(\tilde{Q})} \text{ (in the soft approximation!)}$$

The gluon vertex gets a factor $\Gamma_q(Q')$.

Thus, the 3-clusters multiplicity is $2\Delta_q^2(Q) \int_{Q_{\rm min}}^{Q} \Delta_g(q')\Gamma_g(q', Q)\, dq'$



Same (angular ordering) arguments lead to the following values for the 4-cluster multiplicity diagrams:

$$\Gamma_q(q', Q)\Gamma_q(q'', Q)\,\Delta_{q\bar{q}\,g\,g}, \qquad \Gamma_q(q', Q)\Gamma_q(q'', q')\,\Delta_{q\bar{q}\,g\,g},$$

with $\Delta_{q\bar{q}\,g\,g} = \Delta_q^2(Q)\,\Delta_g(q')\,\Delta_g(q'')$.
The following observation holds to all orders: the Sudakov factors depend only upon the nodal values of the k_T scales q', q'', ... at which branching occours, and on the parton type.

The procedure works because, in an angular ordered shower, the starting evolution scale of a branched soft parton, $E_{\rm soft}\,\theta$, is equal to the k_T.

In CKKW: replace approximate Γ factors by exact matrix elements.

Detailed prescription:

- Consider the cross section $d\sigma_n$ to produce n partons ($n \leqslant N$), all separated by a minimum distance parameter $y_{\min}$, computed with a fixed value of α_s. Generate n and n body kinematics with probability $d\sigma_n$.
- From the given kinematics reconstruct the scheleton, by pairing up recursively partons with smallest y. Only pair up partons that can come from the same splitting process (i.e. $g\,g$, $q\,g$, $q\;\bar{q}$; no $q\,q$, $q'\;\bar{q}$, etc.). Assign to each vertex i of the skeleton the corresponding $q_i = Q\sqrt{y_i}$.
- Associate factors $\Delta(q_i)/\Delta(q_j)$ ($q_i > q_j$) with each intermediate line of the skeleton, a factor $\Delta(q_i)$ with each final line of the skeleton, and $\alpha_s(q_i)/\alpha_s(Q)$ with each node of the skeleton. Compute the product of all this factors and accept the event with a probability equal to this product.

Originally, N (and/or $Q_{\min}$) was assumed to be large enough, so that the result was insensitive to N (i.e., most events had less than N clusters)

Interfacing to a Shower

At this level, we must complete the calculation with a full shower. The calculation was performed by extending the Shower approximation with exact matrix elements, but only for splittings with k_T above $Q_{\min}$. In order to correctly extend the shower, we should:

A) Avoid to generate splittings with $k_T > Q_{\min}$; those were already generated by the matrix elements

B) Include all missing radiation with $k_T < Q_{\min}$

Step A) is achieved by introducing a $\theta(Q_{\min} - k_T)$ in the splitting vertices and Sudakov form factors of the Shower Monte Carlo. In practice, this is achieved by the veto algorithm:

→ At any stage of the generation of a branching starting from a scale t' in the SMC, generate the branching at a scale $t'' < t'$ and generate the z value with the usual method.

→ If $k_T = \sqrt{t}z(1-z) > Q_{\min}$, discard the current branching, set t' to the value t'', and go back to the previous step. Otherwise, continue.

Step B) is more subtle: one should allow branchings from each intermediate and final line of the skeleton that were not included in the ME calculation. The angular ordered SMC should introduce, for any intermediate line initiating at an angle θ' and ending at an angle θ'', radiation with $\theta' > \theta > \theta''$, for any k_T. But only $k_T > Q_{\rm min}$ was provided by the ME.

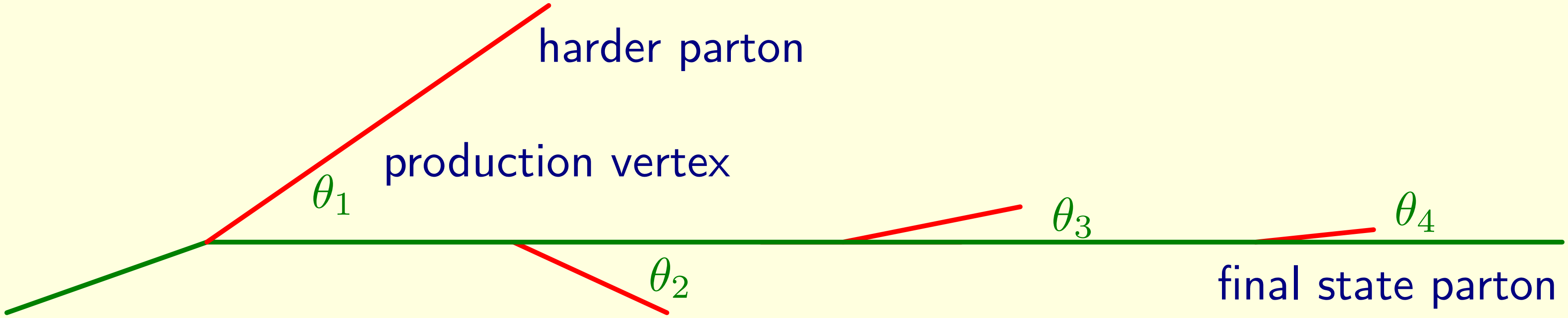
Thus, a truncated shower (P.N. 2004), with the angular ordered bound $\theta' > \theta > \theta''$, and a k_T veto $k_T < Q_{\rm min}$ (as before) should be provided for each intermediate line.

For final state line, a standard vetoed shower (i.e. $\theta' > \theta$, unconstrained from below), should be provided.

CKKW proposed an almost equivalent solution to this problem:

The final state particle are fed into an angular ordered Monte Carlo, their initial showering angle is set equal to the angle at the vertex where the parton was initially produced.

The vertex where the parton is initially produced is found by walking up from the given final state parton in the shower skeleton, skipping vertices where the parton in question is merged with a softer parton, and stopping at the first vertex where this is not the case.

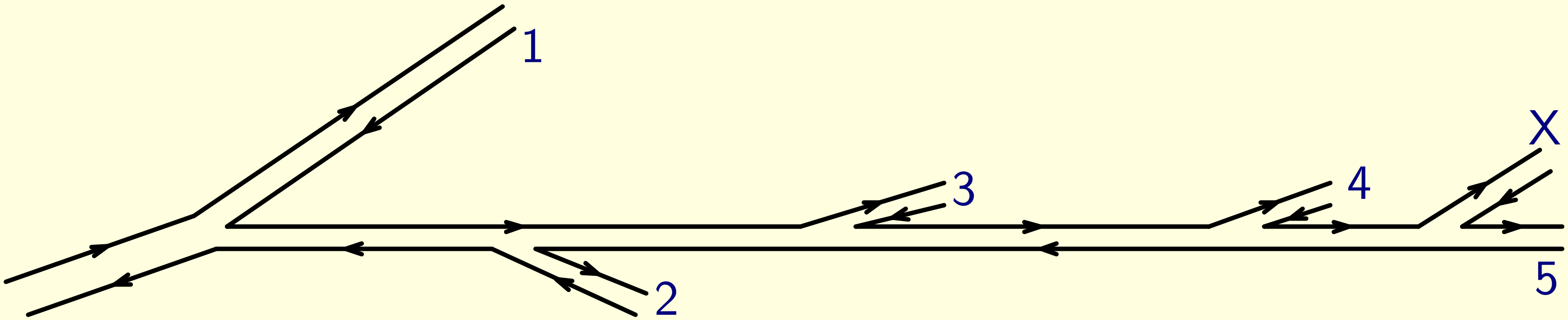


The CKKW prescription provides a single shower from θ_1. The green line from θ_1 has basically constant energy, since radiation from 2, 3, 4 is soft. So, a shower from θ_1 to the minimum is like a shower from θ_1 to θ_2, plus a shower from θ_2 to θ_3 plus a shower from θ_4 to the minimum.
It is then obvious that the CKKW prescription is equivalent, from a kinematical view point, to add a truncated shower to all internal skeleton lines.

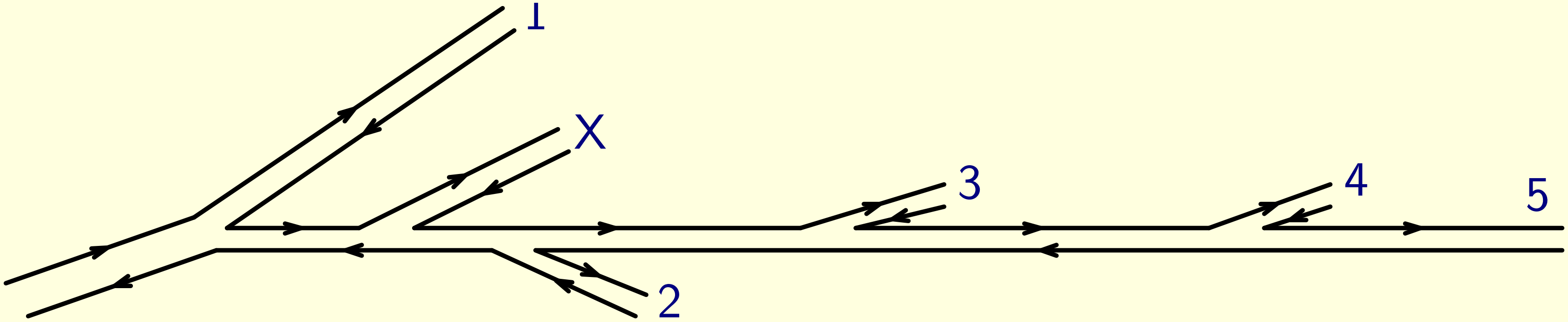
HOWEVER: colour pattern wrong ...

Comparison of colour connections for CKKW and truncated showers

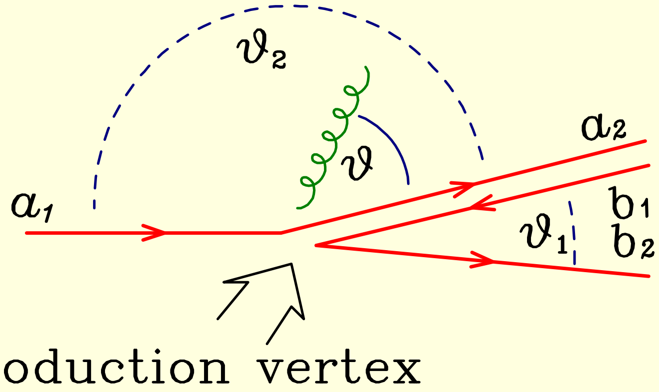
Consider the emission of parton X with $\theta_1 > \theta > \theta_2$.
Colour connection in CKKW:



colour connection with truncated shower:



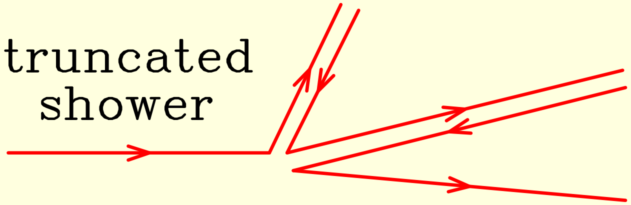
X is close in colour to 4 and 5 in CKKW, to 1 and 3 with truncated showers:
Larger colour gaps with CKKW

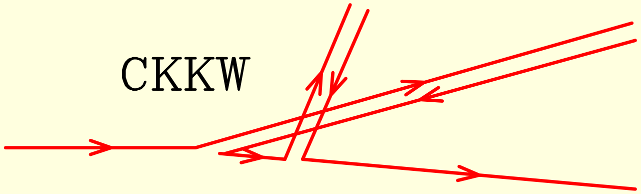


Consider $e^+e^- \to q\,\bar{q}\,g$.
Assume θ_1 small. Consider gluon emission with angle $\theta \gg \theta_1$, $\theta \ll \theta_2$.
Coherence requires that the emission strength is C_F (gluon and quark coherently)

In HERWIG: initial angle for gluon radiation is θ_1 or θ_2 with a 50% probability. Thus (in the above region) strength is $C_A/2 \approx C_F$ (but only in the average!!)

In CKKW: radiation from gluon restricted to $\theta < \theta_1$, quark radiates with angle up to θ_2. Thus only the quark radiates in the above region, with strength C_F. However, the colour connection is incorrect! Large colour gap ...





So: coherent showers are always needed when doing ME-Shower matching with angular ordered showers.

CKKW with finite N

In the original CKKW scheme, N is assumed to be large enough (i.e., almost negiglible amount of final states with N clusters).
Since N is practiccally finite, this means that $Q_{\rm min}$ should be kept large enough.
A practical alternative to this (Mrenna and Richardson, 2003; Schaelicke and Krauss, 2005) is the following:

In the matrix element for N clusters, replace the $Q_{\rm min}$ scale used to compute the Sudakov form factors and the vetoed showers with Q_n, (the $\sqrt{y}Q$ value of the smallest cluster.)

This was, the parton shower will be able to generate $N+1$, $N+2$, etc. clusters with merging scales larger than $Q_{\rm min}$, but the N hardest pairings will be accurate at the matrix element level, while the subsequent ones will be only collinear accurate.

Notice: with this prescription $Q_{\min}$ can be chosen as low as one likes (i.e., even near the shower cutoff). In this limiting case, no subsequent showers will be generated by the Monte Carlo for events with less than N clusters.

In all cases, the scale $Q_{\min}$ and N appear here as the delimiter between the exact matrix element calculation and the shower approach: production of more than N clusters will rely upon the SMC, as well as production of clusters below $Q_{\min}$.

Summary of CKKW

- Provides smooth interface between ME and Shower
- Uses a separation scale $Q_{\rm min}$, but is actually not strictly necessary
- Treats consistently multiple soft emissions in QCD, including interference effects (i.e., it is fully consistent with angular ordering)
- Cancellation of $Q_{\rm min}$ dependence is demonstrated, provided the SMC is of the angular ordered type (i.e. HERWIG like)

In practice, its use has been extended also to SMC of different kind (virtuality ordered, dipoles with k_T ordering, etc.)
If the SMC treats correctly coherence of multilpe soft gluon emission, it should be possible to interface it into a CKKW scheme preserving this accuracy.
If not, only soft emissions above $Q_{\rm min}$, and in number $\leqslant N$, will be correct in the soft limit.

Variants

Several alternatives have been proposed:

- MLM matching (ALPGEN group)
- Pseudo showers (Mrenna and Richardson, 2003)
- CKKW-Lönnblad (Lönnblad, 2002)

mostly to avoid computing explicitly the Sudakov form factors;

It would be interesting to discuss in details the relation of these methods with CKKW.

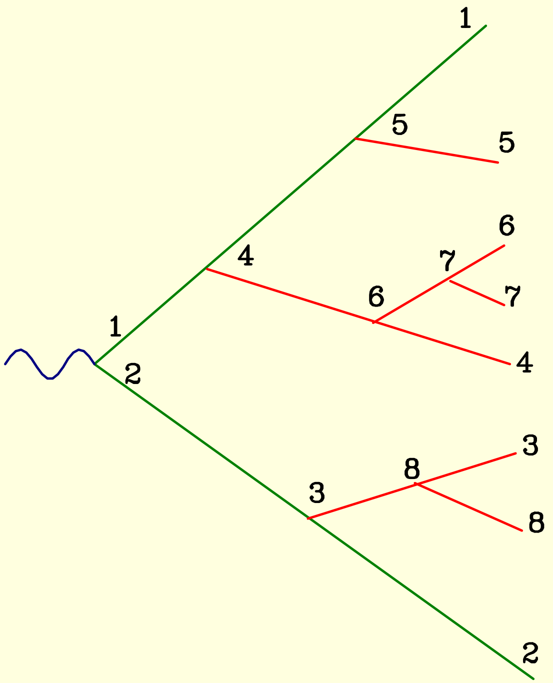
Done it here only for MLM matching ... with a very surprisinig result!

"Theorist" view of MLM matching

MLM: Generate the matrix element kinematics, and reweight with $\alpha_s(q)$ as in CKKW. Start the shower from the given final state, but kill the event if the jets reconstructed after the shower do not match the parton jets.

Is it the same as CKKW?

Let us assume that showering scales are set according to the CKKW method;



green: fermions, red: gluons; straight lines: hardest
The production vertex of each parton (1 to 8) is indicated with the corresponding number.
Looking (for example) at parton 1, herwig builds a shower starting from the wide angle (1,2). In CKKW this shower should be vetoed if $k_T > Q_{\rm min}$. Here it is killed if $k_T > Q_{\rm min}$. The survival probability is $\Delta_q(Q)$, i.e. exactly the Sudakov form factor one needs. So: it seems to be exactly the same, with no need to use vetoed showers and to compute Sudakov form factors ...

Detailed argument:

The probability to undergo a few radiations is

$$\Delta(\theta_1,\theta_2)\frac{d\theta_2}{\theta_2}\frac{\alpha_s(k_T^{(1)})}{2\pi}P(z_1)\Delta(\theta_2,\theta_3)\frac{d\theta_3}{\theta_3}\frac{\alpha_s(k_T^{(1)})}{2\pi}\theta(Q_{\min}>k_T^{(1)},k_T^{(2)})\Delta(\theta_2 E_3)$$

Well, is it really? It cannot happen that further radiation from leg 2 and 3 violates the k_T bound? No, it cannot happen: because of angular ordering, radiation from softer line has always smaller k_T than current splitting (2004,P.N).

It follows that:

The probability that this kind of event do not survive is then $\Delta(Q)$. Dividing the above expression to $\Delta(Q)$ is like subtracting from the exponent in each Sudakov form factor, the region with $k_T > Q_{\min}$. So, we end up with the vetoed shower required in CKKW.

Practical Considerations

In spite of the fact that CKKW is a very well specified procedure, practical implementations often give up several aspects of it. In particular:

- Although formulated for angular ordered showers, it is often applied to other type of shower algorithms (virtuality ordered, k_T ordered dipole showers, etc.) What is lost in these variants is not often clear.

- Even with angular ordered showers (i.e. HERWIG) it is not possible to fix the starting shower scale for each individual parton: this feature is missing in the Les Houches interface. The angle with the colour connected parton is used, with a 50% probability choice for the two colour connected partons of a gluon. This problem is particularly serious in MLM matching, that uses the shower to compute Sudakov form factors

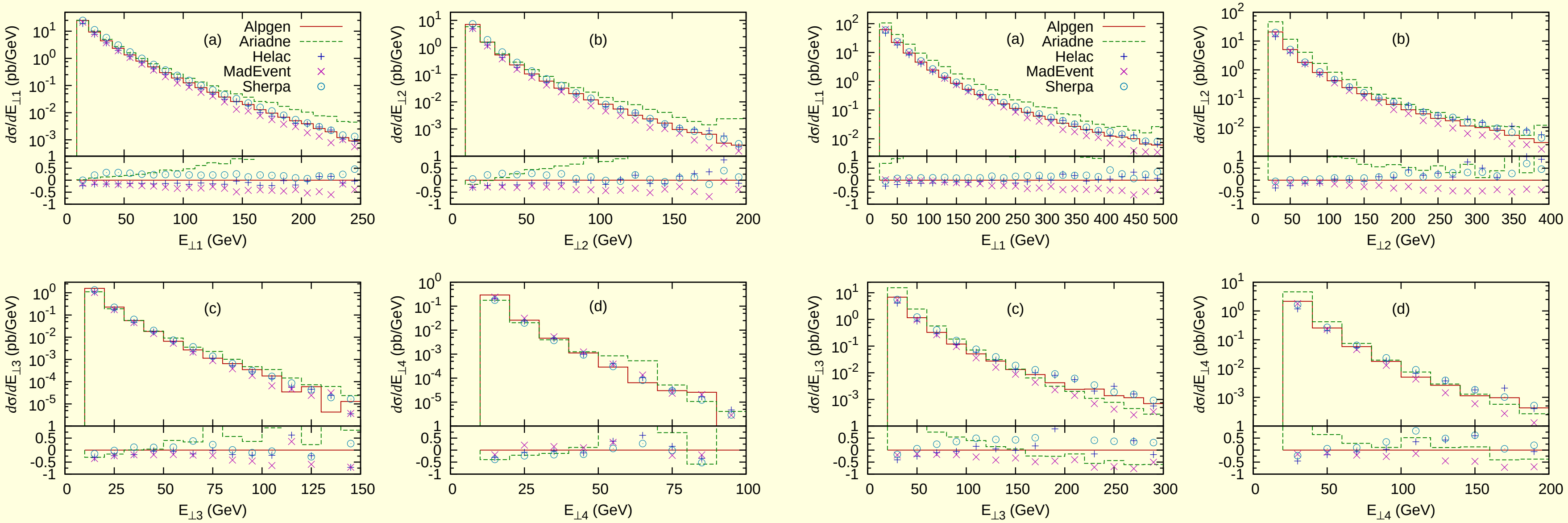
Many approaches introduce variants of the procedure to remedy to the problems created by the imperfect matching.
A critical comparison of the various methods is outside the scope of this lecture ... However

Comparison among different ME generators

(Alwall etal, Jul.07): compare Alpgen,Ariadne,Helac,MadEvent,Sherpa

$W + n\,\mathrm{jets}$, jet E_T spectra

TEVATRON LHC



THE MESSAGE:

good agreement among different ME implementation, in spite of different matching prescriptions (CKKW, MLM, and others)

Some bibliography

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- Lavesson and Lönnblad, arXiv:0712.2966, comparison of various methods in e^+e^-
- P.N., JHEP0411:040,2004,hep-ph/0409146 (POWHEG, trunc. showers)
- Hoeche, Krauss, Schumann, Siegert, JHEP 0905:053,2009 arXiv:0903.1219, on truncated showers

ME matching: some final thoughts

- After 8 years of CKKW, several variants and different implementations
- No exact CKKW seems to exist: problems to solve: truncated showers. Is it so difficult?
- CKKW have studied the double log region with great accuracy; similar studies for variant (i.e. no angular ordered showers) are not as detailed
- Several functional programs already available (invaluable tools for LHC phenomenology)
- It is very desirable that new standards (i.e. Les Houches 2) take into account the needs of ME matching